FM1 Assignment #9 Name: Jerry Jiang

1. List all the subgroups of the symmetric group $(S_3, \circ)$. Use e for the identity and cycle notation otherwise.

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, a = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, b = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$
$$c = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, d = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

∘	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	e	f	d	c	b
b	b	c	d	f	e	a
c	c	b	a	e	f	d
d	d	f	e	a	b	c
f	f	d	c	b	a	e

Non-trivial Subgroups:
$(\{e, a\}, \circ)$
$(\{e, c\}, \circ)$
$(\{e, f\}, \circ)$
$(\{e, b, d\}, \circ)$

2. Find the rank and nullity of the matrix $\begin{pmatrix} 1 & 3 & 1 & 6 & 4 \\ 2 & 6 & 3 & 16 & 11 \\ 3 & 9 & 3 & 18 & 12 \end{pmatrix}$.

$$A = \begin{pmatrix} 1 & 3 & 1 & 6 & 4 \\ 2 & 6 & 3 & 16 & 11 \\ 3 & 9 & 3 & 18 & 12 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 & 0 & 2 & 1 \\ 0 & 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Ax = 0 : \left(\begin{array}{ccccc|c} 1 & 3 & 1 & 6 & 4 & 0 \\ 2 & 6 & 3 & 16 & 11 & 0 \\ 3 & 9 & 3 & 18 & 12 & 0 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 3 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_2 = r, x_4 = s, x_5 = t.$

then $x_1 = -3r - 2s - t, x_3 = -4s - 3t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = r \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -2 \\ 0 \\ -4 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ -3 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{nullity } (A) = \left\{ \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ -4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

row ranking $B: \begin{pmatrix} 1 & 3 & 0 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 4 & 3 \end{pmatrix} \to A: \begin{pmatrix} 1 & 3 & 0 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 4 & 3 \end{pmatrix}$

column ranking $B: \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \to A: \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$

ranking $= 2$.

3. Show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges conditionally.

For $\displaystyle\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

according to the rule of convergence of p-series, $p = \dfrac{1}{2} < 1,$ it diverges

For $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}, \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0.$

when $n > 0, n \leq n + 1$

$\sqrt{n} \leq \sqrt{n+1}$

$\dfrac{1}{\sqrt{n+1}} \leq \dfrac{1}{\sqrt{n}}$

According to the alternating series test, the series converges.

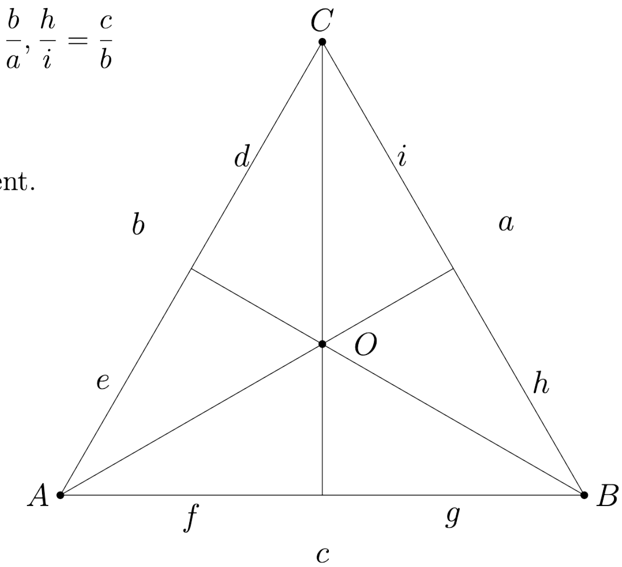
Therefore, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges conditionally.

4. Use the converse of Ceva's theorem to prove that the angle bisectors of a triangle are concurrent.

According to the angle bisector theorem, $\frac{d}{e} = \frac{a}{c}, \frac{f}{g} = \frac{b}{a}, \frac{h}{i} = \frac{c}{b}$

Therefore, $\frac{d}{e} \cdot \frac{f}{g} \cdot \frac{h}{i} = \frac{a}{c} \cdot \frac{b}{a} \cdot \frac{c}{b} = 1.$

So the angle bisectors intersect at O and are concurrent.



5. Consider the locus of a point whose distance from the point $(6, 0)$ is $\frac{3}{2}$ its distance from the line $3x - 8 = 0$.

(a) Find the equation of the locus.

$a = 4, e = \frac{3}{2}, b^2 = 20$, Asymptote: $y = \frac{b}{a} = \pm\frac{\sqrt{5}}{2}$, Equation: $\frac{x^2}{16} - \frac{y^2}{20} = 1.$

(b) Sketch the locus clearly indicating any key features.