$\frac{12}{10}$

Excellent!

1. The minimum degree and maximum degree of a graph are denoted δ and Δ respectively.

   (a) What are δ and Δ for a 3-regular graph?

   $\delta = \Delta = 3$ as   all the vertices in the graph have degrees $= 3$.

   (b) What is δ for any tree with more than one vertex?

   $\delta = 1$.

2. Find $\lim\limits_{x \to 1} \dfrac{\ln x}{\sin \pi x}$.

   $\lim\limits_{x \to 1} \ln x = \lim\limits_{x \to 1} \sin \pi x = 0.$

   $\therefore$ Apply L'Hôpital's Rule. we get $\lim\limits_{x \to 1} \dfrac{\frac{1}{x}}{\cos \pi x \cdot \pi}$

   $= \dfrac{\frac{1}{1}}{\pi \cos \pi}$

   $= \dfrac{1}{-\pi}$

3. Prove that $V_4 \cong \mathbb{Z}_2 \times \mathbb{Z}_2$.

V_4	e	x	y	H
e	e	x	y	H
x	x	e	H	y
y	y	H	e	x
H	H	y	x	e

$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(0,1)$	$(1,1)$	$(0,0)$	$(1,0)$
$(0,1)$	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
$(1,1)$	$(1,0)$	$(0,0)$	$(1,1)$	$(0,1)$
$(0,0)$	$(0,1)$	$(1,1)$	$(0,0)$	$(1,0)$
$(1,0)$	$(1,1)$	$(0,1)$	$(1,0)$	$(0,0)$

$e \longleftrightarrow (0,0)$

$x \longleftrightarrow (0,1)$

$y \longleftrightarrow (1,0)$

$H \longleftrightarrow (1,1)$

4. Determine whether the series $\sum\limits_{n=2}^{\infty} \dfrac{1}{n \ln n}$ converges or diverges.

$\dfrac{1}{n \ln n}$ is continous, decreasing, and positive.

$\int_2^\infty \dfrac{1}{x \ln x} dx$ let $u = \ln x$. $du = \frac{1}{x} dx$.

$\therefore \int_2^\infty \dfrac{1}{x \ln x} dx = \int_2^\infty \frac{1}{u} du = \ln u \Big|_2^\infty = \ln(\ln x) \Big|_2^\infty$

$\therefore \ln(\ln \infty) - \ln(\ln 2) = \infty$

$\therefore \sum\limits_{n=2}^{\infty} \dfrac{1}{n \ln n}$ diverges.   ✓

5. For what values of x is the following subset of $\mathbb{R}^3$ independent?

$$\{ \begin{pmatrix} x \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ x \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ x \end{pmatrix} \}$$

$A = \begin{pmatrix} x & 1 & 2 \\ 1 & x & 2 \\ 1 & 2 & x \end{pmatrix}$

$|A| = x(x^2 - 4) - (x - 2) + 2(2 - x) \neq 0$

$\therefore x^3 - 7x + 6 \neq 0$

$\therefore x = 1$ is a solution of $x^3 - 7x + 6 = 0$.

$\therefore x^3 - 7x + 6 = (x-1)(x-2)(x+3)$

$\therefore (x-1)(x-2)(x+3) \neq 0$

$\therefore x \neq 1, 2, -3$.

✓

/4