FM2 Assignment #2 Name: Jerry Jiang

$9\frac{1}{2}$ / 10 Very good

1. List the subgroups of $\mathbb{Z}_{18}$.

subgroup	generator	order
$\{0\}$	$\langle 0 \rangle$	1
$\{0, 9\}$	$\langle 9 \rangle$	2
$\{0, 6, 12\}$	$\langle 6 \rangle$	3
$\{0, 3, 6, 9, 12, 15\}$	$\langle 3 \rangle$	6
$\{0, 2, 4, 6, 8, 10, 12, 14, 16\}$	$\langle 2 \rangle$	9
$\mathbb{Z}_{18}$	$\langle 1 \rangle$	18

2. Suppose a cyclic group's only proper subgroup has order 7. What is the order of the group?

This indicates that the group's order n only has 1 and 7 as its divisors. Therefore $\underline{\underline{n = 49}}$ and the cyclic group can be $\mathbb{Z}_{49}$ while the only proper subgroup generated by 7 is $\{0, 7, 14, 21, 28, 35, 42\}$.

3. Suppose groups G and G' are isomorphic. Show that if G is Abelian then G' must also be Abelian.

Since $(G, *)$ and $(G', \circ)$ are isomorphic, then there must be $f: G \to G'$ where $f(x * y) = f(x) \circ f(y)$.

$\because$ G is abelian,

$\therefore$ $f(x * y) = f(y * x) = f(y) \circ f(x)$

$\because$ x in G corresponds to $f(x)$ in G' and y in G corresponds to $f(y)$ in G', and $f(x) \circ f(y) = f(y) \circ f(x)$.

$\therefore$ G' is also abelian.

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4. Show that the series $\sum_{n=1}^{\infty} (-1)^n \tan(\frac{1}{n})$ converges conditionally.

① $\sum_{n=1}^{\infty} \tan(\frac{1}{n})$. Limit Comparison Test: when $n \geq 1$, $\tan(\frac{1}{n}) > 0$; $\frac{1}{n} > 0$.

$\lim_{n\to\infty} \frac{\tan(\frac{1}{n})}{(\frac{1}{n})}$ $\quad \lim_{n\to\infty} \tan(\frac{1}{n}) = 0$, $\quad \lim_{n\to\infty} \frac{1}{n} = 0$ (use the L'Hôpital's Rule).

$= \lim_{n\to\infty} \frac{\sec^2 \frac{1}{n} \cdot -n^{-2}}{-n^{-2}}$

$= \lim_{n\to\infty} \sec^2 \frac{1}{n}$

$= \sec^2 0 = 1$

Since $L=1$, $\sum_{n=1}^{\infty} \tan(\frac{1}{n})$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ behave the same.

$\because$ the harmonic series diverges,

$\therefore \sum_{n=1}^{\infty} \tan(\frac{1}{n})$ diverges.

② we have $\tan(\frac{1}{n}) > \tan(\frac{1}{n+1})$ for all $n \geq 1$. so when $n \to \infty$, $0 < \tan(\frac{1}{n+1}) < \tan(\frac{1}{n})$.

Since $\lim_{n\to\infty} \tan(\frac{1}{n}) = \tan 0 = 0$, the alternating series converge.

$\therefore \sum_{n=1}^{\infty} (-1)^n \tan(\frac{1}{n})$ converges conditionally.

5. Let G be a simple graph with p vertices and q edges. Show that if $q > \frac{1}{2}(p-1)(p-2)$ then G is connected.

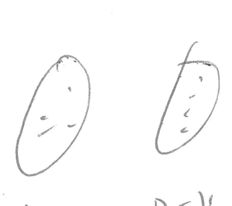
There're p vertices from $a_1, a_2 \cdots$ to a_p.

Let out a_p and connect all the edge available from a_1 to a_{p-1}. $1\frac{1}{2}$

Then number of edges $= \frac{1}{2}(p-1)[(p-1)-1] = \frac{1}{2}(p-1)(p-2)$.

Since $q > \frac{1}{2}(p-1)(p-2)$, there has to be an edge between a_p and one of the $p-1$ remaining vertex. Thus, G is connected.

What if division of vertices was



k $\quad$ p-k

$(k \neq 1)$

instead of

1 $\quad$ p-1

?

$3\frac{1}{2}$

~~设~~ $T = \binom{x}{2} + \binom{p-x}{2}$.