

1. In $\triangle ABC$, $a = 8$, $b = 12$ and $c = 10$. Point L is on side $[BC]$ such that $\angle BAL = \angle CAL$. Find BL .

2. Calculate the values of x for which the determinant $\begin{vmatrix} x & 5 & -1 \\ 1 & 3 & x \\ 1 & 4 & 7 \end{vmatrix}$ is zero.

3. Consider the group $(G, *)$ with identity e . If $x * x = e$ for all $x \in G$, show that $(G, *)$ is Abelian.

4. Use the integral test, clearly stating the conditions for its use, to show that the harmonic series diverges.

5. The simple graph G has the adjacency matrix below. Find the maximum number of edges that can be added to G so that it remains simple and planar. Be sure to justify your answer.

$$A(G) = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}.$$