HL1 Assignment #7 Name: Jerry Jiang

100%

Excellent!!

1. The roots of the equation $x^2 + px + q = 0$ are 5 and -2. Find the values of p and q.

$(x-5)(x+2)=0$

$x^2-3x-10=0$

$\therefore \begin{cases} p=-3 \\ q=-10 \end{cases}$

✓

2. For what values of k does the equation $2x^2 + 5x + k = 0$ have two distinct real roots?

$\Delta = 25-8k > 0$

$\therefore 8k < 25$

$\therefore k < \frac{25}{8}$

✓

3. If $z = a + bi$, find $\mathrm{Re}\left(\frac{z}{z^*}\right)$.

$\frac{z}{z^*}$

$= \frac{a+bi}{a-bi}$

$= \frac{(a+bi)(a+bi)}{(a-bi)(a+bi)}$

$= \frac{a^2-b^2+2abi}{a^2+b^2}$

$= \frac{a^2-b^2}{a^2+b^2} + \frac{2ab}{a^2+b^2} \cdot i$

$\therefore \mathrm{Re}\left(\frac{z}{z^*}\right) = \frac{a^2-b^2}{a^2+b^2}$

✓

11

4. Solve $2\log_7 x - \log_x 7 = 1$.

let $\log_7 x = a$

$2a - \frac{1}{a} = 1$

$2a^2 - 1 - a = 0$

$2a^2 - a - 1 = 0$

$(2a+1)(a-1) = 0$

$\therefore a = -\frac{1}{2}$ or 1

$\therefore$ ① $a = -\frac{1}{2}$ ② $a = 1$ $\therefore x = \frac{\sqrt{7}}{7}$ or 7

$\therefore x = 7^{-\frac{1}{2}} = \frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7}$ $x = 7^1 = 7$

5. A committee of 4 students is to be chosen from 5 boys and 4 girls. In how many ways can this be done if at least two girls must be chosen?

The complement of the requirement is 0 or 1 girl is chosen.

$\underbrace{4 \cdot {}^5C_3}_{\text{1 girl chosen}} + \underbrace{{}^5C_4}_{\text{0 girl chosen}} = 4 \cdot 10 + 5 = 45$

The universal set is to choose 4 people randomly out of ~~5~~ $5+4=9$ people.

${}^9C_5 = \frac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6} \times \cancel{5}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 126$

$\therefore$ The answer is $126 - 45 = 81$

6. Without the calculator solve for the square roots of $3 - 4i$.

let $(a+bi)^2 = 3-4i$ $(a, b \in R)$

$a^2 - b^2 + 2abi = 3 - 4i$

$\therefore \begin{cases} a^2 - b^2 = 3 \\ 2ab = -4 \end{cases}$ $\therefore$ the square roots required is $2-i$ or $-2+i$.

$\therefore \begin{cases} a^2 - b^2 = 3 \\ ab = -2 \end{cases}$

$\therefore b = -\frac{2}{a}$

$\therefore a^2 - \left(-\frac{2}{a}\right)^2 = 3$

$\therefore a^2 - \frac{4}{a^2} = 3$

$\therefore a^4 - 3a^2 - 4 = 0$

$\therefore (a^2-4)(a^2+1) = 0$

$\therefore a^2 = 4,\ a = \pm 2.$

$\because b = -\frac{2}{a}$

$\therefore \begin{cases} a_1 = 2 \end{cases}$ $\begin{cases} a_2 = -2 \end{cases}$

7. Find all values of a, b and c so that $10, a, b, c, 810$ is a geometric sequence.

$$\begin{cases} \frac{810}{10} = r^4 \\ \frac{a}{10} = r \\ \frac{b}{10} = r^2 \\ \frac{c}{10} = r^3 \end{cases}$$

$$\therefore \begin{cases} r = \pm 3 \\ a = 10r \\ b = 10r^2 \\ c = 10r^3 \end{cases}$$

$$\therefore \begin{cases} a_1 = 30 \\ b_1 = 90 \\ c_1 = 270 \end{cases} \quad \begin{cases} a_2 = -30 \\ b_2 = 90 \\ c_2 = -270 \end{cases}$$

8. The coefficient of x^3 in the expansion of $\left(1 + \frac{x}{2}\right)^n$ is 70. Find the coefficient of x^2.

$$1^{n-3} \cdot \left(\frac{x}{2}\right)^3 \cdot \binom{n}{3} = 70x^3$$

$$\frac{x^3}{8} \cdot \frac{n \cdot (n-1) \cdot (n-2)}{3 \cdot 2 \cdot 1} = 70x^3$$

$\therefore$ $n \cdot (n-1) \cdot (n-2) = 3360$

$\because$ n is positive integer

$\therefore$ $n = 16$

$\therefore$ $\left(1 + \frac{x}{2}\right)^{16}$

$\therefore$ $1^{16-2} \cdot \left(\frac{x}{2}\right)^2 \cdot \binom{16}{2}$

$= \frac{x^2}{4} \cdot \frac{16 \cdot 15}{2 \cdot 1}$

$= 30x^2$

$\therefore$ the coefficient is 30.

