$\frac{10}{10}$ Excellent!

FL2 Assignment #1 Name: Jerry Jiang

1. Let X be a set. How many solutions are there to $\{1,2\} \subseteq X \subseteq \{1,2,3,4,5\}$?

$X \supseteq \{1,2\}$, so it's a matter of whether to include 3, 4, and 5.

$2^3 = 8$.

2. The integers 5 and 15 are members of a set of 12 integers that form a group under multiplication modulo 28. List all 12 integers.

$GCD(n, 28) = 1$.

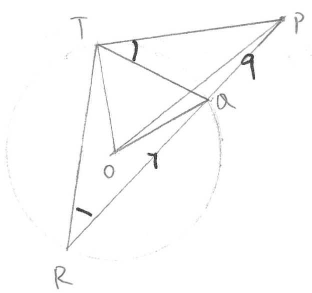
1, 3, 5, 9, 11, 13, 15, 17, 19, 23, 25, 27.

Proof. i. 1 is the only one that can be the identity element.

ii. any number n with factor 2 or 7 will keep having the factor after the operation, so they don't have inverse.

iii. thus only number n that satisfies $GCD(n, 28) = 1$ can be in the group.

3. A point P is outside a circle and 13 cm from its centre. A secant from P cuts the circle at points Q and R so that the external segment $[PQ]$ of the secant is 9 cm and QR is 7 cm. Find the radius of the circle.



In the diagram on the left, PT is tangent to the circle.

Let $\angle PTQ = \alpha$. so $\angle OTQ = 90 - \alpha$.

$\because OT = OQ$, $\therefore \angle TOQ = 180° - \angle OTQ - \angle OQT = 180° - 2\angle OTQ = 180 - (180 - 2\alpha)$

$\therefore \angle TOQ = 2\alpha$. $\therefore \angle TRQ = \alpha = \angle PTQ$

Since $\angle TPR$ is the common angle, $\triangle TQP \sim \triangle RTP$

$\therefore \frac{PT}{PR} = \frac{PQ}{PT} \rightarrow PT^2 = PQ \cdot PR = 9 \cdot (7+9) = 144$

$\therefore PT = 12$.

$\because PO = 13$, $\sqrt{13^2 - 12^2} = 5$

$\therefore TO = 5 =$ radius.

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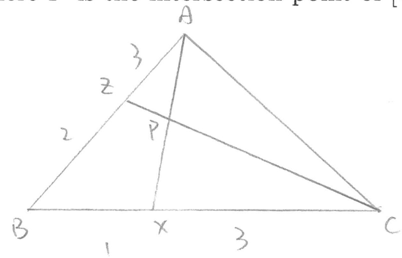
4. In $\triangle ABC$, cevians $[AX]$ and $[CZ]$ are drawn so that $CX : XB = 3 : 1$ and $AZ : ZB = 3 : 2$. Let $k = CP : PZ$, where P is the intersection point of $[CZ]$ and $[AX]$. Find k.



According to the Menelaus's Theorem,

$$\frac{CX}{XB} \cdot \frac{BA}{AZ} \cdot \frac{ZP}{PC} = -1$$

$$\frac{3}{1} \cdot \frac{5}{-3} \cdot \frac{ZP}{PC} = -1$$

$$\therefore \frac{ZP}{PC} = \frac{1}{5}$$

$$\therefore k = \frac{CP}{PZ} = 5$$

5. If H and K are subgroups of G, show that $H \cap K$ is also a subgroup of G.

Let $H \cap K = T$.

① let $a, b \in T$. then $a, b \in H$ and $a, b \in K$

$a \cdot b = c$. since $a, b \in H$ and $a, b \in K$, $a \cdot b$ must be an element in subgroup H&K.

$\therefore c \in H \cap K = T$. closure ✓

② since $e \in H$, $e \in K$, then $e \in H \cap K = T$.

③ for any $a \in H$, $a \in K$, since H&K are subgroups, $a^{-1} \in H$, $a^{-1} \in K$.

$\therefore a^{-1} \in H \cap K = T$.

According to the 3-step subgroup test, T is a subgroup of G.

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