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PORTFOLIO ANALYSIS

Portfolio analysis is one of the most commonly used statistical methodologies in empirical asset pricing. Its objective is to examine the cross-sectional relation between two or more variables. The most frequent application of portfolio analysis is to examine the ability of one or more variables to predict future stock returns. The general approach is to form portfolios of stocks, where the stocks in each portfolio have different levels of the variable or variables posited to predict cross-sectional variation in future returns and to examine the returns of these portfolios.

While the most common application of portfolio analysis is to examine future return predictability, the portfolio methodology can also be employed to understand cross-sectional relations between any set of variables. This is useful for understanding variation in the characteristics of the entities (stocks) across the different portfolios. Thus, in the very general sense, portfolio analysis is useful for understanding the cross-sectional relation between one variable and combinations of other variables.

Perhaps the most important benefit of portfolio analysis is that it is a nonparametric technique. This means that it does not make any assumptions about the nature of the cross-sectional relations between the variables under investigation. Many other methodologies rely on some assumptions regarding the functional form of the relation between the variables being examined. For example, linear regression analysis assumes that the relation between the dependent and independent variables is linear. Portfolio analysis does not require this assumption. In fact, portfolio analysis can be helpful in uncovering nonlinear relations between variables that are quite difficult to detect using parametric techniques. Perhaps the

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main drawback of the technique is that it is difficult to control for a large number of variables when examining the cross-sectional relation of interest. This compares to regression analysis in which it is easy to control for a large number of independent variables in the analysis.

In this chapter, we present and exemplify the details of implementing a portfolio analysis and interpreting the results. There are several different variations of portfolio analysis. While some researchers have implemented variations that are not covered in this chapter, the vast majority of portfolio analyses in empirical asset pricing research follow one of the approaches discussed herein.

Throughout this chapter, we use Y to denote the outcome variable of the portfolio analysis. Y can be thought of as the variable of interest, similar to the dependent variable in a regression analysis. We use X to denote the sort variable or variables. X is analogous to the independent variable or variables in a regression. We refer to Y and X the outcome and sort variables, respectively, to avoid confusion in our presentation of independent and dependent sorts, discussed in Sections 5.2 and 5.3, respectively. We demonstrate the portfolio methodology using the methodology sample described in Section 1.1.

5.1 UNIVARIATE PORTFOLIO ANALYSIS

We begin with the most basic type of portfolio analysis: univariate portfolio analysis. A univariate portfolio analysis has only one sort variable X. The objective of the analysis is to assess the cross-sectional relation between X and the outcome variable Y. A univariate portfolio analysis does not allow us to control for any other effects when examining this relation.

The univariate portfolio analysis procedure has four steps. The first step is to calculate the breakpoints that will be used to divide the sample into portfolios. The second step is to use these breakpoints to form the portfolios. The third step is to calculate the average value of the outcome variable Y within each portfolio for each period t. The fourth step is to examine variation in these average values of Y across the different portfolios.

5.1.1 Breakpoints

The first step in univariate portfolio analysis is to calculate the periodic breakpoints that will be used to group the entities in the sample into portfolios based on values of the sort variable X. Entities with values of X that are less than the first breakpoint will be placed into the first portfolio. Entities with values of X that are between the first and second breakpoints will comprise the second portfolio, etc. Finally, entities with X values higher than the highest breakpoint will be placed in the last portfolio. We denote the number of portfolios to be formed each time period as n_P. The number of breakpoints that need to be calculated each period is therefore $n_P - 1$. The number of portfolios to be formed and, thus, the number of breakpoints to be calculated is the same for all time periods. The value of the kth breakpoint, however, will almost certainly vary from time period to time period. We denote the kth breakpoint for period t as $B_{k,t}$ for $k \in \{1, 2, \dots, n_P - 1\}$.

The breakpoints for period t are determined by percentiles of the time t cross-sectional distribution of the sort variable X. Specifically, letting p_k be the percentile that determines the kth breakpoint, the kth breakpoint for period t is calculated as the p_kth percentile of the values of X across all entities in the sample for which X is available in period t. We therefore define the breakpoints as

$$B_{k,t} = Pctl_{p_k}(\{X_t\}) \tag{5.1}$$

where $Pctl_p(Z)$ is the pth percentile of the set Z and $\{X_t\}$ represents the set of valid values of the sort variable X across all entities i in the sample in time period t. The percentiles, and thus the breakpoints, increase as k increases, giving $0 < p_1 < p_2 < \cdots < p_{n_P-1}$ and $B_{1,t} \leq B_{2,t} \leq \cdots \leq B_{n_P-1,t}$ for all periods t. While the chosen percentiles $(p_1, p_2, \ldots, p_{n_P-1})$ are required to be strictly increasing, this does not necessarily mean that the actual breakpoints, calculated as the chosen percentile values of X, are strictly increasing. In some cases, there may be a large number of entities for which the values of X are the same, causing two or more of the breakpoints to be the same. If the variable X is truly continuous, the probability of this happening should be zero. However, there are examples of variables used in asset pricing research that at first glance would appear to be continuous but actually have many entities for which the value of the variable is the same.

It is worth mentioning here that, in some cases, breakpoints are calculated using only a subset of the entities that are in the sample for the given period t. For example, in research where the entities are stocks, sometimes researchers form breakpoints using only stocks that trade on the New York Stock Exchange, and then use those breakpoints to sort all stocks in the sample (including stocks that trade on other exchanges) into portfolios. Thus, in the previous paragraph as well as for the remainder of Section 5.1.1, when we refer to the sample, what we actually mean is the subset of the full sample that is being used to calculate the breakpoints. In most cases, this subset is the entire sample, but there are many examples where a strict subset is used. It is for this reason that we consider the calculation of breakpoints and the formation of portfolios, two separate steps in the portfolio analysis procedure.

Choosing an appropriate number of portfolios and choosing appropriate percentiles for the breakpoints are important decisions in portfolio analysis. As the entities in the sample will eventually be grouped into portfolios based on the breakpoints, the decision is largely based on trading off the number of entities in each portfolio against the dispersion of the sort variable among the portfolios. As the number of portfolios increases, the number of entities in each portfolio decreases, and vice versa. When the average value of the outcome variable Y for each portfolio is eventually calculated (the average value of Y is the focal point of the portfolio analysis and will be discussed in Section 5.1.3), a small number of entities in each portfolio results in increased noise when using the sample mean value of Y as an estimate of the true mean. Thus, having a large number of entities in each portfolio increases the accuracy of our estimate of the true mean value for each portfolio and is thus desirable. On the other hand, the more entities we group into each portfolio, the smaller the number of portfolios and the smaller the dispersion in the sort variable

X among the portfolios. Decreased dispersion in X across the portfolios can make it more difficult to detect cross-sectional relations between X and Y, as the values of X in the portfolios may not differ substantially if we have too few portfolios.

Most commonly, portfolios are formed using breakpoints that represent evenly spaced percentiles of the cross-sectional distribution of the sort variable. This means that the $n_P - 1$ breakpoints are defined to be the $k \times (1/n_P)$ percentiles of X, where $k \in \{1, \ldots, n_P - 1\}$. For example, if we want to split the sample into five portfolios, we may use the 20th, 40th, 60th, and 80th percentiles of the sort variable as the portfolio breakpoints. While the evenly spaced percentile approach to calculating breakpoints is most common, other approaches have been used. For example, when splitting the sample into only three portfolios, it is common to use the 30th and 70th percentiles of the sort variable as the breakpoints.

In choosing the number of portfolios and breakpoint percentiles, it is important to remember that new portfolios are formed for each time period t. Thus, when assessing the number of entities that fall into each portfolio, it is important to look not only at the average number of entities in the sample during the different time periods t but also at the minimum number of entities in any time period. The number of entities that will put into each portfolio is easily determined by the number of entities in the sample and the percentiles used to calculate the breakpoints. When the breakpoints are determined by equally spaced percentiles, the number of entities in each portfolio for any given time period t will be the number of entities in the sample for that time period divided by the number of portfolios. In the general sense, the minimum number of entities in any portfolio in a given time period t will be the number of entities in the sample during time period t, which we denote n_t, multiplied by the minimum of the lowest percentile, the differences between successive percentiles, and one minus the highest percentile. The exception to these cases is when the sample used to calculate the breakpoints is a strict subset of the set of entities that will be placed in the portfolios. In this case, the minimum number of entities in a portfolio will be higher. Finally, almost all studies use between three and 20 portfolios, with most researchers choosing either five or 10.

To exemplify the calculation of breakpoints in univariate portfolio analysis, we use the methodology sample discussed in Section 1.1 and take β to be the sort variable. Our analysis uses seven portfolios ($n_P = 7$) and thus six breakpoints will be calculated each year. The breakpoints will be the 10th, 20th, 40th, 60th, 80th, and 90th percentiles of β. We choose uneven breakpoints simply to exemplify the flexibility of the portfolio procedure. In some cases, researchers choose to make the distance between the percentiles that determine the breakpoints smaller for the lowest and highest portfolios because doing so can help us understand whether the relation under investigation is stronger for entities with extreme (low or high) values of the sort variables X. It is not uncommon in the empirical finance literature for a cross-sectional phenomenon to be driven by a small number of stocks with extreme values of one of the variables under investigation.

The results of the calculation of the breakpoints are presented in Table 5.1. The table shows that, for example, breakpoints one, two, three, four, five, and six for year

TABLE 5.1 Univariate Breakpoints for β-Sorted Portfolios

This table presents breakpoints for β-sorted portfolios. Each year t, the first ($B_{1,t}$), second ($B_{2,t}$), third ($B_{3,t}$), fourth ($B_{4,t}$), fifth ($B_{5,t}$), and sixth ($B_{6,t}$) breakpoints for portfolios sorted on β are calculated as the 10th, 20th, 40th, 60th, 80th, and 90th percentiles, respectively, of the cross-sectional distribution of β. Each row in the table presents the breakpoints for the year indicated in the first column. The subsequent columns present the values of the breakpoints indicated in the first row.

t	$B_{1,t}$	$B_{2,t}$	$B_{3,t}$	$B_{4,t}$	$B_{5,t}$	$B_{6,t}$
1988	−0.05	0.07	0.29	0.51	0.86	1.11
1989	−0.11	0.05	0.29	0.54	0.89	1.17
1990	−0.06	0.10	0.37	0.68	1.07	1.37
1991	−0.09	0.09	0.38	0.67	1.05	1.33
1992	−0.22	0.08	0.42	0.77	1.23	1.66
1993	−0.21	0.10	0.44	0.73	1.18	1.55
1994	−0.05	0.19	0.52	0.81	1.19	1.56
1995	−0.17	0.10	0.42	0.71	1.16	1.65
1996	0.00	0.19	0.46	0.73	1.14	1.52
1997	−0.00	0.15	0.36	0.59	0.89	1.15
1998	0.13	0.28	0.54	0.79	1.13	1.38
1999	−0.06	0.06	0.24	0.42	0.70	0.99
2000	0.03	0.14	0.37	0.63	1.22	1.79
2001	0.05	0.18	0.46	0.76	1.23	1.75
2002	0.04	0.17	0.48	0.75	1.06	1.37
2003	0.05	0.22	0.54	0.83	1.16	1.46
2004	0.14	0.40	0.81	1.16	1.56	1.96
2005	0.09	0.33	0.80	1.14	1.49	1.74
2006	0.10	0.35	0.82	1.21	1.64	1.94
2007	0.13	0.33	0.75	1.04	1.33	1.54
2008	0.16	0.38	0.74	1.02	1.31	1.54
2009	0.24	0.45	0.84	1.23	1.70	2.06
2010	0.29	0.56	0.92	1.19	1.49	1.72
2011	0.26	0.56	0.99	1.25	1.52	1.73
2012	0.28	0.55	0.91	1.18	1.48	1.75

1988 are −0.05, 0.07, 0.29, 0.51, 0.86, and 1.11, respectively. These are the breakpoints that will be used to sort stocks into portfolios at the end of year 1988. As necessitated by the calculation, the breakpoints are increasing across the columns for each year t.

5.1.2 Portfolio Formation

Having calculated the breakpoints, the next step in univariate portfolio analysis is to group the entities in the sample into portfolios. Each time period t, all entities in the sample with values of the sort variable X that are less than or equal to the first breakpoint, $B_{1,t}$, are put in portfolio one. Portfolio two holds entities with values of X that are greater than or equal to the first breakpoint and less than or equal to the second breakpoint. Portfolio three holds entities with values of X greater than or equal to the second breakpoint and less than or equal to the third breakpoint, and so on. Finally, portfolio n_P holds entities with values of X that are greater than or equal to $B_{n_P-1,t}$. In general, portfolio k holds entities i with period t values of the sort variable, $X_{i,t}$, that are greater than or equal to the $k-1$st breakpoint $B_{k-1,t}$ and less than or equal to the jth breakpoint $B_{k,t}$ for $k \in \{1, \ldots, n_P\}$, where we define $B_{0,t} = -\infty$ and $B_{n_P,t} = \infty$. Thus, letting $P_{k,t}$ be the set of entities in the kth portfolio formed at the end of period t, we have

$$P_{k,t} = \{i | B_{k-1,t} \leq X_{i,t} \leq B_{k,t}\} \tag{5.2}$$

for $k \in \{1, 2, \ldots, n_P\}$. We refer to $P_{k,t}$ as the kth portfolio or portfolio k for period t.

Forming portfolios as such puts all entities with the lowest values of the sort variable $X_{i,t}$ in portfolio one and all entities with the highest values of the $X_{i,t}$ in the last (n_Pth) portfolio, with values of $X_{i,t}$ increasing as the portfolio number increases. As discussed earlier, it is not necessary that the set of entities that is grouped into the portfolios is the same as the set of entities that is used to calculate the breakpoints. Once the breakpoints are calculated, they can be applied to any set of entities, whether it is a superset, subset, or the same set as was used to calculate the breakpoints. We should also point out that when forming the portfolios, if a given entity has a value of X during time period t that is exactly equal to the kth breakpoint, $B_{k,t}$, then this entity is included in both portfolio k and portfolio $k+1$. We define the portfolios in this manner for a good reason. The reason is that, as discussed previously, it is possible that two (or more) consecutive breakpoints have exactly the same value. This occurs when there are a large number of entities with the same value of X in period t. In such situations, if we had defined the portfolios using a strict inequality for either the lower or upper values of X, then one or more of the portfolios would contain no entities. For example, imagine that we are sorting stocks into decile portfolios (the breakpoints are the 10th, 20th, 30th, …, 90th percentiles) based on past returns, but the 30th and 40th percentile of past returns are both zero in the given period t, meaning that breakpoints 3 and 4 would both be zero. If the stocks in the fourth portfolio are those that have returns that are greater than the third breakpoint, which is zero, and less than or equal to the fourth breakpoints, which is also zero, then there would be no stocks in the fourth portfolio. To alleviate this issue, we define the portfolios to be inclusive of entities with values of X that are equal to either the low breakpoint or the high breakpoint that define the portfolio. The ramification of this is that, in this situation, there will be some entities that are included in more than one portfolio. We consider this issue to be minor compared to the issues that arise when a portfolio has no entities. That being said, as a researcher, if such a situation arises in your analysis, special attention should be paid to ensuring that this does not have an important impact on any of the conclusions drawn from the portfolio analysis.

