**Problem 2 Bayesian Network Inference**

(a) [2] What is the probability that there was a bird on the lawn?

P(B) = P(B|S) P(S) + P(B|¬S) P(¬S) = 0.01*0.4 + 0.5*(1-0.4) = **0.304**

(b) [3] If there was a bird on the lawn, what is the probability that the sprinkler was on?

P(S|B) = P (S, B) | P(B) = P(B|S) P(S) / P(B) = 0.01 * 0.4 / 0.304 = **0.013**

(c) [3] If there was a bird on the lawn, what is the probability that the lawn was wet?

P(W|B) = P (W, B) | P(B)

P (W, B) = P (W, B, R, S) + P (W, B, ¬R, S) + P (W, B, R, ¬S) + P (W, B, ¬R, ¬S)

P (W, B, R, S) = P (B|W, R, S) P (W, R, S) = P (B|W, R, S) P (W|R, S) P (R, S) = P(B|S) P (W|R, S) P(R)P(S) = 0.01*1*0.1*0.4 = 0.0004

P (W, B, ¬R, S) = P (B|W, ¬R, S) P (W, ¬R, S) = P (B|W, ¬R, S) P (W|¬R, S) P (¬R, S) = P(B|S) P (W|¬R, S) P(¬R) P(S) = 0.01*1*(1-0.1) *0.4 = 0.0036

P (W, B, R, ¬S) = P (B|W, R, ¬S) P (W, R, ¬S) = P (B|W, R, ¬S) P (W|R, ¬S) P (R, ¬S) = P(B|¬S) P (W|R, ¬S) P(R)P(¬S) = 0.5*1*0.1*(1-0.4) =0.03

P (W, B, ¬R, ¬S) = P (B|W, ¬R, ¬S) P (W, ¬R, ¬S) = P (B|W, ¬R, ¬S) P (W|¬R, ¬S) P (¬R, ¬S) = P(B|¬S) P (W|¬R, ¬S) P(¬R) P(¬S) =0.5*0.1*(1-0.1) *(1-0.4) =0.027

P (W, B) = 0.0004+0.0036+0.03+0.027 = 0.061

P(W|B) = 0.061/0.304 = **0.2006**

(d) [3] If there was a bird on the lawn, what is the probability that the lawn was dry, and it did not rain, and the sprinkler was not on?

P (¬W, ¬R, ¬S|B) = P (¬W, ¬R, ¬S, B) | P(B) = P (B|¬W, ¬R, ¬S) P (¬W, ¬R, ¬S) /P(B) = P(B|¬S) P (¬W|¬R, ¬S) P(¬R) P(¬S)/P(B) = 0.5*(1-0.1) *(1-0.1) *(1-0.4)/0.304 = **0.7993**

(e) [3] What is the probability that the lawn was wet?

P(W) = P (W, R, S, B) + P (W, ¬R, S, B) + P (W, R, ¬S, B) + P (W, ¬R, ¬S, B) + P (W, R, S, ¬B) + P (W, ¬R, S, ¬B) + P (W, R, ¬S, ¬B) + P (W, ¬R, ¬S, ¬B)

P (W, R, S, B) = P (W | R, S, B) P (R, S, B) = P (W | R, S) P (B | S, R) P (S, R) = P (W | R, S) P (B | S) P (S) P (R) = 1*0.01*0.4*0.1 = 0.0004

P (W, ¬R, S, B) = P (W | ¬R, S, B) P (¬R, S, B) = P (W | ¬R, S) P (B | S, ¬R) P (S, ¬R) = P (W | ¬R, S) P (B | S) P (S) P (¬R) = 1*0.01*0.4*(1-0.1) = 0.0036

P (W, R, ¬S, B) = P (W | R, ¬S, B) P (R, ¬S, B) = P (W | R, ¬S) P (B |¬S, R) P (¬S, R) = P (W | R, ¬S) P (B | ¬S) P (¬S) P (R) = 1*0.5*0.6*0.1 = 0.03

P (W, ¬R, ¬S, B) = P (W | ¬R, ¬S, B) P (¬R, ¬S, B) = P (W | ¬R, ¬S) P (B | ¬S, ¬R) P (¬S, ¬R) = P (W | ¬R, ¬S) P (B | ¬S) P (¬S) P (¬R) = 0.1*0.5*0.6*0.9 = 0.027

P (W, R, S, ¬B) = P (W | R, S, ¬B) P (R, S, ¬B) = P (W | R, S) P (¬B | S, R) P (S, R) = P (W | R, S) P (¬B | S) P (S) P (R) = 1*0.99*0.4*0.1 = 0.0396

P (W, ¬R, S, ¬B) = P (W | ¬R, S, ¬B) P (¬R, S, ¬B) = P (W | ¬R, S) P (¬B | S, ¬R) P (S, ¬R) = P (W | ¬R, S) P (¬B | S) P (S) P (¬R) = 1*0.99*0.4*0.9 = 0.3564

P (W, R, ¬S, ¬B) = P (W | R, ¬S, ¬B) P (R, ¬S, ¬B) = P (W | R, ¬S) P (¬B | ¬S, R) P (¬S, R) = P (W | R, ¬S) P (¬B | ¬S) P (¬S) P (R) = 1*0.5*0.6*0.1 = 0.03

P (W, ¬R, ¬S, ¬B) = P (W | ¬R, ¬S, ¬B) P (¬R, ¬S, ¬B) = P (W | ¬R, ¬S) P (¬B | ¬S, ¬R) P (¬S, ¬R) = P (W | ¬R, ¬S) P (¬B | ¬S) P (¬S) P (¬R) = 0.1*0.5*0.6*0.9 = 0.027

P(W) = 0.0004 + 0.0036 + 0.03 + 0.027 + 0.0396 + 0.3564 + 0.03 + 0.027 = **0.514**

(f) [3] If the lawn was wet and the sprinkler was on, what is the probability that it rained?

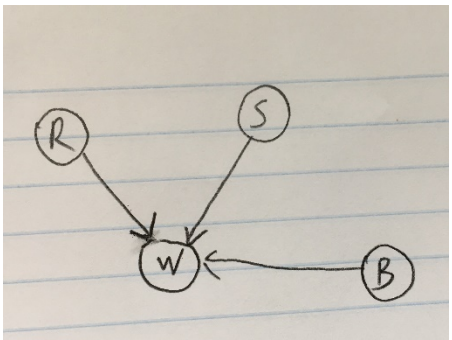
P (R|W, S) = P(R|W) = P (R, W) | P(W) = (P (R, W, S, B) + P (R, W, ¬S, B) + P (R, W, S, ¬B) + P (R, W, ¬S, ¬B)) /P (W)

From part (e):

P (R|W, S) = (0.0004 + 0.03 + 0.0396 + 0.03) / 0.514 = **0.1946**

(g) [3] Suppose we remove the connection between S and B, and then add a directed connection from B to W. Draw this new network. Considering Was the class variable, is this a Naïve Bayes network? Why or why not?

New Network Drawing



Yes. It is the Naïve Bayes Network. In this case, W is the class variable, and R, S, B are the evidence nodes. All the evidence variables are conditionally independent of each other given the class variable, because there are no connections between any evidence nodes.