

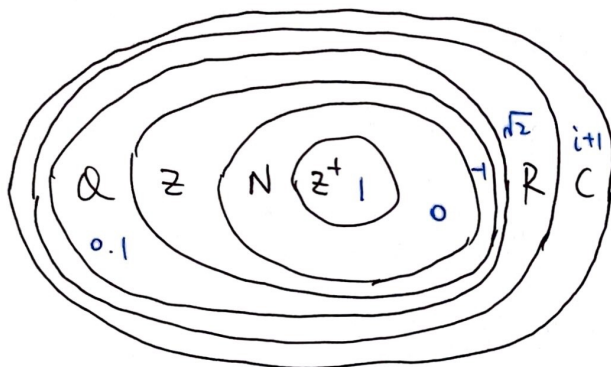
1. Write down the augmented matrix for the following system of equations.

$$\begin{aligned} x + y - 2z &= 0 \\ x - y &= 1 \\ y + 2z &= 2 \end{aligned}$$

Excellent work 10/10. One small point, it is conventional to draw the universal set as a rectangle.

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 1 & 2 & 2 \end{array} \right)$$

2. Draw a Venn diagram to show the relationship between the sets \mathbb{Z}^+ , \mathbb{N} , \mathbb{Z} , \mathbb{Q} , \mathbb{R} , \mathbb{C} with \mathbb{C} as the universal set. Put one representative number in each region of the diagram.

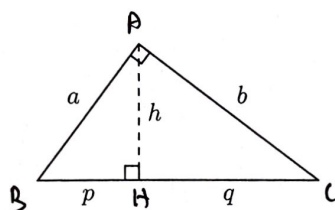


3. Euclid's theorem for proportional segments in a right triangle states that h is the geometric mean of p and q , a is the geometric mean of p and $p+q$, and b is the geometric mean of q and $p+q$. Use Euclid's theorem to prove Pythagoras's theorem.

Euclid's theorem



$$\begin{cases} h^2 = pq & (1) \\ a^2 = p(p+q) & (2) \\ b^2 = q(p+q) & (3) \end{cases}$$



$$(2) + (3) \Rightarrow$$

$$a^2 + b^2 = (p+q)(p+q) = (p+q)^2$$

\therefore in right $\triangle ABC$,

$$AB^2 = a^2, AC^2 = b^2,$$

$$BC^2 = (p+q)^2,$$

$$\therefore a^2 + b^2 = (p+q)^2$$

\therefore Pythagoras's theorem is proved.

4. Section 1.4 of T4 uses the binomial theorem to prove the cardinality of the power set of a set with n elements is 2^n . Prove this result without the use of the binomial theorem.

$$n(A) = n,$$

then in set A , there's element:

	A_1	A_2	A_3	\dots	A_{n-1}	A_n
1				\dots		
0				\dots		

when creating the element in the power set of A , we have to decide whether or not to include $A_1, A_2, A_3, \dots, A_n$, which produce a binary string like $0000 \dots 001$, which only include A_n , or $1111 \dots 111$,

which is equivalent to the set A itself.

The binary string has 2^n different possibilities, thus 2^n elements in the power set of A .

5. Two ladders, one of length 10 feet and the other of length 15 feet are resting between two walls as indicated in the diagram below. If the ladders cross at a point 4 feet above the ground, how far apart are the walls? Give your answer correct to three significant figures.

$$\triangle AOB \sim \triangle DOC$$

$$\therefore \frac{AO}{BO} = \frac{DO}{CO}$$

$$\therefore \frac{10-a}{b} = \frac{a}{15-b}$$

$$\therefore ab = 150 - 15a - 10b + ab$$

$$\therefore 15a + 10b = 150,$$

$$3a + 2b = 30.$$

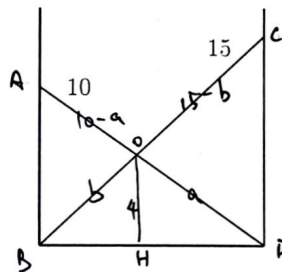
$$\therefore b = \frac{3}{2}(10-a)$$

$$\therefore \frac{OD}{OH} = \frac{AO}{AB}.$$

$$\therefore \frac{a}{4} = \frac{10}{AB}$$

$$\therefore AB = \frac{40}{a}.$$

$$\text{similarly, } CD = \frac{40}{10-a}.$$



$$\therefore BD^2 = AD^2 - AB^2 \\ = BC^2 - CD^2$$

$$\therefore 100 - \frac{(40)^2}{a^2} = 225 - \frac{(40)^2}{(10-a)^2}$$

according to the GDC,

when $a = 6.8307$, the two functions intersect.

$$BD^2 = 65.708257.$$

$$\therefore BD = 8.11 \text{ (3 s.f.)}.$$