Ex 12F p273 1, 5, 9.

Ex 12G 1-16.

HL1 Assignment #20

Name: Jerry Jiang.

98%

Excellent!

1. Dividing $2x^3 + 5x^2 + ax + 7$ by x+3 gives a remainder of 16. What is the value of a?

$f(-3) = 2\cdot(-27) + 5\cdot 9 - 3a + 7 = 16$

$-54 + 45 + 7 - 3a = 16$

$3a = -18$

$a = -6$ ✓

2. Without the calculator solve $8^{2x+1} = 16^{2x-3}$.

$2^{6x+3} = 2^{8x-12}$

$6x + 3 = 8x - 12$

$2x = 15$

$x = \frac{15}{2}$ ✓

3. A curve has equation $y = x^3 + px^2 + px$. For what values of p does this curve have no stationary points?

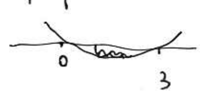
$y' = 3x^2 + 2px + p$

$\Delta = 4p^2 - 4p\cdot 3 = 4p^2 - 12p < 0$

$\therefore \quad p^2 < 3p$

$p^2 - 3p < 0$

$p(p-3) < 0$



✓

$\therefore \quad 0 < p < 3$

4. A sector of a circle has perimeter 24 cm. Use calculus to find the maximum area of the sector.

$C = 2r + \frac{\theta}{2\pi} \cdot 2\pi r$

$= 2r + \theta r.$

$= 24$

$\therefore r = \frac{24}{\theta + 2}$

$A = \frac{\theta}{2\pi} \cdot \pi r^2$

$= \frac{288\theta}{(\theta+2)^2}$

$A' = 288 \cdot \frac{(\theta+2)^2 - \theta \cdot 2(\theta+2) \cdot 1}{(\theta+2)^4}$

$= \frac{288(-\theta^2+4)}{(\theta+2)^4}$

if $A' = 0$, then:

$\theta^2 = 4$

$\therefore \theta = \pm 2$

$\because \theta \neq -2$

$\therefore \theta = 2.$

✓

$\therefore A_{max} = \frac{288 \times 2}{4^2}$

$= 36 \text{ cm}^2$

Justify max.

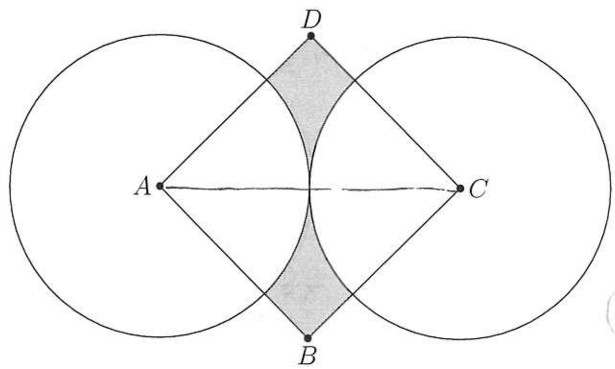
5. The circles with centres A and C each have radius 8 cm and are intersected by the square $ABCD$. Find the area of the shaded region.

$A_{shaded} = \left(\frac{8 \times 2}{\sqrt{2}}\right)^2 - \frac{1}{2} \cdot \pi \cdot 8^2$

$= \frac{256}{2} - 32\pi$

$= 128 - 32\pi \quad (\text{cm}^2)$

✓



6. When the binomial $(2 + ax)^{10}$ is expanded, the coefficient of the term in x^3 is 414 720. Find the value of a.

$2^7 \cdot (ax)^3 \cdot \binom{10}{3}$

$= 2^7 \cdot a^3 \cdot x^3 \cdot \frac{10 \times \cancel{9} \times \cancel{8}}{\cancel{3 \times 2 \times 1}}$ 3×4

$= \underbrace{2^9 \cdot 3 \cdot 10 \cdot a^3}_{\begin{array}{c}\shortparallel \\ 414720\end{array}} \; x^3$

$\therefore a^3 = 27$

$\therefore a = 3$

✓

11

7. A fair tetrahedral die is thrown three times. If event R is the sum of the three scores is 9 and event S is the product of the three scores is 16, determine whether events R and S are independent.

