FM2 Assignment #14 Name: Jerry Jiang 9/10 V. ...

1. Write the Maclaurin series for e^x, $\cos x$ and $\sin x$ in sigma notation.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \checkmark$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \checkmark$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \checkmark$$

2. Find the n-th degree Taylor polynomial for $\frac{1}{x}$ about $x = 1$ and write the polynomial in sigma notation.

$$P_n(x) = 1 - (x-1) + \frac{2}{2!}(x-1)^2 - \frac{6}{3!}(x-1)^3 + \cdots$$

$$P_n(x) = \sum_{n=0}^{\infty} (-1)^n (x-1)^n$$

1½

3. Use the alternating series estimation theorem to find an interval centre 0 throughout which $\cos x$ can be approximated by $1 - \frac{1}{2}x^2$ to three decimal places.

$f(x) = \cos x$

$P_2(x) = 1 - \frac{x^2}{2}$

$\therefore \left| \cos x - \left(1 - \frac{x^2}{2}\right) \right| < \frac{x^4}{4!}$

In order for it to be an appropriate estimation, $\frac{x^4}{4!} \le 0.0005$.

$\therefore x^4 \le 0.012$ $\left(\sqrt[4]{0.012} = 0.330975 \text{ (6 s.f.)}\right)$.

$\therefore x \in [-0.330975, 0.330975]$. ✓

5½

4. Show that the power series $\sum_{n=1}^{\infty} \frac{n^2}{n!}(x-1)^n$ converges for all $x \in \mathbb{R}$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{(n+1)^2}{(n+1)!}(x-1)^{n+1}}{\frac{n^2}{n!}(x-1)^n}\right| = \left|\frac{n+1}{n^2}(x-1)\right| = \left|\left(\frac{1}{n}+\frac{1}{n^2}\right)(x-1)\right|$$

$$\therefore \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\left(\frac{1}{n}+\frac{1}{n^2}\right)(x-1)\right| = 0\,(x-1) = 0 < 1.$$

According to the ratio test, the series converges for all $x \in \mathbb{R}$.

5. Let $f(x) = e^x \sin x$. Show that $f''(x) = 2(f'(x) - f(x))$. Hence find the fifth degree Maclaurin polynomial for f.

$$f'(x) = e^x(\sin x + \cos x)$$

$$f''(x) = 2e^x \cos x$$

$$\begin{aligned} f''(x) &= 2e^x\cos x = 2e^x[(\sin x+\cos x)-\sin x] \\ &= 2[e^x(\sin x+\cos x) - e^x \sin x] \\ &= 2(f'(x)-f(x)). \end{aligned}$$

Therefore, $f'''(x) = 2f''(x) - 2f'(x)$.

$$\begin{aligned} &= 4f'(x) - 4f(x) - 2f'(x) \\ &= 2f'(x) - 4f(x) \end{aligned}$$

$$\begin{aligned} f''''(x) &= 2f''(x) - 4f'(x) \\ &= 4f'(x) - 4f(x) - 4f'(x) \\ &= -4f(x) \end{aligned}$$

$$f'''''(x) = -4f'(x).$$

Note that $f(0)=0$, $f'(0)=1$

$$P_5(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f''''(0)}{4!}x^4 + \frac{f'''''(0)}{5!}x^5$$

$$P_5(x) = 0 + 2x + \frac{2}{2}x^2 + \frac{2}{6}x^3 + 0 + \frac{-4}{120}x^5$$

$$\therefore P_5(x) = 2x + x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5.$$

JL

$1\frac{1}{2}$

$3\frac{1}{2}$

mode → SEQ

↓

y=

↓

nmin = 0

↓

$u(n) = 2u_{n-1} - 2u_{n-2}$

↓

$u(n\min) = \{1, 0\} \Rightarrow u_1 = 1, u_0 = 0.$

↓

table