When the set of entities used to calculate the breakpoints is the same as the set of entities that are grouped into portfolios, the number of entities in each of the portfolios should be approximately dictated by the percentiles used to calculate the breakpoints and the number of stocks in the sample during the given period t. We say "approximately" because, as mentioned earlier, if the value of X for a given entity i is exactly

TABLE 5.2 Number of Stocks per Portfolio

This table presents the number of stocks in each of the portfolios formed in each year during the sample period. The column labeled t indicates the year. The subsequent columns, labeled $n_{k,t}$ for $k \in \{1, 2, \ldots, 7\}$ present the number of stocks in the kth portfolio.

t	$n_{1,t}$	$n_{2,t}$	$n_{3,t}$	$n_{4,t}$	$n_{5,t}$	$n_{6,t}$	$n_{7,t}$
1988	569	569	1138	1138	1138	569	569
1989	552	552	1104	1103	1104	552	552
1990	541	541	1082	1081	1082	541	541
1991	531	530	1060	1061	1061	529	531
1992	539	539	1078	1077	1078	539	539
1993	567	567	1134	1134	1134	567	567
1994	615	615	1229	1230	1229	615	615
1995	629	629	1257	1258	1257	629	629
1996	659	658	1317	1318	1316	659	659
1997	687	687	1373	1373	1373	687	687
1998	661	661	1321	1322	1321	661	661
1999	610	610	1219	1219	1219	610	610
2000	591	590	1180	1180	1180	589	591
2001	551	551	1101	1102	1101	551	551
2002	510	510	1020	1019	1020	510	510
2003	474	474	947	947	947	474	474
2004	458	457	915	914	915	457	458
2005	450	449	899	899	898	450	450
2006	446	445	890	891	890	445	446
2007	434	433	866	866	866	433	434
2008	427	426	853	853	852	426	427
2009	398	398	795	795	795	398	398
2010	381	380	761	761	761	380	381
2011	369	368	736	736	736	368	369
2012	355	354	709	709	709	354	355

equal to one of the breakpoints, then because the portfolios are constructed to be inclusive of the breakpoints at both the low and high ends of X, the entity i will be held in more than one portfolio.

Table 5.2 presents the number of stocks in each of the portfolios for each year t in our example sample. As expected, as the 10th percentile is used to calculate the first breakpoint, approximately 10% of the stocks. Similarly, the second, sixth, and seventh portfolios each hold approximately 10% of the stocks in each cross section. Portfolios three, four, and five each hold approximately 20% of the stocks in the sample.

5.1.3 Average Portfolio Values

The third step in univariate portfolio analysis is to calculate the average value of the outcome variable Y for each of the n_P portfolios in each time period t. In many cases, instead of taking the simple average of the outcome variable values, it is desirable to weight the entities within each portfolio according to some other variable $W_{i,t}$. The most commonly used weight variable is market capitalization. In cases where market capitalization is used as the weight variable, the average is referred to as the value-weighted average.[1] When a simple average is desired, the values of the $W_{i,t}$ are set to one ($W_{i,t} = 1 \; \forall i, t$). In this case, we refer to the portfolios as equal-weighted portfolios. This is equivalent to giving a weight of $1/n_t$ to each entity. Thus, in its general form, the average value of the outcome variable for portfolio k in period t is defined as

$$\overline{Y}_{k,t} = \frac{\sum_{i \in P_{k,t}} W_{i,t} Y_{i,t}}{\sum_{i \in P_{k,t}} W_{i,t}} \tag{5.3}$$

for $k \in \{1, \ldots, n_P\}$. The summations in equation (5.3) are taken over all entities i in the kth portfolio for time period t ($P_{k,t}$).

A few details are worthy of discussion. Given that the grouping of entities into portfolios was performed without any consideration for whether a value of Y is available for each of the entities in the portfolio, it is possible, and in practice quite common, that there will be some entities i in any given portfolio $P_{k,t}$ for which the value of Y is not available. In this case, the set over which the average is taken should be the set of entities i in $P_{k,t}$ for which a value of Y is available. A similar consideration arises when the portfolio weights are not equal but are determined by some other variable W. In this case, the summation is taken over all entities i for which values of both W and Y are available.

In addition to calculating the average value of the outcome variable ($\overline{Y}$) for each portfolio, we also calculate the difference in average values between portfolio n_P and portfolio one. For each period t, we define the difference in the average outcome variable between the highest and lowest portfolios to be

$$\overline{Y}_{Diff,t} = \overline{Y}_{n_P,t} - \overline{Y}_{1,t}. \tag{5.4}$$

This value represents the difference in the average value of the outcome variable Y for entities with high values of the sort variable compared to those with low values of the sort variable. This difference in averages is the primary value used to detect a cross-sectional relation between the sort variable and the outcome variable,

[1] Value-weighting is most appropriate when the entities in the analysis are stocks. In such cases, the results of equal-weighted analyses are indicative of phenomena for the average stock. The results of value-weighted analyses account for the importance, from the point of view of the stock market as a whole, of each individual stock relative to the other stocks in the given portfolio. When the outcome variables (Y) is the future stock return, the results of value-weighted analyses are generally considered to be more indicative of return that an investor would have realized by implementing the portfolio in question. The reason for this is that value-weighted portfolios have large weights on stocks with large market capitalizations, which tend to be highly liquid. The returns of equal-weighted portfolios are potentially driven by the low-market capitalization stocks in the portfolio, which are more expensive to trade. The result is often that the average return indicated by the portfolio analysis cannot be realized by an actual investor because of transaction costs.

which is the main objective of portfolio analysis. We frequently refer to the difference between the average value of the n_Pth portfolio and the average value of the first portfolio as the average value of the difference portfolio.

Turning to our example, we use the one-year-ahead excess stock return (r_{t+1}) as our outcome variable. Because r_{t+1} represents the excess return of the stock in the year after the calculation of β (the sort variable), the average excess stock returns represent the excess returns that would have been realized by an investor who, at the end of year t, created the portfolios as described previously and held the portfolios without further trading for the entirety of year $t + 1$. To be specific, the timing of the portfolio formation is as follows. At the end of each year t, we calculate β for each stock and form seven different portfolios as described earlier. We then enter into positions as indicated by these portfolios. The prices paid for each of the stocks are the prices as of the close of the last trading day during year t. We hold the portfolios unchanged until the end of year $t + 1$, at which point all portfolios are liquidated at the closing prices on the last trading day of year $t + 1$. We repeat the procedure for each year t. We refer to the year t as the portfolio formation period and the year $t + 1$ as the portfolio holding period. Because r_{t+1} is the excess stock return, we assume that all positions are financed by borrowing at the risk-free rate.

Table 5.3 presents the average equal-weighted portfolio excess returns for each of the seven portfolios as well as for the difference between portfolio seven and portfolio one. To make the timing clear, in the table, we present both the portfolio formation year (column labeled t) and the portfolio holding year (column labeled $t + 1$). As can be seen from the table, the portfolio that holds stocks in the lowest decile of β (portfolio 1) as of the end of 1988 generated an excess return of −0.97% during 1989. Similarly, portfolios two through seven generated excess returns of 1.12%, 2.12%, 6.77%, 3.18%, 9.04%, and 8.96%. The difference in excess return between portfolios seven and one is 9.93% (8.96% − [−0.97%]). The corresponding values for portfolios formed at the end of (held during) year 1989 through 2011 (1990 through 2012) are also presented. As the return data for year 2013 are not available in the version of the Center for Research in Security Prices (CRSP) database used to construct the methodology sample, we cannot determine the 2013 excess returns of the portfolios that are formed at the end of year 2012.

We now repeat the analysis using value-weighted portfolios. Thus, the weights in each of the portfolios are determined by the market capitalization (*MktCap*) measured as of the end of the portfolio formation year t. Table 5.4 presents the average portfolio excess returns for the value-weighted portfolios. As can be seen from the results, the weighting scheme can have a substantial impact of the average portfolio returns. This will become much more apparent in the analyses presented throughout Part II.

5.1.4 Summarizing the Results

The main objective of portfolio analysis is to determine whether there is a cross-sectional relation between the sort variable X and the outcome variable Y. To do so, we begin by calculating the time-series means of the period average values of the outcome variable, $\overline{Y}_{k,t}$, for each of the n_P portfolios as well as for the difference

TABLE 5.3 Univariate Portfolio Equal-Weighted Excess Returns
This table presents the one-year-ahead excess returns of the equal-weighted portfolios formed by sorting on β. The column labeled t indicates the portfolio formation year. The column labeled $t + 1$ indicates the portfolio holding year. The columns labeled 1 through 7 show the excess returns of the seven β-sorted portfolios. The column labeled 7-1 presents the difference between the return of portfolio seven and that of portfolio one.

t	$t + 1$	1	2	3	4	5	6	7	7-1
1988	1989	−0.97	1.12	2.12	6.77	3.18	9.04	8.96	9.93
1989	1990	−30.21	−29.09	−28.72	−29.81	−27.85	−26.85	−25.75	4.45
1990	1991	56.81	28.99	36.22	40.42	54.51	64.44	66.49	9.68
1991	1992	55.37	30.93	29.45	21.99	19.16	15.95	20.12	−35.26
1992	1993	36.07	27.60	22.98	24.03	19.78	15.39	7.86	−28.21
1993	1994	−4.51	−4.47	−5.55	−4.16	−5.88	−10.42	−4.74	−0.23
1994	1995	28.38	21.81	24.95	29.62	26.82	23.90	37.46	9.08
1995	1996	21.04	16.14	18.46	14.16	12.32	11.73	9.07	−11.97
1996	1997	22.72	39.12	28.28	20.44	18.00	6.13	−7.58	−30.30
1997	1998	−6.68	−9.02	−7.01	−10.42	−6.52	−6.60	0.13	6.81
1998	1999	14.69	10.32	19.71	23.15	38.11	61.77	93.44	78.74
1999	2000	−11.18	−4.54	−0.96	−1.23	−3.50	−10.26	−31.33	−20.16
2000	2001	37.64	28.96	27.08	28.23	22.53	−1.75	−22.09	−59.73
2001	2002	11.60	11.08	−0.27	−8.09	−22.56	−36.19	−53.83	−65.43
2002	2003	76.69	68.50	85.65	64.78	63.70	76.56	86.90	10.22
2003	2004	27.56	21.07	25.82	21.34	15.75	12.89	−0.10	−27.65
2004	2005	6.06	5.52	2.97	3.40	3.11	−3.15	−10.24	−16.31
2005	2006	8.85	17.13	13.32	10.62	6.87	12.98	12.26	3.41
2006	2007	−13.52	−13.06	−6.97	−7.65	−6.57	−2.60	−4.19	9.33
2007	2008	−42.54	−42.30	−41.36	−39.18	−40.75	−45.34	−44.46	−1.92
2008	2009	57.06	65.73	64.76	59.46	55.19	60.47	73.60	16.55
2009	2010	20.31	20.19	23.14	22.98	33.39	30.27	36.77	16.46
2010	2011	−3.87	−5.50	−1.04	−4.24	−7.11	−14.11	−16.48	−12.61
2011	2012	27.95	27.11	16.22	20.33	16.10	17.83	18.05	−9.89

portfolio. We define these average values as

$$\overline{Y}_k = \frac{\sum_{t=1}^{T} \overline{Y}_{k,t}}{T} \tag{5.5}$$

and

$$\overline{Y}_{Diff} = \frac{\sum_{t=1}^{T} \overline{Y}_{Diff,t}}{T} \tag{5.6}$$

where $t = 1$ indicates the first period in the sample and T is the number of periods in the sample.

The time-series means serve as estimates of the true average values of the outcome variable for entities in each of the portfolios in the average time period. Similarly, the

TABLE 5.4 Univariate Portfolio Value-Weighted Excess Returns
This table presents the one-year-ahead excess returns of the value-weighted portfolios formed by sorting on β. The column labeled t indicates the portfolio formation year. The column labeled $t+1$ indicates the portfolio holding year. The columns labeled 1 through 7 show the excess returns of the seven β-sorted portfolios. The column labeled 7-1 presents the difference between the return of portfolio seven and that of portfolio one.

t	$t+1$	1	2	3	4	5	6	7	7-1
1988	1989	−2.98	11.14	9.04	16.29	20.25	29.16	18.21	21.19
1989	1990	−28.42	−33.78	−18.81	−19.12	−16.14	−11.51	−11.88	16.53
1990	1991	−0.45	18.50	12.28	17.79	22.08	31.14	51.51	51.96
1991	1992	−1.18	22.64	14.17	8.47	5.44	0.43	12.45	13.62
1992	1993	21.22	18.77	14.44	9.45	7.33	5.64	3.35	−17.87
1993	1994	−14.71	−7.26	−3.17	−4.07	−2.86	−9.35	1.52	16.23
1994	1995	17.44	21.81	25.41	32.54	31.73	28.45	29.02	11.57
1995	1996	5.38	17.64	18.34	14.51	13.71	18.02	29.65	24.27
1996	1997	3.37	34.35	26.16	21.17	27.71	27.54	22.22	18.85
1997	1998	−7.37	−6.09	−0.99	−0.63	12.21	16.51	31.82	39.20
1998	1999	−17.35	−17.55	−9.51	1.20	0.22	31.34	57.98	75.33
1999	2000	−23.95	−1.23	8.74	14.82	5.78	0.02	−26.46	−2.50
2000	2001	−11.02	3.53	−14.41	−9.48	−5.48	−2.79	−52.09	−41.08
2001	2002	−3.26	−14.06	−14.71	−13.39	−18.92	−31.44	−46.53	−43.27
2002	2003	61.15	41.66	22.03	23.18	23.98	28.01	55.03	−6.12
2003	2004	10.16	26.60	18.88	16.89	9.60	8.39	−4.06	−14.22
2004	2005	6.56	0.85	2.75	2.57	3.44	3.26	1.44	−5.12
2005	2006	4.83	15.65	13.43	9.53	7.75	11.26	11.11	6.28
2006	2007	−13.27	12.99	−1.88	−1.57	7.57	7.66	1.51	14.78
2007	2008	−36.55	−22.48	−24.84	−35.30	−40.15	−52.41	−60.52	−23.97
2008	2009	11.32	29.50	12.89	35.12	26.63	40.36	35.67	24.35
2009	2010	8.67	9.59	12.54	17.22	24.11	25.04	19.59	10.92
2010	2011	3.48	14.68	7.41	4.64	−8.02	−20.42	−20.61	−24.09
2011	2012	26.83	8.12	15.10	18.62	19.19	16.04	35.77	8.94

time-series mean of the difference portfolio estimates the difference, in the average time period, of the average value of the outcome variable for entities in the n_Pth portfolio compared to those in the first portfolio.

