HL1 Assignment #2

1. Look up the meanings of the floor and

d cei	ling func	tions. He	1100	1			
x	0.6	π	-4.3	7			, wen
x	1	4	-4	7		²	74
$x \rfloor$	0	3	-5	7			

2. Solve the equation $\log_3(\log_2 x) = 2$.

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 $\log_2 x = 9$
 $x = 512$

3. Determine x so that $\log_x 2 + \log_x 4 + \log_x 8 = 1$.

$$\frac{\log x^{2} + \log x^{4} + \log x^{8} = 1}{\log x} + \frac{\log 4}{\log x} + \frac{\log 8}{\log x} = 1$$

$$\frac{\log 2 + \log 4 + \log 8}{\log x} = 1$$

$$\frac{\log 2 + \log 4 + \log 8}{\log x} = 1$$

$$\frac{\log 64}{\log x} = 1$$

$$\log 64 = \log x$$

$$x = 64$$



4. If 2, 2 + y, 2 + 4y are the first three terms of a geometric sequence, what is the non-zero value of y?

5. Solve the equation $12^{2x+1} = 2^{3x+7} \times 3^{3x-4}$.

$$|2^{2x+1}| = (2^{2} \times 3)^{2x+1}$$

$$= 2^{4x+2} \times 3^{2x+1}$$

$$= 2^{3x+7} \times 3^{3x-4}$$

6. For a certain arithmetic series $S_{21}=546,\,S_{22}=660$ and d=8. Find $S_{23}.$

$$U_{22} = S_{22} - S_{21} = 114$$

 $U_{23} = U_{22} + d = 122$
 $S_{23} = U_{23} + S_{22} = 782$



7. The curve $y = ax^r$ passes through the points (2,1) and (32,4). Calculate the value of r.

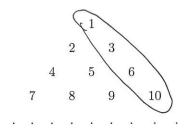
$$\begin{cases} \alpha \cdot 2^r = 1 & 0 \\ \alpha \cdot 32^r = 4 & 2 \end{cases}$$

(D-Q):

$$\frac{a \cdot 32^r}{a \cdot 2^r} = 4$$

$$r = \frac{1}{2}$$

8. Find the sum of the elements in the 100th row of the following triangular array.



1. | looth row: 4951
$$\rightarrow$$
 5050.
2. $S = \frac{(5050+4951) \times 100}{2}$

9. Prove log 3 is irrational.

let's prove by contradiction. if lop3 is rational, we can write it as:

10° won't have factor of 3 3 won't have factor of 2005

so a, b are not integers if the fails.

equation is expected to established. In conclusion, log 3 Thus, our hypothesis is irrational. 10 = 3b

be made, hypothesis

10. The four positive numbers a, b, a + b and ab are consecutive terms in a geometric sequence. Find the value of a.

$$\frac{ab}{a} = b = r^{3}$$
 $\frac{ab}{b} = a = r^{2}$

i. a, b, a+b, ab
 r^{2} , r^{3} , r^{2} + r^{3} , r^{5}

$$\therefore a=r^2 = \frac{3+\sqrt{5}}{2}$$



Solutions to HL1 Assignment #2

1. The completed table is as follows.

\boldsymbol{x}	0.6	π	-4.3	7
$\lceil x \rceil$	1	4	-4	7
$\lfloor x \rfloor$	0	3	-5	7

- 2. We have $\log_2 x = 3^2 = 9$. So $x = 2^9 = 512$.
- 3. We have $\log_x(2 \times 4 \times 8) = 1 \Leftrightarrow \log_x 64 = 1$. So x = 64.
- 4. Since the sequence is geometric $(2+y)^2 = 2(2+4y) \Leftrightarrow y^2 4y = 0$, which has solutions y = 0 and y = 4. Hence the non-zero solution is y = 4.
- 5. Since $12 = 2^2 \cdot 3$, we have $2^{4x+2} \cdot 3^{2x+1} = 2^{3x+7} \cdot 3^{3x-4} \Leftrightarrow 2^{x-5} = 3^{x-5} \Leftrightarrow (2/3)^{x-5} = 1$, from which we conclude x = 5.
- 6. Recall $S_{22} = S_{21} + u_{22}$. So $u_{22} = 660 546 = 114$. Since d = 8, we conclude $u_{23} = 114 + 8 = 122$. So $S_{23} = 660 + 122 = 782$.
- 7. Substitution gives $1 = a \cdot 2^r$ and $4 = a \cdot 32^r$. Division gives $4 = 16^r$. So r = 1/2.
- 8. The number of numbers in the first 99 rows is $1 + 2 + 3 + \cdots + 99 = 4950$. So the first number in the 100th row is 4951. Hence the sum of the numbers in the 100th row is

$$4951 + 4952 + \dots + 5050 = \frac{100}{2}(4951 + 5050) = 500050.$$

- 9. We first note that $\log 3$ is positive. Now suppose $\log 3$ is not irrational, that is $\log 3$ is rational. Then $\log 3 = p/q$ for some positive integers p and q. So $10^{p/q} = 3 \Leftrightarrow 10^p = 3^q$. But this is a contradiction as the LHS is divisible by 2 but the RHS is not. Hence what we supposed must be false and therefore $\log 3$ is irrational.
- 10. Let the common ratio be r. Then we have r = b/a and $r = (a+b)/b = a/b+1 \Leftrightarrow r = 1/r+1 \Leftrightarrow r^2-r-1=0$, which has positive root $r = (1+\sqrt{5})/2$. Next we also have $r = ab/(a+b) = a/(1/r+1) \Leftrightarrow 1+r=a$. So $a = (3+\sqrt{5})/2$.