

1. Give the reduced row echelon form of the augmented matrix $\left(\begin{array}{cc|c} 2 & 1 & 7 \\ 4 & -2 & 6 \end{array}\right)$. Hence solve the corresponding system.

$$\left(\begin{array}{cc|c} 2 & 1 & 7 \\ 4 & -2 & 6 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} -2 & -1 & -7 \\ -4 & 2 & -6 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} -4 & -2 & -14 \\ -4 & 2 & -6 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} -4 & -2 & -14 \\ 0 & 4 & 8 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cc|c} -8 & 0 & -20 \\ 0 & 4 & 8 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 0 & \frac{5}{2} \\ 0 & 1 & 2 \end{array}\right)$$

$$\therefore (x_1, x_2) = \left(\frac{5}{2}, 2\right)$$

2. Use De Morgan's laws to prove $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$.

$$\begin{aligned} (A \cup B) \setminus (A \cap B) &= (A \cup B) \cap (A \cap B)' \\ &= (A \cup B) \cap (A' \cup B') \\ &= [(A \cup B) \cap A'] \cup [(A \cup B) \cap B'] \\ &= (B \setminus A) \cup (A \setminus B) \\ &= (A \setminus B) \cup (B \setminus A). \end{aligned}$$

3. In the diagram $AB = 4$, $BD = 5$, $AC = 3$ and $CE = 9$. Prove the quadrilateral $CEDB$ is cyclic.

$$\frac{AC}{AB} = \frac{3}{4}, \quad \frac{AD}{AE} = \frac{9}{12} = \frac{3}{4}.$$

$$\therefore \frac{AC}{AB} = \frac{AD}{AE}$$

$\therefore \angle A$ is the common angle,

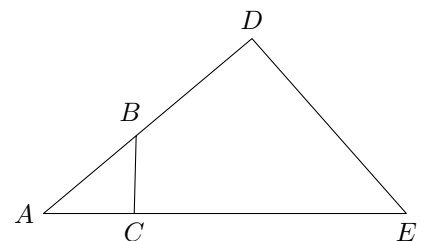
$$\therefore \triangle ABC \sim \triangle AED$$

$$\therefore \angle E = \angle ABC$$

$$\therefore \angle ABC + \angle DBC = 180^\circ$$

$$\therefore \angle E + \angle DBC = 180^\circ.$$

$\therefore CEDB$ is cyclic.



4. Let $G = (V, E)$ be an r -regular graph. Prove that either $|V|$ or r is even.

- any edge, either a loop or a line between two vertices, add 2 degrees to the total degree number.
so $E \cdot 2 = V \cdot r$.
- since V , E , and r are all integers, either $|V|$ or r must be even in order to provide the factor 2 to the equation.

5. Use L'Hôpital's rule to find $\lim_{x \rightarrow 0} \frac{\tan 3x - 3 \tan x}{\sin 3x - 3 \sin x}$.

$$\text{let } A = \lim_{x \rightarrow 0} \frac{\tan 3x - 3 \tan x}{\sin 3x - 3 \sin x}$$

$$\therefore \lim_{x \rightarrow 0} \tan 3x - 3 \tan x = \lim_{x \rightarrow 0} \sin 3x - 3 \sin x = 0$$

$$\therefore A = \lim_{x \rightarrow 0} \frac{(\tan 3x - 3 \tan x)'}{(\sin 3x - 3 \sin x)'} = \lim_{x \rightarrow 0} \frac{3 \sec^2 3x - 3 \sec^2 x}{3 \cos 3x - 3 \cos x} = \lim_{x \rightarrow 0} \frac{\sec^2 3x - \sec^2 x}{\cos 3x - \cos x}$$

$$\therefore \lim_{x \rightarrow 0} \sec^2 3x - \sec^2 x = \lim_{x \rightarrow 0} \cos 3x - \cos x = 0$$

$$\therefore A = \lim_{x \rightarrow 0} \frac{(\sec^2 3x - \sec^2 x)'}{(\cos 3x - \cos x)'} = \lim_{x \rightarrow 0} \frac{6 \sec^2 3x \tan 3x - 2 \sec^2 x \tan x}{-3 \sin 3x + \sin x}$$

$$\therefore \lim_{x \rightarrow 0} 6 \sec^2 3x \tan 3x - 2 \sec^2 x \tan x = \lim_{x \rightarrow 0} -3 \sin 3x + \sin x = 0$$

$$\begin{aligned} \therefore A &= \lim_{x \rightarrow 0} \frac{(6 \sec^2 3x \tan 3x - 2 \sec^2 x \tan x)'}{(-3 \sin 3x + \sin x)'} = \lim_{x \rightarrow 0} \frac{36 \sec^2 3x \tan^2 3x + 18 \sec^4 3x - 4 \sec^2 x \tan^2 x - 2 \sec^4 x}{-9 \cos 3x + \cos x} \\ &= \frac{18 - 2}{-8} \\ &= -2. \end{aligned}$$