10/10 Excellent!

FM2 Assignment #8 Name: Jerry Jiang

1. Give an example of a relation that is symmetric and transitive but not reflexive.

Here's an example of how the idea of relation can be applied to genetics.
In the case of autosomal recessive disorder, only by having two recessive allele, genetic information that determines a trait of a person, can a person get the disease. There're 2 types of alleles: D and d, where D is a dominant normal allele and d is a recessive disordered allele.
Now let relation △ define on set $S = \{D, d\}$ with rule: $x \triangle y$ if the combination of allele x and y show the disease.

- if $x \triangle y$, which means combination of x and y shows the disease, have the only possibility of $x = y = d$. Since if $x = D$ or $y = D$, then the disease is hidden. Therefore, $d \triangle d$, so $y \triangle x$. it's symmetric.
- if $x \triangle y$, $y \triangle z$, then due to similar reasoning, $x = y = z = d$. then $x \triangle z$. it's transitive.
- In terms of reflexive, though $d \triangle d$, $D \not\triangle D$ as having two normal dominant allele won't show the disease. it's not reflexive.

Therefore, relation △ is symmetric, transitive, but not reflexive.

Though the "product is odd" relation would work here, it would be interesting to see something else here:

Interesting application!

2. Evaluate the improper integral $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$.

Let $x = 2\sin u$. when $x = 0$, $u = 0$; when $x = 2$, $u = \frac{\pi}{2}$.

$dx = 2\cos u\,du$. $x^2 = 4\sin^2 u$. $4 - x^2 = 4\cos^2 u$.

$$\therefore \int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{4\cos^2 u}} \cdot 2\cos u\,du = \int_0^{\frac{\pi}{2}} \frac{2\cos u}{2\cos u}\,du = u\Big|_0^{\frac{\pi}{2}}$$

$$\therefore \int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx = u\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 0 = \frac{\pi}{2}.$$

✓

3. Determine the kernel of the group homomorphism $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (4x + 2y, 2x + y)$.

$e' = (0,0)$, therefore $\begin{cases} 4x + 2y = 0 \\ 2x + y = 0 \end{cases} \to y = -2x$

$\therefore \ker(f) = \{(x, -2x) \mid x \in \mathbb{R}\}$.

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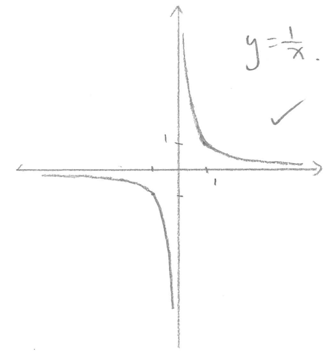
4. The relation $\sim$ is defined on $\mathbb{R}^2$ by $(x_1, y_1) \sim (x_2, y_2)$ if $x_1y_1 = x_2y_2$. Show that $\sim$ is an equivalence relation and graph the equivalence class $[(1, 1)]$.

- reflexive: $x_1y_1 = a$, then $x_1y_1 = a = x_1y_1$. $(x_1,y_1) \sim (x_1,y_1)$.
- symmetric: $(x_1,y_1) \sim (x_2,y_2)$. $x_1y_1 = x_2y_2 = a$, then $x_2y_2 = x_1y_1 = a$.

  $(x_2,y_2) \sim (x_1,y_1)$.
- transitive: $(x_1,y_1) \sim (x_2,y_2)$, $(x_2,y_2) \sim (x_3,y_3)$. then $x_1y_1 = x_2y_2 = a$.

  $a = x_2y_2 = x_3y_3$. Therefore $x_1y_1 = a = x_3y_3$. $(x_1,y_1) \sim (x_3,y_3)$.

for $(1,1)$, $x \cdot y = 1$. So the graph is $y = \frac{1}{x}$.



5. The convergent sequence $u_n = \dfrac{e^n + 2^n}{2e^n}$ has limit L. Find the smallest value of n for which $|u_n - L| < 0.001$.

$$u_n = \frac{1}{2} + \frac{1}{2}\left(\frac{2}{e}\right)^n.$$

$\because \left|\frac{2}{e}\right| < 1$

$\therefore u_\infty = \frac{1}{2} = L.$

$$|u_n - L| = \left|\frac{1}{2}\left(\frac{2}{e}\right)^n\right| = \frac{1}{2}\left(\frac{2}{e}\right)^n$$

$\therefore \frac{1}{2}\left(\frac{2}{e}\right)^n < 0.001$

$\therefore n > \log_{\frac{2}{e}} 0.002$

$\therefore n > 20.3.$

$\therefore n_{min} = 21.$

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