HL1 Test #6

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100%

Bravo!!

1. The events A and B have probabilities $P(A) = 0.3$ and $P(B) = 0.4$. Find $P(A \cup B)$ if

   (a) A and B are mutually exclusive.

   $P(A \cup B) = p(A) + p(B) = 0.7.$ ✓

   (b) A and B are independent.

   $P(A \cup B) = p(A) + p(B) - p(A \cap B)$

   $= 0.3 + 0.4 - 0.12$ ✓

   $= 0.58$

2. The second term of an arithmetic series is 7 and the seventh term is 22. Find the sum of the first twenty terms.

   $\begin{cases} a + d = 7 \\ a + 6d = 22 \end{cases}$

   $\begin{cases} d = 3 \\ a = 4 \end{cases}$

   $S_{20} = \dfrac{(4 + 4 + 19 \times 3) \times 20}{2}$

   $= 10 \times (8 + 57)$

   $= 650$ ✓

3. The graph of $y = a\cos(b(x + c)) + d$ is shown below. Find the values of a, b, c and d.

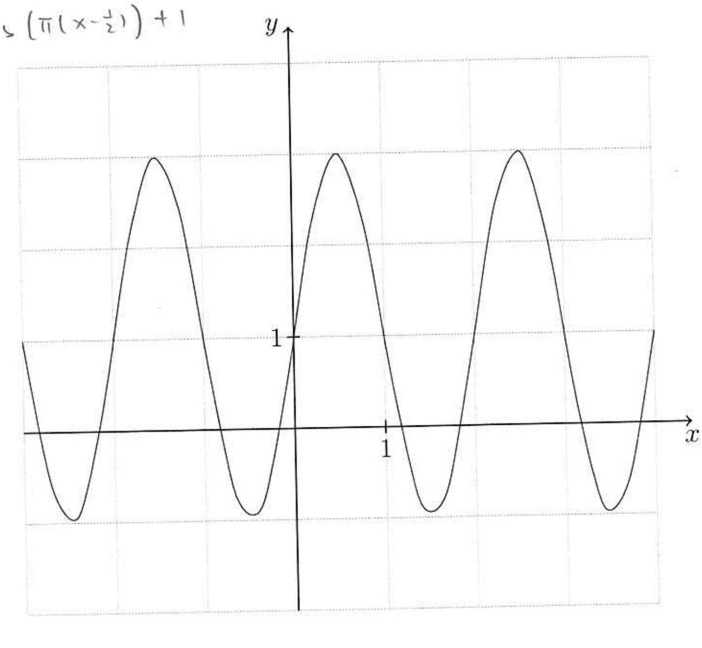
   $y = 2\cos(\pi(x - \frac{1}{2})) + 1$ ✓

   $a = 2$

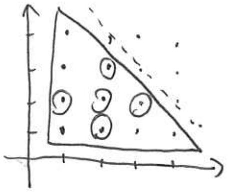
   $b = \frac{2\pi}{2} = \pi$ ✓

   $c = -\frac{1}{2}$ ✓

   $d = 1$ ✓



4. Two fair tetrahedral dice are thrown. Find the probability that at least one die is a 2 given that sum is less than 6.



$P = \frac{5}{10} = \frac{1}{2}$

(2,1) (1,2)

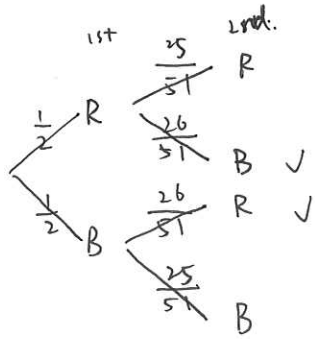
(2,2)

(3,2)

(2,3)

✓

5. Two cards are drawn without replacement from a well shuffled pack. Using a clearly labelled tree diagram find the probability that the cards have different colours.



