HL1 Assignment #1

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100%

Excellent !!

1. How many terms are there in the arithmetic sequence $\frac{1}{3}, \frac{5}{3}, \ldots, 31$?

$a = \frac{1}{3} \quad d = \frac{4}{3}$

$31 = \frac{1}{3} + (n-1) \cdot \frac{4}{3}$

$n-1 = \frac{92}{3} \cdot \frac{3}{4} = 23$

$n = 24$

2. In an arithmetic sequence $u_5 = 83$ and $u_{16} = -16$. Find the first three terms of the arithmetic sequence.

$\begin{cases} a + 4d = 83 \\ a + 15d = -16 \end{cases}$

$11d = -99$

$\begin{cases} d = -9 \\ a = 119 \end{cases}$

$\therefore U_n = 119 - 9(n-1)$

$U_1 = 119$

$U_2 = 110$

$U_3 = 101$

3. Find the sum of the series $\sum_{n=1}^{4} n^n$.

$\sum_{n=1}^{4} n^n = 1^1 + 2^2 + 3^3 + 4^4$

$= 1 + 4 + 27 + 256$

$= 288$

4. A geometric sequence has second term -6 and fifth term 162. Find its general term.

$$\begin{cases} a \cdot r = -6 \\ a \cdot r^4 = 162 \end{cases}$$

$$r^3 = -\frac{162}{6} = -\frac{81}{3} = -27$$

$$\begin{cases} r = -3 \\ a = 2 \end{cases}$$

$$U_n = 2 \cdot (-3)^{n-1}$$

5. If the population of a country increases by 3% each year, how many years will it take for population to double?

let population be p. year-number be t

$$p \cdot 1.03^t \geqslant 2p.$$

$$1.03^t \geqslant 2$$

$$t \geqslant 23.45$$

so it will take 24 years to double the population.

6. The sum of the first n positive integers is 4950. Find n.

$$\frac{(1+n) \cdot n}{2} = 4950$$

$$n^2 + n - 9900 = 0$$

$$(n+100)(n-99) = 0$$

$$n_1 = -100 \; (\times).$$

$$n_2 = 99.$$

so $n = 99$.

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7. Complete the following table of partial sums for the series of Fibonacci numbers. Make a conjecture about S_n.

n	1	2	3	4	5	6	7	8	9	10
S_n	1	2	4	7	12	20	33	54	88	143

1 1 2 3 5 8 13 21 34 55 89

$$S_n = \frac{1}{\sqrt{5}}\left[\left(\frac{\sqrt{5}+1}{2}\right)^{n+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+2}\right] - 1$$

but how to prove $\sum_{n=1}^{a} \frac{1}{\sqrt{5}}\left[\left(\frac{\sqrt{5}+1}{2}\right)^{n} - \left(\frac{1-\sqrt{5}}{2}\right)^{n}\right] = \frac{1}{\sqrt{5}}\left[\left(\frac{\sqrt{5}+1}{2}\right)^{a+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{a+2}\right] - 1$

yeah I prove it. see the added paper. Yeah!

8. Let x and y be two non-negative real numbers. The *arithmetic mean* and *geometric mean* of x and y are defined to be $A = (x+y)/2$ and $G = \sqrt{xy}$ respectively. Prove that $A \geq G$.

$A = \frac{x+y}{2}$

$G = \sqrt{xy}$

$G^2 = xy$

we have $x^2 - 2xy + y^2 = (x-y)^2 \geq 0$

so $x^2 + y^2 \geq 2xy$

so $x^2 + 2xy + y^2 \geq 4xy$

$(x+y)^2 \geq 4xy$

if we take the root of the inequation.

$x+y \geq 2\sqrt{xy}$ $(x, y \geq 0)$

so we finally get

$\frac{x+y}{2} \geq \sqrt{xy}$,

which is $A \geq G$

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9. A sequence is said to be in *harmonic progression* if the reciprocals of the terms are in arithmetic progression. The eighth term of a harmonic progression is 3/26 and the twenty second term is 1/18. Find the hundredth term of this harmonic progression.

