FM1 Assignment #4 Name: Jerry Jiang

1. Find the element in the second row and third column of the matrix M^2 where $M = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$.

$$M^2 = \begin{pmatrix} 30 & 36 & 42 \\ 66 & 81 & 96 \\ 102 & 126 & 150 \end{pmatrix}$$

$\therefore {M^2}_{2,3} = 96.$

2. Find the radius of the circle in the diagram.

given that:

$EF = 2, AE = 3, ED = 6.$

$\because FD = 2 + 6 = 8$

$\therefore CF = CD = 4$

$\therefore EC = BO = 2$

let $BE = OC = a$

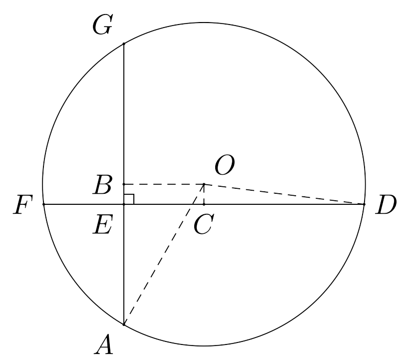
$\because OA^2 = OD^2$

$\therefore OB^2 + BA^2 = OC^2 + CD^2$

$\therefore 2^2 + (3 + a)^2 = a^2 = 4^2$

$\therefore a = \frac{1}{2}$

$$\therefore r = \sqrt{16 + \frac{1}{4}}$$
$$= \frac{\sqrt{65}}{2}$$



3. The *symmetric difference* of two sets A and B is $A \triangle B = (A \setminus B) \cup (B \setminus A)$. Construct the Cayley table for the sets $\emptyset, A, A', U$ under the operation of symmetric difference. Does $(\{\emptyset, A, A', U\}, \triangle)$ form a group?

$\triangle$	$\emptyset$	A	A'	U
$\emptyset$	$\emptyset$	A	A'	U
A	A	$\emptyset$	U	A'
A'	A'	U	$\emptyset$	A
U	U	A'	A	$\emptyset$

it's a group since:

- it's closed with $\emptyset, A, A', U$.
- it's associative.
- with identity element $\emptyset$.
- inverse with itself.

4. Prove that a simple graph with more than one vertex contains two vertices of the same degree.

Let's first make a hypothesis that it's possible for a simple graph with $V > 1$ to have vertices with no repeating degree numbers.

In a simple graph with n vertices, there are at most $\frac{n \cdot (n-1)}{2}$ edges, which is when it's complete. If all the vertices are wanted to have different degrees, they start from $0, 1, 2, \ldots, n-1$, and these add up to $\frac{n \cdot (n-1)}{2}$, the sum that's at least required for all vertices to have different degree numbers and also the largest that a simple graph can provide. So this case seems to be the only one to be able to satisfy the hypothesis.

However in the case, a vertex has degree 0, which means it's isolated; while another vertex has degree $n-1$, which means it's connected to all other vertices in the graph as the graph only has n vertices. This contradict itself and the hypothesis fails. So finally, we get to the conclusion that a simple graph with more than one vertex contains two vertices of the same degree.

5. Determine the values of p for which the integral $\int_1^\infty \frac{1}{x^p}\,dx$ converges and when it does so find its value.

1. $p \neq 1$

$$\int_1^\infty \frac{1}{x^p}\,dx$$
$$= \lim_{a\to\infty} \int_1^a x^{-p}\,dx$$
$$= \lim_{a\to\infty} \left[\frac{x^{-p+1}}{1-p}\right]_1^a$$
$$= \frac{\lim_{a\to\infty} a^{-p+1} - 1}{1-p} = A$$

i. $1 - p > 0, p < 1$

$$A = \frac{\infty - 1}{1-p} = \infty, \text{diverges.}$$

ii. $1 - p < 0, p > 1$

$$A = \frac{0-1}{1-p} = \frac{1}{p-1}, \text{converges.}$$

2. $p = 1$

$$\int_1^\infty \frac{1}{x}\,dx$$
$$= [\ln x]_1^\infty$$
$$= \ln\infty - 0$$
$$= \infty, \text{diverges.}$$