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**HL1** Assignment #2 Name: Jerry Jiang

100%

1. Look up the meanings of the *floor* and *ceiling* functions. Hence complete the following table.

| $x$ | 0.6 | $\pi$ | $-4.3$ | 7 |
|---|---|---|---|---|
| $\lceil x \rceil$ | 1 | 4 | -4 | 7 |
| $\lfloor x \rfloor$ | 0 | 3 | -5 | 7 |

Excellent!

2. Solve the equation $\log_3(\log_2 x) = 2$.

$\log_3(\log_2 x) = 2.$

$\log_2 x = 9$

$x = 512$

3. Determine $x$ so that $\log_x 2 + \log_x 4 + \log_x 8 = 1$.

$\log_x 2 + \log_x 4 + \log_x 8 = 1$

$$\frac{\log 2}{\log x} + \frac{\log 4}{\log x} + \frac{\log 8}{\log x} = 1$$

$$\frac{\log 2 + \log 4 + \log 8}{\log x} = 1$$

$$\frac{\log 64}{\log x} = 1$$

$\log 64 = \log x$

$x = 64$

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4. If $2, 2+y, 2+4y$ are the first three terms of a geometric sequence, what is the non-zero value of $y$?

$u_1 = 2$

$u_2 = 2+y$

$u_3 = 2+4y$

$r = \frac{u_2}{u_1} = \frac{u_3}{u_2}$

$r^2 = \frac{u_3}{u_1}$

$\therefore \left(\frac{2+y}{2}\right)^2 = \frac{2+4y}{2}$

$\therefore \frac{y^2+4y+4}{4} = \frac{2+4y}{2}$

$\therefore 2y^2+8y+8 = 8+16y$

$\therefore 2y^2-8y=0$

$\therefore y^2-4y=0$

$\therefore y(y-4)=0$

$y_1 = 0$ $(r=1, \times)$

$y_2 = 4$

$\therefore y = 4$

5. Solve the equation $12^{2x+1} = 2^{3x+7} \times 3^{3x-4}$.

$12^{2x+1} = (2^2 \times 3)^{2x+1}$

$= 2^{4x+2} \times 3^{2x+1}$

$= 2^{3x+7} \times 3^{3x-4}$

$\begin{cases} 4x+2 = 3x+7 \\ 2x+1 = 3x-4 \end{cases}$

$\therefore x = 5$

6. For a certain arithmetic series $S_{21} = 546$, $S_{22} = 660$ and $d = 8$. Find $S_{23}$.

$U_{22} = S_{22} - S_{21} = 114$

$U_{23} = U_{22} + d = 122$

$S_{23} = U_{23} + S_{22} = 782$

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7. The curve $y = ax^r$ passes through the points $(2, 1)$ and $(32, 4)$. Calculate the value of $r$.

$$\begin{cases} a \cdot 2^r = 1 & ① \\ a \cdot 32^r = 4 & ② \end{cases}$$

① ÷ ②:

$$\frac{a \cdot 32^r}{a \cdot 2^r} = 4$$

$\therefore\ 2^{4r} = 4$

$\therefore\ 4r = 2$

$\therefore\ r = \frac{1}{2}$

8. Find the sum of the elements in the 100th row of the following triangular array.

1

2 3

4 5 6

7 8 9 10

. . . . . . . . .

1, 3, 6, 10.

$n=1$ $1 = 1$

$n=2$ $3 = 1+2$

$n=3$ $6 = 1+2+3$

$n=4$ $10 = 1+2+3+4$

$n=?$ $? = 1+2+\cdots+n$

$n=1$ 1 number

$n=2$ 2

$n=3$ 3

$n=4$ 4

$\therefore\ 1+2+\cdots+100$

$= \frac{(100+1) \times 100}{2}$

$= 5050$

$\therefore$ 100th row: $4951 \to 5050$.

$\therefore\ S = \frac{(5050+4951) \times 100}{2}$

$= 500050$

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9. Prove $\log 3$ is irrational.

let's prove by contradiction.