$R:\ 1,4,4 \to 3 \qquad \therefore P(R) = \frac{10}{64}$ ✓

$\quad 2,3,4 \to 6$

$\quad 3,3,3 \to 1$

$S:\ 1,4,4 \to 3 \qquad \therefore P(S) = \frac{6}{64}$ ✓

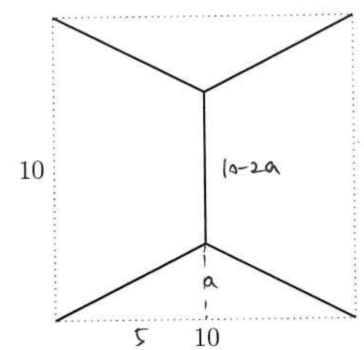
$\quad 2,2,4 \to 3$

$R \cap S:\ 1,4,4 \to 3 \qquad \therefore P(R \cap S) = \frac{3}{64}$ ✓

$\because P(R) \cdot P(S) \neq P(R \cap S)$

$\therefore$ not independent

8. The following shape is made from wire. It has both vertical and horizontal lines of symmetry. The ends of the shape are at the vertices of a square with a side length of 10. Find the minimum length of the wire.



$C = 10 - 2a + 4\sqrt{a^2+25}$

$C' = -2 + 4 \cdot \frac{1}{2}(a^2+25)^{-\frac{1}{2}} \cdot 2a$

$= -2 + \frac{4a}{\sqrt{a^2+25}}$

$=0$

then $\frac{a}{\sqrt{a^2+25}} = \frac{2}{4} = \frac{1}{2}$

$\therefore 2a = \sqrt{a^2+25}$

$4a^2 = a^2 + 25$

$a = \pm \frac{\sqrt{3}}{3} \cdot 5$ (negative ×)

$\therefore C_{min} = 10 - 2 \cdot \frac{5\sqrt{3}}{3} + 4\sqrt{\frac{25+75}{3}}$

$= 10 - \frac{10}{3}\sqrt{3} + \frac{10}{3}\sqrt{3} \cdot 4$

$= 10 + 10\sqrt{3}$

Justify 说明这个 $f'(x)=0$ 产生的 min 而不是 max.

min.

4

9

9. The curve $y = \dfrac{ax-b}{x^2-1}$ where $a, b \in \mathbb{R}$ has a stationary point at $(3, 1)$. Sketch the curve indicating any key features.

$$y' = \frac{a(x^2-1)-(ax-b)(2x)}{(x^2-1)^2}$$

$$= \frac{ax^2-a-2ax^2+2bx}{(x^2-1)^2}$$

$y' = 0$

then $-ax^2+2bx-a=0$

$-a\cdot 9 + 6b - a = 0$

① $6b = 10a$

$(3,1) \to \dfrac{3a-b}{8} = 1$

$\therefore$ ② $b = 3a-8$

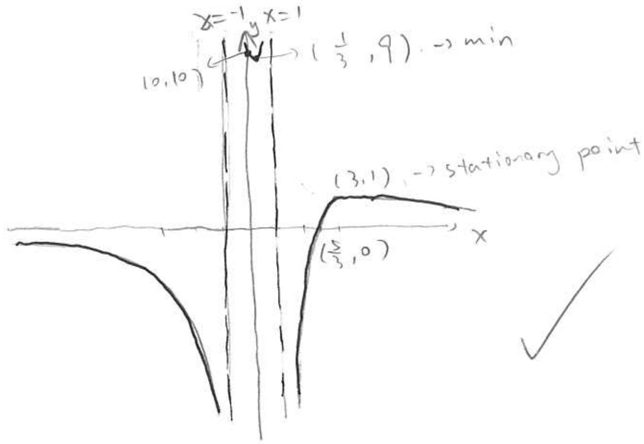
① & ② $\Rightarrow a = 6$

$b = 10$

$$\therefore y = \frac{6x-10}{x^2-1}$$

$$y' = \frac{-6x^2+20x-6}{(x^2-1)^2}$$

$y' = 0 \Rightarrow x_1 = \frac{1}{3}, x_2 = 3$



10. Research the sum of an infinite geometric series. Hence find the sums of the two possible infinite geometric series with first term 18 and third term 8.