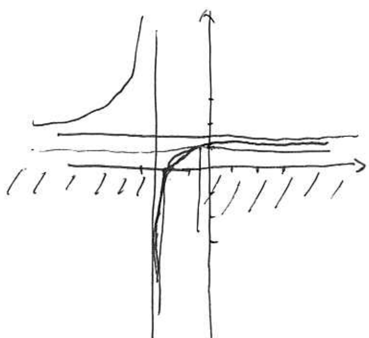
$P = \frac{1}{2} \cdot \frac{26}{51} \cdot 2 = \frac{26}{51}$

✓

R : red.

B : black.

6. Solve $\dfrac{3x+6}{2x+5} \geq 1$ expressing your answer in interval notation.



$x = -\frac{5}{2}$

$y = \frac{3}{2}$

$(-2, 0)$

$(0, \frac{6}{5})$

$y = \frac{3}{2}$

$x = -\frac{5}{2}$

$\frac{3x+6}{2x+5} = 1$

$3x + 6 = 2x + 5$

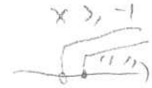
$\underline{x = -1}$

① $2x + 5 > 0$

$x > -\frac{5}{2}$

$3x + 6 \geq 2x + 5$

$x \geq -1$



② $2x + 5 < 0$

$x < -\frac{5}{2}$

$3x + 6 \leq 2x + 5$

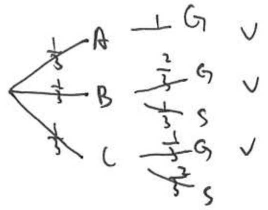
$x \leq -1$

∴ according to the graph,

$x \in \left]-\infty, -\frac{5}{2}\right[\cup \left[-1, +\infty\right[$.

✓

7. A desk has three drawers. Drawer A contains three gold coins, drawer B contains two gold coins and one silver coin, and drawer C contains one gold coin and two silver coins. A drawer is chosen at random and from it a coin is chosen at random. Given that the chosen coin is gold, find the probability that drawer C was chosen.



$$P(C|G) = \frac{\frac{1}{9}}{\frac{1}{3}+\frac{2}{9}+\frac{1}{9}}$$

$$= \frac{1}{3+2+1}$$

$$= \frac{1}{6}$$

$$P(C|G) = P(G|C)\cdot\frac{P(C)}{P(G)}$$

$$= \frac{1}{3}\cdot\frac{\frac{1}{3}}{\frac{2}{3}}$$

$$= \frac{1}{6}$$

8. Find the *exact* value of x in the diagram.

mark the two angles as θ.

$$x^2 = 1^2+4^2-2\cdot1\cdot4\cdot\cos\theta$$

$$= 3^2+4^2-2\cdot3\cdot4\cdot\cos\theta$$

$$\therefore\ 17-8\cos\theta = 25-24\cos\theta$$

$$\therefore\ 16\cos\theta = 8$$

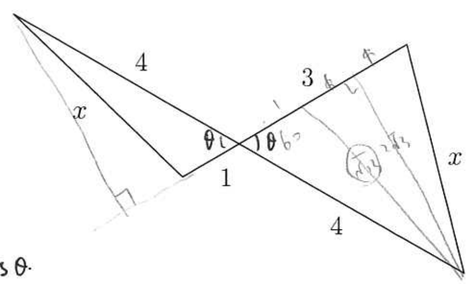
$$\cos\theta = \frac{1}{2}$$

$$\because\ \theta < 90°$$

$$\therefore\ \theta = 60°.$$

$$\therefore\ x = \sqrt{17-4}$$

$$= \sqrt{13}$$



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9. Without the calculator solve $\log_3(2\sin x) = \log_9(\cos 2x + 2)$ for $0 \le x < 2\pi$.

$\log_3(2\sin x) = \frac{1}{2}\log_3(\cos 2x+2)$

$\log_3(2\sin x)^2 = \log_3(\cos 2x+2)$

$4\sin^2 x = \cos^2 x - \sin^2 x + 2$

$5\sin^2 x = 1 - \sin^2 x + 2$

$6\sin^2 x = 3$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

$\therefore\ x = \frac{\pi}{4}, \frac{3}{4}\pi, \underbrace{\frac{5}{4}\pi, \frac{7}{4}\pi}_{(\times)}$

However,

$\begin{cases} 2\sin x > 0 & \rightarrow \sin x > 0. \rightarrow x \in]0, \frac{\pi}{2}[. \\ \cos 2x + 2 > 0 & \rightarrow \cos 2x > -2 \rightarrow \text{v}. \end{cases}$

$\therefore\ x = \frac{\pi}{4}, \frac{3}{4}\pi.$

10. Events A and B are such that $P(A) = \frac{1}{2}$, $P(A \mid B) = \frac{3}{7}$ and $P(A \mid B') = \frac{2}{3}$. Find $P(B \mid A)$.

