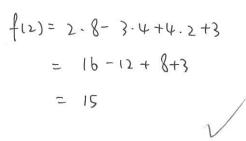
1. What is the remainder when  $2x^3 - 3x^2 + 4x + 3$  is divided by x - 2?





2. Solve  $\cos(\theta + 70^{\circ}) = 0.5$  for  $0^{\circ} \le \theta < 360^{\circ}$ .

3. If z = a + bi and  $\frac{z}{z^*} = c + di$ , prove  $c^2 + d^2 = 1$ .

$$\frac{8}{8} = \frac{a+bi}{a-bi} = \frac{(a+bi)^2}{a^2+b^2} = (+di)$$

$$2^{2} \cdot C = \frac{a^{2} - b^{2}}{a^{2} + b^{2}}$$
  $d = \frac{2ab}{a^{2} + b^{2}}$ 

$$-1 c^{2} = \frac{a^{4} + b^{4} - 2a^{2}b^{2}}{(a^{2} + b^{2})^{2}} d^{2} = \frac{4a^{2}b^{2}}{(a^{2} + b^{2})^{2}}$$

$$(a^{2}+b^{2})^{2} = \frac{(a^{2}+b^{2})^{2}}{(a^{2}+b^{2})^{2}} = 1$$

4. Expand and simplify 
$$(3-\sqrt{2})^4$$
. Give your answer in the form  $a+b\sqrt{2}$  where  $a,b\in\mathbb{Z}$ .

$$(3-\sqrt{2})^{4}$$
=  $(11-6\sqrt{2})^{2}$   
=  $121+72-132\sqrt{2}$   
=  $193-132\sqrt{2}$ 

5. Find the value of 
$$(1^2 + 3^2 + 5^2 + \dots + 99^2) - (2^2 + 4^2 + 6^2 + \dots + 100^2) + (4 + 8 + 12 + \dots + 200)$$
.

$$\zeta = -\left[ (100^{2} - 99^{4}) + \dots + (12^{2} - 1^{2}) \right] + \frac{104.50}{2}$$

$$= -\left[ (100 + 99 + \dots + 12 + 1) \right] + 5|00$$

$$= 5|00 - 5050$$

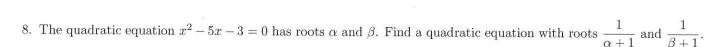
$$= 50$$

6. For what values of m is the line 
$$y = mx + 5$$
 tangent to the parabola  $y = 4 - x^2$ ?

$$\begin{cases} y = mx + s \\ y = 4 - x^{2} \\ 2 - x^{2} + mx + 1 = 0 \\ 0 = m^{2} - 4 = 0 \\ 2 - m = \pm 2. \end{cases}$$

7. One solution of  $z^3 + bz^2 + 34z - 40 = 0$  is z = 3 + i. If  $b \in \mathbb{R}$  find the value of b.

according to the conjugate root theorem,



$$\frac{1}{\alpha + 1} + \frac{1}{\beta + 1} \qquad \frac{1}{\alpha + 1} \cdot \frac{1}{\beta + 1}$$

$$= \frac{\alpha + \beta + 1 + 1}{(\alpha + 1)(\beta + 1)} = \frac{1}{(\alpha + 1)(\beta + 1)}$$

$$= \frac{5+2}{\Delta\beta+\alpha+\beta+1} = \frac{1}{\Delta\beta+\alpha+\beta+1}$$

$$=\frac{7}{-3+5+1}$$
  $=\frac{1}{-3+5+1}$ 



9. The point (a, b) is the point on the curve  $y = x^2$  that is closest to (6, 0). Calculate the value of a.

2. 
$$b^{3}c^{3} = -\frac{1}{21b}$$
2.  $b^{3}$ ,  $c^{3}$  are roots of

 $t^{2} - (b^{3} + c^{3})t + b^{3}c^{3} = 0$ .