5.1.5 Interpreting the Results

In addition to calculating the time-series means for each of the portfolios, we frequently want to test whether the time-series mean for each of the portfolios differs from some null hypothesis mean value. That value is often zero. Most importantly, we want to examine whether the time-series mean of the difference portfolio is statistically distinguishable from zero. A statistically nonzero mean for the difference portfolio is evidence that, in the average time period, a cross-sectional relation exists between the sort variable and the outcome variable. To make such an assessment, for each of the n_P portfolios, as well as the difference portfolio, we calculate standard errors, t-statistics, and p-values for the test with null hypothesis that the time-series mean of the average portfolio outcome variable value is equal to zero. Because for each portfolio the portfolio average values ($\overline{Y}_{k,t}$) represent a time series, the standard errors are frequently adjusted following Newey and West (1987). The details of the Newey and West (1987) adjustment are discussed in Section 1.3. Most researchers use a 5% level of significance to determine whether a test rejects or fails to reject the null hypothesis.[2] Thus, t-statistics greater than 2.00 (approximately) in magnitude, or p-values less than 0.05, result in rejection of the null hypothesis that the time-series mean is equal to zero. In addition to examining whether the time-series mean for the difference portfolio is statistically distinguishable from zero, researchers frequently examine the average values of Y across the n_P portfolios ($\overline{Y}_k$, $k \in \{1, 2, \ldots, n_P\}$) for monotonicity. If a monotonic or near monotonic pattern arises, it is a strong indication that the results of the difference portfolio are not spurious.[3]

The results for our example are presented in Table 5.5. The row labeled Average shows the time-series average of the annual portfolio excess returns for portfolios 1 through 7 as well as for the difference portfolio (column labeled 7-1). The rows labeled Standard error, t-statistic, and p-value present the standard error of the estimated mean portfolio excess return, adjusted following Newey and West (1987) using six lags, and the corresponding t-statistics and p-values, respectively.

The results indicate that the average excess returns for portfolios 1 through 7 are 16.47%, 13.89%, 14.55%, 12.79%, 11.99%, 10.92%, and 10.43%, respectively. Each of these average returns is found to be highly statistically significant, as the corresponding t-statistics range from 3.43 for portfolio 7 to 6.73 for portfolio 4, and all p-values are very close to zero. This indicates that in the average year, each of these seven portfolios produces positive excess returns. This is not surprising because stocks are known to generate average returns that are higher than the return on the risk-free security. The average return of the difference portfolio, presented in the column labeled 7-1, is −6.04%. This difference is not statistically distinguishable from zero as the t-statistic is −1.31 and the p-value is 0.20. Thus, our portfolio analysis fails to detect a cross-sectional relation between β and one-year-ahead excess stock returns (r_{t+1}).

We do not examine the results for the value-weighted portfolio analysis here because the procedure for generating the results is identical to that for the equal-weighted portfolios. However, for many of the analyses in Part II of this text, both equal-weighted and value-weighted portfolio results will be investigated.

[2]Harvey, Liu, and Zhu (2015) argue that due to data mining and the large amount of research examining the cross section of expected returns, a 5% level of significance is too low a threshold and argue in favor of using much more stringent requirements for accepting empirical results as evident of true economic phenomena.
[3]Patton and Timmermann (2010) develop a statistical test of monotonicity.

TABLE 5.5 Univariate Portfolio Equal-Weighted Excess Returns Summary
This table presents the results of a univariate portfolio analysis of the relation between beta (β) and future stock returns (r_{t+1}). The row labeled Average presents the equal-weighted average annual return for each of the portfolios. The row labeled Standard error presents the standard error of the estimated mean portfolio return. Standard errors are adjusted following Newey and West (1987) using six lags. The row labeled t-statistic presents the t-statistic (in parentheses) for the test with null hypothesis that the average portfolio excess return is equal to zero. The row labeled p-value presents the two-sided p-value for the test with null hypothesis that the average portfolio excess return is equal to zero. The columns labeled 1 through 7 show the excess returns of the seven β-sorted portfolios. The column labeled 7-1 presents the results for the difference between the return of portfolio seven and that of portfolio one.

	1	2	3	4	5	6	7	7-1
Average	16.47	13.89	14.55	12.79	11.99	10.92	10.43	−6.04
Standard error	3.62	2.42	2.50	1.90	1.83	1.80	3.04	4.61
t-statistic	4.55	5.74	5.83	6.73	6.57	6.06	3.43	−1.31
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20

5.1.6 Presenting the Results

There are many different approaches to presenting the results of one or more portfolio analyses. Exactly which approach is chosen depends on the objective of the analysis. Here, we discuss some of the most common approaches to presenting portfolio analysis results.

Single Portfolio Analysis

We begin with a presentation of the results analyzing the relation between β and r_{t+1} discussed throughout this chapter. While the results of this analysis are well summarized by Table 5.5, several of the results in Table 5.5 are redundant, as the standard error, t-statistic, and p-value all contain essentially the same information. Thus, only one of these values, most commonly the t-statistic, is presented. Furthermore, t-statistics are frequently presented in parentheses to enhance the appearance of the presentation. Thus, the results of the portfolio analysis may be presented as in Table 5.6. Only the average excess return and the corresponding Newey and West (1987) adjusted (six lags) t-statistics are displayed.

Multiple Analyses, Same Sort Variable, Different Outcome Variables

Frequently, we want to examine the cross-sectional relation between the sort variable X and many different outcome variables Y. To do this, we repeat the univariate portfolio analysis for each outcome variable Y. Notice that the breakpoints step does not need to be repeated as the sort variable has not changed. We can then present the results of all of these portfolio analyses in one table. Often, it is not of particular interest to examine whether the average value of the outcome variable in any of the n_P portfolios is equal to zero. For example, we know that all stocks have a positive

TABLE 5.6 β-Sorted Portfolio Excess Returns

This table presents the results of a univariate portfolio analysis of the relation between beta (β) and future stock returns (r_{t+1}). The table shows that average excess return for each of the seven portfolios as well as for the long–short zero-cost portfolio, that is, long stocks in the seventh portfolio and short stocks in the first portfolio. Newey and West (1987) t-statistics, adjusted using six lags, testing the null hypothesis that the average portfolio excess return is equal to zero, are shown in parentheses.

1	2	3	4	5	6	7	7-1
16.47	13.89	14.55	12.79	11.99	10.92	10.43	−6.04
(4.55)	(5.74)	(5.83)	(6.73)	(6.57)	(6.06)	(3.43)	(−1.31)

market capitalization. Thus, testing whether the average market capitalization of a certain set of stocks is not of interest. We may only be interested in whether the average value of the difference portfolio is equal to zero, as nonzero differences indicate a cross-sectional relation between the sort variable X and the outcome variable Y. Therefore, sometimes the only t-statistic presented is that of the difference portfolio. The objective of such analyses is often to understand the complexion of each of the portfolios formed by sorting on the variable X.

To exemplify this, Table 5.7 presents the results of a portfolio analysis using the same β-sorted portfolios but taking each of β, *MktCap*, and *BM* to be the outcome variable. In this analysis, it is worth noting that the values of the outcome variables β, *MktCap*, and *BM* are measured contemporaneously with β. Thus, for each portfolio, we have the full 25 years of average values of these variables instead of the 24 years that we had when using future excess returns. Here, t-statistics for the difference portfolio are reported in a separate column at the end of the table instead of in parentheses under the average value.

Because the portfolios are formed by sorting on β, the average value of β is monotonically increasing across the seven portfolios and the time-series mean of the differences in average β between portfolios seven and one of 2.16 is highly statistically

TABLE 5.7 Univariate Portfolio Average Values of β, *MktCap*, and *BM*

This table presents the average values of β, *MktCap*, and *BM* for each of the β-sorted portfolios. The first column of the table indicates the variable for which the average value is being calculated. The columns labeled 1 through 7 present the time-series average of annual portfolio mean values of the given variable. The column labeled 7-1 presents the average difference between portfolios 7 and 1. The column labeled 7-1 t presents the t-statistic, adjusted following Newey and West (1987) using six lags, testing the null hypothesis that the average of the difference portfolio is equal to zero.

Outcome Variable	1	2	3	4	5	6	7	7-1	7-1 t-statistic
β	−0.22	0.14	0.41	0.71	1.03	1.37	1.94	2.16	21.08
MktCap	153	1065	2307	2572	2529	2519	2523	2369	3.98
BM	0.94	0.96	0.77	0.71	0.64	0.53	0.51	−0.42	−3.69

significant with a t-statistic of 21.08. The table indicates that stocks with low β tend to be small market capitalization stocks, as the average *MktCap* of stocks in the first portfolio is only \$153 million. The average market capitalization increases monotonically through portfolio four, and the average market capitalization of portfolios four through seven are very similar, each being a little higher than \$2.5 billion. The average difference in market capitalization between the stocks in portfolio seven and those in portfolio one of more than \$2.3 billion is highly statistically significant with a t-statistic of 3.98. Finally, the portfolios exhibit a nearly monotonically decreasing (the exception is portfolio one) pattern in average book-to-market ratio (*BM*). The average difference between portfolio seven and portfolio one of −0.42 is highly statistically significant (t-statistic = −3.69). In summary, the portfolio analysis detects a positive relation between β and *MktCap* and a negative relation between β and *BM*. It is worth noting that the direction of the relations uncovered by the portfolio analyses is consistent with the results of the correlation analysis presented in Table 3.2.

Multiple Analyses, Different Sort Variables, Same Outcome Variable

Sometimes, we want to present the results of portfolio analyses with different sort variables X but with the same outcome variable Y. This is often the case when we are examining the ability of many different variables to predict future stock returns.

Table 5.8 presents an example of how the results of such portfolio analyses can be presented. The table shows the average excess returns and the associated t-statistics for portfolios sorted on each of β, *MktCap*, and *BM*. The results for the portfolios formed by sorting on β are identical to those presented in Table 5.5. The results for the portfolios sorted on *MktCap* indicate a strong negative relation between market capitalization and future stock returns as the average return of the difference portfolio is −20.89% per year with a t-statistic of −4.80. The results also indicate a strong positive relation between book-to-market ratio (*BM*) and future stock returns as the average return of the difference portfolio is 17.59% per year (t-statistic = 8.28). In both the *MktCap* and *BM* cases, the average excess returns are nearly monotonic across the seven portfolios. These results, known as the size and value effects, respectively, will be discussed in detail in Chapters 9 and 10, respectively.

In general, there is no one correct way to present the results of portfolio analyses. The exact format of the presentation should be chosen to highlight the focal results of the analysis. The above examples are indicative of some of the most common presentation formats.

5.1.7 Analyzing Returns

When the entities in the sample are securities and the outcome variable Y measures the returns of the securities, the average values $\overline{Y}_{k,t}$ represent the returns of the portfolios that hold long positions in each of the portfolio's securities.[4] In such cases, it is usually desirable to perform some additional analyses that are intended to examine

[4]It is worth noting here that Asparouhova, Bessembinder, and Kalcheva (2013) find that deviations from fundamental values can produce biases in the estimates of expected returns generated by portfolio analyses.

TABLE 5.8 Average Returns of Portfolios Sorted on β, *MktCap*, and *BM*
This table presents the average excess returns of equal-weighted portfolios formed by sorting on each of β, *MktCap*, and *BM*. The first column of the table indicates the sort variable. The columns labeled 1 through 7 present the time-series average of annual one-year-ahead excess portfolio returns. The column labeled 7-1 presents the average difference in return between portfolios 7 and 1. t-statistics testing the null hypothesis that the average portfolio return is equal to zero, adjusted following Newey and West (1987) using six lags, are presented in parentheses.

Sort Variable	1	2	3	4	5	6	7	7-1
β	16.47	13.89	14.55	12.79	11.99	10.92	10.43	−6.04
	(4.55)	(5.74)	(5.83)	(6.73)	(6.57)	(6.06)	(3.43)	(−1.31)
MktCap	29.08	16.85	12.16	10.12	8.47	9.25	8.19	−20.89
	(7.00)	(5.48)	(4.55)	(5.23)	(5.97)	(5.93)	(4.32)	(−4.80)
BM	7.61	6.65	11.06	13.55	13.74	17.50	25.21	17.59
	(3.47)	(4.21)	(6.67)	(7.43)	(6.51)	(6.37)	(8.77)	(8.28)

whether patterns in the average portfolio returns are driven by cross-sectional variation in portfolio sensitivities to systematic risk factors. Stated alternatively, we want to examine whether after controlling for sensitivity of the portfolios to systematic risk factors, the patterns in the average portfolio returns persist.[5]

There are three very common models of risk-adjustment that are used throughout the finance literature. While the objective of this chapter is not to discuss these models in detail, we will provide a brief overview. The risk models will be discussed in detail in Part II. The first model, based on the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966) and known as the one-factor market model, is designed to adjust the portfolio returns for the effect of the overall stock market return. The specification of the one-factor market model is

$$r_{p,t} = \alpha + \beta_{MKT} MKT_t + \epsilon_t \tag{5.7}$$

where $r_{p,t}$ is the excess return of the portfolio and MKT_t is the excess return on the market factor mimicking portfolio during the period t.

The second risk model, originally proposed by Fama and French (1993) and known as the Fama and French (FF) three-factor model, uses two additional risk factors that proxy for the returns associated with the size (see Chapter 9) and value (see Chapter 10) effects.[6] The size effect refers to the fact that stocks with small market capitalizations have, on average and in the long run, outperformed stocks with large market capitalizations. The time series of returns associated with taking one unit of size factor risk are proxied by the returns of a zero-cost portfolio that is long small capitalization stocks and short large capitalization stocks. This zero-cost portfolio and its returns are referred to as *SMB* for "small minus big." The value effect refers to the

[5] Ang (2014) provides a comprehensive overview of factor investing.

[6] A recent paper by Fama and French (2015) proposes a five-factor model that includes the FFC factors along with factors based on profitability and investment. Another article, Hou, Xue, and Zhang (2015), proposes a similar model that excludes the value (*SMB*) factor.

fact that stocks with high book-to-market ratios (value stocks) have historically outperformed stocks with low book-to-market ratios (growth stocks). The time series of returns associated with taking one unit of value factor risk are proxied by the returns of a zero-cost portfolio, that is, long high book-to-market ratio stocks and short low book-to-market ratio stocks. The value portfolio and its returns are denoted *HML* for "high minus low." Therefore, the FF model is

$$r_{p,t} = \alpha + \beta_{MKT} MKT_t + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \epsilon_t. \tag{5.8}$$

where SMB_t and HML_t are the returns of the size and value factor mimicking portfolios, respectively, during time period t.

The third commonly used risk model augments the FF model with an additional factor that accounts for the momentum phenomenon documented by Jegadeesh and Titman (1993) and Carhart (1997). This model is known as the Fama, French, and Carhart (FFC) four-factor model. The momentum factor, denoted *MOM* for "momentum" (sometimes researchers refer to this factor using *UMD* for "up minus down"), represents the returns of a portfolio that is long stocks with the highest recent performance and short stocks with the lowest recent performance, where recent performance is defined as the return of the stock over the 11-month period beginning 12 months ago and ending one month ago. The FFC model can be written as

$$\begin{aligned} r_{p,t} = \alpha &+ \beta_{MKT} MKT_t + \beta_{SMB} SMB_t \\ &+ \beta_{HML} HML_t + \beta_{MOM} MOM_t + \epsilon_t. \end{aligned} \tag{5.9}$$

As each of the regressions used to risk-adjust the portfolio returns is a time-series regression, the Newey and West (1987) adjustment is usually applied. The result of the regression is a set of coefficients (intercept and slopes), as well as the corresponding standard errors, t-statistics, and p-values. The intercept coefficient (α) is interpreted as the average excess return of the portfolio that is not due to sensitivity to any of the factors included in the chosen factor model. This value is frequently referred to as the portfolio's alpha, Jensen (1968)'s alpha, or average abnormal return. To examine whether the portfolio generates statistically significant average abnormal returns, we use the t-statistic and/or p-value associated with the intercept coefficient.

Each slope coefficient is an estimate of the portfolio's sensitivity to the corresponding factor. The coefficients, as well as the associated inferential statistics, can be used to determine which factor or factors are related to the returns of the portfolio in question.

