HL1 Assignment #8 Name: Jerry Jiang

100%

Excellent!!

1. Find the angle of inclination for a line perpendicular to $2x + 3y = 5$.

$3y = -2x+5$

$y = -\frac{2}{3}x + \frac{5}{3}$

$y = \frac{3}{2}x + b$

$\alpha = \arctan(\frac{3}{2})$

$= 56.3^\circ$

↑ 3 s.f. in IB

2. Find the coefficient of x^5 in the expansion of $(2-3x)^8$.

$(-3x)^5 \cdot 2^3 \cdot \binom{8}{3}$

$= -3^5 \cdot x^5 \cdot 8 \cdot \frac{8\cdot 7\cdot \cancel{6}}{\cancel{3\cdot 2\cdot 1}}$

$= -108864x^5$

$\therefore$ the coefficient is -108864.

3. Solve $(2+i)z - (2-4i) = 3 - i$ without the use of a calculator.

$(2+i)z = 3 - i + 2 - 4i$

$z = \frac{5-5i}{2+i}$

$= \frac{(5-5i)(2-i)}{5}$

$= (1-i)(2-i)$

$= 1-3i$

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4. At a party 300 handshakes were exchanged. Each person at the party shook hands exactly once with each of the others. Find the number of people at the party.

$$\frac{n(n-1)}{2} = 300$$

$$n(n-1) = 600$$

$$n^2 - n - 600 = 0$$

$$(n-25)(n+24) = 0$$

$$n_1 = 25$$

$$n_2 = -24 \text{ (inadmissible)}$$

$\therefore$ 25 people.

5. A vertical line divides the triangle with vertices $O(0,0)$, $A(9,0)$ and $B(8,4)$ into two regions of equal area. Find the equation of the line.

$$S = \frac{1}{2} \cdot 9 \cdot 4$$
$$= 18$$

$$l_{OB}: y = \frac{1}{2}x$$

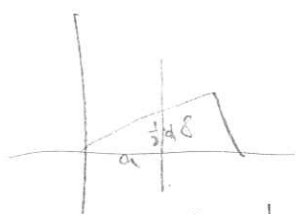
$$\begin{cases} l: x = a \\ l_{OB}: y = \frac{1}{2}x \end{cases} \Rightarrow \left(a, \frac{1}{2}a\right)$$

$$\therefore \frac{1}{2} \cdot a \cdot \left(\frac{1}{2}a\right) = 18 \cdot \frac{1}{2}$$

$$\therefore a^2 = 72 \cdot \frac{1}{2}$$

$$a = \pm 6$$
$$(a = -6 \text{ inadmissible})$$

$$\therefore l: x = 6$$



$$a \cdot \frac{1}{2}a \cdot \frac{1}{2} = 9$$

6. Solve $\log_2(x+1) - \log_4(3x-1) = 0.5$.

$$\log_2(x+1) - \log_2(3x-1)^{\frac{1}{2}} = 0.5$$

$$\log_2 \frac{x+1}{\sqrt{3x-1}} = 0.5$$

$$\frac{x+1}{\sqrt{3x-1}} = \sqrt{2}$$

$$x+1 = \sqrt{6x-2}$$

$$x^2+2x+1 = 6x-2$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x_1 = 3$$

$$x_2 = 1$$

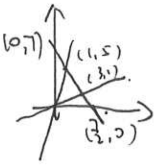
$$\begin{cases} x+1>0 \Rightarrow x>-1 \\ 3x-1>0 \Rightarrow x>\frac{1}{3} \end{cases}$$

$$\therefore x > \frac{1}{3}$$

$$\therefore x_1 = 1$$
$$x_2 = 3$$

7. Find the area of the triangle formed by the lines $5x - y = 0$, $x - 3y = 0$ and $2x + y - 7 = 0$.

$$\begin{cases} y = 5x \\ y = \frac{1}{3}x \\ y = -2x+7 \end{cases}$$



$$\begin{cases} l: y=5x \\ l: y=-2x+7 \end{cases} \quad (1,5)$$

$$\begin{cases} l: y=\frac{1}{3}x \\ l: y=-2x+7 \end{cases} \quad (3,1)$$

$$\begin{cases} l: x=0 \\ l: y=-2x+7 \end{cases} \quad (0,7)$$

$$\begin{cases} l: y=0 \\ l: y=-2x+7 \end{cases} \quad (\tfrac{7}{2},0)$$

$$S = \frac{7}{2}\cdot 7\cdot\frac{1}{2} - 7\cdot 1\cdot\frac{1}{2} - \frac{1}{2}\cdot 1\cdot\frac{7}{2}$$

