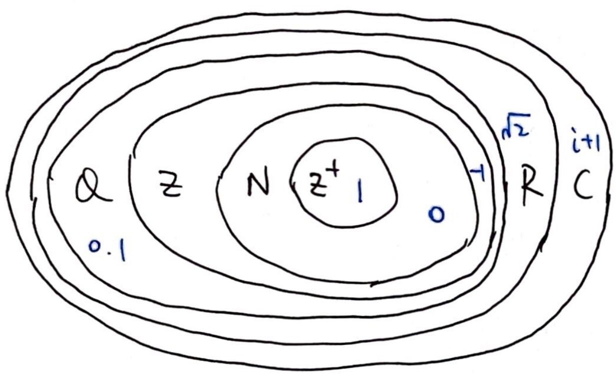
FM1 Assignment #1 Name: Jerry Jiang

1. Write down the augmented matrix for the following system of equations.

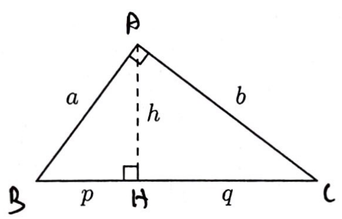
$$\begin{aligned} x + y - 2z &= 0 \\ x - y \phantom{{}-2z} &= 1 \\ y + 2z &= 2 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 1 & 2 & 2 \end{array}\right).$$

2. Draw a Venn diagram to show the relationship between the sets $\mathbb{Z}^+$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$ with $\mathbb{C}$ as the universal set. Put one representative number in each region of the diagram.



3. Euclid's theorem for proportional segments in a right triangle states that h is the geometric mean of p and q, a is the geometric mean of p and $p + q$, and b is the geometric mean of q and $p + q$. Use Euclid's theorem to prove Pythagoras's theorem.



Euclid's theorem

$\Updownarrow$

$$\begin{cases} h^2 = pq & \text{①} \\ a^2 = p(p+q) & \text{②} \\ b^2 = q(q+p) & \text{③} \end{cases}$$

② + ③ $\Rightarrow$

$$a^2 + b^2 = (p+q)(p+q) = (p+q)^2$$

$\therefore$ in right $\triangle ABC$,

$AB^2 = a^2$, $AC^2 = b^2$,

$BC^2 = (p+q)^2$,

$\because a^2 + b^2 = (p+q)^2$

$\therefore$ Pythagoras's theorem is proved.

4. Section 1.4 of T4 uses the binomial theorem to prove the cardinality of the power set of a set with n elements is 2^n. Prove this result without the use of the binomial theorem.

$n(A) = n$,

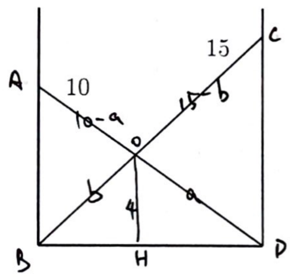
Then in set A, there's element:

	A_1	A_2	A_3	$\cdots$	A_{n-1}	A_n
1				$\cdots$		
0				$\cdots$		

when creating the element in the power set of A, we have to decide whether or not to include $A_1, A_2, A_3 \cdots A_n$, which produce a binary string like $0000 \cdots 001$, which only include A_n, or $1111 \cdots 111$, which is equivalent to the set A itself.

The binary string has 2^n different possibilities, thus 2^n elements in the power set of A.

5. Two ladders, one of length 10 feet and the other of length 15 feet are resting between two walls as indicated in the diagram below. If the ladders cross at a point 4 feet above the ground, how far apart are the walls? Give your answer correct to three significant figures.



$\triangle AOB \sim \triangle DOC$

$\therefore \dfrac{AO}{BO} = \dfrac{DO}{CO}$

$\therefore \dfrac{10-a}{b} = \dfrac{a}{15-b}$

$\therefore ab = 150 - 15a - 10b + ab$

$\therefore 15a + 10b = 150$,

$3a + 2b = 30$.

$\therefore b = \frac{3}{2}(10-a)$

$\because \dfrac{OD}{OH} = \dfrac{AO}{AB}$.

$\therefore \dfrac{a}{4} = \dfrac{10}{AB}$

$\therefore AB = \dfrac{40}{a}$.

Similarly, $CD = \dfrac{40}{10-a}$.

$\because BD^2 = AD^2 - AB^2$

$= BC^2 - CD^2$

$\therefore 100 - \dfrac{(40)^2}{a^2} = 225 - \dfrac{(40)^2}{(10-a)^2}$

according to the GDC,

when $a = 6.8307$, the two functions intersect.

$BD^2 = 65.708257$.

$\therefore BD = 8.11$ (3 s.f.).