HL1 Assignment #19 Name: Jerry Jiang

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1. The graph of the derivative of the function f is drawn below. State the values of x where the function f has a

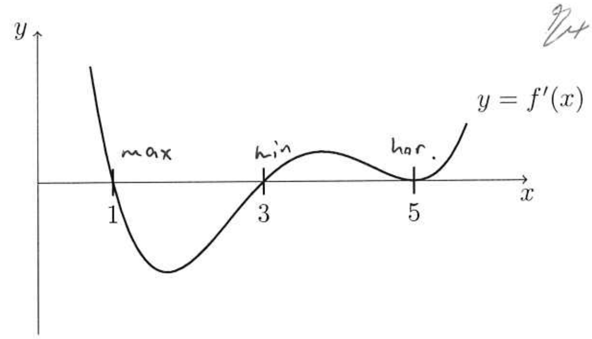
Excellent!

(a) local minimum;

3.

(b) local maximum.

1.



2. The tangent to the curve $y = \sqrt{x}$ at the point $(4, 2)$ meets the x-axis at the point Q. Find the coordinates of Q.

$f(x) = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f'(4) = \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{1}{4}$

$\therefore\ l: y = \frac{1}{4}x + b$

$\frac{1}{4} \cdot 4 + b = 2$

$\therefore\ b = 1$

$\therefore\ l: y = \frac{1}{4}x + 1$

$\because\ y = 0$

$\therefore\ x = -4$

$\therefore\ Q(-4, 0)$.

3. Consider the four integers a, b, c, d where $a \le b \le c \le d$. If the mean of the four integers is 4, the mode 3, the median 3 and the range 6, find the values of a, b, c and d.

$b + c = 6$

$a + b + c + d = 16$

$d - a = 6$

$\therefore \begin{cases} a + d = 10 \\ a - d = -6 \end{cases}$

$\begin{cases} a = 2 \\ d = 8 \end{cases}$

$\because$ mod $= 3$,

$\therefore\ c = b = 3$.

$\therefore\ a = 2,\ b = 3,\ c = 3,\ d = 8$.

4. If $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{5}{12}$, find $P(A' \mid B')$.

	A	A'	
B	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{1}{3}$
B'	$\frac{1}{12}$	$\frac{7}{12}$	$\frac{2}{3}$
	$\frac{1}{6}$	$\frac{5}{6}$	1

$P(A \cap B) = \frac{1}{6} + \frac{1}{3} - \frac{5}{12} = \frac{1}{12}$

$$\therefore P(A' | B') = \frac{P(A' \cap B')}{P(B')} = \frac{\frac{7}{12}}{\frac{2}{3}} = \frac{21}{24} = \frac{7}{8}$$

5. Solve $\dfrac{2x}{|x-1|} < 1$ giving your answer in interval notation.

$|x-1| > 0$

$\therefore 2x < |x-1|$

① $x \leq 1$

$2x < 1 - x$

$x < \frac{1}{3}$

$\therefore x < \frac{1}{3}$

② $x > 1$

$2x < x - 1$

$x < -1$

$\therefore \phi$

$\therefore x < \frac{1}{3}$

$\therefore x \in]-\infty, \frac{1}{3}[.$

6. Use the first derivative test to determine the nature of the stationary points for the curve $y = 15x^3 - x^5$.

let $f(x) = 15x^3 - x^5$

$\therefore f'(x) = 45x^2 - 5x^4$

have $f'(x) = 0$

$\therefore 45x^2 - 5x^4 = 0$

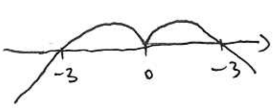
① $x = 0$ ✓

② $x \neq 0$

$45 = 5x^2$

$x^2 = 9$

$x = \pm 3.$



x	$\to$	-3	$\to$	0	$\to$	$+3$	$\to$
$f'(x)$	$-$	0	$+$	0	$+$	0	$-$
$f(x)$	$\searrow$	min	$\nearrow$		$\nearrow$	max	$\searrow$

rising point of inflection

y-coords? 4

/4

7. A classroom has twelve empty chairs arranged in three rows of four chairs. Three students enter the room and randomly choose a seat. Find the probability that exactly one of the rows is empty.

choose the empty row : 3.

choose the lonely guy : 3.

choose the row with 2 guy: 2

number probability of lonely : 4. ←

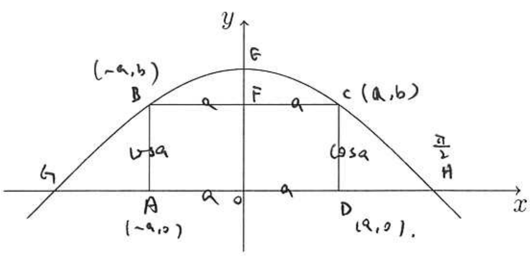
number probability of 2 guy: 6 x 2. ←

not probability.

$$p = \frac{3 \times \cancel{3} \times 2 \times \cancel{4} \times 6 \times 2}{\cancel{12} \times 11 \times 10}$$

$$= \frac{36}{55}$$

8. A rectangle is drawn as depicted inside the central arch of the cosine curve. Find the maximum area of the rectangle giving your answer to three significant figures.