$$= \frac{49}{4} - \frac{7}{2} - \frac{7}{4}$$

$$= \frac{49-14-7}{4}$$

$$= \frac{35-7}{4}$$

$$= \frac{28}{4}$$

$$= 7$$

✓

8. Find all values of c such that the line $y = x + c$ is tangent to the circle $x^2 + y^2 = 8$.

$$y^2 = 8 - x^2$$

$$\therefore y_1 = \sqrt{8-x^2}$$

$$y_2 = -\sqrt{8-x^2}$$

① $$\begin{cases} y = x+c \\ y = \sqrt{8-x^2} \end{cases}$$

$$x^2+2cx+c^2 = 8-x^2$$

$$2x^2+2cx+c^2-8=0$$

$$\Delta = 4c^2-8(c^2-8)$$

$$= 4c^2-8c^2+64$$

$$= -4c^2+64=0$$

$$c^2=16$$

$$c=\pm 4.$$

② $$\begin{cases} y = x+c \\ y = -\sqrt{8-x^2} \end{cases}$$

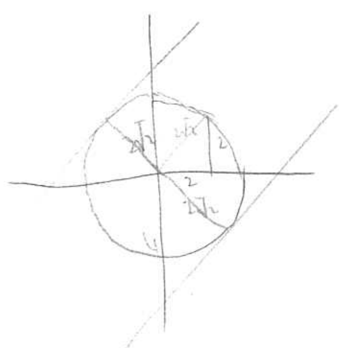
$$x^2+2cx+c^2=8-x^2$$

$$c = \pm 4.$$

$$\therefore c_1 = 4$$

$$c_2 = -4$$

✓



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9. The unit circle $x^2 + y^2 = 1$ and the parabola $y = kx^2 - 1$ intersect in 3 points. What are the possible values of k?

$$\begin{cases} y^2 = (kx^2-1)^2 \\ \quad = k^2x^4 - 2kx^2 + 1 \\ x^2 + y^2 = 1 \end{cases}$$

$x^2 + k^2x^4 - 2kx^2 + \cancel{1} = \cancel{1}$

$x^4k^2 + x^2(1-2k) = 0$

$x^2(k^2x^2 + 1 - 2k) = 0$

$\downarrow$

$x_1 = x_2 = 0$. same point. $(0, -1)$

So $k^2x^2 + 1 - 2k = 0$

have two distinct root

$x^2 = \frac{2k-1}{k^2}$

$\therefore \frac{2k-1}{k^2} > 0.$

$\because k^2 > 0$

$\therefore 2k - 1 > 0.$

$\therefore k > \frac{1}{2}$

10. The point A is on the line $4x + 3y - 48 = 0$ and the point B is on the line $x + 3y + 10 = 0$. If the midpoint of $[AB]$ is $(4, 2)$, find the coordinates of A and B.

$3y = -4x + 48$ $\quad$ $3y = -x - 10$

$y = -\frac{4}{3}x + 16$ $\quad$ $y = -\frac{1}{3}x - \frac{10}{3}$

$A(a, -\frac{4}{3}a + 16)$ $\quad$ $B(b, -\frac{1}{3}b - \frac{10}{3})$

$$\begin{cases} \frac{a+b}{2} = 4 \\ \frac{-\frac{4}{3}a + 16 - \frac{1}{3}b - \frac{10}{3}}{2} = 2 \end{cases}$$

$\Downarrow$

$$\begin{cases} a = 6 \\ b = 2 \end{cases}$$

$\therefore A(6, 8)$

$B(2, -4)$

Solutions to HL1 Assignment #8

1. For the given line, we have $m = -2/3$. So $m_\perp = 3/2$ with an angle of inclination of $56.3°$ (3 s.f.).
2. The required coefficient is $\binom{8}{5}2^3(-3)^5 = -108\,864$.
3. We have $z = (5-5i)/(2+i) = (5-5i)(2-i)/5 = 1-3i$.
4. Let the number of people be n. We must solve $\binom{n}{2} = 300 \Leftrightarrow n(n-1) = 600$. By inspection $n = 25$.
5. Let the equation of the vertical line be $x = k$. The area of $\triangle OAB = \frac{1}{2} \times 9 \times 4 = 18$. So we must solve $\frac{1}{2} \times k \times (\frac{1}{2}k) = 9 \Leftrightarrow k = 6$.
6. We have $\log_2(x+1) - \frac{1}{2}\log_2(3x-1) = 0.5 \Leftrightarrow \log_2[(x+1)^2/(3x-1)] = 1$. Whence the quadratic equation $x^2 - 4x + 3 = 0$, which has solutions $x = 1$ and $x = 3$.
7. The vertices of this triangle are $(0,0)$, $(3,1)$ and $(1,5)$. The area of this triangle is 7.
8. Substitution gives the quadratic equation $x^2 + (x+c)^2 = 8$. For tangency, we want only one solution for x in this equation. Hence the discriminant, which in this case is $64 - 4c^2$, must be 0. Solving gives $c = \pm 4$.
9. Substitution gives $x^2 + (kx^2-1)^2 = 1 \Leftrightarrow x^2(k^2x^2 + 1 - 2k) = 0$. So one solution, a double root in fact is $x = 0$, the other two roots must come from the quadratic equation $k^2x + 1 - 2k = 0$. The discriminant here is $\Delta = 4(2k-1)k^2$. Solving $\Delta > 0$ gives $k > 0.5$. (Question: What does the double root at $x = 0$ imply geometrically?)
10. Let $A = (x_1, y_1)$ and $B = (x_2, y_2)$. We have $x_1 + x_2 = 8$ and $y_1 + y_2 = 4$. Substituting for y_1 and y_2 gives the equation $4x_1 + x^2 = 26$. Solving simultaneously for x_1 and x_2 gives $x_1 = 6$ and $x_2 = 2$. Hence $A = (6,8)$ and $B = (2,-4)$.