FM1 Assignment #3 Name: Jerry Jiang

1. Give the reduced row echelon form of the augmented matrix $\left(\begin{array}{cc|c} 2 & 1 & 7 \\ 4 & -2 & 6 \end{array}\right)$. Hence solve the corresponding system.

$$\left(\begin{array}{cc|c} 2 & 1 & 7 \\ 4 & -2 & 6 \end{array}\right) \sim \left(\begin{array}{cc|c} -2 & -1 & -7 \\ -4 & 2 & -6 \end{array}\right) \sim \left(\begin{array}{cc|c} -4 & -2 & -14 \\ -4 & 2 & -6 \end{array}\right) \sim \left(\begin{array}{cc|c} -4 & -2 & -14 \\ 0 & 4 & 8 \end{array}\right)$$

$$\sim \left(\begin{array}{cc|c} -8 & 0 & -20 \\ 0 & 4 & 8 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & \frac{5}{2} \\ 0 & 1 & 2 \end{array}\right)$$

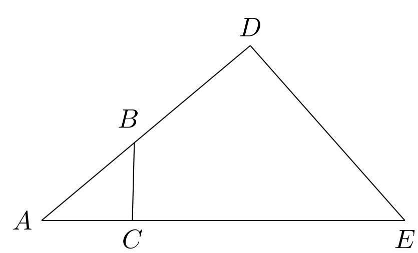
$\therefore \quad (x_1, x_2) = \left(\frac{5}{2}, 2\right)$

Excellent work 10/10. In question 4, the result that the sum of the degrees of the vertices is twice the number of edges is called the handshaking lemma. So by the handshaking lemma |V| times r = 2 times |E| and the result follows.

2. Use De Morgan's laws to prove $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$.

$$\begin{aligned} (A \cup B) \setminus (A \cap B) &= (A \cup B) \cap (A \cap B)' \\ &= (A \cup B) \cap (A' \cup B') \\ &= [(A \cup B) \cap A'] \cup [(A \cup B) \cap B'] \\ &= (B \setminus A) \cup (A \setminus B) \\ &= (A \setminus B) \cup (B \setminus A). \end{aligned}$$

3. In the diagram $AB = 4$, $BD = 5$, $AC = 3$ and $CE = 9$. Prove the quadrilateral $CEDB$ is cyclic.



$\frac{AC}{AB} = \frac{3}{4}, \quad \frac{AD}{AE} = \frac{9}{12} = \frac{3}{4}.$

$\therefore \frac{AC}{AB} = \frac{AD}{AE}$

$\because \angle A$ is the common angle,

$\therefore \triangle ABC \sim \triangle AED$

$\therefore \angle E = \angle ABC$

$\because \angle ABC + \angle DBC = 180^\circ$

$\therefore \angle E + \angle DBC = 180^\circ.$

$\therefore$ CEDB is cyclic.

4. Let $G = (V, E)$ be an r-regular graph. Prove that either $|V|$ or r is even.

- any edge, either a loop or a line between two vertices, add 2 degrees to the total degree number.

  so $E \cdot 2 = V \cdot r$.

- since V, E, and r are all integers, either $|V|$ or r must be even in order to provide the factor 2 to the equation.

5. Use L'Hôpital's rule to find $\lim_{x \to 0} \frac{\tan 3x - 3\tan x}{\sin 3x - 3\sin x}$.

let $A = \lim_{x \to 0} \frac{\tan 3x - 3\tan x}{\sin 3x - 3\sin x}$

$\because \lim_{x \to 0} \tan 3x - 3\tan x = \lim_{x \to 0} \sin 3x - 3\sin x = 0$

$\therefore A = \lim_{x \to 0} \frac{(\tan 3x - 3\tan x)'}{(\sin 3x - 3\sin x)'} = \lim_{x \to 0} \frac{3\sec^2 3x - 3\sec^2 x}{3\cos 3x - 3\cos x} = \lim_{x \to 0} \frac{\sec^2 3x - \sec^2 x}{\cos 3x - \cos x}$

$\because \lim_{x \to 0} \sec^2 3x - \sec^2 x = \lim_{x \to 0} \cos 3x - \cos x = 0$

$\therefore A = \lim_{x \to 0} \frac{(\sec^2 3x - \sec^2 x)'}{(\cos 3x - \cos x)'} = \lim_{x \to 0} \frac{6\sec^2 3x \tan 3x - 2\sec^2 x \tan x}{-3\sin 3x + \sin x}$

$\because \lim_{x \to 0} 6\sec^2 3x \tan 3x - 2\sec^2 x \tan x = \lim_{x \to 0} -3\sin 3x + \sin x = 0$

$\therefore A = \lim_{x \to 0} \frac{(6\sec^2 3x \tan 3x - 2\sec^2 x \tan x)'}{(-3\sin 3x + \sin x)'} = \lim_{x \to 0} \frac{36\sec^2 3x \tan^2 3x + 18\sec^4 3x - 4\sec^2 x \tan^2 x - 2\sec^4 x}{-9\cos 3x + \cos x}$

$= \frac{18 - 2}{-8}$

$= -2.$