OE = 1

let OD = FC = FB = OA = a. $(0 < a < \frac{\pi}{2})$

CD = AB = b

∵ C(a, b).

∴ b = cos a

∴ A = 2 cos a · a

∴ A' = 2 · [-sin a · a + 1 · cos a]

= 2 cos a - 2 sin a · a

let A' = 0.

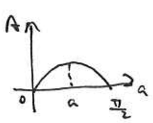
then cos a - sin a · a = 0

1 - a tan a = 0

a tan a = 1

tan a = $\frac{1}{a}$

∴ a = 0.86033 →



∴ A = 1.12 (3.s.f.).

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9. The lengths of the sides of triangle ABC are $x-2$, x and $x+2$. The largest angle is $120°$. Find $\sin A + \sin B + \sin C$ giving your answer in surd form.

according to the rule of sine,

$$\frac{x+2}{\sin 120°} = \frac{x}{\sin \alpha} = \frac{x-2}{\sin(60°-\alpha)}$$

we get $x = 5$. by solving it.

$\therefore \sin A = \sin 120° = \frac{\sqrt{3}}{2}$

$\sin B = \frac{\sqrt{3}}{2} \cdot \frac{5}{7} = \frac{5\sqrt{3}}{14}$

$\sin C = \frac{\sqrt{3}}{2} \cdot \frac{3}{7} = \frac{3\sqrt{3}}{14}$

$\therefore \sin A + \sin B + \sin C = \frac{15\sqrt{3}}{14}$

10. The equation $3z^3 + (2-3i)z^2 + (6+2ai)z + 4 = 0$ where $a \in \mathbb{R}$ has only one real root. Find the value of a.

$(3z^3 + 2z^2 + 6z + 4) + i(-3z^2 + 2az) = 0$

$a \in R$ satisfy the equation,

so since there's a real root,

$i(-3z^2 + 2az)$ is complex,

while $(3z^3 + 2z^2 + 6z + 4)$ is real,

so there's no way for this to happen unless:

$$\begin{cases} -3z^2 + 2az = 0 \\ 3z^3 + 2z^2 + 6z + 4 = 0 \end{cases}$$

$\therefore \underbrace{(z^2+2)}_{>0}(3z+2) = 0$

$\therefore z = -\frac{2}{3}$

$\therefore a = -1$

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Solutions to HL1 Assignment #19

1. a) 3 b) 1

2. Here $y' = \frac{1}{2}x^{-1/2}$. So $m_T = y'(4) = \frac{1}{4}$. Hence $T : y - 2 = \frac{1}{4}(x-4)$, giving $Q = (-4, 0)$.

3. We have $a + b + c + d = 16$, $b + c = 6$ and $d - a = 6$. From which $a = 2$ and $d = 8$. Since the mode is 3 we conclude $b = c = 3$.

4. Here $P(A \cap B) = \frac{1}{6} + \frac{1}{3} - \frac{5}{12} = \frac{1}{12}$. So $P(A' \mid B') = \frac{7}{12} / \frac{8}{12} = \frac{7}{8}$. (A Venn diagram or table of outcomes is helpful in the solution of this problem.)

5. We need to solve $|x - 1| > 2x$. By considering the cases $x \geq 1$ and $x < 1$, or by drawing the graphs of $y = |x - 1|$ and $y = 2x$, we find $x \in \left]-\infty, \frac{1}{3}\right[$.

6. Here $y' = 45x^2 - 5x^5$. Solving $y' = 0$ gives $x = 0, \pm 3$. Constructing a table, which you should do, we find by the first derivative test that $(-3, -162)$ is a local minimum, $(0, 0)$ is a horizontal (stationary) point of inflection and $(3, 162)$ is a local maximum.

7. Let A be the event that only the first row is empty and E the event that exactly one of the rows is empty. Now $n(U) = \binom{12}{3} = 220$ and $n(E) = 3 \times n(A)$. Since $n(A) = \binom{8}{3} - 2\binom{4}{3}$, we conclude $P(E) = \frac{36}{55}$.

8. Here $A = 2x \cos x$, $x \in \left]0, \frac{\pi}{2}\right[$. Using the maximum tool on the GDC we find $A_{\max} = 1.12$ (3 s.f.).

9. By the cosine rule we have $(x + 2)^2 = x^2 + (x - 2)^2 - 2x(x - 2)\cos 120°$, whence $x = 5$. Next using the sine rule we find $\sin A + \sin B + \sin C = \frac{1}{14}(3\sqrt{3} + 5\sqrt{3} + 7\sqrt{3}) = \frac{15}{14}\sqrt{3}$.

10. Suppose the real root is x. Then we must have $3x^3 + 2x^2 + 6x + 4 = 0$ and $3x^2 - 2ax = 0$. From the first equation $x = -\frac{2}{3}$ and from the second $2a = 3x$ as $x \neq 0$. Hence $a = -1$.