FM1 Assignment #7 Name: Jerry Jiang

1. Use the inverse matrix method without the aid of the calculator to solve the system $\begin{cases} x + 3y = 7 \\ 4x - y = 2 \end{cases}$.

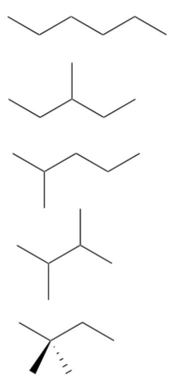
$\begin{pmatrix} 1 & 3 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 3 \\ 4 & -1 \end{pmatrix}$, $|A| = -1 - 12 = -13$

So $A^{-1} = -\frac{1}{13} \begin{pmatrix} -1 & -3 \\ -4 & 1 \end{pmatrix}$

Therefore, $\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{13} \begin{pmatrix} -1 & -3 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

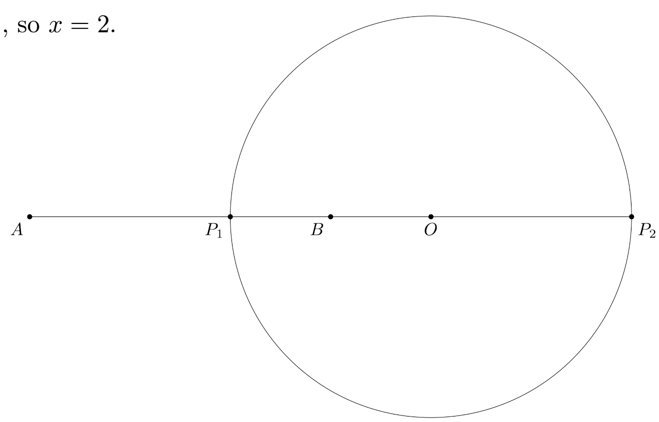
2. Draw all the non-isomorphic trees on six vertices. How many isomers does hexane (C_6H_{14}) have?



A very very little point, the C and H in the parenthesis should not be in math mode.

3. Let $A = (0, 0)$ and $B = (6, 0)$. If $AP : PB = 2 : 1$, show that the locus of P is a circle and find its centre and radius.

According to the Apollonius' Cirlce Theorem, since $\frac{AP}{PB} = 2$, the locus of P is a circle.
Let $OB = x$, then $P_2A = 2P_2B$, $6 + x + x + 2 = 2(x + x + 2)$, so $x = 2$.
$OP_1 = 4$ with O at $(8, 0)$.



4. Use the limit comparison test to determine whether the series $\sum_{n=1}^{\infty} \frac{3n^2 - n}{\sqrt{n^6 + n^3}}$ converges or diverges.

$$\lim_{n\to\infty} \frac{3n^2 - n}{\sqrt{n^6 + n^3}}$$
$$= \lim_{n\to\infty} \frac{\frac{3n^2-n}{n^2}}{\sqrt{\frac{n^6+n^3}{n^4}}}$$
$$= \lim_{n\to\infty} \frac{3 - \frac{1}{n}}{\sqrt{n^2 + \frac{1}{n}}}$$
$$= \lim_{n\to\infty} \frac{3}{n}$$

Since $\lim_{n\to\infty} \frac{\frac{3n^2-n}{\sqrt{n^6+n^3}}}{\frac{3}{n}} = 1$, and the harmonic series $\sum_{n=1}^{\infty} \frac{3}{n}$ diverges, according to the limit comparison test, the series $\sum_{n=1}^{\infty} \frac{3n^2-n}{\sqrt{n^6+n^3}}$ diverges.

5. Consider the symmetric group $(S_4, \circ)$. Let A be the set of elements in S_4 that commute with $(12)(3)(4)$.

(a) There are four elements in A. Write them down.

P1 = (1)(2)(3)(4)

P2 = (1)(2)(34)

P3 = (12)(3)(4)

P4 = (12)(34)

(b) Construct the operation table for $(A, \circ)$. Does $(A, \circ)$ form a group? Be sure to justify your answer.

$(A, \circ)$ is an abelian group with associativity, identity element P1, elements inverse with themselves, and closed within P1, P2, P3, and P4.

$\circ$	P1	P2	P3	P4
P1	P1	P2	P3	P4
P2	P2	P1	P4	P3
P3	P3	P4	P1	P2
P4	P4	P3	P2	P1