FM2 Assignment #19 Name: Jerry Jiang $\frac{9\frac{1}{2}}{10}$ Very good

1. The linear transformation that maps (x, y) to $(x + ky, y)$ is called a horizontal shear with shear factor k. If M is the matrix for a horizontal shear with shear factor 1, find M^{2019}.

$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$. $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

$(x, y) \xrightarrow{T} (x + y, y) \xrightarrow{T} (x + 2y, y) \xrightarrow{T} (x + 3y, y) \cdots$

$\therefore M^{2019} = \begin{pmatrix} 1 & 2019 \\ 0 & 1 \end{pmatrix}$. ✓

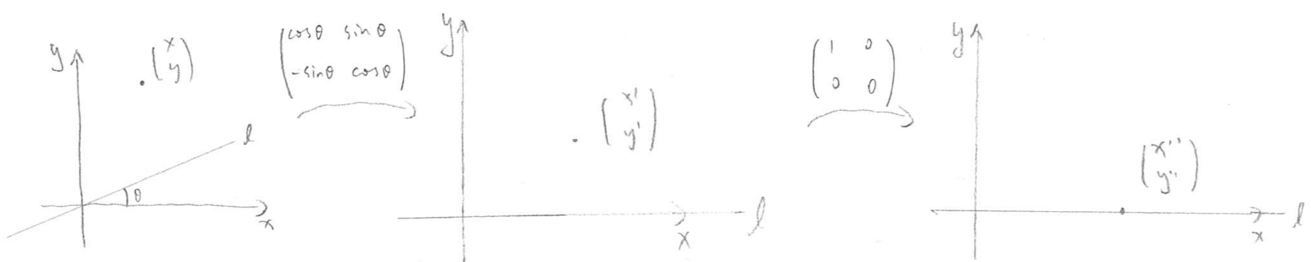
2. Find T^{-1} for the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + 3y, 2x + 5y)$.

$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 3y \\ 2x + 5y \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$.

$\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}^{-1} = \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix}$.

$T^{-1}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$. ✓

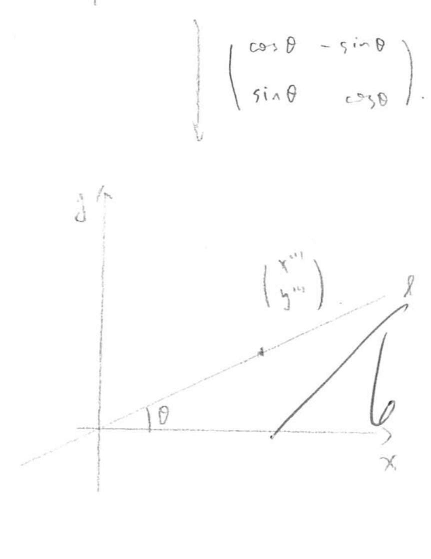
3. Find the matrix for projection onto the line $y = (\tan\theta)x$. Describe the kernel of this transformation.



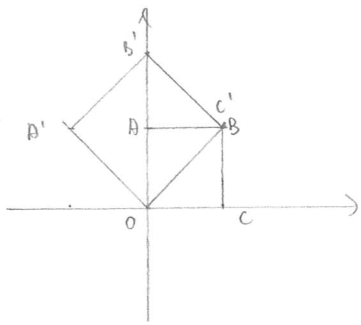
$M = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$= \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$ ✓

$\ker(T) = \left\langle \begin{pmatrix} \tan\theta \\ -1 \end{pmatrix} \right\rangle$.



4. Draw the image of the unit square under the transformation with matrix M= $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Hence write M as the product of a dilation (enlargement) matrix and a rotation matrix.



$$M = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}.$$

✓

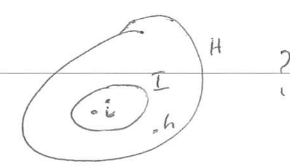
$G \neq A \cup B$ if $A \subseteq B$ or $B \subseteq A$. Then proceed as you have done.

5. Prove that a group cannot be the union of two of its proper subgroups.

Let the two subgroups be H, I. $H \cup I = J$. ← which is?

① $J \neq H$. $J \neq I$.

← what if



?

Then there must be $h \in H$, $i \in I$ that $h \notin I$, $i \notin H$.

But $h, i \in J$, and $hi \notin J$ since neither H or I contain hi. why?

So J doesn't satisfy closure and isn't a group.

② $J = H$ or $J = I$.

Then since H or I are proper subgroups, J can't be the entire group G H, I are subgroups of as that's the definition of proper subgroups.

$1\frac{1}{2}$

Therefore, a group cannot be the union of two of its proper subgroups.

$3\frac{1}{2}$.