HL1 Assignment #3

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100%

Excellent!

1. Solve the exponential equation $4^x = 5000$. Give your answer correct to 3 significant figures.

$x = \log_4 5000 = 6.14$ (3sf).

2. Simplify $\log_2 3 \times \log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8$.

$\log_2 3 \times \log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8$

$= \frac{\cancel{\log 3}}{\log 2} \times \frac{\cancel{\log 4}}{\cancel{\log 3}} \times \frac{\cancel{\log 5}}{\cancel{\log 4}} \times \frac{\cancel{\log 6}}{\cancel{\log 5}} \times \frac{\cancel{\log 7}}{\cancel{\log 6}} \times \frac{\log 8}{\cancel{\log 7}}$

$= \log_2 8$

$= 3.$

3. Find the sum of all the numbers in the table below.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

$S = (1 + \cdots + 10)(1 + \cdots + 10)$

$= 55^2$

$= 3025$

4. A triangle has sides of length 3, 5 and 7. Find the size of the largest angle.

a relatively long side has a relatively large corresponding angle.

so the largest angle is the opposite angle of the side with length of 7.

Let that angle be a°. ($a \in (0, 180)$).

According to law of cosine,

$7^2 = 3^2 + 5^2 - 2 \cdot 3 \cdot 5 \cdot \cos a$

$\cos a = -\frac{1}{2}$

$\arccos(-\frac{1}{2}) = 120^\circ$

$\therefore$ The value of the largest angle is 120°.

5. A bacteria culture increases by 4% every minute. How many minutes will it take for the culture to triple in size?

let the required time be t. ($t \in Z^+$).

$(1 + 4\%)^t \geqslant 3$

$1.04^t \geqslant 3.$

$t \geqslant \log_{1.04} 3.$

$t \geqslant 28.01$

$\therefore$ It will take 29 minutes for the culture to triple in size.

6. Figure 1 has one unit square, figure 2 has 5 unit squares, and so on. How many unit squares does figure 100 have?

1 0+1   4   5 1+4   8   13 4+9   12   25 9+16

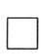

Fig. 1

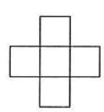

Fig. 2

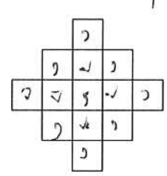

Fig. 3

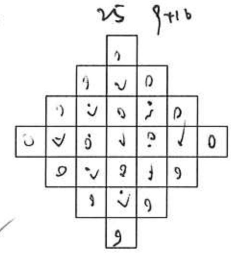

Fig. 4

$U_1 = 1^2 + 0^2$

$U_2 = 2^2 + 1^2$

$U_3 = 3^2 + 2^2$

$U_4 = 4^2 + 3^2$

$U_n = n^2 + (n-1)^2$

$\therefore U_{100} = 100^2 + 99^2$

$= 19801$

7. The curve $y = a^x$ passes through the points $(2,5)$ and $(5,b)$. Find the value of ab.

$$\begin{cases} a^2 = 5 \\ a^5 = b \end{cases}$$

$a = \pm\sqrt{5}$

$\because a > 0, a \neq 1$

$\therefore a = \sqrt{5}$

$b = a^5 = 25\sqrt{5}$

$\therefore ab = \sqrt{5} \cdot 25\sqrt{5}$

$= 125.$

8. Solve the equation $9^x + 3^x = 20$ for $x \in \mathbb{R}$.

$(3^x)^2 + 3^x - 20 = 0$

$(3^x + 5)(3^x - 4) = 0$

$\therefore 3^x = -5$ or 4

$\because 3^x > 0$

$\therefore 3^x = 4$

$\therefore x = \log_3 4 \approx 1.26$ (3 s.f.)