$18 \cdot a^2 = 8$

$a = \pm\frac{2}{3}$

$$S = \frac{18(1-a^{\infty})}{1-a}$$

$$\therefore S_1 + S_2 = \frac{18\left[1-\left(\frac{2}{3}\right)^{\infty}\right]}{1-\frac{2}{3}} + \frac{18\left[1-\left(-\frac{2}{3}\right)^{\infty}\right]}{1+\frac{2}{3}}$$

$$= \frac{18}{\frac{1}{3}} + \frac{18}{\frac{5}{3}}$$

$$= \boxed{54 + \frac{54}{5}} \Rightarrow S_1 = 54. \quad S_2 = \frac{54}{5}$$

$$= \frac{324}{5}$$

10

Solutions to HL1 Assignment #20

1. By the remainder theorem $p(-3) = -54 + 45 - 3a + 7 = 16$, whence $a = -6$.

2. We have $8 \cdot 2^{6x} = 2^{8x-2}$, whence $2x = 15$ or $x = \frac{15}{2}$.

3. Here $y' = 3x^2 + 2px + p$. Notice y' is quadratic with $\Delta = 4p^2 - 12p$. Solving $\Delta < 0$ gives $p \in]0, 3[$.

4. Denote the radius of the sector by r, the central angle's radian measure by θ and the area by A. Then we have $2r + r\theta = 24$ and $A = \frac{1}{2}r^2\theta$, whence $A = 12r - r^2$, $0 < r < 12$. Next $A' = 12 - 2r$ and solving $A' = 0$ gives $r = 6$. Since $A'' = -2 < 0$ for all r, we conclude $r = 6$ gives the maximum area of $36\,\text{cm}^2$.

5. The shaded area can be thought of as a square of side length $8\sqrt{2}\,\text{cm}$ minus a half circle of radius $8\,\text{cm}$. This gives the area as $32(4 - \pi)\,\text{cm}^2$.

6. By the binomial theorem $\binom{10}{3} \cdot 2^7 \cdot a^3 = 414720$, whence $a = 3$.

7. Considering a sample space of 64 ordered triples we have $n(R) = 10$, $n(S) = 6$ and $n(R \cap S) = 3$. So $P(R \mid S) = \frac{3}{6} \neq \frac{10}{64} = P(R)$, whence events R and *are* not independent.

8. Denote the length of the central leg by x, the length of an oblique leg by y and the total length by t. Then $t = x + 4y$ and $y^2 = 25 + \frac{1}{4}(10 - x)^2$, $0 < x < 10$. Then we conclude

$$t = x + 2\sqrt{10^2 + (10 - x)^2}$$

. Using the GDC gives $t_{\text{min}} = 27.3$ (3 s.f).

9. By the quotient rule $y' = \frac{a(x^2-1)-2x(ax-b)}{(x^2-1)^2}$. Next $y(3) = 1$ and $y'(3) = 0$, whence $-10a + 6b = 0$ and $3a - b = 8$. Solving simultaneously gives $a = 6$ and $b = 10$.

10. Here $r^2 = \frac{4}{9}$, whence $r = \pm\frac{2}{3}$. So the required sums are

$$S_\infty = \frac{18}{1 - \frac{2}{3}} = 54 \quad \text{and} \quad S'_\infty = \frac{18}{1 + \frac{2}{3}} = \frac{54}{5}.$$