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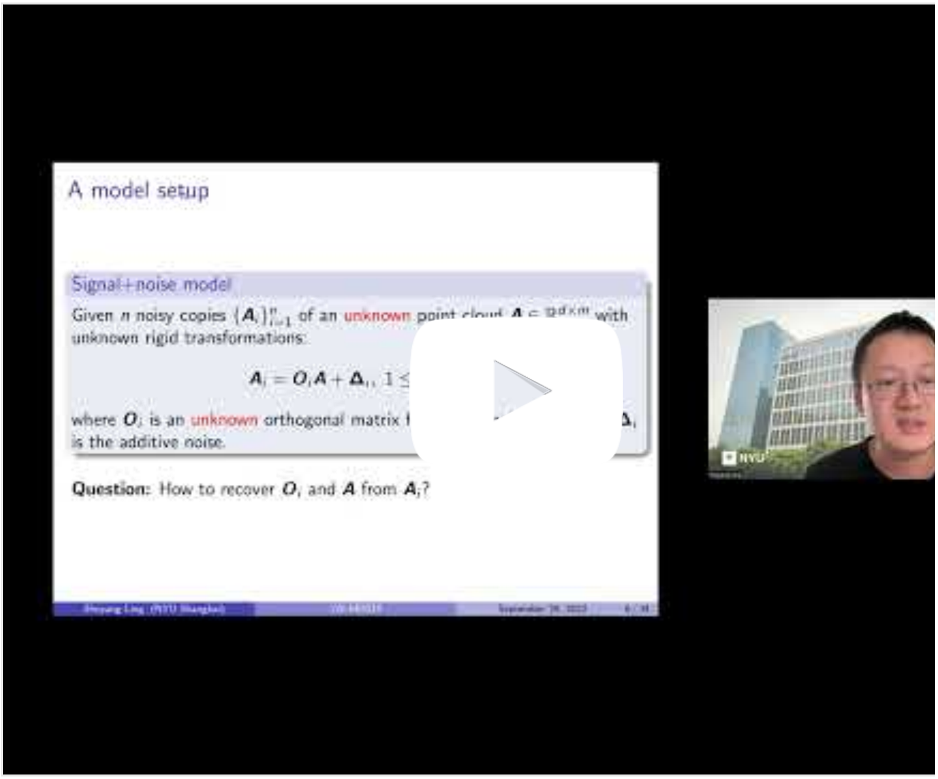

Description

Near-Optimal Bounds for Generalized Orthogonal Procrustes Problem via Generalized Power Method Abstract: Given multiple point clouds, how to find the rigid transform (rotation, reflection, and shifting) such that these point clouds are well aligned? This problem, known as the generalized orthogonal Procrustes problem (GOPP), has found numerous applications in statistics, computer vision, and imaging science. While one commonly-used method is finding the least squares estimator, it is generally an NP-hard problem to obtain the least squares estimator exactly due to the notorious nonconvexity. In this work, we apply the semidefinite programming (SDP) relaxation and the generalized power method to solve this generalized orthogonal Procrustes problem. In particular, we assume the data are generated from a signal-plus-noise model: each observed point cloud is a noisy copy of the same unknown point cloud transformed by an unknown orthogonal matrix and also corrupted by additive Gaussian noise. We show that the generalized power method (equivalently alternating minimization algorithm) with spectral initialization converges to the unique global optimum to the SDP relaxation, provided that the signal-to-noise ratio is high. Moreover, this limiting point is exactly the least squares estimator and also the maximum likelihood estimator. In addition, we derive a block-wise estimation error for each orthogonal matrix and the underlying point cloud. Our theoretical bound is near-optimal in terms of the information-theoretic limit (only loose by a factor of the dimension and a log factor). Our results significantly improve the state-of-the-art results on the tightness of the SDP relaxation for the generalized orthogonal Procrustes problem, an open problem posed by Bandeira, Khoo, and Singer in 2014.