FM2 Assignment #11 Name: Jerry Jiang $\frac{10}{10}$ Excellent

1. Let $f(x) = \tan x$. Observe that $f(0) = f(\pi)$ but there is there no $c \in]0, \pi[$ such that $f'(c) = 0$. Explain why this does not contradict Rolle's theorem.

f(x) isn't continuous and differentiable at $\frac{\pi}{2}$. ✓

2. Let $f(x) = x + |x|$. Prove that f is continuous but not differentiable at $x = 0$.

- $f(0) = 0 + |0| = 0$.

  $\lim_{x\to 0} f(x) = \lim_{x\to 0} x + \lim_{x\to 0} |x| = 0 + |0| = 0$.

  continuous. ✓. ✓

- $f'(0) = \lim_{h\to 0} \frac{(0+h)+|0+h|-0-|0|}{h} = \lim_{h\to 0} \frac{h+|h|}{h}$

  if $h\to 0^-$, $f'(x) = \frac{0}{h} = 0$; if $h \to 0^+$, $f'(x) = \frac{2h}{h} = 2$. ✓

  not differentiable. ✓.

3. Use the mean value theorem to prove the inequality $|\sin a - \sin b| \leq |a - b|$ for all $a, b \in \mathbb{R}$.

$f(x) = \sin x$ is continuous and differentiable for all real inputs.

We can apply the MVT so that there's a real c that satisfies:

$f(a) - f(b) = f'(c)(a-b)$ where a, b can be any number in $\mathbb{R}$.

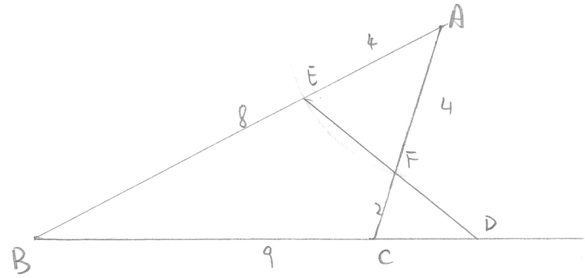
Therefore, $\sin a - \sin b = \cos c\,(a-b)$, so we know that $|\sin a - \sin b| = |\cos c|\,|a-b|$

$0 \leq |\cos c| \leq 1$, $|\sin a - \sin b| \leq |a-b|$. ✓

$\frac{}{6}$

4. In $\triangle ABC$, $a = 9$, $b = 6$ and $c = 12$. A circle with centre A and radius 4 meets sides $[AB]$ and $[AC]$ at E and F respectively. The secant (EF) meets (BC) at D. Use Menelaus's theorem to calculate the length CD.

$$\frac{AE}{EB} \cdot \frac{BD}{DC} \cdot \frac{CF}{FA} = -1$$



$$\frac{4}{8} \cdot \frac{9+a}{-a} \cdot \frac{2}{4} = -1$$

$\therefore$ a = length of $CD = 3$.

5. Verify that $f(x) = 2x^4 - 3x^2 - x + 5$ satisfies the hypotheses of the mean value theorem on the interval $[0, 1]$ and find all numbers c that satisfy the conclusion of the mean value theorem.

$f'(x) = 8x^3 - 6x - 1$ and all $c \in [0,1]$ have $f'(c)$.

Since they're all differentiable, we can for sure apply MVT.

$$f'(c) = \frac{f(1) - f(0)}{1 - 0} = \frac{3-5}{1} = -2.$$

$\therefore$ $8c^3 - 6c - 1 = -2$, $8c^3 - 6c + 1 = 0$.

According to polysmlt 2, $c_1 = -0.940$ (3.s.f.).

$c_2 = 0.766$ (3s.f.)

$c_3 = 0.174$ (3s.f.)

Since $c \in [0,1]$, $c = 0.766$ or 0.174 (3s.f.).