FM2 Assignment #18 Name: Jerry Jiang $9\frac{1}{2}/10$ Very good

1. Find $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \cos t\, dt$.

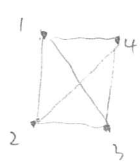
Let $a = x^2$.

$$\frac{d}{dx}\int_0^a \cos t\, dt = \cos a \cdot a' = \cos x^2 \cdot 2x = 2x\cos x^2$$

2. Find $\dfrac{d}{dx}\displaystyle\int_x^{x^2} \cos t\, dt$.

$$\frac{d}{dx}\int_x^{x^2} \cos t\, dt = \frac{d}{dx}\int_0^{x^2} \cos t\, dt - \frac{d}{dx}\int_0^{x} \cos t\, dt$$

$$= 2x\cos x^2 - \cos x$$



3. For each of the following either explain why the graph cannot exist or draw a graph with the given property.

(a) A bipartite graph which contains K_4.

Consider the K_4 graph above. Since vertex 1 is connected to 2, they're on different side of the partition. Similarly, 1 and 3 are also on different side. However, 2 is connected to 3, indicating they're on different side as well. Thus, it's impossible to do partition into 2 parts and contains K_4 in a bipartite graph.

(b) A simple planar bipartite graph with 7 vertices and 11 edges.

There're no ~~circuits~~ cycles of ~~degree~~ length 3 in a bipartite graph.
also known as a triangle

$$\left.\begin{array}{l} 2e \geq 4f \\ 2+e-v=f \end{array}\right\} \Rightarrow 2e \geq 4(2+e-v), \quad e \leq 2v-4$$ as long as it's simple, planar.

However, $11 > 2\times 7-4$, so the graph can't exist.

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4. Show that for small x, $x \csc x \approx [1-(x^2/6 - x^4/120)]^{-1}$. Expand the RHS by the binomial expansion and conclude that

$$\csc x \approx \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360}.$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - O(x^7).$$

$$\csc x = \frac{1}{x - \frac{x^3}{6} + \frac{x^5}{120} - O(x^7)}$$

$$x \csc x = \frac{1}{1 - \frac{x^2}{6} + \frac{x^4}{120} - O(x^6)}$$ and $O(x^6)$ has minor influence when x is small.

$$x\csc x = [1-(\tfrac{x^2}{6} - \tfrac{x^4}{120})]^{-1} = 1 + \binom{-1}{1}\left[-\left(\tfrac{x^2}{6}-\tfrac{x^4}{120}\right)\right] + \binom{-2}{1}\left[-\left(\tfrac{x^2}{6}-\tfrac{x^4}{120}\right)\right]^2 + \cdots$$

$$= 1 + \frac{x^2}{6} - \frac{x^4}{120} + \frac{x^4}{36} + O(x^6)$$

$$= 1 + \frac{x^2}{6} + \frac{7x^4}{360}$$

Therefore, $\csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360}$.

5. Let $f\colon G \to H$ be a group homomorphism with $K = \ker(f)$.

(a) Show that $gkg^{-1} \in K$ for all $g \in G$ and $k \in K$.

$$f(gkg^{-1}) = f(g)\circ f(k) \circ f(g^{-1})$$
$$= g' \circ e' \circ (g^{-1})'$$
$$= g' \circ (g^{-1})'$$
$$= e'$$

$\therefore gkg^{-1} \in K$.

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(b) Deduce that each left coset of K in G is also a right coset.

$$f(g^{-1}kg) = f(g^{-1})\circ f(k)\circ f(g)$$, where $g \in G$, $k \in K$.
$$= (g^{-1})' \circ e' \circ g'$$
$$= e'$$

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$\therefore g^{-1}kg \in K$. for all $g \in G$.

$\therefore$ Left coset of K can be written as $g(g^{-1}kg) = (gg^{-1})\cdot kg = kg$, which is the right coset of K in G.

Not a coset, but an element of a coset.

Left coset of K in G containing g, $\{gk \mid k \in K\}$, which is a set not an element.

Elements in the left coset of K can be written as ... which is elements in the right coset of K in G.