To exemplify the use of risk-adjustment in portfolio analysis, we adjust the returns of the β-sorted portfolios used previously in this chapter. Table 5.9 presents the estimated alphas (α) and factor sensitivities (β_{MKT}, β_{SMB}, β_{HML}, and β_{MOM}) for each of the seven portfolios as well as for the difference portfolio.

The section of Table 5.9 corresponding to the excess return (Model = Excess Return) replicates the results in Table 5.3. Notice that each of the seven portfolios generates large and highly statistically significant average excess returns, as the

TABLE 5.9 β-Sorted Portfolio Risk-Adjusted Results

This table presents the risk-adjusted alphas and factor sensitivities for the β-sorted portfolios. Each year t, all stocks in the sample are sorted into seven portfolios based on an ascending sort of β with breakpoints set to the 10th, 20th, 40th, 60th, 80th, and 90th percentiles of β in the given year. The equal-weighted average one-year-ahead excess portfolio returns are then calculated. The table presents the average excess returns (Model = Excess return) for each of the seven portfolios as well as for the zero-cost portfolio that is long the seventh portfolio and short the first portfolio. Also presented are the alphas (Coefficient = α) and factor sensitivities (Coefficient = β_{MKT}, β_{SMB}, β_{HML}, and β_{UMD}) for each of the portfolios using the CAPM (Model = CAPM), Fama and French (1993) three-factor model (Model = FF), and Fama and French (1993) and Carhart (1997) four-factor model (Model = FFC). t-statistics, adjusted following Newey and West (1987) using six lags, are presented in parentheses.

Model	Coefficient	1	2	3	4	5	6	7	7-1
Excess return	Excess return	16.47	13.89	14.55	12.79	11.99	10.92	10.43	−6.04
		(4.55)	(5.74)	(5.83)	(6.73)	(6.57)	(6.06)	(3.43)	(−1.31)
CAPM	α	8.89	6.78	6.75	5.32	3.58	0.45	−2.49	−11.38
		(1.41)	(1.32)	(1.33)	(1.31)	(1.13)	(0.19)	(−1.11)	(−1.72)
	β_{MKT}	1.02	0.96	1.05	1.01	1.13	1.41	1.74	0.72
		(3.30)	(3.05)	(3.80)	(3.75)	(5.20)	(7.63)	(8.70)	(1.89)
FF	α	3.44	1.57	2.20	1.44	0.86	−0.77	−1.32	−4.76
		(1.36)	(1.01)	(1.79)	(1.39)	(0.69)	(−0.53)	(−0.69)	(−1.82)
	β_{MKT}	1.12	1.08	1.13	1.06	1.12	1.31	1.50	0.38
		(7.55)	(6.11)	(7.64)	(7.72)	(12.01)	(11.55)	(11.20)	(1.83)
	β_{SMB}	1.53	1.17	1.35	1.25	1.37	1.47	1.64	0.12
		(8.98)	(6.99)	(7.55)	(10.29)	(29.78)	(11.01)	(9.86)	(0.41)
	β_{HML}	0.71	0.76	0.57	0.46	0.18	−0.16	−0.72	−1.43
		(7.36)	(13.40)	(6.39)	(5.19)	(1.66)	(−1.45)	(−6.25)	(−11.97)
FFC	α	5.54	6.21	6.49	4.54	2.37	−0.01	−0.12	−5.66
		(2.43)	(4.91)	(5.16)	(5.53)	(2.35)	(−0.01)	(−0.04)	(−1.14)
	β_{MKT}	1.06	0.95	1.00	0.97	1.07	1.29	1.47	0.41
		(8.59)	(9.30)	(18.20)	(13.77)	(16.98)	(15.76)	(11.21)	(1.62)
	β_{SMB}	1.42	0.92	1.12	1.08	1.29	1.43	1.58	0.16
		(6.57)	(3.94)	(5.47)	(6.29)	(13.85)	(8.25)	(7.61)	(0.69)
	β_{HML}	0.66	0.65	0.47	0.38	0.14	−0.18	−0.75	−1.41
		(5.27)	(9.15)	(7.96)	(7.38)	(1.53)	(−1.79)	(−6.96)	(−11.95)
	β_{MOM}	−0.16	−0.36	−0.33	−0.24	−0.12	−0.06	−0.09	0.07
		(−1.91)	(−3.38)	(−5.83)	(−2.90)	(−2.05)	(−0.51)	(−0.44)	(0.29)

t-statistics for each portfolio are positive and substantially greater than 2.00. This indicates that on average, each of the portfolios generates positive excess returns. This is not surprising because each of these portfolios is a portfolio of stocks. It is well known that, on average, stocks generate returns that are higher than the return of the risk-free security. Therefore, it is not surprising that portfolios comprised a large number of stocks exhibit similar behavior. The average return of the difference portfolio of −6.04% per year is statistically indistinguishable from zero.

When the returns are adjusted for exposure to the market factor using the CAPM risk model (Model = CAPM), the results indicate that none of the seven portfolios generates abnormal returns that are statistically distinguishable from zero as all t-statistics are substantially less than 2.00 in magnitude. This indicates that the excess returns generated by the portfolios are a manifestation of the portfolios' exposures to the market factor. After controlling for this, the average abnormal return of each of the portfolios is statistically insignificant. The table also presents the sensitivities of each of the portfolios to the market factor. The results indicate that all portfolios have a positive and statistically significant sensitivity to the market portfolio. Furthermore, the sensitivities are nearly monotonically increasing from portfolios 1 to 7. This is not surprising given that the portfolios were formed by sorting on β, which measures stock-level sensitivity to the market portfolio. The abnormal return of the difference portfolio remains statistically insignificant when using the CAPM risk model, and the sensitivity of the difference portfolio to the market portfolio of 0.72 is marginally statistically significant.

The remainder of Table 5.9 presents the results for the FF and FFC models. While some of these results are interesting, the objective here is to discuss the implementation and interpretation of portfolio analysis, not to examine the economic implications of these results. This will be done in Part II of this text.

In almost all cases, only a subset of the results in Table 5.9 are presented in a research article. Unless the factor sensitivities are of particular interest, they are usually not reported. Furthermore, frequently only results from one of the risk models are shown. The FFC model is the most common choice.

At this point, the reader may be wondering why, throughout this chapter, we have used the excess stock return, which is equal to the return on the stock minus the return on the risk-free security, instead of simply the stock return itself (without subtracting the risk-free security return). The reason for this is that the excess return represents the additional return that was realized by forgoing the certain return associated with the risk-free security in favor of a risky return. Asset pricing theory dictates that to be willing to take the risk associated with a given security, investors demand that, on average, the return of that security is greater than the risk-free rate. A major objective of empirical asset pricing research is to understand exactly which risks investors care about and how much of an average return, in excess of the return on the risk-free security, investors require to entice them to take such risk. It is for this reason that the CAPM, FF, and FFC (as well as all other) risk factor models are based on excess returns and not raw returns. Thus, when analyzing the time series of the returns of the n_P portfolios, it is important to make sure that the excess portfolio returns, not the raw portfolio returns, are used as the dependent variable in the factor regression. For this reason, it is advisable to use the excess stock return, not the raw stock return, as the outcome variable Y. Regardless of whether the excess return or the raw return is used as the outcome variable Y, the difference between the (excess or raw) return of the n_Pth portfolio and the first portfolio should be interpreted as an excess return. In fact, this difference will be the same regardless of whether the excess or raw return is used. Despite the fact that this difference in returns is interpreted as an excess return, it is commonly referred to as the return of the difference portfolio, the difference portfolio return, or the high minus low portfolio return. We frequently adopt this terminology throughout this book.

5.2 BIVARIATE INDEPENDENT-SORT ANALYSIS

The previous section presented the simplest form of portfolio analysis, designed to assess the cross-sectional relation between two variables without accounting for the effects of any other variables. In this section, we present bivariate portfolio analysis. Bivariate portfolio analysis is very similar to univariate portfolio analysis, except in bivariate portfolio analysis there are two sort variables.

There are two types of sorting procedures, independent and dependent, that are commonly employed in bivariate portfolio analysis. In the present section, we discuss independent-sort analysis. In Section 5.3, we discuss dependent-sort analysis.

Bivariate independent-sort portfolio analysis is designed to assess the cross-sectional relations between two sort variables, which we refer to as $X1$ and $X2$, and an outcome variable Y.

5.2.1 Breakpoints

As the name implies, in bivariate independent-sort portfolio analysis, portfolios are formed by sorting on two variables independently. Thus, in each period, two sets of breakpoints will be calculated. The first set of breakpoints corresponds to values of the first sort variable $X1$. The second set of breakpoints corresponds to values of the second sort variable $X2$ and is calculated completely independently of the breakpoints for $X1$. Thus, the name *independent* sort. The fact that the sorts are independent means it makes no difference which sort variable is considered the first sort variable, and which is considered the second. Switching the order will have no effect on the results of the analysis.

The first step in a bivariate independent-sort portfolio analysis is to sort all entities in the sample into groups according to each of the sort variables. We use the term "groups" here to differentiate the groups that are formed by independent univariate sorts of the entities from the eventual portfolios that are formed. The portfolios will represent intersections of groups from sorts based on the first and second sort variables. Letting n_{P1} represent the number of groups that will be created based on the first sort variable and n_{P2} be the number of groups that will be created based on the second sort variable, the number of portfolios that will be formed is $n_{P1} \times n_{P2}$. There are therefore n_{P1} and n_{P2} breakpoints for the first and second sort variables, respectively.

The breakpoints for each of the two sort variables are calculated in exactly the same way as for a univariate portfolio analysis. The breakpoints (percentiles used to calculate the breakpoints) used to form the groups for the first sort variable are denoted $B1_{j,t}$ $(p1_j)$ for $j \in \{1, 2, \ldots, n_{P1} - 1\}$, and the breakpoints (percentiles) for the second sort variable are $B2_{k,t}$ $(p2_k)$ for $k \in \{1, 2, \ldots, n_{P2} - 1\}$. The actual breakpoints are calculated as

$$B1_{j,t} = Pctl_{p1_j}(\{X1_t\}) \tag{5.10}$$

and

$$B2_{k,t} = Pctl_{p2_k}(\{X2_t\}) \tag{5.11}$$

where $Pctl_p(Z)$ is the pth percentile of the set Z and $\{X1_t\}$ and $\{X2_t\}$ are the set of available values of $X1$ and $X2$, respectively, in period t. It should be noted that frequently the set of entities in the sample for which values of $X1$ are available may differ from those for which $X2$ is available. In this case, the researcher must decide whether the breakpoints are formed using only entities for which valid values of both variables are available or whether the breakpoints are formed using all available data for each variable. In some cases, the set of entities used to calculate the breakpoints for $X1$ may be different than the set of entities used to calculate the $X2$ breakpoints. Furthermore, neither of these sets of entities is necessarily the same as the set of entities that will eventually be grouped into the portfolios. As in the univariate analysis, breakpoints are calculated for each time period t. Throughout this text, in our bivariate portfolio analyses, we use only entities for which valid values of both $X1$ and $X2$ are available when calculating breakpoints for bivariate (both independent-sort and dependent-sort) portfolio analyses.

We exemplify the calculation of breakpoints for the bivariate independent-sort portfolio analysis using beta ($X1 = \beta$) and market capitalization ($X2 = MktCap$) as our sort variables. We divide the sample into three groups based on β ($n_{P1} = 3$) and four groups based on *MktCap* ($n_{P2} = 4$). We use the 30th and 70th percentiles to calculate the β breakpoints, and the 25th, 50th, and 75th percentiles to calculate the *MktCap* breakpoints.

The annual breakpoints for this analysis are shown in Table 5.10. The table shows that, in year 1988, the first β breakpoint is 0.18 and the second β breakpoint is 0.66. The first, second, and third *MktCap* breakpoints are 9.65, 34.83, and 159.85, respectively. The breakpoints for other years are presented in the subsequent rows of the table.

As discussed in Section 5.1.1, the decision of how many groups to form based on each of the sort variables, and therefore how many total portfolios to use, is based on trade-offs between the number of stocks in each portfolio and dispersion among the portfolios of the sort variables. In bivariate independent-sort portfolio analysis, there is one additional criterion that may factor into the decision of how many breakpoints to use, as well as what percentiles of each sort variable to use as breakpoints. If the sort variables are highly positively correlated, then this may result in a large number of entities being put into the portfolio that holds entities with high values of both sort variables as well as the portfolio holding entities with low values of both sort variables. Portfolios that hold entities with low values of one sort variable and high values of the other will contain relatively fewer entities. The situation is reversed when the sort variables are negatively correlated. The more extreme the correlation between the two sort variables, the more exacerbated this effect will be. The number of groups to form based on each sort variable, therefore, should take this correlation into account and ensure that for each time period during the sample, each of the $n_{P1} \times n_{P2}$ portfolios contains a sufficient number of entities. This will become more clear shortly when we discuss portfolio formation. Apart

TABLE 5.10 Bivariate Independent-Sort Breakpoints

This table presents the breakpoints for a bivariate independent-sort portfolio analysis The first sort variable is β and the second sort variable is *MktCap*. The sample is split into three groups (and thus two breakpoints) based on the 30th and 70th percentiles of β, and four groups (and thus three breakpoints) based on the 25th, 50th, and 75th percentiles of *MktCap*. The column labeled t indicates the year for which the breakpoints are calculated. The columns labeled $B1_{1,t}$ and $B1_{2,t}$ present the first and second β breakpoints, respectively. The columns labeled $B2_{1,t}$, $B2_{2,t}$, and $B3_{3,t}$ present the first, second, and third *MktCap* breakpoints, respectively.

t	$B1_{1,t}$	$B1_{2,t}$	$B2_{1,t}$	$B2_{2,t}$	$B2_{3,t}$
1988	0.18	0.66	9.65	34.83	159.85
1989	0.17	0.70	9.77	37.04	184.14
1990	0.23	0.86	6.53	25.90	149.54
1991	0.24	0.85	9.70	41.56	223.70
1992	0.26	0.97	16.21	62.88	284.60
1993	0.29	0.92	22.48	78.32	345.67
1994	0.36	0.97	20.72	72.17	304.26
1995	0.27	0.90	26.03	91.38	382.85
1996	0.32	0.90	28.54	102.21	438.55
1997	0.26	0.72	32.62	119.62	521.35
1998	0.41	0.94	28.78	106.66	509.71
1999	0.15	0.53	33.79	128.21	623.06
2000	0.25	0.85	24.36	102.25	610.68
2001	0.31	0.95	34.09	142.62	717.07
2002	0.33	0.89	33.28	130.81	635.43
2003	0.38	0.97	70.06	270.53	1054.81
2004	0.63	1.36	90.64	334.90	1308.59
2005	0.60	1.30	96.99	349.11	1410.58
2006	0.61	1.40	108.61	406.64	1592.96
2007	0.56	1.18	92.95	352.91	1513.62
2008	0.57	1.14	38.11	197.13	879.62
2009	0.65	1.45	68.54	307.83	1352.40
2010	0.76	1.33	95.47	420.44	1850.99
2011	0.82	1.38	80.78	393.16	1771.85
2012	0.76	1.31	105.41	485.94	2034.15

from the additional consideration relating to correlation among the sort variables, the factors impacting the decision of how many groups to use for each variable (and thus the number of portfolios), and the choice of breakpoint percentiles, are similar in bivariate independent-sort portfolio analysis to those discussed in the univariate analysis (Section 5.1.1). We do not repeat the discussion here.