10

9. Find the integer values of x such that $15^x - 27 \times 5^x - 25 \times 3^x + 675 = 0$.

$$15^x - 27 \times 5^x - 25 \times 3^x + 675 = 0$$

$$(5^x - 25)(3^x - 27) = 0.$$

$$\therefore 5^x = 25 \text{ or } 3^x = 27$$

$$\therefore x = 2 \text{ or } 3$$

1
2
4
5
8
10

$2^6 \times 5^6$

$7 \times 7 = 49$

$48.$

500

1000 × 1000

10. Find the sum of the base 10 logarithms of the positive divisors of 1 000 000. That is, evaluate

$$S = \log 1 + \log 2 + \log 4 + \log 5 + \log 8 + \ldots + \log 1\,000\,000.$$

$$= \log 1 \cdot 2 \cdot 4 \cdot 5 \cdots 1000000.$$

$1000000 = 2^6 \cdot 5^6$

$1 = 2^0 \cdot 5^0$

$2 = 2^1 \cdot 5^0$

$4 = 2^2 \cdot 5^0$

$5 = 2^0 \cdot 5^1$

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2^n	5^n
0	0
1	1
2	2
3	3
4	4
5	5
6	6

$7 \times 7 = 49$ numbers.

$1 \times 10^6 = 10^6$

$2 \times (5 \times 10^5) = 10^6$

$4 \times (2.5 \times 10^5) = 10^6$

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$1000 \times 1000 = 10^6$

$(49 - 1) \div 2 = 24$

1000 only 1, others are paired.

$$\therefore S = \log\left((10^6)^{24} \cdot 10^3\right)$$

$$= \log 10^{6 \times 24 + 3}$$

$$= 6 \times 24 + 3$$

$$= 147.$$

10

Solutions to HL1 Assignment #3

1. Here $x = \dfrac{\log 5000}{\log 4} = 6.14$ (3s.f.).

2. Using the change of base formula we conclude $\log_2 3 \times \log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8$ is

$$\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} = \frac{\log 8}{\log 2} = \log_2 8 = 3.$$

3. Let $S = 1 + 2 + 3 + \cdots + 10$. Then the required sum is

$$1 \times S + 2 \times S + 3 \times S + \cdots + 10 \times S = S(1+ = 1 + 2 + 3 + \cdots + 10) = S^2.$$

Since $S = 55$, we conclude the required sum is 3025.

4. First note the largest angle θ is opposite the largest side. Now by the cosine rule

$$\cos\theta = \frac{3^2 + 5^2 - 7^2}{2 \cdot 3 \cdot 5} = -\frac{1}{2}.$$

So $\theta = 120°$ and this is the largest angle in the triangle.

5. Let t be the time for the bacterial culture to triple. We must solve the exponential equation $1.04^t = 3$, which has solution $t = \log 3 / \log 1.04 = 28.0$ (3 s.f.).

6. Thinking of the figure as a large pyramid on top of a smaller upside down pyramid, we have

$$S_{100} = (1 + 3 + 5 + \cdots + 199) + (1 + 3 + 5 + \cdots + 197) = 100^2 + 99^2 = 19801.$$

7. Substitution gives $5 = a^2$ and $b = a^5$, or equivalently $a^6 = 125$ and $a^5 = b$. Division gives $a = 125/b$, which then gives $ab = 125$.

8. Letting $y = 3^x$ gives the quadratic equation $y^2 + y - 20 = 0$, which has roots $y = -5$ and $y = 4$. So $3^x = 4$ or $3^x = -5$. Only the first equation has a solution and this gives $x = 1.26$ (3 s.f.).

9. Factoring gives $5^x(3^x - 27) - 25(3^x - 27) = (5^x - 25)(3^x - 27) = 0$. So $x = 2$ or $x = 3$.

10. First observe that $1\,000\,000 = 2^6 \times 5^6$. So $1\,000\,000$ has $7 \times 7 = 49$ positive divisors. Next let the sum of this series of 49 terms be S. Reversing the series and adding gives

$$\begin{aligned} 2S &= (\log 1 + \log 1\,000\,000) + (\log 2 + \log 500\,000) + (\log 4 + \log 250\,000) + \cdots + (\log 1\,000\,000 + \log 1) \\ &= 49 \times \log 1\,000\,000 \\ &= 49 \times 6 = 294. \end{aligned}$$

Hence the sum of the series is $S = 147$.