 $t^{2} - 3t - \frac{1}{21b} = 0$ 
2.  $21bt^{2} - b48t - 1 = 0$ 
2.  $0 = 4207b8$ 

$$= 487 \times 2^{5} \times 3^{3}$$

$$= 648 + 12 \sqrt{2922}$$
2.  $b^{2} = 3\sqrt{\frac{648 + 12 \sqrt{2922}}{432}}$ 
2.  $c = 3\sqrt{\frac{648 - 12 \sqrt{2922}}{432}}$ 
3.  $a = b + c$ 

10. If  $\log_{(\tan \theta + \cot \theta)} \cos \theta = k$  where  $k \in \mathbb{R}$ , find an expression for  $\log_{\tan \theta} \sin \theta$  in terms of k.

let 
$$\tan \theta = a$$
,  $\cos \theta = b$ 

$$\frac{1}{2} + \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\frac{1}{2} - a = \frac{\sqrt{1-b^2}}{b}$$

$$\frac{1}{2} - a = \frac{1}{ab^2}$$

$$\frac{1}{2} - a = \frac{1}{ab^2}$$

$$\frac{1}{ab^2}b=k$$

$$- \left(\frac{1}{ab^2}\right)^k = b$$

$$-1$$
.  $a^{-k} = b^{1+2k}$ 

$$= \frac{\log a \cdot ab}{\log a + \log ab}$$

$$= \frac{1 + \frac{-k}{1 + 2k}}{2k + 1}$$



1. a= 3/- + 3/-

:. a = 3/648+12/12/22 + 3/648-12/12/22

## Solutions to HL1 Assignment #11

- 1. Let  $p(x) = 2x^3 3x^2 + 4x + 3$ . By the remainder theorem R = p(2) = 15.
- 2. Here  $\cos(\theta + 70^{\circ}) = \cos(60^{\circ})$ . So  $\theta + 70^{\circ} = 60^{\circ} + n360^{\circ}$  or  $\theta + 70^{\circ} = -60^{\circ} + n360^{\circ}$ , whence  $\theta = -10^{\circ} + n360^{\circ}$  or  $\theta = -130^{\circ} + n360^{\circ}$ . For the given interval we conclude  $\theta = 230^{\circ}, 350^{\circ}$ .
- 3. Notice  $c^2 + d^2 = \frac{z}{z^*} \cdot \left(\frac{z}{z^*}\right)^* = \frac{z}{z^*} \cdot \frac{z^*}{z} = 1$ .
- 4. By the binomial theorem  $(3 \sqrt{2})^4 = 3^4 \binom{4}{1} \cdot 3^3 \cdot \sqrt{2} + \dots + (\sqrt{2})^4$ , which simplifies to  $193 132\sqrt{2}$ .
- 5. Notice  $(1^2+3^2+5^2+\cdots+99^2)-(2^2+4^2+6^2+\cdots+100^2)+(4+8+12+\cdots+200)=(4+8+12+\cdots+200)-(2^2-1^2)+(4^2-3^2)+(6^2-5^2)+\cdots+(100^2-99^2)]$ , which is  $(4+8+12+\cdots+200)-(3+7+11+\cdots+199)$ , and this is simply  $\underbrace{1+1+1+1+\cdots+1}_{50 \text{ times}}=50$ .
- 6. The line intersects the parablola when  $mx + 5 = 4 x^2$ , or equivalently  $x^2 + mx + 1 = 0$ . Tangency occurs when  $\Delta = 0$ . That is when  $m^2 4 = 0$ , giving  $m = \pm 2$ .
- 7. We have the roots as  $z_{1,2} = 3 \pm i$  and  $z_3$ . We are given  $z_1 \cdot z_2 \cdot z_3 = 40$ , so  $z_3 = 4$ . Lastly,  $b = -(z_1 + z_2 + z_3) = -10$ .
- 8. A quadratic equation with roots  $\alpha + 1$  and  $\beta + 1$  is  $(x 1)^2 5(x 1) 3 = 0$ , or equivalently  $x^2 7x + 3 = 0$ . Hence the quadratic equation  $3x^2 7x + 1 = 0$  solves our problem.
- 9. Notice that  $b = a^2$ . Next the square of the distance between the points is  $d^2 = (a-6)^2 + (a^2-0)^2 = a^4 + a^2 12a + 36$ . Since d is non-negative the value of a which gives the minimum value of  $d^2$  is also the value of a that gives the minimum value of d. Using the minimum tool on the GDC gives a = 1.33 (3 s.f.).
- 10. In exponential form we are given  $(\tan \theta + \cot \theta)^k = \cos \theta$ , from which we obtain

$$\left(\frac{\sec^2\theta}{\tan\theta}\right)^k = \cos\theta \quad \Leftrightarrow \quad \left(\frac{1}{\tan\theta}\right)^k = \cos^{2k+1}\theta.$$

Multiplying both sides of the second form by  $\tan^{2k+1}\theta$  gives  $\tan^{k+1}\theta = \sin^{2k+1}\theta$ , whence

$$\log_{\tan\theta}\sin\theta = \frac{k+1}{2k+1}.$$