1. Use l'Hôpital's rule to evaluate $\lim_{x\to 1} \frac{\arctan x - \pi/4}{x-1}$.

Apply L'Hôpital's rule since
$$\lim_{x\to 1} \arctan x - \frac{\pi}{4} = \lim_{x\to 1} x-1 = 0$$
.

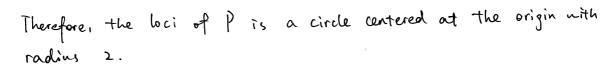
$$\lim_{X \to 1} \frac{\arctan X - \frac{1}{4}}{X - 1} = \lim_{X \to 1} \frac{\frac{1}{1 + 1}}{1} = \frac{1}{1 + 1} = \frac{1}{2}.$$

Therefore,
$$\lim_{x\to 1} \frac{\arctan x - \frac{\pi}{4}}{x-1} = \frac{1}{2}$$

2. Let A = (-1,0) and B = (1,0). Find the locus of a point P that moves so that $PA^2 + PB^2 = 10$.

$$2a^{2} + 2 + 2b^{2} = 10$$
.

$$a^2 + b^2 = 4 = 2^2$$



3. The third degree Taylor polynomial for the function f centred at 1 is $4 - (x - 1) + 3(x - 1)^2 - 5(x - 1)^3$.

(a) Write down the value of f''(1). $\int_{-1}^{1} (x) \approx \int_{3}^{3} (x) = \int_{-1}^{1} (x) + \int_$

(a) Write down the value of
$$f''(1)$$
.

$$f''(1) = 6$$

(b) Approximate f'(1.2).

$$f'(x) = -1 + 6(x-1) - 15(x-1)^2$$

4. The sequence $\{u_n\}$ is defined recursively by $u_1 = 2$ and $u_{n+1} = \frac{1}{2}(u_n + 4)$. Use mathematical induction to show that $\{u_n\}$ is an increasing sequence bounded above by 4. What is the limit of the sequence?

We need to prove Uncum, <4 for all n & Zt.

- 0 for n=1, u,=2 <4, u,= \$(2+4)=3. u, < u2 <4.
- 3 for n=m, um <4. we claim that um < um <4.

 proof: um +1 = \frac{1}{2} (um +4) < \frac{1}{2} (4+4) = 4, so Um +1 <4.

Um+1 = 1 (Um+4) > 1 (Um+Um) = Um, so Um+1>Um.

Therefore, $Um \subset Umt_1 \subset \mathcal{H}$, the segmence is increasing and bounded above by 4. • Let $Um_1 = f(x) = \frac{1}{2}(x+4)$ where x = Um. the segmence has limit L. $\lim_{n\to\infty} Um_1 = L$. $\lim_{n\to\infty} Um_1 = \lim_{n\to\infty} \frac{1}{2}(x+4) = \frac{1}{2}[\lim_{n\to\infty} x_n + 4] = \frac{1}{2}[L+4]$

: L= 1 (L+4) , L= 4.

Therefore, the limit of the sequence is 4.

5. The function f has derivatives of all orders for all real numbers. The third degree Taylor polynomial for f centred at 2 is $7 - 9(x-2)^2 - 3(x-2)^3$. If $|f^{(4)}(x)| \le 6$ for all x in the open interval]0, 2[, show that f(0) must be negative.

let x=0, a=2.

$$f(0) = 7 - 36 + 24 + \frac{2}{3} f^{(4)}(c) = -5 + \frac{2}{3} f^{(4)}(c)$$

Since
$$|f^{(4)}(x)| \le 6$$
, $f(0) = -5 + \frac{1}{3} f^{(4)}(c) \le -5 + \frac{1}{3} \times 6 = -5 + 4 = -1 < 0$.

Therefore, f(0) LO.

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