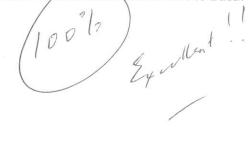
1. Five people including Ali and Baba sit in a row. How many ways can this be done if Ali refuses to sit next to Baba?

Fother sit together. Dethey don't sit together.

There's only 2 possibilities.



2. The constant term in the expansion of $\left(2x-\frac{1}{x}\right)^6$ is an integer. Find its value.

$$= -190$$

$$= -8 \times_3 \cdot -\frac{x_2}{1} \cdot \frac{244}{4}$$

$$= 8 \times_3 \cdot -\frac{x_2}{1} \cdot \frac{244}{4}$$

$$= 8 \times_3 \cdot -\frac{x_2}{1} \cdot \frac{244}{4}$$



3. Binary(base-2) is computer friendly but not very human friendly. Computer scientists therefore often find it convenient to use *hexadecimal*(base-16) instead of binary. Write the binary number 111100101100 in hexadecimal.

$$= 2^{\circ} (4+8) + 2^{4} \cdot 2 + 2^{8} \cdot 15 = 16^{\circ} \cdot 12 + 16^{2} \cdot 15$$

$$= 2^{\circ} (4+8) + 2^{4} \cdot 2 + 2^{8} \cdot 15 = 16^{\circ} \cdot 12 + 16^{2} \cdot 2 + 16^{2} \cdot 15$$

4. By adding the same constant to each of the numbers 60, 100, and 150, a geometric sequence is obtained. Find the common ratio for the sequence.

into ratio for the sequence.

$$\frac{190+x}{60+x} = r$$

$$\frac{150+x}{60+x} = r^{2}$$

$$\frac{150+x}{160} = r^{2}$$

$$\frac{150+x}{160}$$

5. Find the sum of the coefficients in the expansion of $(3-4x)^5$.

$$S = {5 \choose 0} \cdot 3^{5} + {5 \choose 1} \cdot 3^{4} \cdot [-4] + {5 \choose 2} \cdot 3^{3} \cdot (-4)^{7} + {5 \choose 3} \cdot 3^{2} \cdot (-4)^{3} + {5 \choose 4} \cdot 3 \cdot [-4]^{4} + {5 \choose 5} \cdot (-4)^{5}$$

$$= 3^{5} * - S \cdot 4 \cdot 3^{4} + (0 \cdot 3^{3} \cdot 16 * -10 \cdot 9 \cdot 64 + 5 \cdot 3 \cdot 4^{4} * - 4^{5}$$

$$= -1 \quad (1 \text{ assume that } 3^{5} \text{ the constant coefficient is considered in the sum})$$

6. Solve $\log_2 x - \log_8 x = 4$.

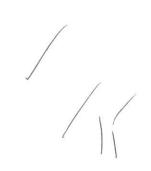
$$|og_2 x - \frac{1}{3}|og_2 x = 4$$

$$\frac{2}{3}|og_2 x = 4$$

$$|og_2 x = 6$$

$$x = 2^{6}$$

$$= 64$$



7. Complete the row of Pascal's triangle that begins 1 7 21. Hence give the exact value of 1.01^7 .

1 7 21 35 35 21 7 1

(similar rule when doing 11", add to overflow to the higher digits)

8. Four men and five women stand in a line. How many ways can this be done if no two men are adjacent? nomen have become separation, providing b potential space for men.



9. Find the sum of the series $1 \times 1! + 2 \times 2! + 3 \times 3! + \cdots + 99 \times 99!$

$$S = (|x|! + 1!) + (2x2! + 2!) + \cdots + (99 \times 99! + 99!) - (1! + 2! + \cdots + 99!)$$

$$= (2! + 3! + \cdots + (2x2! + 2!) + \cdots + (99 \times 99! + 99!) - (1! + 2! + \cdots + 99!)$$

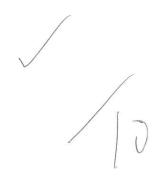
10. Prove that $\sqrt{3}$ is irrational.

let's assume that 13 can's rational,

so we can have
$$\bar{t}_s = \frac{P}{q} \left(p, q + 2, q \neq 0, \gcd(p, q) = 1 \right)$$

so our hypothesis fails

i. 13 is irrational



Solutions to HL1 Assignment #6

- 1. The required number is $5! 2 \times 4! = 72$.
- 2. The required term is $\binom{6}{3}(2x)^3(-1/x)^3 = -160$.
- 3. The simplest approach is to start from the right and break the binary number into blocks of 4 digits filling the final block with leading zeros if necessary. We then convert each block into its hexadecimal equivalent. Proceeding in this way gives $1111\,0010\,1100_2 = F2C_{16}$.
- 4. Let the constant be k. Then we know $(100 + k)^2 = (60 + k)(150 + k)$. A little algebra gives k = 100. So the common ratio is 200/160 = 1.25.
- 5. The sum of the coefficients is most easily found by substituting x = 1, giving the sum as $(3-4)^5 = -1$.
- 6. Letting $= \log_2 x$, gives $y \frac{1}{3}y = 4$. Hence y = 6 and so x = 64.
- 7. The row is 1 7 21 35 35 21 7 1. So the exact value of 1.01^7 is 1.07213535210701.
- 8. Consider the string $W_1 W_2 W_3 W_4 W_5$, where the W_i are the women and each _ is a possible space for a man. We then have the required number as $\binom{6}{4} \times 4! \times 5! = 43\,200$.
- 9. Notice $(n+1)! n! = n \times n!$. So our series is the same as

$$(2!-1!)+(3!-2!)+(4!-3!)+(5!-4!)+\cdots+(99!-98!)+(100!-99!),$$

which collapses to 100! - 1! and this is the required sum.

10. Suppose to the contrary that $\sqrt{3}$ is rational. Then $\sqrt{3} = p/q$ for integers p and q with $q \neq 0$. It follows that $p^2 = 3q^2$, but this is a contradiction since p^2 will contain the factor 3 an even number of times in its prime factorization while $3q^2$ will contain the factor 3 an odd number of times in its prime factorization. Hence what we supposed is false and $\sqrt{3}$ must therefore be irrational.