FM1 Assignment #9 Name: Jerry Jiang

1. List all the subgroups of the symmetric group $(S_3, \circ)$. Use e for the identity and cycle notation otherwise.

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, a = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, b = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$
$$c = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, d = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

∘	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	e	f	d	c	b
b	b	c	d	f	e	a
c	c	b	a	e	f	d
d	d	f	e	a	b	c
f	f	d	c	b	a	e

Non-trivial Subgroups:

$(\{e, a\}, \circ)$

$(\{e, c\}, \circ)$

$(\{e, f\}, \circ)$

$(\{e, b, d\}, \circ)$

Very good work 9/10. In question 1, since the question asks for all subgroups you should list the trivial subgroup (e,\circ) and the group itself (S_3,\circ). Also for question 1, you used matrix notation rather than cycle notation as the question stated. For question 2, you have given a basis for the null space, a basis for the row space and a basis for the column space. However, the nullity is the dimension of the null space and the rank (not ranking) is the dimension of the column space, which always equals the dimension of the row space.

2. Find the rank and nullity of the matrix $\begin{pmatrix} 1 & 3 & 1 & 6 & 4 \\ 2 & 6 & 3 & 16 & 11 \\ 3 & 9 & 3 & 18 & 12 \end{pmatrix}$.

$$A = \begin{pmatrix} 1 & 3 & 1 & 6 & 4 \\ 2 & 6 & 3 & 16 & 11 \\ 3 & 9 & 3 & 18 & 12 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 & 0 & 2 & 1 \\ 0 & 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Ax = 0 : \left(\begin{array}{ccccc|c} 1 & 3 & 1 & 6 & 4 & 0 \\ 2 & 6 & 3 & 16 & 11 & 0 \\ 3 & 9 & 3 & 18 & 12 & 0 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 3 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_2 = r, x_4 = s, x_5 = t.$

then $x_1 = -3r - 2s - t, x_3 = -4s - 3t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = r \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -2 \\ 0 \\ -4 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ -3 \\ 0 \\ 1 \end{pmatrix}$$

$$\text{nullity } (A) = \left\{ \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ -4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\text{row ranking } B : \begin{pmatrix} 1 & 3 & 0 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 4 & 3 \end{pmatrix} \to A : \begin{pmatrix} 1 & 3 & 0 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 4 & 3 \end{pmatrix}$$

$$\text{column ranking } B : \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \to A : \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$$

ranking $= 2$.

3. Show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges conditionally.

For $\displaystyle\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

according to the rule of convergence of p-series, $p = \frac{1}{2} < 1$, it diverges

For $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}, \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0.$

when $n > 0, n \leq n + 1$

$\sqrt{n} \leq \sqrt{n+1}$

$$\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$$

According to the alternating series test, the series converges.

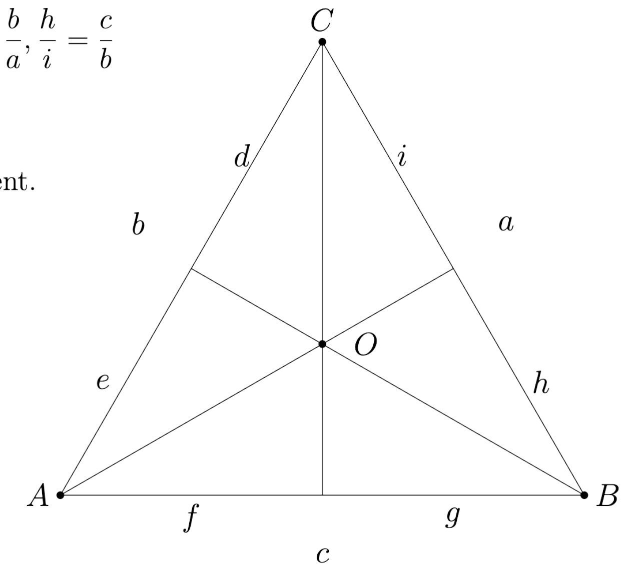
Therefore, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges conditionally.

4. Use the converse of Ceva's theorem to prove that the angle bisectors of a triangle are concurrent.

According to the angle bisector theorem, $\dfrac{d}{e} = \dfrac{a}{c}, \dfrac{f}{g} = \dfrac{b}{a}, \dfrac{h}{i} = \dfrac{c}{b}$

Therefore, $\dfrac{d}{e} \cdot \dfrac{f}{g} \cdot \dfrac{h}{i} = \dfrac{a}{c} \cdot \dfrac{b}{a} \cdot \dfrac{c}{b} = 1.$

So the angle bisectors intersect at O and are concurrent.



5. Consider the locus of a point whose distance from the point $(6, 0)$ is $\frac{3}{2}$ its distance from the line $3x - 8 = 0$.

(a) Find the equation of the locus.

$a = 4, e = \dfrac{3}{2}, b^2 = 20,$ Asymptote: $y = \dfrac{b}{a} = \pm\dfrac{\sqrt{5}}{2}$, Equation: $\dfrac{x^2}{16} - \dfrac{y^2}{20} = 1.$

(b) Sketch the locus clearly indicating any key features.