1. Solve the equation ${}^{n}P_{2} = 9900$.

$$\frac{N!}{(n-2)!} = 9900$$

$$N(n-1) = 9900$$

$$N^{2} - N - 9900 = 0$$

$$(N - (00)(N+99) = 0$$

$$N_{1} = (00)$$

$$N_{2} = -99(in admissible)$$

n=100

2. Find all values of x so that $3^{x^2-1} = (\sqrt{3})^{126}$.

$$3^{x^{2}-1} = \frac{126}{2}$$

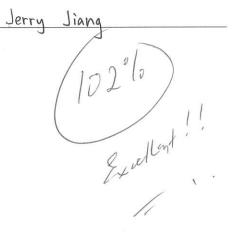
$$x^{2}-1 = \frac{126}{2}$$

$$x^{2} = \frac{126+2}{2} = \frac{128}{2} = 64$$

$$x = \pm 8$$

3. If $A = 5^x + 5^{-x}$ and $B = 5^x - 5^{-x}$, find the value of $A^2 - B^2$.

$$A^{2} - B^{2}$$
= $(A+B)(A-B)$
= $(5^{x}+5^{x}+5^{x}-5^{x})[5^{x}+5^{x}-5^{x}]$
= $2.5^{x} \cdot 2.5^{x}$
= 4.5°



4. Let $S = \{n \in \mathbb{Z} \mid 1 \le n \le 9\}$. How many four element subsets of S contain two odd and two even numbers?

$$\begin{array}{lll}
50 & 5 & 4 & 4 \\
& = \frac{5x4}{2x+} \times \frac{4x3}{2x+} \\
& = 60 & (= 5p_3)
\end{array}$$

- 1. 60 subsets meet the requirment.
- 5. What positive integer n satisfies $\log(225!) \log(223!) = 1 + \log(n!)$?

6. Solve the equations x + 2y = 5 and $4^x = 8^y$ simultaneously.

7. Solve
$$\log_2(9x+5) - \log_2(x^2-1) = 2$$
.

8. The coefficient of x^2 in the expansion of $(1+2x)^n$ is 264. Find the value of n.

$${\binom{n}{2} \cdot \binom{n-2}{2} \cdot 4} = 264$$

$$= \frac{n(n-1)}{2} \cdot 4$$

$$= 2n(n-1)$$

$$= 264$$

$$\therefore n(n-1) = 132$$

$$\therefore n^2 - n - 132 = 0$$

$$(n-12)(n+11) = 0$$

$$n_1 = 12 \quad n_2 = -11 \text{ (paadmissible)}$$

$$1 \cdot n = 12$$



9. Solve
$$x\sqrt{x} = x^{\sqrt{x}}$$
 where $x > 0$.

$$-1. \quad \alpha = \frac{3}{2}$$

$$1. \quad \sqrt{1} \times = \frac{3}{2}$$

$$a^{3} = a^{2}$$
 $2a = 3$ or $a = 1$
 $a = \frac{3}{2}$
 $Ax = \frac{3}{2}$
 $x = \frac{9}{4}$ or $x = 1$



10. Given that $(1+x)^6(1+mx)^5 = 1 + nx + 415x^2 + \cdots + m^5x^{11}$, find the possible values of m and n.

$$\begin{cases} nX = bX + 5mX \\ (4.5X^{2} = 1.(\frac{5}{2})(mx)^{2} + 1.(\frac{b}{2})x^{2} + (bx).(5mx) \end{cases}$$

$$(m+8)(m-5)=0$$

$$2 - \begin{cases} m_1 = -8 \\ n_2 = 31 \end{cases}$$

$$m_2 = 31$$





Solutions to HL1 Assignment #5

- 1. We must solve n(n-1) = 9900. By inspection n = 100.
- 2. We conclude $x^2 1 = 63$. So $x = \pm 8$.
- 3. We have $A^2 B^2 = (A + B)(A B) = (2 \times 5^x)(2 \times 5^{-x}) = 4$.
- 4. The required number is $\binom{5}{2} \times \binom{4}{2} = 10 \times 6 = 60$.
- 5. We have $\log(225 \times 224) = \log(10 \times n!)$, whence n! = 5040. So n = 7.
- 6. We must solve x + 2y = 5 and 2x = 3y simultaneously. We conclude x = 15/7 and y = 10/7.
- 7. We first note that any solution must satisfy x > 1. First we have $(9x + 5)/(x^2 1) = 4$, from which we arrive at the quadratic equation $4x^2 9x 9 = 0$. The admissible solution is x = 3.
- 8. The x^2 term is $\binom{n}{2}(2x)^2$. So we must solve 2n(n-1)=264, whence n=12.
- 9. By inspection x=1 is a solution. If $x \neq 1$, then we can take \log_x of both sides giving $1+\frac{1}{2}=\sqrt{x}$, whence $x=\frac{9}{4}$. We conclude x=1 or $x=\frac{9}{4}$.
- 10. Expanding gives $(1+6x+15x^2+\cdots+x^6)(1+5mx+10m^2x^2+\cdots+m^5x^5)=1+(5m+6)x+(10m^2+30m+15)x^2+\cdots+m^5x^{11}$. Equating coefficients we conclude 5m+6=n and $10m^2+30m+15=415$. Solving simultaneously gives m=-8 and n=-34 or m=5 and n=31.