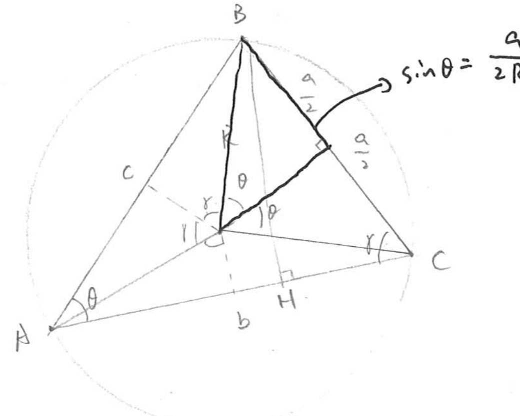
FM2 Assignment #20 Name: Jerry Jiang $\frac{9}{10}$

1. Must a linear transformation of the plane that preserves areas also preserve lengths?

No, $T\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$ doesn't. ✓

2. Denote the area and circumcircle radius of $\triangle ABC$ by $[ABC]$ and R respectively. Prove that $[ABC] = abc/4R$.

$$[ABC] = AC \cdot BH \cdot \frac{1}{2}$$
$$= b \cdot c \cdot \sin\theta \cdot \frac{1}{2}$$
$$= \frac{1}{2} bc \cdot \frac{a}{2R}$$
$$= \frac{abc}{4R}$$
✓



3. The smiley face on the right is transformed. Match the matrices with the transformed smiley faces.

A-F	image	A-F	image	A-F	image
D ✓		C ✓		E ✓	
A		B		F	

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$$A = \begin{pmatrix}1&-1\\1&1\end{pmatrix},\quad B = \begin{pmatrix}1&2\\0&1\end{pmatrix},\quad C = \begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad D = \begin{pmatrix}1&-1\\0&-1\end{pmatrix},\quad E = \begin{pmatrix}-1&0\\0&1\end{pmatrix},\quad F = \frac{1}{2}\begin{pmatrix}0&1\\-1&0\end{pmatrix}.$$

4. Give a non-identity matrix with the property that $A^T = A^{-1}$. Show that if $A^T = A^{-1}$ then $\det A = \pm 1$. Does the converse hold?

$A^T = A^{-1}$, $\quad A \cdot A^T = A \cdot A^{-1} = I$. $\quad = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

missing

$\because \det(AB) = \det(A) \times \det(B)$,

$\therefore \det(A) \cdot \det(A^T) = \det(A \cdot A^T) = \det(I) = 1$

$\because \det(A) = \det(A^T)$

$\therefore \det(A)^2 = 1$

$\therefore \det(A) = \pm 1$. ✓

$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$.

The converse doesn't hold as $A \cdot A^T = I \Rightarrow \det(A) \times \det(A^T) = 1$ but doesn't work the other way round.

Provide evidence, such as a counterexample.

5. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(2x)^n}{\ln(n+1)}$.

Let $U_n = \frac{(2x)^n}{\ln(n+1)}$.

Ratio Test: $\left| \frac{U_{n+1}}{U_n} \right| = \left| \frac{\frac{(2x)^{n+1}}{\ln(n+2)}}{\frac{(2x)^n}{\ln(n+1)}} \right| = \left| \frac{\ln(n+1)}{\ln(n+2)} \cdot (2x) \right| = |2x|$ when $n \to \infty$.

So $|2x| < 1$, $|x| < \frac{1}{2}$. ✓

✓

- when $x = \frac{1}{2}$, $\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$ diverges as it's term by term larger than harmonic series.

- when $x = -\frac{1}{2}$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ converges conditionally, as $\frac{1}{\ln(n+1)} > \frac{1}{\ln(n+2)}$. ✓

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Hence, the interval of convergence is $[-\frac{1}{2}, \frac{1}{2}[$.

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