5.2.2 Portfolio Formation

As with the univariate portfolio analysis, the next step in bivariate portfolio analysis is to form the periodic portfolios. As mentioned earlier, if there are n_{P1} groups based on the first sort variable $X1$ and n_{P2} groups based on the second sort variable $X2$, then there will be $n_{P1} \times n_{P2}$ portfolios each time period. The portfolios for period t are denoted $P_{j,k,t}$, where the first subscript indicates the group of the first sort variable and the second subscript indicates that of the second sort variable. In general, the portfolios are defined as

$$P_{j,k,t} = \{i | B1_{j-1,t} \leq X1_{i,t} \leq B1_{j,t}\} \cap \{i | B2_{k-1,t} \leq X2_{i,t} \leq B2_{k,t}\} \tag{5.12}$$

for $j \in \{1, 2, \ldots, n_{P1}\}$, $k \in \{1, 2, \ldots, n_{P2}\}$, where $B1_{0,t} = B2_{0,t} = -\infty$, $B1_{n_{P1},t} = B2_{n_{P2},t} = \infty$, and $\cap$ is the intersection operator. Thus, for a given entity i to be held in portfolio $P_{j,k,t}$, the entity must have a value of $X1$ in period t that is between the $j-1$st and jth (inclusive) period t breakpoints for the first sort variable, and also have a period t value of $X2$ between the $k-1$st and kth (inclusive) period t breakpoints for the second sort variable.

In a bivariate independent-sort portfolio analysis, the percentage of entities held by each of the portfolios will likely not reflect the percentiles used to calculate the breakpoints. The reason is that the sort variables are likely to have nonzero correlation. As each portfolio represents the intersection of sets formed based on the independent sorts, positive correlation between $X1$ and $X2$ results in portfolios that contain entities with high (or low) values of both sort variables having a disproportionately large number of entities, while those portfolios comprised entities with low values of one sort variable and high values of the other contains fewer entities. The opposite is the case when the sort variables are negatively correlated. As discussed in Section 5.1.2, when the breakpoint sample and the full sample are not the same, in addition to the correlation effect, the number of entities in each portfolio will also depend on how the sort variables are distributed in the different samples. However, when the breakpoint sample and the sample grouped into portfolios are the same, the total number of entities in all portfolios (across all n_{P2} groups of $X2$) that correspond to the jth group of $X1$ will reflect the percentiles used to calculate the breakpoints based on the first sort variable $X1$. The same can be said for the total number of entities in the set of portfolios corresponding to a particular group of the second sort variable $X2$.

The number of stocks in each annual portfolio for our example is presented in Table 5.11. At this point, it is worth reminding ourselves that the correlation analysis presented in Table 3.3 as well as the portfolio analysis presented in Table 5.7 indicate a positive cross-sectional relation between β and *MktCap*. This manifests in portfolios that hold entities with high values of β and high values of *MktCap* having a large number of stocks. Similarly, portfolios comprised entities with low values of both sort variables have a large number of stocks. On the other hand, portfolios that hold entities with low values of one of the sort variables and high values of the other contain relatively few stocks. For example, in 1988, the table shows that the portfolio holding low β (β 1) and low *MktCap* (*MktCap* 1) stocks has 736 stocks and the portfolio holding high β (β 3) and high *MktCap* (*MktCap* 4) stocks has 788 stocks. On the other hand, the portfolio comprised high β (β 3) and low *MktCap* (*MktCap* 1) stocks has only 217 such stocks, and the portfolio containing low β (β 1) and high

TABLE 5.11 Bivariate Independent-Sort Number of Stocks per Portfolio

This table presents the number of stocks in each of the 12 portfolios formed by sorting independently into three β groups and four *MktCap* groups. The columns labeled t indicate the year of portfolio formation. The columns labeled β 1, β 2, and β 3 indicate the β group. The rows labeled *MktCap* 1, *MktCap* 2, *MktCap* 3, and *MktCap* 4 indicate the *MktCap* groups.

t		β 1	β 2	β 3	t		β 1	β 2	β 3
1988	*MktCap* 1	736	468	217	1998	*MktCap* 1	842	557	252
	MktCap 2	539	585	297		*MktCap* 2	622	667	361
	MktCap 3	335	683	403		*MktCap* 3	368	709	574
	MktCap 4	95	538	788		*MktCap* 4	149	708	794
1989	*MktCap* 1	736	419	224	1999	*MktCap* 1	785	484	253
	MktCap 2	537	574	268		*MktCap* 2	635	608	279
	MktCap 3	298	663	418		*MktCap* 3	338	729	455
	MktCap 4	84	549	746		*MktCap* 4	69	613	840
1990	*MktCap* 1	725	439	187	2000	*MktCap* 1	609	522	342
	MktCap 2	507	574	269		*MktCap* 2	663	455	354
	MktCap 3	287	604	459		*MktCap* 3	290	640	542
	MktCap 4	102	543	706		*MktCap* 4	205	738	530
1991	*MktCap* 1	763	393	169	2001	*MktCap* 1	686	465	225
	MktCap 2	508	570	247		*MktCap* 2	612	459	305
	MktCap 3	254	606	465		*MktCap* 3	190	647	539
	MktCap 4	65	551	709		*MktCap* 4	163	631	582
1992	*MktCap* 1	681	428	237	2002	*MktCap* 1	759	387	126
	MktCap 2	507	530	309		*MktCap* 2	649	397	225
	MktCap 3	306	562	478		*MktCap* 3	93	641	537
	MktCap 4	121	634	591		*MktCap* 4	25	609	638
1993	*MktCap* 1	738	409	270	2003	*MktCap* 1	835	275	73
	MktCap 2	531	579	307		*MktCap* 2	488	454	241
	MktCap 3	330	611	475		*MktCap* 3	51	587	545
	MktCap 4	102	667	648		*MktCap* 4	46	577	560
1994	*MktCap* 1	767	437	333	2004	*MktCap* 1	772	265	106
	MktCap 2	596	593	348		*MktCap* 2	405	390	348
	MktCap 3	352	684	501		*MktCap* 3	55	485	603
	MktCap 4	130	744	663		*MktCap* 4	140	688	315
1995	*MktCap* 1	784	430	358	2005	*MktCap* 1	834	231	58
	MktCap 2	584	595	392		*MktCap* 2	400	434	289
	MktCap 3	373	698	500		*MktCap* 3	33	429	662
	MktCap 4	145	791	636		*MktCap* 4	81	704	339
1996	*MktCap* 1	792	518	336	2006	*MktCap* 1	816	251	46
	MktCap 2	623	581	441		*MktCap* 2	357	427	329
	MktCap 3	421	676	548		*MktCap* 3	19	467	626
	MktCap 4	139	857	650		*MktCap* 4	144	635	334
1997	*MktCap* 1	865	571	280	2007	*MktCap* 1	808	236	39
	MktCap 2	688	654	374		*MktCap* 2	390	351	341
	MktCap 3	415	779	520		*MktCap* 3	28	451	604
	MktCap 4	91	740	885		*MktCap* 4	74	693	316

TABLE 5.11 (*Continued*)

t		β 1	β 2	β 3	t		β 1	β 2	β 3
2008	*MktCap* 1	742	247	77	2011	*MktCap* 1	721	152	48
	MktCap 2	429	289	347		*MktCap* 2	155	337	428
	MktCap 3	43	548	474		*MktCap* 3	49	460	411
	MktCap 4	65	620	381		*MktCap* 4	180	523	218
2009	*MktCap* 1	690	219	85	2012	*MktCap* 1	662	151	74
	MktCap 2	264	322	408		*MktCap* 2	135	396	355
	MktCap 3	75	509	409		*MktCap* 3	85	444	356
	MktCap 4	164	539	291		*MktCap* 4	182	426	279
2010	*MktCap* 1	715	184	52					
	MktCap 2	158	398	395					
	MktCap 3	69	472	410					
	MktCap 4	199	468	284					

MktCap (*MktCap* 3) stocks contains only 95 stocks. Similar patterns are observed in most years, although the patterns are not always as perfect as they are in 1988.

5.2.3 Average Portfolio Values

Having created the portfolios, the next step is to calculate, for each time period t, the average value of the outcome variable Y for each of the $n_{P1} \times n_{P2}$ portfolios. As was discussed in Section 5.1.3, the average values can be either equal-weighted or weighted according to some weight field W, which is quite often market capitalization (value-weighted). Thus, the average value of the outcome variable for portfolio $P_{j,k,t}$ is

$$\overline{Y}_{j,k,t} = \frac{\sum_{i \in P_{j,k,t}} W_{i,t} Y_{i,t}}{\sum_{i \in P_{j,k,t}} W_{i,t}} \tag{5.13}$$

for $j \in \{1, 2, \ldots, n_{P1}\}$ and $k \in \{1, 2, \ldots, n_{P2}\}$, where the summations in both the numerator and denominator are taken over all entities in portfolio $P_{j,k,t}$. If no weighting field is used, then $W_{i,t} = 1$ for all i and t. This is exactly the same as in the univariate case.

In addition to calculating the average values of Y for each of the portfolios, for each of the n_{P1} groups of the first sort variable $X1$, we calculate the difference in average Y value of the portfolio that holds the entities with the highest and lowest values of the second sort variable $X2$. Thus, for each time period t, we have

$$\overline{Y}_{j,\,Diff,\,t} = \overline{Y}_{j,n_{P2},t} - \overline{Y}_{j,1,t} \tag{5.14}$$

for $j \in \{1, \ldots, n_{P1}\}$. Similarly, for each group of the second sort variable $X2$, we calculate the difference in average Y value between the portfolio with the highest and lowest values of the first sort variable $X1$, giving

$$\overline{Y}_{Diff,k,t} = \overline{Y}_{n_{P1},k,t} - \overline{Y}_{1,k,t} \tag{5.15}$$

for $k \in \{1, \ldots, n_{P2}\}$.

In addition to calculating differences, in bivariate independent-sort portfolio analysis, we can also calculate the average value of the outcome variables, across all groups of one of the sort variables and within a given group of the other sort variable. Thus, the average value of $\overline{Y}$ across all groups of sort variable $X1$ and within the kth group of sort variable $X2$ is defined as

$$\overline{Y}_{Avg,k,t} = \frac{\sum_{j=1}^{n_{P1}} \overline{Y}_{j,k,t}}{n_{P1}} \tag{5.16}$$

This calculation can be performed not only for $k \in \{1, 2, \ldots, n_{P2}\}$ but also for the difference between the high and low sort variable two portfolios ($k = Diff$). Similarly, we calculate the average value of $\overline{Y}$ across all groups of the second sort variable $X2$ and within the jth group of the first sort variable $X1$, giving

$$\overline{Y}_{j,Avg,t} = \frac{\sum_{k=1}^{n_{P2}} \overline{Y}_{j,k,t}}{n_{P2}}. \tag{5.17}$$

for $j \in \{1, 2, \ldots, n_{P1}, Diff\}$.

There are two more values that may be calculated each month. The first is the average of the averages. There are many ways that this can be calculated. The first is to take the average across the n_{P1} averages for groups formed on the first sort variable ($\overline{Y}_{j,Avg,t}$, $j \in \{1, 2, \ldots, n_{P1}\}$). The second is to take the average across the n_{P2} averages for groups formed on the second sort variable ($\overline{Y}_{Avg,k,t}$, $k \in \{1, 2, \ldots, n_{P2}\}$). The third is to take the average of all $n_{P1} \times n_{P2}$ portfolio average values ($\overline{Y}_{j,k,t}$, $j \in \{1, 2, \ldots, n_{P1}\}$ and $k \in \{1, 2, \ldots, n_{P2}\}$). Each of these approaches yields the same result. We therefore have

$$\overline{Y}_{Avg,Avg,t} = \frac{\sum_{j=1}^{n_{P1}} \overline{Y}_{j,Avg,t}}{n_{P1}} = \frac{\sum_{k=1}^{n_{P2}} \overline{Y}_{Avg,k,t}}{n_{P2}} = \frac{\sum_{j=1}^{n_{P1}} \sum_{k=1}^{n_{P2}} \overline{Y}_{j,k,t}}{n_{P1} \times n_{P2}}. \tag{5.18}$$

Finally, we come to the difference in the differences. The difference in differences can be used to examine how the average value of Y relates to $X1$ for high values of $X2$ compared to how the average value of Y relates to $X1$ for low values of $X2$. Alternatively, the difference in differences portfolio can be interpreted as indicating how the average value of Y relates to $X2$ for high values of $X1$ compared to how the average values of Y relates to $X2$ for low values of $X1$. This is most easily seen graphically.

In Table 5.12, we illustrate the calculation of the difference in the differences between the average values of Y. To simplify notation, we let $A = Y_{1,1,t}$, $B = Y_{n_{P1},1,t}$,

TABLE 5.12 Average Value for the Difference in Difference Portfolio
This diagram describes how the difference in difference portfolio for a bivariate-sort portfolio analysis is constructed.

	$X1\ 1$	$\cdots$	$X1\ n_{P1}$	$X1\ Diff$
$X2\ 1$	$\overline{Y}_{1,1,t}$ A	$\cdots$	$\overline{Y}_{n_{P1},1,t}$ B	$\overline{Y}_{Diff,1,t}$ $B-A$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$X2\ n_{P2}$	$\overline{Y}_{1,n_{P2},t}$ C	$\cdots$	$\overline{Y}_{n_{P1},n_{P2},t}$ D	$\overline{Y}_{Diff,n_{P2},t}$ $D-C$
$X2\ Diff$	$\overline{Y}_{1,Diff,t}$ $C-A$	$\cdots$	$\overline{Y}_{n_{P1},Diff,t}$ $D-B$	$(D-C)-(B-A)$ $=$ $(D-B)-(C-A)$ $=$ $D-C-B+A$

$C = \overline{Y}_{1,n_{P2},t}$, and $D = \overline{Y}_{n_{P1},n_{P2},t}$. These values correspond to the interior values (within the square) of Table 5.12. The difference in average Y values between entities with the highest values of $X1$ ($X1\ n_{P1}$) and the lowest values of $X1$ ($X1\ 1$), in the group that corresponds to the lowest values of $X2$ ($X2\ 1$), is therefore $B - A = \overline{Y}_{Diff,1,t}$. Similarly, the corresponding value for the group with the highest values of $X2$ is $D - C = \overline{Y}_{Diff,n_{P2},t}$. These values are indicated in the right-center portion of Table 5.12. Going in the other direction, the difference in average Y between entities with high $X2$ and low $X2$ values among entities with low $X1$ values is $C - A = \overline{Y}_{1,Diff,t}$, and the corresponding value for entities with high values of $X1$ is $D - B = \overline{Y}_{n_{P1},Diff,t}$. These differences are shown in the center-bottom part of the table. Thus, if we take the difference along the $X2$ dimension in $X1$ differences, we get $\overline{Y}_{Diff,n_{P2},t} - \overline{Y}_{Diff,1,t} = (D - C) - (B - A) = D - C - B + A$. If we take the difference along the $X1$ dimension in $X2$ differences, we get the same result. $\overline{Y}_{n_{P1},Diff,t} - \overline{Y}_{1,Diff,t} = (D - B) - (C - A) = D - C - B + A$.

