Assignment 6

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3 Half-3SAT

Proof. Half-3SAT is clearly in NP since a truth assignment satisfying half of the clauses can serve as a natural certificate. Now let's show it is also NP-hard. To show this, we find a reduction from 3-SAT.

Given a 3-SAT formula φ with m clauses, we define φ_t which contains $4m$ clauses. The first m clauses in φ_t are just φ. Next m clauses in φ_t are all of the form $x \vee \neg x \vee y$, so these clauses are always true whatever the truth table is. The following $2m$ clauses are all the same: $u \vee v \vee w$, where u, v, w are newly created variables. This makes sure that these $2m$ clauses are either all satisfied or all unsatisfied.

Now suppose φ is satisfiable, then a satisfying assignment for φ together with $u = v = w = 0$ forms an assignment which satisfied $2m$ clauses in φ_t. If φ is not satisfiable, then the number of clauses satisfied in φ_t is less than $2m$ or at least $3m$ for any assignment. This completes our proof. $\square$

6 SIP problem

Proof. SIP is in NP since a set T which satisfies the conditions is a natural certificate. Now let's prove it is NP-hard by showing a reduction from 3-SAT.

Given a 3-SAT formula $\varphi = \bigwedge_{i=1}^{m} (l_{i1} \vee l_{i2} \vee l_{i3})$ which contains n distinct variables $x_1, x_2, \ldots, x_n$. We construct $A_1, \ldots, A_m$ such that $A_i = \{l_{i1}, l_{i2}, l_{i3}\}$ and $B_1, \ldots, B_n$ such that $B_i = \{x_i, \neg x_i\}$. We claim that such a T exists if and only if φ is satisfiable.

If φ is satisfiable, it is easy to check that the set T, which contains all variables which are assigned to be true and the negation of all variables which are assigned to be false, satisfies our conditions. Conversely, if such a T exists, then it contains at most one element in $B_i = \{x_i, \neg x_i\}$. If it contains x_i then we assign $x_i = 1$, otherwise we assign $x_i = 0$. This will result in satisfying assignment for φ since $|T \cap A_i| \geq 1$ for all i, meaning that every clause has a literal contained in T and therefore is satisfied. $\square$