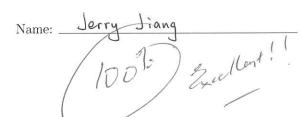
1. Simplify $a^{1/\ln a}$.





2. Suppose that the function f is even and everywhere differentiable. Show that the derivative f' is odd.

3. Fill in the table giving exact answers or answers correct to four decimal places.

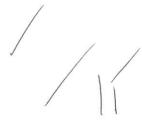
n	1	100	10 000	1 000 000
$(1+\frac{2}{n})^n$	3	7.2446	7.3876	7.3890

Give the exact value of the limit $\lim_{n\to\infty} (1+\frac{2}{n})^n$ and justify your answer.

There's
$$f(x) = \ln x$$
, $f'(x) = \frac{1}{x}$. $f'(x) = 1$

$$f'(u) = \lim_{n \to \infty} \cdot \ln\left(1 + \frac{2}{n}\right)^{\frac{1}{2} \cdot n} = 1$$

$$\lim_{N\to\infty} \left(\left| +\frac{2}{N} \right|^{\frac{1}{2} \cdot N} \right) = \ell$$



4. Find the derivative of the function $f(x) = x^x$.

$$f'(x) = (e^{\ln x \cdot x})'$$

$$= e^{\ln x \cdot x} \cdot (\ln x \cdot x)'$$

$$= e^{\ln x \cdot x} \cdot (\frac{1}{x} \cdot x + \ln x)$$

$$= x^{x} (1 + \ln x)$$

5. The normal to $y = \ln x$ at the point P has y-intercept $1 + e^2$. Find the coordinates of P.

$$y' = \frac{1}{x} = f'(x)$$
 $P(a, lna)$
 $f'(a) = \frac{1}{a}$
 $i' : (: y = -a x + 1 + e^2)$
 $lna = -a^2 + 1 + e^2$
 $a = e$

6. Each side of an equilateral triangle is increasing at a rate of $3\,\mathrm{cm}\,\mathrm{s}^{-1}$. At what rate is the area of the triangle increasing when each side is 10 cm long?

$$A = \frac{\sqrt{3}}{4} (\alpha)^{2} \qquad \alpha = 3t \qquad \frac{\sqrt{3}}{\sqrt{3}} = 3$$

$$\frac{\sqrt{3}A}{\sqrt{3}} = A' = \frac{\sqrt{3}}{2} \alpha$$



7. Find the coordinates of any stationary points on the curve $y = \sin(\pi \sin x)$ given $x \in [0, \pi]$.

1 cos x 20

2 (05 (TISINX) =0

$$T_1 \leq 1 \leq x \leq \frac{T}{2} + kT$$

$$\chi = \frac{1}{2} \int \frac{1}{2} = \chi$$

 $(\frac{\pi}{b}, 0)$

$$(\frac{\pi}{6}, 1)$$

8. Prove $\lim_{x\to\infty} \frac{\lfloor x\rfloor}{x} = 1$ where $\lfloor x\rfloor$ is the floor of x.

=
$$\lim_{x\to\infty} \left(\frac{x}{x} - \frac{a}{x} \right)$$



9. Find the values of k so that $y = e^x \cos kx$ satisfies the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ for all values of x.

10. For what value of k does $y = \sin(\ln x)$ satisfy the differential equation $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = k$? Hence find a solution to the differential equation

$$\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) = x\frac{dy}{dx} - y + 5.$$

0 - sinfluxt-cos (Inx) + cos (tax) + sintlux)=k

$$\frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = x \frac{dy}{dx} - y + 5$$

$$\frac{d^2y}{dx^2} + x \frac{dy}{dx} + y - 5 = 0$$

$$\frac{d^2y}{dx^2} + x \frac{dy}{dx} + y - 5 = 0$$

$$\frac{d^2y}{dx^2} + x \frac{dy}{dx} + y - 5 = 0$$



Solutions to HL1 Assignment #22

- 1. $a^{1/\ln a} = (e^{\ln a})^{1/\ln a} = e^1 = e$.
- 2. If f is even then f(x) = f(-x). Since f is differentiable we have by the chain rule $f'(x) = f'(-x) \cdot (-1) = -f(-x)$, whence f' is odd.
- 3. The table entries are 3, 7.2446, 7.3876 and 7.3890. Now $\lim_{n\to\infty} (1+\frac{2}{n})^n = \lim_{n\to\infty} [(1+\frac{2}{n})^{n/2}]^2 = e^2$.
- 4. First $x^x = e^{x \ln x}$. So $f'(x) = e^{x \ln x} (\ln x + 1) = x^x (\ln x + 1)$.
- 5. Let the coordinates of P be $(a, \ln a)$. Then $m_N = -a$. So $N: y \ln a = -a(x a)$. Solving when x = 0 and $y = 1 + e^2$ gives a = e. Hence P has coordinates (e, 1).
- 6. Let the side length of the triangle be x cm and the area be A cm². Then $A = \frac{1}{2}x^2 \sin 60^\circ = \frac{\sqrt{3}}{4}x^2$. By the chain rule $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$, so

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2}x \times \frac{dx}{dt} = 15\sqrt{3}.$$

So the area is increasing at a rate of $15\sqrt{3}$ cm² when the side is 10 cm.

- 7. Here $y' = \cos(\pi \sin x) \cdot \pi \cos x$. Solving y' = 0 gives $\cos x = 0$ or $\pi \sin x = \frac{\pi}{2}$. Hence the stationary points For $x \in [0, \pi]$ are $(\frac{\pi}{2}, 0)$, $(\frac{\pi}{6}, 1)$ and $(\frac{5\pi}{6}, 1)$.
- 8. First observe that $x-1 < |x| \le x$. So for x > 0

$$\frac{x-1}{x} < \frac{\lfloor x \rfloor}{x} \le \frac{x}{x}.$$

Since $\lim_{x\to\infty} \frac{x-1}{x} = 1$ and $\lim_{x\to\infty} \frac{x}{x} = 1$, we conclude by the squeeze theorem that $\lim_{x\to\infty} \frac{\lfloor x \rfloor}{x} = 1$ also.

9. To simplify the algebra let $C = \cos kx$ and $S = \sin kx$. Then $y' = e^x(C - kS)$ and $y'' = e^x(C(1 - k^2) - 2kS)$. So

$$y'' - 2y' + 2y = e^{x}[C(1 - k^{2}) - 2kS - 2(C - kS) + 2C] = e^{x}C(1 - k^{2}).$$

We conclude $k = \pm 1$.

10. Let $C=\cos(\ln x)$ and $S=\sin(\ln x)$. Then $y'=C\cdot\frac{1}{x}$ and $y''=-\frac{S+C}{x^2}$. So $x^2y''+xy'+y=-S-C+C+S=0$, whence k=0. Next notice

$$\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) = x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx}.$$

So our differential equation can be written as $x^2 + xy' + y = 5$. We conclude $y = \sin(\ln x) + 5$ is a solution.