Putting all of this together, we get

$$\begin{aligned}\overline{Y}_{Diff,Diff,t} &= \overline{Y}_{n_{P1},n_{P2},t} - \overline{Y}_{n_{P1},1,t} - \overline{Y}_{1,n_{P2},t} + \overline{Y}_{1,1,t} \\ &= \overline{Y}_{Diff,n_{P2},t} - \overline{Y}_{Diff,1,t} \\ &= \overline{Y}_{n_{P1},Diff,t} - \overline{Y}_{1,Diff,t}.\end{aligned} \tag{5.19}$$

To help understand this difference in differences portfolio, consider the case when $\overline{Y}_{j,k,t}$ represents the return of the j,kth portfolio in time period t. Then, the difference in difference portfolio can be thought of as the return associated with going long portfolios D and A in equal dollar amounts and short portfolios C and D in the same dollar amounts. In a more general sense, this value indicates how the relation between $X1$ and Y changes across different levels of $X2$. Similarly, it indicates how the relation between $X2$ and Y changes across different levels of $X1$.

We exemplify calculation of the mean dependent variable using the one-year-ahead stock excess return (r_{t+1}) as the outcome variable. We use equal-weighted portfolios in this analysis, thus $W_{i,t}$ is one for all i and t. Table 5.13 presents the average one-year-ahead stock excess returns for each of the $n_{P1} \times n_{P2}$ portfolios, as well as for the difference and average portfolios. The table indicates the portfolio formation year (t) as well as the portfolio holding year ($t + 1$). Taking the year $t = 1988$ and $t + 1 = 1989$ as an example, the table shows that the portfolio comprised low-β (β 1) and low-*MktCap* (*MktCap* 1) stocks generated an excess return of 1.13%. The portfolios comprised high-β and low-*MktCap* stocks generated an excess return of 1.69%. The difference between the high-β and low-β portfolio returns for low-*MktCap* stocks was therefore 0.56% (1.69% − 1.13%). The average of the low-*MktCap* portfolios generated a return equal to 0.62%. This process can be repeated for all four *MktCap* groups and performed analogously for the three β groups. For the average market capitalization group (*MktCap* Avg), the difference in excess return between the high-β and low-β (β Diff) portfolio is 0.05% ((0.56% + (−8.47%) + 0.97% + 7.15%)/4). For the average β group (β Avg), the difference in excess return between the high-*MktCap* and low-*MktCap* (*MktCap* Diff) portfolio is 13.20% ((8.80%+15.40%+15.39%)/3). Moving to the difference of the differences portfolio (β Diff, *MktCap* Diff), the excess return of this portfolio is 6.59% (17.08% − 1.69% − 9.92% + 1.13% = 7.15% − 0.56% = 15.39% − 8.80%). Finally, the average of the average (β Avg, *MktCap* Avg) portfolio generates an excess return of 3.57%.

5.2.4 Summarizing the Results

Having calculated, for each period t, the average values of Y for each of the portfolios as well as for the difference and average portfolios, the final calculation required to complete the bivariate dependent-sort portfolio analysis is the time-series means of the periodic average values along with the corresponding standard errors, t-statistics, and p-values for each of the portfolios. This is done in exactly the same manner as for the univariate portfolio analysis. Usually, the standard errors, and therefore t-statistics and p-values, are Newey and West (1987) adjusted.

Table 5.14 presents the time-series averages of the annual portfolio excess returns for the β- and *MktCap*-sorted portfolios used in our example. As the periodic values represent portfolio excess returns, we adjust the excess returns for risk using the FF and FFC models. We choose to use only one risk model here to save space. We also present only the average excess returns, alphas, and corresponding Newey and West (1987) adjusted (six lags) t-statistics. We omit the standard errors, p-values, as well as all of the sensitivity coefficients. The table shows that the average annual excess return for the portfolio that holds low-*MktCap* (*MktCap* 1) and low-β (β 1) stocks is 20.96%, with a t-statistic of 6.12. The FFC alpha of this portfolio is 11.96% per year (t-statistic = 5.05). Within the low-β group (β 1), the difference in average excess return between the high-*MktCap* and low-*MktCap* portfolios is −14.51% per year (t-statistic = −4.81) and the corresponding FFC alpha is −12.87% per year (t-statistic = −3.83). The average excess return of the difference of the differences portfolio (β Diff, *MktCap* Diff) is 1.95% per year (t-statistic = 0.60) and the alpha of this portfolio is 1.56% per year (t-statistic = 0.43).

TABLE 5.13 Bivariate Independent-Sort Portfolio Excess Returns

This table presents the equal-weighted excess returns for each of the 12 portfolios formed by sorting independently into three β groups and four *MktCap* groups, as well as for the difference and average portfolios. The columns labeled $t/t+1$ indicate the year of portfolio formation (t) and the portfolio holding period ($t+1$). The columns labeled β 1, β 2, β 3, β Diff, and β Avg indicate the β groups. The rows labeled *MktCap* 1, *MktCap* 2, *MktCap* 3, *MktCap* 4, *MktCap* Diff, and *MktCap* Avg indicate the *MktCap* groups.

$t/t+1$		β 1	β 2	β 3	β Diff	β Avg
1988/1989	*MktCap* 1	1.13	−0.96	1.69	0.56	0.62
	MktCap 2	−2.02	−1.49	−10.49	−8.47	−4.67
	MktCap 3	4.48	3.62	5.45	0.97	4.52
	MktCap 4	9.92	14.45	17.08	7.15	13.82
	MktCap Diff	8.80	15.40	15.39	6.59	13.20
	MktCap Avg	3.38	3.90	3.43	0.05	3.57
1989/1990	*MktCap* 1	−27.14	−31.81	−15.01	12.14	−24.65
	MktCap 2	−32.32	−37.25	−39.78	−7.46	−36.45
	MktCap 3	−27.72	−31.88	−32.50	−4.78	−30.70
	MktCap 4	−24.15	−20.04	−19.69	4.46	−21.29
	MktCap Diff	2.99	11.77	−4.69	−7.68	3.36
	MktCap Avg	−27.83	−30.25	−26.75	1.09	−28.28
1990/1991	*MktCap* 1	54.21	64.97	106.14	51.92	75.11
	MktCap 2	38.00	44.27	73.81	35.80	52.03
	MktCap 3	22.96	41.38	53.57	30.60	39.30
	MktCap 4	15.71	28.61	47.77	32.06	30.69
	MktCap Diff	−38.50	−36.37	−58.37	−19.86	−44.41
	MktCap Avg	32.72	44.81	70.32	37.60	49.28
1991/1992	*MktCap* 1	56.71	41.73	64.00	7.30	54.15
	MktCap 2	24.66	25.09	18.36	−6.30	22.71
	MktCap 3	24.28	18.93	10.83	−13.45	18.01
	MktCap 4	16.09	14.16	9.64	−6.45	13.29
	MktCap Diff	−40.62	−27.57	−54.37	−13.75	−40.85
	MktCap Avg	30.43	24.98	25.71	−4.73	27.04
1992/1993	*MktCap* 1	42.33	46.82	35.10	−7.23	41.42
	MktCap 2	21.13	25.05	13.77	−7.37	19.98
	MktCap 3	18.02	15.32	5.55	−12.47	12.96
	MktCap 4	15.89	11.82	11.96	−3.93	13.22
	MktCap Diff	−26.44	−35.00	−23.14	3.30	−28.19
	MktCap Avg	24.34	24.75	16.59	−7.75	21.90
1993/1994	*MktCap* 1	−1.88	0.31	−7.36	−5.48	−2.98
	MktCap 2	−8.69	−4.96	−8.41	0.28	−7.35
	MktCap 3	−5.71	−4.23	−10.38	−4.68	−6.77
	MktCap 4	−4.93	−6.19	−6.00	−1.07	−5.71
	MktCap Diff	−3.05	−6.50	1.36	4.41	−2.73
	MktCap Avg	−5.30	−3.77	−8.04	−2.74	−5.70
1994/1995	*MktCap* 1	28.18	29.95	26.75	−1.43	28.29
	MktCap 2	25.19	34.46	35.39	10.20	31.68
	MktCap 3	22.53	26.73	29.23	6.71	26.17
	MktCap 4	19.72	23.67	24.42	4.70	22.60
	MktCap Diff	−8.46	−6.28	−2.33	6.13	−5.69
	MktCap Avg	23.90	28.71	28.95	5.04	27.19

(*continued*)

TABLE 5.13 (***Continued***)

$t/t+1$		β 1	β 2	β 3	β Diff	β Avg
1995/1996	*MktCap* 1	24.86	15.18	15.79	−9.07	18.61
	MktCap 2	13.64	12.10	8.62	−5.01	11.45
	MktCap 3	19.98	16.12	7.56	−12.42	14.56
	MktCap 4	15.87	15.41	9.42	−6.45	13.57
	MktCap Diff	−8.98	0.23	−6.36	2.62	−5.04
	MktCap Avg	18.59	14.70	10.35	−8.24	14.55
1996/1997	*MktCap* 1	27.39	15.21	0.38	−27.01	14.33
	MktCap 2	34.26	21.36	0.33	−33.93	18.65
	MktCap 3	33.10	24.20	−0.22	−33.32	19.02
	MktCap 4	24.78	24.34	14.43	−10.35	21.18
	MktCap Diff	−2.62	9.14	14.05	16.66	6.86
	MktCap Avg	29.88	21.28	3.73	−26.15	18.30
1997/1998	*MktCap* 1	−6.19	−7.08	−9.26	−3.07	−7.51
	MktCap 2	−12.35	−13.48	−18.68	−6.32	−14.84
	MktCap 3	−5.25	−9.09	−10.07	−4.82	−8.13
	MktCap 4	−2.88	−2.48	5.52	8.40	0.05
	MktCap Diff	3.31	4.59	14.79	11.47	7.56
	MktCap Avg	−6.67	−8.03	−8.12	−1.45	−7.61
1998/1999	*MktCap* 1	32.76	66.89	114.46	81.70	71.37
	MktCap 2	9.58	38.21	79.21	69.63	42.33
	MktCap 3	−8.57	17.84	56.12	64.69	21.80
	MktCap 4	−14.56	−4.59	46.49	61.05	9.11
	MktCap Diff	−47.32	−71.48	−67.97	−20.65	−62.26
	MktCap Avg	4.80	29.59	74.07	69.27	36.15
1999/2000	*MktCap* 1	−6.74	−14.10	−33.47	−26.74	−18.10
	MktCap 2	−8.91	−10.52	−26.62	−17.71	−15.35
	MktCap 3	−0.22	4.16	−13.24	−13.01	−3.10
	MktCap 4	7.68	13.11	−8.71	−16.39	4.03
	MktCap Diff	14.42	27.20	24.76	10.34	22.13
	MktCap Avg	−2.05	−1.84	−20.51	−18.46	−8.13
2000/2001	*MktCap* 1	44.30	53.06	20.51	−23.79	39.29
	MktCap 2	29.88	43.64	0.82	−29.06	24.78
	MktCap 3	31.84	22.49	0.29	−31.55	18.21
	MktCap 4	−2.73	4.54	−20.47	−17.74	−6.22
	MktCap Diff	−47.03	−48.52	−40.98	6.05	−45.51
	MktCap Avg	25.82	30.93	0.29	−25.53	19.01
2001/2002	*MktCap* 1	7.03	−6.41	−31.57	−38.60	−10.32
	MktCap 2	14.58	−8.38	−41.64	−56.22	−11.82
	MktCap 3	3.14	−10.19	−42.42	−45.56	−16.49
	MktCap 4	−7.97	−8.65	−38.66	−30.69	−18.43
	MktCap Diff	−15.01	−2.24	−7.09	7.91	−8.11
	MktCap Avg	4.19	−8.41	−38.57	−42.77	−14.26
2002/2003	*MktCap* 1	104.77	132.30	184.10	79.34	140.39
	MktCap 2	56.30	100.00	108.86	52.56	88.39
	MktCap 3	36.95	45.07	71.55	34.61	51.19
	MktCap 4	23.00	30.87	49.36	26.36	34.41
	MktCap Diff	−81.76	−101.43	−134.74	−52.98	−105.98
	MktCap Avg	55.25	77.06	103.47	48.22	78.59

TABLE 5.13 (***Continued***)

$t/t+1$		β 1	β 2	β 3	β Diff	β Avg
2003/2004	*MktCap* 1	28.52	24.02	23.09	−5.44	25.21
	MktCap 2	20.52	17.48	7.45	−13.07	15.15
	MktCap 3	23.70	22.21	6.76	−16.94	17.56
	MktCap 4	27.28	19.84	10.89	−16.39	19.34
	MktCap Diff	−1.24	−4.18	−12.19	−10.96	−5.87
	MktCap Avg	25.01	20.89	12.05	−12.96	19.31
2004/2005	*MktCap* 1	2.77	3.03	−28.68	−31.45	−7.63
	MktCap 2	4.73	−2.27	−7.45	−12.19	−1.66
	MktCap 3	14.58	4.23	−0.63	−15.21	6.06
	MktCap 4	3.06	7.56	4.59	1.53	5.07
	MktCap Diff	0.29	4.53	33.27	32.98	12.70
	MktCap Avg	6.28	3.14	−8.04	−14.33	0.46
2005/2006	*MktCap* 1	13.00	13.66	−5.47	−18.47	7.06
	MktCap 2	14.45	13.71	11.06	−3.39	13.08
	MktCap 3	11.97	9.42	11.62	−0.35	11.00
	MktCap 4	9.24	9.24	8.07	−1.17	8.85
	MktCap Diff	−3.76	−4.41	13.54	17.30	1.79
	MktCap Avg	12.17	11.51	6.32	−5.85	10.00
2006/2007	*MktCap* 1	−12.82	−10.28	−28.89	−16.07	−17.33
	MktCap 2	−11.36	−12.18	−11.42	−0.06	−11.65
	MktCap 3	−12.44	−9.85	−5.79	6.65	−9.36
	MktCap 4	−0.45	−0.98	8.37	8.82	2.32
	MktCap Diff	12.37	9.30	37.26	24.89	19.64
	MktCap Avg	−9.27	−8.32	−9.43	−0.16	−9.01
2007/2008	*MktCap* 1	−46.58	−49.30	−56.63	−10.05	−50.84
	MktCap 2	−38.65	−48.25	−45.66	−7.01	−44.19
	MktCap 3	−37.78	−34.51	−37.15	0.63	−36.48
	MktCap 4	−32.35	−34.83	−50.83	−18.48	−39.34
	MktCap Diff	14.22	14.47	5.79	−8.43	11.50
	MktCap Avg	−38.84	−41.72	−47.57	−8.73	−42.71
2008/2009	*MktCap* 1	94.97	156.50	150.77	55.80	134.08
	MktCap 2	29.40	49.10	87.00	57.60	55.17
	MktCap 3	15.41	37.43	51.26	35.85	34.70
	MktCap 4	16.16	32.86	48.35	32.19	32.46
	MktCap Diff	−78.81	−123.64	−102.42	−23.61	−101.62
	MktCap Avg	38.99	68.97	84.35	45.36	64.10
2009/2010	*MktCap* 1	22.10	34.09	30.45	8.36	28.88
	MktCap 2	20.57	25.63	37.38	16.81	27.86
	MktCap 3	13.03	24.95	34.39	21.36	24.12
	MktCap 4	17.52	23.59	29.29	11.77	23.46
	MktCap Diff	−4.58	−10.50	−1.17	3.41	−5.41
	MktCap Avg	18.30	27.07	32.88	14.57	26.08
2010/2011	*MktCap* 1	−8.37	−19.22	−39.11	−30.73	−22.23
	MktCap 2	−5.42	−8.15	−15.23	−9.82	−9.60
	MktCap 3	4.75	−0.11	−7.95	−12.70	−1.11
	MktCap 4	10.67	1.62	−10.53	−21.20	0.59
	MktCap Diff	19.04	20.84	28.58	9.54	22.82
	MktCap Avg	0.41	−6.47	−18.21	−18.61	−8.09

(*continued*)

TABLE 5.13 (*Continued*)

$t/t+1$		β 1	β 2	β 3	β Diff	β Avg
2011/2012	*MktCap* 1	27.77	9.70	−9.06	−36.83	9.47
	MktCap 2	28.61	19.69	19.03	−9.59	22.44
	MktCap 3	9.71	17.22	20.34	10.64	15.76
	MktCap 4	12.35	17.85	16.67	4.32	15.62
	MktCap Diff	−15.42	8.16	25.73	41.15	6.15
	MktCap Avg	19.61	16.11	11.74	−7.87	15.82

TABLE 5.14 Bivariate Independent-Sort Portfolio Excess and Abnormal Returns
This table presents the average excess returns (rows labeled Excess Return) and FFC alphas (rows labeled FFC α) for portfolios formed by grouping all stocks into three β groups and four *MktCap* groups. The numbers in parentheses are t-statistics, adjusted following Newey and West (1987) using six lags, testing the null hypothesis that the time-series average of the portfolio's excess return or FFC alpha is equal to zero.