10

9. If 2^{2018} is multiplied out, it has n digits. Find the value of n.

$\because$ if we want to write $\overline{abcd}$ in scientific form,

we have $\overline{a.bcd} \times 10^3$, which can also be written in $\overline{a.bcd} \times 10^{\lfloor \log \overline{abcd} \rfloor}$

$\therefore$ we find out that $\log_{10} x$ function can help write the scientific form of a number, thus lead to the number of digits in that number

$\because \lfloor \log \overline{abcd} \rfloor = 3$,

$\therefore$ number of digits $= \lfloor \log \overline{abcd} \rfloor + 1$

$\therefore n = \lfloor \log 2^{2018} \rfloor + 1$

$= \lfloor 2018 \cdot \log 2 \rfloor + 1$

$= \lfloor 607.48 \rfloor + 1$

$= 608$

✓

10. The roots of $x^2 + cx + d = 0$ are a and b and the roots of $x^2 + ax + b = 0$ are c and d. If a, b, c and d are nonzero, find the value of $a + b + c + d$.

we can get Vieta's Theorem from the quadratic equation.

Vieta theorem: in $ax^2 + bx + c = 0$ $(a \neq 0)$, the root x_1 & x_2 have the following relationship with the coefficient:

$x_1 + x_2 = -\frac{b}{a}$ $\qquad x_1 \cdot x_2 = \frac{c}{a}$.

$\therefore \begin{cases} -c = a + b \text{ ①} \\ d = a \cdot b \text{ ②} \end{cases} \quad \begin{cases} -a = c + d \text{ ③} \\ b = c \cdot d \text{ ④} \end{cases}$

① − ③:

$a + b + c - a - c - d = 0$

$\therefore b - d = 0$

$\therefore b = d$

✓

② & ④:

$\because d = a \cdot b$

$b = c \cdot b$

$b = d$

$\therefore a = c = 1$

① + ③:

$a + b + c + d = -a - c$

$= -1 - 1$

$= -2$

/10

Solutions to HL1 Assignment #7

1. The sum of the roots is 3, so $p = -3$. The product of the roots is -10, so $q = -10$.

2. Here $\Delta = 25 - 8k$. We want $\Delta > 0$, so $k < 25/8$.

3. Now $\dfrac{z}{z^*} = \dfrac{(a+bi)(a+bi)}{a^2+b^2} = \dfrac{a^2-b^2+2abi}{a^2+b^2}$. So the real part is $\dfrac{a^2-b^2}{a^2+b^2}$.

4. Letting $t = \log_7 x$, gives $2t - 1/t = 1 \Leftrightarrow 2t^2 - t - 1 = 0 \Leftrightarrow t = -0.5$ or $t = 1$. So $x = 1/\sqrt{7}$ or $x = 7$.

5. This is $\binom{4}{2} \times \binom{5}{2} + \binom{4}{3} \times \binom{5}{1} + \binom{4}{4} \times \binom{5}{0} = 81$.

6. Suppose $(a+bi)^2 = 3 - 4i$ where $a, b \in \mathbb{R}$. Then $a^2 - b^2 = 3$ and $ab = -2$. Solving simultaneously gives $a = 2$ and $b = -1$, or $a = -2$ and $b = 1$. So the square roots of $3 - 4i$ are $\pm(2 - i)$.

7. Let the ratio be r. Then the sequence is $10, 10r, 10r^2, 10r^3, 810$. We conclude $10r^4 = 810 \Leftrightarrow r = \pm 3$. So $a = \pm 30$, $b = 90$, $c = \pm 270$. (This solution only considers real values of r but we could also consider r to be complex in which case we would also have $r = \pm 3i$ and the associated values of a, b and c.)

8. We are given $\binom{n}{3}\left(\frac{x}{2}\right)^3 = 70$. Solving we have $n = 16$. So the x^2 term is $\binom{16}{2}\left(\frac{x}{2}\right)^2$. Hence the required coefficient is 30.

9. We have $2^{2018} = 10^{\log 2^{2018}} = 10^{2018 \log 2} = 10^{607.5}$. So 2^{2018} has 608 digits when multiplied out.

10. By the sum of roots formula we have $a + b = -c$ and $c + d = -a$. So $c + d = b + c \Leftrightarrow b = d$. By the product of the roots formula we have $ab = d$ and $cd = b$. It follows that $a = c = 1$. So $a+b+c+d = -2$.