$8^{th}: \frac{26}{3}$

$22^{th}: 18$

$$\begin{cases} a+7d=\frac{26}{3} \\ a+21d=18 \end{cases}$$

$14d = \frac{28}{3}$

$$\begin{cases} d=\frac{2}{3} \\ a=\frac{12}{3} \end{cases}$$

$U_n = \frac{12}{3} + \frac{2}{3}(n-1)$

$U_{100} = \frac{12}{3} + \frac{198}{3}$

$= \frac{210}{3} = 70$

So the 100^{th} term of the harmonic progression is $\frac{1}{70}$.

✓

10. The interior angles of a convex polygon, measured in degrees, form an arithmetic sequence. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.

let there be n sides.

the sum of the interior angles are $180°(n-2)$.

~~$120 + 5(n-1) = 180(n-1)$~~

the biggest angle of the polygon is $120+5(n-1)$

the sum of the interior angles can also be presented as

$$\frac{\{120+[120+5(n-1)]\}\times n}{2} = 180(n-2)$$

$n^2-25n+144=0$

$(n-9)(n-16)=0$

$n_1=9 \quad n_2=16$

$a_1 = 120 + 5\times(9-1) = 160$ (✓)

$a_2 = 120 + 5\times(16-1) = 195$ (✗) (convex).

∴ this is a 9-sided polygon. ✓

$120n + 120n + 5(n-1)n = 360n - 720$

$5n^2 - 125n + 720 = 0$

$n^2 - 25n + 144 = 0$

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Solutions to HL1 Assignment #1

1. Let the number of terms be n. Then we have $31 = 1/3 + (n-1) \times 4/3$. Solving gives $n = 24$.

2. Here $a + 4d = 83$ and $a + 15d = -16$. Solving simultaneously gives $a = 119$ and $d = -9$. Hence the first three terms of the sequence are $119, 110, 101$.

3. Let the sum of the series be S. We have $S = 1^1 + 2^2 + 3^3 + 4^4 = 288$.

4. Here $ar = -6$ and $ar^4 = 162$. Solving simultaneously gives $a = 2$ and $r = -3$. Hence $u_n = 2 \times (-3)^{n-1}$.

5. Let the initial population be P_0 and let n be the number of years for the population to double. We must solve $2P_0 = P_0(1.03)^n$, or equivalently $1.03^n = 2$. The solution is $n = 23.4$ (3 s.f.).

6. We must solve $1 + 2 + 3 + \cdots + n = 4950$, which is equivalent to $n(n+1) = 9900$. So $n = 99$.

7. The completed table is below. We conjecture $S_n = \sum_{k=1}^{n} f_k = f_{n+2} - 1$.

n	1	2	3	4	5	6	7	8	9	10
S_n	1	2	4	7	12	20	33	54	88	143

8. Observe that $(\sqrt{x} - \sqrt{y})^2 \geq 0$ for all $x, y \geq 0$. So $x + y \geq 2\sqrt{xy} \Leftrightarrow (x+y)/2 \geq \sqrt{xy}$. That is, $A \geq G$.

9. Let the general terms of the harmonic sequence and its associated arithmetic sequence be h_n and u_n respectively. We have $h_8 = 3/26$ and $h_{22} = 1/18$. So $u_8 = 26/3$ and $u_{22} = 18$. Hence $a + 7d = 26/3$ and $a + 21d = 18$. Solving simultaneously gives $a = 4$ and $d = 2/3$. Hence $u_{100} = 70$ and we conclude $h_{100} = 1/70$.

10. Let the number of sides of the polygon be n. We have

$$\frac{n}{2}[240 + 5(n-1)] = 180(n-2),$$

which is equivalent to $n^2 - 25n + 144 = 0$. So $n = 9$ or $n = 16$. However, $n = 16$ is inadmissible as a convex polygon has no angle of $180°$ or more. So the number of sides of the polygon is 9.