

Problem 1 Conditional Independence and Joint Probability

(a)

At least $3^4 - 1 = 80$ values needed to uniquely specify the joint probability table.

In general, $3^n - 1$ values are needed for n 3-value random variables.

(b)

A: 2 B: $2 \cdot 3 = 6$ C: $2 \cdot 3 = 6$ D: $2 \cdot 3 = 6$

Total = $2 + 6 + 6 + 6 = 20$

In this Bayesian Network, at least 20 values need to determine the joint probability table.

(c)

In the network given in (b), A and D are not independent. Because the information could flow between A and D if B or C is not observed.

A and D are conditionally independent given B and C. Once B or C is determined, the information cannot be transmitted between A and D.

(d)

The minimum number of values needed to specify the joint probability table over all possible networks containing four Boolean random variables is 7.

(e)

