Homework 4

Solution 1. 1.

$$\mathbf{S} = \mathbf{X}\mathbf{X}^T \tag{1}$$

The principal components $\{\mathbf{u}_i\}$ are eigenvectors of $\mathbf{S}$, which are vectors such that: $\mathbf{S}\mathbf{u}_i = \lambda_i \mathbf{u}_i$.

$\mathbf{X} = \mathbf{U}\mathbf{S}'\mathbf{V}^T$ so,

$$\begin{aligned} \mathbf{S} &= \mathbf{X}\mathbf{X}^T & (2) \\ &= \mathbf{U}\mathbf{S}'\mathbf{V}^T\mathbf{V}\mathbf{S}'\mathbf{U}^T & (3) \\ &= \mathbf{U}\mathbf{S}'^2\mathbf{U} & (4) \end{aligned}$$

Therefore, the columns of $\mathbf{U}$ are eigenvectors of $\mathbf{S}$.

2. Assume that the complexity of SVD is $O(ND\min(N, D))$ and the complexity of solving eigenvector problem is $O(D^3)$, we should use SVD.

Solution 2. *Proof.* .

a. Answer the following questions about the conditional independence structure in the model:

- **i.** **school cancellation** and **roads salted**
- **ii.** None
- **iii.** None
- **iv.** **temperature**

In the following, we abbreviate **temperature** as T, **snow** as S, **roads salted** as R, **school cancellation** as C

b. $p(T, S, R, C) = p(T) \cdot p(S|T) \cdot p(R|S) \cdot p(C|S, R)$

c.

$$
\begin{aligned}
p(C=true|S=light) &= \frac{p(C=true,S=light)}{p(S=light)} \\
&= \frac{\sum_{T,R} p(T,R,C=true,S=light)}{\sum_{T,R,C} p(T,R,C,S=light)} \\
&= \frac{\sum_{T,R} p(T)p(S=light|T)p(R|S=light)p(C=true|S=light,R)}{\sum_{T,R,C} p(T)p(S=light|T)p(R|S=light)p(C|S=light,R)} \\
&= \frac{\sum_{T} p(T)p(S=light|T)\sum_{R} p(R|S=light)p(C=true|S=light,R)}{\sum_{T} p(T)p(S=light|T)\sum_{R} p(R|S=light)\sum_{C} p(C|S=light,R)} \\
&= \frac{\sum_{T} p(T)p(S=light|T)\sum_{R} p(R|S=light)p(C=true|S=light,R)}{\sum_{T} p(T)p(S=light|T)} \\
&= \sum_{R} p(R|S=light)p(C=true|S=light,R) \\
&= 0.9*0.2+0.1*0.4 \\
&= 0.22
\end{aligned}
$$

$$p(C=false|S=light)=0.78$$

$\square$