FM1 Assignment #2 Name: Jerry Jiang

1. Let $A = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid 0 \leq a + b \leq 5\}$ and $B = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid b = a^2\}$. List the elements of $A \cap B$.

$B = \{(0,0), (\pm1,1), (\pm2,4), (\pm3,9), (\pm4,16) \cdots\}$

what does the $\times$ in $\mathbb{Z} \times \mathbb{Z}$ mean?

$A \cap B = \{(0,0), (1,1), (-1,1), (-2,4)\}$.

ZxZ is the Cartesian product of Z with Z. The x is the symbol for the operation of Cartesian product, which is an operation between two sets.

2. List the four possible reduced row echelon forms for a 2×2 matrix.

$$\left(\begin{array}{cc|c} 1 & 0 & m \\ 0 & 1 & n \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & 0 & m \\ 0 & 0 & n \end{array}\right)$$

what about $\left(\begin{array}{cc|c} 1 & a & m \\ 0 & 0 & n \end{array}\right)$?

that also counts right?

$$\left(\begin{array}{cc|c} 0 & 1 & m \\ 0 & 0 & n \end{array}\right)$$

$$\left(\begin{array}{cc|c} 0 & 0 & m \\ 0 & 0 & n \end{array}\right)$$

Tha matrix you mention is one of the four forms I had in mind. When a=0 it includes one of your matrices in your list. Also you needn't give augmented matrices here. The forms on the left of the bar are sufficient.

3. Let the radius of the circumcircle of $\triangle ABC$ be R. Prove $\dfrac{a}{\sin A} = 2R$ using the diagram below. (Can you see how this result can be used to prove the sine rule?)

$\angle BAC = \angle BA'C$

$\therefore \sin A = \sin A'$

when BA' pass O,

$\angle BCA' = 90°$

$\therefore \sin A' = \dfrac{BC}{2R} = \dfrac{a}{2R}$

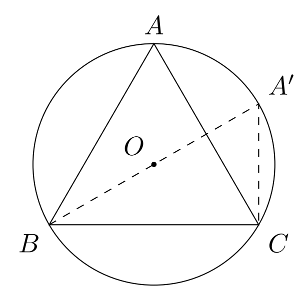
$\therefore \sin A = \dfrac{a}{2R}$

$\therefore \dfrac{a}{\sin A} = 2R$.

Similarly,

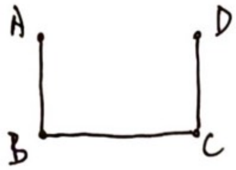
$2R = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$,

and that's the sine rule.



4. The *degree sequence* of a graph is the non-increasing list of its vertex degrees. A sequence is called *graphic* if there is a simple graph whose degree sequence is that sequence.

(a) Draw a simple graph to show that the sequence 2, 2, 1, 1 is graphic.



$\deg(B) = \deg(C) = 2.$

$\deg(A) = \deg(D) = 1.$

Very good 9/10.
In question 4(b) you confused simple and complete. Simple means no loops or multiple edges while a complete graph is a simple graph in which each pair of vertices is adjacent.

(b) Explain why the sequence 4, 3, 2, 1, 1 is not graphic.

there're 5 vertices, if it's simple, which means all vertices are connected to each other, there're at most $\binom{5}{2}$ edges. since $4+3+2+1+1 = 11 > \binom{5}{2}$, the graph is not simple, thus the sequence is not graphic.

5. (a) Explain why $(1+x)^n > 1 + nx$ for $x > 0$ and $n > 1$.

- $(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n-2}x^{n-2} + \binom{n}{n-1}x^{n-1} + x^n$

  $= 1 + nx + \cdots + nx^{n-1} + x^n$
- since $x>0$, $n>1$, $nx^{n-1} + x^n > 0$,
- so $(1+x)^n > 1+nx$.

(b) Hence deduce that $r^n \to \infty$ as $n \to \infty$ when $r > 1$.

- let $r = 1+x$, then $x = r-1$.
- from (a), we have $1+nx < (1+x)^n$, so we have:

  $1+n(r-1) < r^n < \infty$.
- $\lim_{n\to\infty} 1+n(r-1) = \infty$ when $r>1$; $\infty \le \lim_{n\to\infty} r^n \le \infty$, $\lim_{n\to\infty} r^n = \infty$.
- therefore when $r>1$, as $n\to\infty$, $r^n \to \infty$.

(c) Hence show that $r^n \to 0$ as $n \to \infty$ when $0 < r < 1$.

from (b), we have $\lim_{n\to\infty} k^n = \infty$ when $k>1$.

we know $0 < \frac{1}{k} < 1$. let $r = \frac{1}{k}$, $0<r<1$.

$$\lim_{n\to\infty} \left(\frac{1}{k}\right)^n = \frac{\lim_{n\to\infty} 1^n}{\lim_{n\to\infty} k^n} = \frac{1}{\infty} = 0.$$

$\therefore \lim_{n\to\infty} r^n = 0$, which means, as $n\to\infty$, $r^n \to 0$.