HL1 Assignment #16 Name: Jerry Jiang

1. Dividing $2x^3 + 5x^2 + ax + 7$ by x+3 gives a remainder of 16. What is the value of a?

$f(-3) = -54 + 45 - 3a + 7 = 16$

$-3a = 18$

$a = -6$

100%

Excellent!

2. Let $u = 2 + 3i$ and $v = 5 + mi$ where $m \in \mathbb{R}$. If $uv = 22 + 7i$ what is the value of m?

$10 + 2mi + 15i - 3m = 22 + 7i$

$(10 - 3m) + (15 + 2m)i = 22 + 7i$

$\therefore 10 - 3m = 22$

$\therefore m = -4$

3. A die is biased so that the probability of throwing an odd number is twice the probability of throwing an even number. If each of the odd numbers is equally likely and each of the even numbers is equally likely, what is the the probability of throwing a 3?

$P(3) = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

4. The equation $x^2 - 18x + c = 0$ has roots k and $2k$. Find the values of c and k.

$3k = 18$

$k = 6$

$c = 2k^2 = 72$

5. Solve $5^{x+3} = 7^{x-1}$, giving your answer to 3 significant figures.

$\log 5^{x+3} = \log 7^{x-1}$

$(x+3) \log 5 = (x-1) \log 7$

$\frac{x+3}{x-1} = \log_5 7 \approx 1.209062$

$x + 3 = 1.209062x - 1.209062$

$0.209062x = 4.209062$

$x = 20.1$ (3s.f.).

6. Nine tickets numbered 1 through 9 are placed in a hat. Five tickets are chosen at random and without replacement from the hat. Find the probability that at least one ticket is numbered odd.

there're 5 odd and 4 even from 1 to 9.

so at least one ticket is odd:

$P(\text{odd}) = 1$

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7. An arithmetic sequence has first term 7. The n^{th} term is 84 and the $(3n)^{\text{th}}$ term is 245. Find the value of n.

$(n-1) \times d = 84-7 = 77$

$(3n-n) \times d = 245-84 = 161$

$2nd = 161$

$nd - d = 77$

$\therefore nd = 80.5$

$\therefore d = 3.5$

$\therefore n = 23$ ✓

8. Let $p(x) = x^4 + x^3 + 3x^2 + ax + b$. If $x^2 - x + 4$ is a factor of $p(x)$ what are the values of a and b?

let $p(x) = (x^2 - x + 4)(x^2 + mx + n)$

$\therefore \underline{x^4} + \underline{mx^3} + \underline{nx^2} - \underline{x^3} - \underline{mx^2} - \underline{nx} + \underline{4x^2} + \underline{4mx} + \underline{4n} = p(x)$

$\therefore x^4 + (m-1)x^3 + (n-m+4)x^2 + (4m-n)x + 4n = p(x)$

$\therefore \begin{cases} m = 2 \\ n = 1 \end{cases}$

$\therefore \begin{cases} a = 8-1 = 7 \\ b = 4n = 4 \end{cases}$ ✓

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9. Two friends are invited to a dinner party. The dining table is a long, narrow table with six chairs on each side and no chairs at the ends. The seating plan has been organised randomly. What is the probability that the two friends will either be sitting next to each other or directly opposite each other?

$$P = \frac{2! \times 5 \times 2 + 2! \times 6}{12 \times 11} = \frac{8}{33}$$ ✓

10. Research the pigeonhole principle. Use the principle to explain why any subset of size five from the set $S = \{1, 2, 3, \ldots, 8\}$ contains two elements whose sum is 9.

$1 + 8 = 9$

$2 + 7 = 9$

$3 + 6 = 9$

$4 + 5 = 9$.

there're 4 pigeonholes and 5 pigeons.

So at least one of the pairs can be satisfied.

which means it must contain two elements whose sum is 9.

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Solutions to HL1 Assignment #16

1. By the remainder theorem $p(-3) = -54 + 45 - 3a + 7 = 16$, whence $a = -6$.

2. Equating real and imaginary parts we have $10 - 3m = 22$ and $15 + 2m = 7$, whence $m = -4$.

3. Let A be the event of an odd number and T the event of a three. We are given $P(A) = 2P(A')$. So $P(A) = \frac{2}{3}$, whence $P(T) = \frac{2}{9}$.

4. By Vieta's formulae, $3k = 18$ and $2k^2 = c$, whence $k = 6$ and $c = 72$.

5. We have $125 \cdot 5^x = \frac{1}{7} \cdot 7^x$, or equivalently $1.4^x = 875$, whence $x = 20.1$ to 3 significant figures.

6. Let A be the event of at least one ticket numbered odd. Then A' is the event of all tickets numbered even. Since there are only four even numbers we conclude $P(A') = 0$. Hence $P(A) = 0$.

7. We have $84 = 7 + (n-1)d$ and $245 = 7 + (3n-1)d$, or equivalently $77 = (n-1)d$ and $238 = (3n-1)d$, whence $n = 23$.

8. Dividing gives $x^4 + x^3 + 3x^2 + ax + b = (x^2 - x + 4)(x^2 + 2x + 1) + (a-8)x + (b-4)$. So $a = 7$ and $b = 4$ for a remainder of 0.

9. Let A be the event that the first friend sits at an end position and let B be the event that the second friend sits beside or opposite the first friend. Then using a tree diagram or otherwise we have

$$P(\text{friendly position}) = P(A \cap B) + P(A' \cap B) = \frac{1}{3} \cdot \frac{2}{11} + \frac{2}{3} \cdot \frac{3}{11} = \frac{8}{33}.$$

10. Partition the set S into the four subsets $\{1, 8\}$, $\{2, 7\}$, $\{3, 6\}$, $\{4, 8\}$. By the pigeonhole principle, two of the elements in any subset of five elements of S must lie in one of the four subsets. And hence two elements in any subset of size five must have a sum of 9.