If $\log 3$ is rational,

we can write it as:

$\log 3 = \frac{a}{b}$ $\left(a \cdot b \neq 0,\ a,b \in \mathbb{Z},\ \cancel{\gcd(a,b)=1},\ a>0\right)$

$\therefore 10^{\frac{a}{b}} = 3$

$\therefore 10^a = 3^b$

① $b>0$.

if $a, b$ are integers,

$10^a$ won't have factor of 3

$3^b$ won't have factor of 2 or 5

So $a, b$ are not integers if the equation is expected to established.

Thus, our hypothesis fails.

② $b<0$

$10^a = 3^b$

$3^b$ is a fraction.

$10^a$ is a integer.

The equality can't be made, hypothesis fails.

In conclusion, $\log 3$ is irrational.

10. The four positive numbers $a$, $b$, $a+b$ and $ab$ are consecutive terms in a geometric sequence. Find the value of $a$.

$\frac{ab}{a} = b = r^3$

$\frac{ab}{b} = a = r^2$

$\therefore a, b, a+b, ab$

$\Downarrow$

$r^2, r^3, r^2+r^3, r^5$

$\therefore r^2 + r^3 = r^4$

$\because r \neq 0$

$\therefore 1 + r = r^2$

$\therefore r^2 - r - 1 = 0$

$\Delta = 5$

$r = \frac{1 \pm \sqrt{5}}{2}$

① $r = \frac{1+\sqrt{5}}{2}$ (v)

② $r = \frac{1-\sqrt{5}}{2}$ (x) ( $b, ab < 0$ ).

$\therefore a = r^2 = \frac{3+\sqrt{5}}{2}$

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# Solutions to HL1 Assignment #2

1. The completed table is as follows.

| $x$ | 0.6 | $\pi$ | $-4.3$ | 7 |
|---|---|---|---|---|
| $\lceil x \rceil$ | 1 | 4 | $-4$ | 7 |
| $\lfloor x \rfloor$ | 0 | 3 | $-5$ | 7 |

2. We have $\log_2 x = 3^2 = 9$. So $x = 2^9 = 512$.

3. We have $\log_x(2 \times 4 \times 8) = 1 \Leftrightarrow \log_x 64 = 1$. So $x = 64$.

4. Since the sequence is geometric $(2+y)^2 = 2(2+4y) \Leftrightarrow y^2 - 4y = 0$, which has solutions $y = 0$ and $y = 4$. Hence the non-zero solution is $y = 4$.

5. Since $12 = 2^2 \cdot 3$, we have $2^{4x+2} \cdot 3^{2x+1} = 2^{3x+7} \cdot 3^{3x-4} \Leftrightarrow 2^{x-5} = 3^{x-5} \Leftrightarrow (2/3)^{x-5} = 1$, from which we conclude $x = 5$.

6. Recall $S_{22} = S_{21} + u_{22}$. So $u_{22} = 660 - 546 = 114$. Since $d = 8$, we conclude $u_{23} = 114 + 8 = 122$. So $S_{23} = 660 + 122 = 782$.

7. Substitution gives $1 = a \cdot 2^r$ and $4 = a \cdot 32^r$. Division gives $4 = 16^r$. So $r = 1/2$.

8. The number of numbers in the first 99 rows is $1 + 2 + 3 + \cdots + 99 = 4950$. So the first number in the 100th row is 4951. Hence the sum of the numbers in the 100th row is

$$4951 + 4952 + \cdots + 5050 = \frac{100}{2}(4951 + 5050) = 500\,050.$$

9. We first note that $\log 3$ is positive. Now suppose $\log 3$ is not irrational, that is $\log 3$ is rational. Then $\log 3 = p/q$ for some positive integers $p$ and $q$. So $10^{p/q} = 3 \Leftrightarrow 10^p = 3^q$. But this is a contradiction as the LHS is divisible by 2 but the RHS is not. Hence what we supposed must be false and therefore $\log 3$ is irrational.

10. Let the common ratio be $r$. Then we have $r = b/a$ and $r = (a+b)/b = a/b + 1 \Leftrightarrow r = 1/r + 1 \Leftrightarrow r^2 - r - 1 = 0$, which has positive root $r = (1+\sqrt{5})/2$. Next we also have $r = ab/(a+b) = a/(1/r+1) \Leftrightarrow 1 + r = a$. So $a = (3+\sqrt{5})/2$.