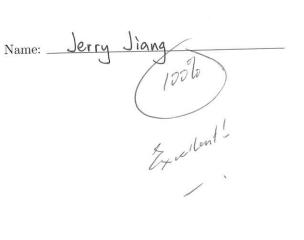
1. How many positive divisors does 1071 have?

$$|071 = 3^{2} \times 7 \times 1$$
  
 $(2+1) \times (1+1) \times (1+1)$   
 $= 3 \times 2 \times 2$   
 $= 12$ 



2. Find integers x and y such that  $3^x 6^y = 24$ .

$$3^{x} \cdot (2 \cdot 3)^{y} = 24$$

$$= 3^{x+y} \cdot 2^{y}$$

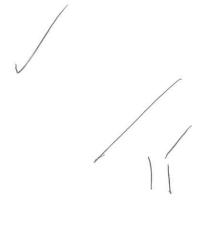
$$24 = 2^{3} \cdot 3$$

$$\begin{cases} x+y=1 \\ y=3 \end{cases}$$

$$\begin{cases} x=-2 \\ y=3 \end{cases}$$



3. Let  $U=\{n\in\mathbb{Z}\mid 1\leq n\leq 2018\},\ A=\{n\in U\mid n\text{ is odd}\}\ \text{and }B=\{n\in U\mid n\text{ is a multiple of }3\}.$  Find  $n(A\cap B')$ . The question is equivalent to : In positive integers from 1 to 2018, inclusive, find the number of numbers which is neither multiple of 2 nor multiple of 3.



4. Find the first term of the geometric sequence  $6, 6\sqrt{2}, 12, 12\sqrt{2}, \dots$  that exceeds 2018.

$$r=\sqrt{2}$$
 $a_1 = b$ .

 $a_1 = b$ .

 $a_1 = b$ .

 $a_1 = b$ .

 $a_2 = b \cdot \sqrt{2}$ 
 $a_1 = b \cdot \sqrt{2}$ 
 $a_2 = b \cdot \sqrt{2}$ 
 $a_3 = b \cdot \sqrt{2}$ 
 $a_4 = b \cdot \sqrt{2}$ 
 $a_5 = b$ 

5. Find the sum of the first 20 terms of the sequence defined recursively by  $u_n = 9/u_{n-1}$  and  $u_1 = 9$ .

$$U_{2} = \frac{q}{q} = 1$$

$$U_{3} = \frac{q}{1} = 9$$

$$U_{4} = \frac{q}{q} = 1$$

$$U_{5} = \frac{q}{1} = 9$$

6. What is the units digit of the sum  $1! + 2! + 3! + 4! + \cdots + 2018!$ ?

7. Solve 
$$\log_2(x+1) - \log_4(3x-1) = 0.5$$
.

$$|0f_{2}(x+1) - \frac{1}{2}|0f_{2}(3x-1) = 0.5.$$

$$|0f_{2}(x+1) - |0f_{2}(3x-1)^{\frac{1}{2}} = 0.5.$$

$$|0f_{2}(x+1) - |0f_{2}(3x-1)^{\frac{1}{2}} = 0.5.$$

$$|(x+1)| = \sqrt{2}$$

$$|(x+1)|^{\frac{1}{2}} = \sqrt{2$$

X2=1

8. Is there a prime number p that satisfies the inequality  $2018! + 2 \le p \le 2018! + 2018$ ? Justify your answer.

NO.

2018! to is the multiple of 2 since 2(1+ 1x3x4x-x x 2018)
2018! to is the multiple of 3 since 3 (1+ 1x2x4x-x 2018)
2018! to is the multiple of 4 since 4(1+ 1x2x3x5-...x2018)
2018! to17 is the multiple of 2017 since 2017 (1+ 1x2x3x-...x2018x2018)
2018! to18 is the multiple of 2018 since 2018 (1+ 2017!)

So the 2017 integers, starting from 2018! to and end with 2018! to18
are all composite numbers

So there is no presible p that satisfies the inequality above as a prime number.



9. Solve 
$$x^{\log x} = \frac{x^3}{100}$$
.

$$|Of_{\mathcal{K}}(X^{\log^{3}x})| = |Of_{\mathcal{K}}(x^{3})|$$

$$|Of_{\mathcal{K}}| = |Of_{\mathcal{K}}|$$

$$|Of_{\mathcal{K}}| = |Of_{\mathcal{K}}(x^{3})|$$

$$|Of_{\mathcal{K}$$

10. The number 1071 is equal to a060 in base b. Find the values of a and b.

$$6 \times 6 + a \cdot 6^3 = [07], (a, b \in 2^4, 1 \le a, b \le 9)$$
  
 $6 (6 + a, b^2) = [07]$ 

.. the integer divisor of 1071 in range 1 to 9

- . since there's a 'b' in 'aobo', the base be have to be no less than?.

0 
$$b = 7$$
. 3  $b = 9$   
 $6 + a \cdot 49 = 153$   $6 + 8 | a = 119$   
 $a = 1.40$   
 $a = 3$  (in a dmissable)

## Solutions to HL1 Assignment #4

- 1. Since  $1071 = 3^27^117^1$ , we have  $\tau(1071) = 3 \cdot 2 \cdot 2 = 12$ .
- 2. We have  $3^x 6^y = 3^x 2^y 3^y = 2^y 3^{x+y} = 24 = 2^3 3^1$ . So y = 3 and x + y = 1. We conclude x = -2, y = 3.
- 3. Observe  $A \cap B = \{3, 9, 15, \dots, 2013\}$ . So  $n(A \cap B) = 336$ . Since n(A) = 1009, we conclude  $n(A \cap B') = n(A) n(A \cap B) = 1009 336 = 673$ .
- 4. Here  $u_n = 6 \times (\sqrt{2})^{n-1}$ . Solving  $u_n > 2018$  gives  $n_{\min} = 18$ . So the first term to exceed 2018 is  $u_{18} = 1536\sqrt{2}$ .
- 5. The sequence to 20 terms is  $1, 9, 1, 9, 1, 9, \dots, 9$ . So  $S_{20} = 10 \times (1+9) = 100$ .
- 6. Observe that the units digit for n! when n > 4 is 0. So the units digit of this sum is the same as the units digit of 1! + 2! + 3! + 4!, which is 3.
- 7. We first note that any solution must satisfy x > 1/3. Next changing base gives  $\log_2(x+1) \frac{1}{2}\log_2(3x-1) = 0.5$ , which gives in turn

$$\log_2 \frac{(x+1)^2}{3x-1} = 1.$$

So  $(x+1)^2 = 2(3x-1) \Leftrightarrow x^2-4x+3=0 \Leftrightarrow x=1 \text{ or } x=3, \text{ and both of these solutions are valid.}$ 

- 8. There is no such prime number since 2018! + k where  $2 \le k \le 2018$  is divisible by k.
- 9. Taking logs of both sides we have  $\log x \times \log x = 3\log x 2$ . Letting  $y = \log x$  gives the quadratic equation  $y^2 3y + 2 = 0$ , whence y = 1 or y = 2. So x = 10 or x = 100.
- 10. Recall  $1071 = 3^2 \times 7 \times 17$ . Now 1071 in base b has 0 as its units digit, so b must be a divisor of 1071. Next a060 contains a 6, so the base b cannot be 3. Also a base of 17 or more would not give a four digit representation for 1071 as  $17^3 > 1017$ . So our remaining candidates for b are 7 and 9. Since  $1071 = 1420_9$ , we conclude b = 7. Lastly  $1071 = 3060_7$ , so a = 3.