	Coefficient	β 1	β 2	β 3	β Diff	β Avg
MktCap 1	Excess return	20.96	23.68	21.20	0.24	21.95
		(6.12)	(6.81)	(3.62)	(0.06)	(5.66)
	FFC α	11.96	18.63	9.90	−2.06	13.50
		(5.05)	(5.46)	(1.31)	(−0.38)	(3.44)
MktCap 2	Excess return	11.07	13.45	11.49	0.41	12.00
		(4.13)	(4.03)	(4.98)	(0.13)	(5.20)
	FFC α	0.18	1.42	0.92	0.74	0.84
		(0.08)	(0.45)	(0.43)	(0.16)	(0.65)
MktCap 3	Excess return	8.86	10.48	8.51	−0.36	9.28
		(3.01)	(5.50)	(5.18)	(−0.11)	(5.52)
	FFC α	−0.27	1.33	−1.14	−0.87	−0.02
		(−0.09)	(1.21)	(−0.88)	(−0.20)	(−0.03)
MktCap 4	Excess return	6.45	8.99	8.64	2.19	8.03
		(3.85)	(6.74)	(4.00)	(1.17)	(5.37)
	FFC α	−0.91	1.78	−1.41	−0.50	−0.18
		(−0.81)	(3.23)	(−0.94)	(−0.24)	(−0.25)
MktCap Diff	Excess return	−14.51	−14.69	−12.55	1.95	−13.92
		(−4.81)	(−4.37)	(−2.16)	(0.60)	(−3.67)
	FFC α	−12.87	−16.85	−11.31	1.56	−13.68
		(−3.83)	(−5.23)	(−1.57)	(0.43)	(−3.29)
MktCap Avg	Excess return	11.84	14.15	12.46	0.62	12.82
		(4.90)	(6.59)	(5.14)	(0.25)	(6.29)
	FFC α	2.74	5.79	2.07	−0.67	3.53
		(1.72)	(5.12)	(0.84)	(−0.19)	(3.00)

5.2.5 Interpreting the Results

In most cases, the focal results of the portfolio analysis are the differences between the portfolios that contain high and low values of a given variable. The differences in average Y values between portfolios with high values of $X1$ and low values of $X1$ ($X1$ Diff portfolios) indicate whether a cross-sectional relation between $X1$ and Y exists after controlling for the effect of $X2$. The logic is reversed to examine the cross-sectional relation between $X2$ and Y after controlling for $X1$ ($X2$ Diff portfolios).

Turning to our example in Table 5.14, the results indicate that among low-β (β 1) stocks, high-*MktCap* stocks have significantly lower average returns than low-*MktCap* stocks, since the *MktCap* difference portfolio (*MktCap* Diff) generates an average excess return of −14.51% (t-statistic = −4.81) and an FFC alpha of −12.87% (t-statistic = −3.83) per year. Similar results are obtained for stocks with moderate levels of β (β 2), as the *MktCap* Diff portfolio generates an average annual excess return of −14.69% and FFC alpha of −16.85%, both of which are highly statistically significant. For high-β stocks (β 3), the average excess return of the *MktCap* Diff portfolio of −12.55% (t-statistic = −2.16) is statistically significant, but after adjusting for factor sensitivities using the FFC model, the abnormal return of −11.31% is no longer statistically significant. Examination of the relation between β and future stock returns presents no evidence of such a relation after controlling for the effect of *MktCap* as, within each of the four *MktCap* groups, the average return differences and FFC alphas between the portfolios comprised high-β stocks and low-β stocks (β Diff) are economically small and statistically insignificant, as all associated t-statistics are well below 2.00 in magnitude.

In some cases, looking at the $X1$ difference portfolio within the different groups of $X2$ may give differing indications for different $X2$ groups. For example, in some cases, there may be a statistically significant relation between $X1$ and Y among entities with low values of $X2$, but this relation may not exist, or may even take the opposite sign, for entities with high $X2$ values. For this reason, it is instructive to examine the $X1$ difference portfolio for the average $X2$ group. The results for this portfolio indicate whether, for the average group of $X2$, there is a relation between $X1$ and Y. Furthermore, it is frequently of interest to examine whether any detected relation between $X1$ and Y is driven by entities with low values of $X1$ or by entities with high values of $X1$. To do this, we can examine the average $X1$ portfolio in the low $X2$ group and the average $X1$ portfolio in the high $X2$ group. If the average $X1$ portfolio in the low (high) $X2$ group generates statistically significant results but the average $X1$ portfolio in the high (low) $X2$ group does not, this may indicate that the relation is being driven by entities with low (high) values of $X2$. If the average $X1$ portfolio for both high and low $X2$ groups are statistically significant but with opposite signs, it indicates that the difference is driven by entities with both high and low values of $X2$. Obviously, the roles of $X1$ and $X2$ can be reversed to examine the relation between $X2$ and Y.

Table 5.14 shows that, in our example, for the average β group (β Avg), the difference in annual returns between the high-*MktCap* portfolio and the low-*MktCap* portfolio (*MktCap* Diff) is −13.92% with a corresponding t-statistic of −3.67. The FFC alpha of this portfolio is −13.68% per year with a t-statistic of −3.29. However, if we examine the alphas of portfolios with different levels of *MktCap* for the average β (β Avg) portfolio, we see that only entities with low-*MktCap* (*MktCap* 1) generate statistically significant abnormal returns relative to the FFC model. The FFC alpha of this portfolio is 13.50% per year with a corresponding t-statistic of

3.44. The abnormal returns for the average β group for stocks in *MktCap* groups two through four of 0.84% (t-statistic = 0.65), −0.02% (t-statistic = −0.03), and −0.18% (t-statistic = −0.25) per year, respectively, are all statistically indistinguishable from zero. Similar patterns are also observed within each of the three β groups. The results therefore indicate that the negative abnormal returns of the portfolios that take long positions in high-*MktCap* stocks and short positions in low-*MktCap* stocks are driven by the low-*MktCap* stocks. A similar analysis examining the relation between β and portfolio excess returns and alphas shows that for the average *MktCap* (*MktCap* Avg) group, the difference in excess return and FFC alpha between the high-β (β 3) and low-β (β 1) portfolios are 0.62% and −0.67%, respectively, per year, both of which are statistically insignificant. This is not surprising given that we failed to detect a relation between β and future returns in any of the *MktCap* groups.

In some cases, it may be of interest to researchers to examine the difference of the differences portfolio. This portfolio indicates whether the relation between $X1$ and Y changes for the high and low $X2$ groups. It also indicates whether the relation between $X2$ and Y changes for different groups of $X1$. A positive (negative) and statistically significant result for the difference of the differences portfolio indicates that the relation between $X1$ and Y is more positive (negative) for entities with high levels of $X2$ than for entities with low levels of $X2$. The same can be said reversing the roles of $X1$ and $X2$.

Examining our example results, we find no evidence that the relation between *MktCap* and future returns is different for stocks with different levels of β, because the β Diff, *MktCap* Diff portfolio does not generate statistically significant excess or abnormal returns. Similarly, the same result provides no evidence that the relation between β and future stock returns is different for stocks with high *MktCap* compared to stocks with low *MktCap*.

Analysis of the results for the average of the averages portfolio is rarely, if ever, undertaken in the empirical asset pricing literature. However, it can roughly be interpreted as indicative of the average Y value if all categories of entities are given equal weight, where a category corresponds to an intersection of the $X1$ and $X2$ groups. When the Y variable is the excess return, this portfolio indicates the returns associated with a long-only portfolio that gives higher weights to entities in categories that have fewer entities. Thus, it may have application in assessing the effects of different weighting schemes on portfolio returns.

5.2.6 Presenting the Results

There are many ways that the results of bivariate independent-sort portfolio analyses can be presented. Here, we describe a few of these. As always, the optimal approach depends largely on which elements of the analysis are to be emphasized.

Single Analysis

One common approach is to present only the average values ($\overline{Y}$) for each of the $n_{P1} \times n_{P2}$ portfolios, and to present both values of $\overline{Y}$ and the associated t-statistics for the difference and average portfolios. Sometimes, results for the average portfolios are not shown. When the outcome variable Y is a return variable, frequently the alphas relative to a factor model are shown instead of the average returns or excess returns.

TABLE 5.15 Bivariate Independent-Sort Portfolio Results
This table presents the average abnormal returns relative to the FFC model for portfolios sorted independently into three β groups and four *MktCap*. The breakpoints for the β portfolios are the 30th and 70th percentiles. The breakpoints for the *MktCap* portfolios are the 25th, 50th, and 75th percentiles. Table values indicate the alpha relative to the FFC model with corresponding t-statistics in parentheses.

	β 1	β 2	β 3	β 3-1	β Avg
MktCap 1	11.96	18.63	9.90	−2.06	13.50
				(−0.38)	(3.44)
MktCap 2	0.18	1.42	0.92	0.74	0.84
				(0.16)	(0.65)
MktCap 3	−0.27	1.33	−1.14	−0.87	−0.02
				(−0.20)	(−0.03)
MktCap 4	−0.91	1.78	−1.41	−0.50	−0.18
				(−0.24)	(−0.25)
MktCap 4-1	−12.87	−16.85	−11.31	1.56	−13.68
	(−3.83)	(−5.23)	(−1.57)	(0.43)	(−3.29)
MktCap Avg	2.74	5.79	2.07	−0.67	
	(1.72)	(5.12)	(0.84)	(−0.19)	

In Table 5.15, we present the results of our example bivariate independent-sort portfolio analysis. We show only the FFC alpha for the 12 portfolios formed by sorting on β and *MktCap*. We also show the FFC alpha and the associated t-statistics for the difference and average portfolios, although frequently researchers present the average returns or excess returns and only present alphas for the difference portfolios. As it is rarely if ever used by researchers, we will not show any results for the average of the averages portfolio.

Multiple Analyses, Same Relation of Interest

Another common approach is to present only the results for the difference or average portfolios. This is frequently the case when the objective of the analysis is to examine the relation between one of the sort variables (say $X2$) and Y while controlling for the other sort variable ($X1$), but we are not interested in the relation between $X1$ and Y when controlling for $X2$. This presentation style also allows the researcher to present results for more than one bivariate portfolio analysis while minimizing the amount of space required to do so. In exemplifying each of these approaches to presenting the results of bivariate portfolio analyses, we present not only the results for our analysis using β and *MktCap* as the sort variables, but also the results of a similar analysis using *BM* and *MktCap* as the sort variables. In this second analysis, we once again use the 30th and 70th percentiles of *BM* to calculate the *BM* breakpoints and the 25th, 50th, and 75th percentiles of *MktCap* for the *MktCap* breakpoints.

Table 5.16 gives an example of how the results of only the difference portfolios may be presented. The column labeled Control indicates the first sort variable ($X1$) used in the sorting procedure. Thus, the results for the analysis sorting on β and

TABLE 5.16 Bivariate Independent-Sort Portfolio Results—Differences
This table presents the average abnormal returns relative to the FFC model for long–short zero-cost portfolios that are long stocks in the highest quartile of *MktCap* and short stocks in the lowest quartile of *MktCap*. The portfolios are formed by sorting all stocks independently into groups based on β and *MktCap*. The breakpoints used to form the β groups are the 30th and 70th percentiles of β. Table values indicate the alpha relative to the FFC model with the corresponding t-statistics in parentheses.

Control	Coefficient	1	2	3	Avg	3-1
β	Excess return	−14.51	−14.69	−12.55	−13.92	1.95
		(−4.81)	(−4.37)	(−2.16)	(−3.67)	(0.60)
	FFC α	−12.87	−16.85	−11.31	−13.68	1.56
		(−3.83)	(−5.23)	(−1.57)	(−3.29)	(0.43)
BM	Excess return	−7.74	−13.31	−18.18	−13.08	−10.44
		(−2.17)	(−3.41)	(−6.57)	(−3.97)	(−6.53)
	FFC α	−6.19	−11.24	−22.04	−13.15	−15.85
		(−2.23)	(−3.00)	(−8.32)	(−4.73)	(−7.75)

MktCap are presented in the rows where the control variable is β, and the results for the analysis sorting on *BM* and *MktCap* are shown in the portion of the table where the control variable is *BM*. The columns labeled 1, 2, 3, Avg, and 3-1 indicate the control variable portfolio. Thus, for example, the column labeled 2 indicates the results for portfolios in the middle group of the given control variable (β or *BM*). The table presents the average excess returns and FFC alphas for the long–short zero-cost portfolio that is long stocks in the fourth quartile of *MktCap* and short stocks in the first *MktCap* quartile (*MktCap* Diff portfolio). The FFC alpha results when the control variable is β are therefore identical to those in the row labeled *MktCap* 4-1 in Table 5.15. In some cases, it is not necessary or important to present both the average excess returns and FFC alphas. In fact, when the outcome variable is not a return, the FFC alpha does not exist and therefore cannot be presented. We present both the average excess returns and the FFC alphas here for completeness.

The results in Table 5.16 indicate that the negative relation between *MktCap* and future stock returns persists after controlling for β and after controlling for *BM*. Furthermore, as mentioned previously, presenting the results for the *MktCap* difference portfolio within each control variable group allows us to see that this result is robust across all levels of both β and *BM* because, within each group of each control variable, this relation is detected. We should stress here that neither of these analyses controls simultaneously for both β and *BM*. Each analysis includes only one control variable. The column labeled Avg shows that for the average β group, the negative relation between *MktCap* and future excess returns persists. This is not surprising given that this relation exists within each of the individual β groups. A similar result holds when controlling for *BM*. Finally, the column labeled 3-1 presents the results for the difference in differences portfolio. The results fail to detect any difference in the relation between *MktCap* and future excess returns among stocks with low values of β compared to stocks with high values of β. However, the results indicate a strong difference in the relation between *MktCap* and future stock returns among stocks with differing levels of *BM*, because the average excess return of −10.44% per year for the difference in differences portfolio is highly statistically significant with a t-statistic of −6.53. Adjusting the returns of this portfolio for risk only strengthens this result, as the FFC alpha for this portfolio is −15.85% per year (t-statistic = −7.75). These results indicate that the negative relation between *MktCap* and future stock returns is much stronger for stocks with high values of *BM* than for low-*BM* stocks.