	A	A'	
B	3a		7a
B'	0.5-3a		1-7a
	0.5	0.5	1

let $p(B) = 7a$,

then $p(A\cap B) = 3a$.

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\begin{tabular}{c|c|c|c}
~ & A & A' & ~ \\
\hline
B & 3a & ~ & 7a \\
\hline
B' & 0.5-3a & ~ & 1-7a \\
\hline
~ & 0.5 & 0.5 & 1 \\
\end{tabular}
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$p(A \mid B') = \frac{p(A\cap B')}{p(B')} = \frac{0.5-3a}{1-7a} = \frac{2}{3}$

$\therefore\ 2 - 14a = 1.5 - 9a$

$\therefore\ 0.5 = 5a$

$\therefore\ a = 0.1$

	A	A'	
B	0.3	0.4	0.7
B'	~~0.4~~ 0.2	0.1	0.3
	0.5	0.5	1

$p(B \mid A) = \frac{p(A\cap B)}{p(A)} = \frac{0.3}{0.5} = \frac{3}{5}$

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Solutions to HL1 Test #6

1. (a) 0.7 (b) 0.58

2. Since $a + d = 7$ and $a + 6d = 22$, we conclude $d = 3$ and $a = 4$. Hence $S_{20} = 10(8 + 19 \cdot 3) = 650$.

3. $a = 2$, $b = \pi$, $c = -\frac{1}{2}$, $d = 1$

4. Let T be the event of a 2 and L the event of a sum less than 6. Using a lattice diagram, which you should draw, we then have $P(T \mid L) = \frac{5}{10}$.

5. Let D be the event of different colours. Using a tree diagram, which you should draw, we have $P(D) = 2 \cdot \frac{1}{2} \cdot \frac{\cancel{35}\,26}{51} = \frac{\cancel{25}\,26}{51}$.

6. From the graph of $y = \frac{3x+6}{2x+5}$ we conclude $x \in]-\infty, -\frac{5}{2}[\cup [-1, \infty[$, or if you prefer $x \in \mathbb{R} \setminus [-\frac{5}{2}, -1[$.

7. Using a tree diagram, a Venn diagram or by Bayes' theorem $P(C \mid G) = \dfrac{\frac{1}{3} \cdot \frac{1}{3}}{\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{3}} = \frac{1}{6}$.

8. By applying the cosine rule to both triangles we obtain $x^2 = 4^2 + 1^2 - 2 \cdot 4 \cdot 1 \cdot \cos\theta$ and $x^2 = 4^2 + 3^2 - 2 \cdot 4 \cdot 3 \cdot \cos\theta$. Solving simultaneously gives $x = \sqrt{13}$.

9. After agreeing on a base of 3 we find $\log_3(2\sin x)^2 = \log_3(\cos 2x + 2)$. Letting $s = \sin x$ gives $4s^2 = (1 - 2s^2) + 2$, whence $s^2 = 0.5$. As $s > 0$, we conclud $x = \frac{\pi}{4}, \frac{3\pi}{4}$.

10. Now $P(A) = P(B \cap A) + P(B' \cap A) = P(B) \cdot P(A \mid B) + P(B') \cdot P(A \mid B')$. So

$$\frac{1}{2} = P(B) \cdot \frac{3}{7} + (1 - P(B)) \cdot \frac{2}{3},$$

whence $P(B) = 0.7$. So

$$P(B \mid A) = \frac{P(B)}{P(A)} \cdot P(A \mid B) = \frac{0.7}{0.5} \cdot \frac{3}{7} = \frac{3}{5}.$$