While Table 5.16 allows us to examine the relation between *MktCap* and future excess returns among stocks with differing levels of β or *BM*, it does not give us an idea of whether it is the low-*MktCap* or high-*MktCap* stocks that are generating the results. Another way of presenting the results is to show only the results that correspond to the average $X1$ (β or *BM* in this example) portfolio for each level of $X2$ (*MktCap* in this example). This allows the reader to get an understanding of how the average value of Y varies with $X2$ after controlling for $X1$. This presentation style is frequently desirable when a univariate sort portfolio analysis indicates a relation between $X2$ and Y but the bivariate independent-sort portfolio analysis indicates that after controlling for $X1$, this relation disappears. This is because by showing results for each $X2$ group, it emphasizes the fact that there is no difference in average Y values for high and low values of $X2$ (after controlling for $X1$). Alternatively, this approach is useful when there is no univariate relation between $X2$ and Y but a relation appears after controlling for $X1$. In our example, we find a negative univariate relation between *MktCap* and future stock returns, but we do not find that controlling for β or *BM* explains this relation. Nonetheless, we exemplify this presentation style in Table 5.17.

Multiple Analyses, Different Relations of Interest

The results in Tables 5.16 and 5.17 examine the relation between *MktCap* and future stock returns after controlling for each of β and *BM*. Sometimes, however, we may

TABLE 5.17 Bivariate Independent-Sort Portfolio Results—Averages
This table presents the average abnormal returns relative to the FFC model for portfolios formed by sorting independently on β and *MktCap*. The table shows the portfolio FFC alphas and the associated Newey and West (1987) adjusted t-statistics calculated using six lags (in parentheses) for the average β group within each group of *MktCap*.

Control	Coefficient	*MktCap* 1	*MktCap* 2	*MktCap* 3	*MktCap* 4	*MktCap* 4-1
β	Excess return	21.95	12.00	9.28	8.03	−13.92
		(5.66)	(5.20)	(5.52)	(5.37)	(−3.67)
	FFC α	13.50	0.84	−0.02	−0.18	−13.68
		(3.44)	(0.65)	(−0.03)	(−0.25)	(−3.29)
BM	Excess return	22.04	11.78	9.27	8.97	−13.08
		(6.48)	(5.59)	(6.70)	(6.76)	(−3.97)
	FFC α	13.98	1.76	0.56	0.82	−13.15
		(5.00)	(1.31)	(1.33)	(1.21)	(−4.73)

TABLE 5.18 Bivariate Independent-Sort Portfolio Results—Differences

This table presents the average excess returns and FFC alphas for portfolios formed by sorting independently on β and a second sort variable, which is either *MktCap* or *BM*. The table shows the average excess returns and FFC alphas, along with the associated Newey and West (1987) adjusted t-statistics calculated using six lags (in parentheses), for the difference between the portfolios with high and low values of the second sort variable (*MktCap* or *BM*). The first column indicates the second sort variable. The remaining columns correspond to different β groups, as indicated in the header.

Sort Variable	Coefficient	β 1	β 2	β 3	β Avg	β 3-1
MktCap	Excess return	−14.51	−14.69	−12.55	−13.92	1.95
		(−4.81)	(−4.37)	(−2.16)	(−3.67)	(0.60)
	FFC α	−12.87	−16.85	−11.31	−13.68	1.56
		(−3.83)	(−5.23)	(−1.57)	(−3.29)	(0.43)
BM	Excess return	10.21	10.52	11.79	10.84	1.58
		(3.74)	(3.90)	(4.34)	(5.46)	(0.55)
	FFC α	8.93	8.97	7.72	8.54	−1.21
		(3.91)	(2.74)	(2.71)	(3.68)	(−0.35)

want to examine the relation between several different variables, perhaps *MktCap* and *BM*, after controlling for another variable, say β. In this case, we can generate tables that look similar to Tables 5.16 and 5.17, except instead of the first column of the table indicating the control variable, it will indicate the variable whose relation with the outcome variable Y is of interest.

In Table 5.18, we present the results of two bivariate portfolio analyses. The first is the same analysis that we have been using throughout this section that takes β and *MktCap* to be the sort variables. The second takes β and *BM* to be the sort variables. The relations of interest are those between future stock returns and each of *MktCap* and *BM*. Table 5.18 presents the average excess returns and FFC alphas for the difference portfolios for each of the sort variables of interest (*MktCap* and *BM*) within each of the β groups.

The results indicate a negative relation between *MktCap* and future stock returns after controlling for β. This is the same result as was presented in Tables 5.15 and 5.16. The table also detects a strong positive relation between *BM* and future stock returns after controlling for β. These results indicate that after controlling for β, the positive relation between *BM* and future stock returns detected in the univariate analysis (see Table 5.8) persists. The last column in the table indicates that this relation appears to be quite similar among stocks with low and high values of β.

Finally, in Table 5.19, we present the results of the same two portfolio analyses as were presented in Table 5.18. The only difference here is that, instead of presenting the results for the difference portfolios, we present the results for the average portfolios across each group of the control variable, $X1$ (β in this example), and within each group of the sort variables of interest, $X2$ (*MktCap* and *BM* in this example). Thus, the columns labeled 1 through 4 show results for the average (across the β groups) portfolio within the given group of the indicated sort variable (*MktCap* or *BM*). The

TABLE 5.19 Bivariate Independent-Sort Portfolio Results—Averages

This table presents the average excess returns and FFC alphas for portfolios formed by sorting independently on β and a second sort variable, which is either *MktCap* or *BM*. The table shows the average excess returns and FFC alphas, along with the associated Newey and West (1987) adjusted t-statistics calculated using six lags (in parentheses), for the difference between the portfolios with high and low values of the second sort variable (*MktCap* or *BM*). The first column indicates the second sort variable. The remaining columns correspond to different β groups, as indicated in the header.

Sort Variable	Coefficient	1	2	3	4	4-1
MktCap	Excess return	21.95	12.00	9.28	8.03	−13.92
		(5.66)	(5.20)	(5.52)	(5.37)	(−3.67)
	FFC α	13.50	0.84	−0.02	−0.18	−13.68
		(3.44)	(0.65)	(−0.03)	(−0.25)	(−3.29)
BM	Excess return	8.92	12.63	12.98	19.76	10.84
		(4.92)	(7.75)	(6.35)	(7.88)	(5.46)
	FFC α	1.41	4.37	4.28	9.95	8.54
		(0.88)	(3.90)	(2.32)	(5.41)	(3.68)

column labeled 4-1 presents results for the difference in these averages (which is the same as the average difference) between portfolios in the fourth and first groups of the indicated sort variable.

5.3 BIVARIATE DEPENDENT-SORT ANALYSIS

Bivariate dependent-sort portfolio analysis is similar to its independent-sort counterpart in that the portfolios are formed by sorting entities based on values of two sort variables $X1$ and $X2$. The only difference between the dependent-sort and independent-sort analyses is that in the dependent-sort analysis, breakpoints for the second sort variable are formed within each group of the first sort variable. For this reason, dependent-sort analysis is used when the objective is to understand the relation between $X2$ and Y conditional on $X1$. The relation between $X1$ and Y is not examined in dependent-sort analysis. $X1$ is used only as a control variable.

5.3.1 Breakpoints

Calculation of the breakpoints for the bivariate dependent-sort portfolio analysis begins exactly the same way as in the independent-sort analysis. In the dependent-sort analysis, however, it is extremely important to distinguish which independent variable is the control variable, $X1$, and which variable is part of the relation of interest, $X2$, as unlike independent-sort analysis, in the dependent-sort analysis, the order of sorting is critically important.

The dependent-sort portfolio procedure begins by calculating breakpoints for the first sort variable ($X1$, the control variable). Letting n_{P1} be the number of groups based on $X1$, and $p1_j, j \in \{1, \ldots, n_{P1} - 1\}$ be the percentiles used to calculate the breakpoints, the breakpoints for $X1$ are calculated exactly as described in Section 5.2 and equation (5.10). As always, breakpoints may be calculated using a different set of entities than the set that will eventually be sorted into portfolios.

Having calculated the breakpoints for the first sort variable $X1$ for each time period t, the entities are divided into n_{P1} groups based on the breakpoints $B1_{j,t}$. The next step is what differentiates dependent-sort portfolio analysis from independent-sort portfolio analysis. In dependent-sort analysis, the second sort, based on values of $X2$, is done separately for each of the n_{P1} groups of entities created by the breakpoints for the first sort variable. Thus, the breakpoints that determine how the sample is divided into portfolios based on the second independent variable $X2$ will be different for each of the n_{P1} groups formed by sorting on the first sort variable. We therefore define the breakpoints for the second sort variable as

$$B2_{j,k,t} = Pctl_{p2_k}(\{X2_t | B1_{j-1,t} \leq X1_t \leq B1_{j,t}\}) \tag{5.20}$$

where $j \in \{1, \ldots, n_{P1}\}$, $k \in \{1, \ldots, n_{P2} - 1\}$, $p2_k$ is the percentile for the kth breakpoint based on the second sort variable, n_{P2} is the number of groups to be formed based on the second sort variable $X2$, $B1_{0,t} = -\infty$, $B1_{n_{P1},t} = \infty$, and $\{X2_t | B1_{j-1,t} \leq X1_t \leq B1_{j,t}\}$ is the set of values of $X2$ across all entities in the sample with values of $X1$ that are between $B1_{j-1,t}$ and $B1_{j,t}$ inclusive. Thus, for each of the n_{P1} groups of entities formed on $X1$, there will be $n_{P2} - 1$ breakpoints for the second sort variable $X2$.

Before proceeding to an example, a brief discussion of the choice of the number of groups to use for each of the sort variables is warranted. As always, the objective is to find a reasonable balance between the number of entities in each portfolio and the number of portfolios. In dependent-sort portfolio analysis, there is one major difference in choosing the number of groups compared to independent-sort portfolio analysis. In dependent-sort analysis, because sorting based on the second sort variable $X2$ is done within each group of entities formed by the first sort, correlation between the sort variables does not play a role in determining an appropriate number of breakpoints. As long as there are sufficient entities in each group formed by sorting on the first sort variable $X1$ to form n_{P2} groups of entities when sorting based on the chosen percentiles of $X2$, the dependent-sort analysis should provide an accurate assessment of the relation between $X2$ and Y.

We exemplify the bivariate dependent-sort procedure by calculating breakpoints for portfolios formed using β as the control variable, $X1$, and *MktCap* as the sort variable of interest, $X2$. As in previous analyses, the β breakpoints are the 30th and 70th percentiles and the *MktCap* breakpoints are the 25th, 50th, and 75th percentiles. Table 5.20 shows the breakpoints for each year during our sample period. The table shows that in 1988, the β breakpoints are 0.18 (column $B1_{1,t}$) and 0.66 (column $B1_{2,t}$). These breakpoints are identical to the β breakpoints used in the independent-sort portfolio analysis (see Table 5.10). As β is the first sort variable, these are the breakpoints that are used to group the stocks according to β, regardless of the level of *MktCap*. Within the set of stocks with the lowest values of β ($\beta \leq 0.18$), we see that the first, second, and third *MktCap* breakpoints are 4.57, 12.36, and 35.38, respectively. Thus, the portfolio $P_{1,1}$, which holds low-β and low-*MktCap* stocks, comprised

TABLE 5.20 Bivariate Dependent-Sort Breakpoints

This table presents the breakpoints for portfolios formed by sorting all stocks in the sample into three groups based on the 30th and 70th percentiles of β, and then, within each β group, into four groups based on the 25th, 50th, and 75th percentiles of *MktCap* among only stocks in the given β groups. The columns labeled t indicates the year of the breakpoints. The columns labeled $B1_{1,t}$ and $B1_{2,t}$ present the β breakpoints. The columns labeled $B2_{1,k,t}$, $B2_{2,k,t}$, and $B2_{3,k,t}$ indicate the kth *MktCap* breakpoint for stocks in the first, second, and third β group, respectively, where k is indicated in the columns labeled k.

t	k	$B2_{1,k,t}$	$B1_{1,t}$	$B2_{2,k,t}$	$B1_{2,t}$	$B2_{3,k,t}$	t	k	$B2_{1,k,t}$	$B1_{1,t}$	$B2_{2,k,t}$	$B1_{2,t}$	$B2_{3,k,t}$
1988	1	4.57	0.18	12.49	0.66	25.31	2001	1	16.71	0.31	45.87	0.95	91.00
	2	12.36		39.76		126.79		2	44.75		231.86		350.97
	3	35.38		142.60		796.57		3	113.32		886.00		1162.14
1989	1	4.29	0.17	14.05	0.70	26.72	2002	1	12.80	0.33	56.33	0.89	140.84
	2	12.05		46.16		135.74		2	33.43		234.04		415.02
	3	33.94		183.14		929.26		3	69.34		851.28		1455.45
1990	1	2.74	0.23	8.62	0.86	21.50	2003	1	26.39	0.38	151.58	0.97	302.73
	2	8.40		30.99		98.69		2	57.59		415.27		714.81
	3	24.50		151.58		583.68		3	110.03		1480.06		2090.83
1991	1	3.76	0.24	14.27	0.85	38.90	2004	1	34.44	0.63	203.14	1.36	250.79
	2	10.60		55.37		169.80		2	75.35		702.06		562.28
	3	31.59		242.53		909.57		3	151.62		2530.95		1211.46
1992	1	8.16	0.26	23.08	0.97	38.17	2005	1	34.51	0.60	211.76	1.30	344.26
	2	22.05		79.97		140.75		2	72.25		748.77		680.42
	3	67.79		384.71		681.30		3	135.07		3277.05		1414.81
1993	1	11.03	0.29	34.12	0.92	43.02	2006	1	37.80	0.61	226.34	1.40	382.07
	2	28.81		101.77		180.90		2	82.29		778.31		734.76
	3	79.28		480.49		789.78		3	161.21		2900.79		1588.50
1994	1	11.60	0.36	30.21	0.97	35.30	2007	1	33.25	0.56	208.68	1.18	316.60
	2	26.86		105.28		144.17		2	67.62		848.39		607.85
	3	76.27		439.81		694.79		3	127.17		3253.81		1474.95
1995	1	13.60	0.27	41.60	0.90	38.33	2008	1	12.40	0.57	136.70	1.14	137.59
	2	34.46		145.19		152.81		2	31.09		475.72		356.81
	3	105.74		611.37		739.47		3	63.84		1611.62		1122.37
1996	1	14.99	0.32	40.72	0.90	49.92	2009	1	22.53	0.65	180.11	1.45	185.19
	2	41.62		163.43		177.91		2	54.22		632.27		427.60
	3	120.22		692.85		769.25		3	149.12		2220.07		1311.04
1997	1	15.50	0.26	42.75	0.72	80.59	2010	1	26.98	0.76	219.47	1.33	263.99
	2	44.11		155.84		346.42		2	61.05		721.38		609.64
	3	116.47		603.98		1524.24		3	259.80		2455.43		1828.61
1998	1	14.04	0.41	37.38	0.94	74.16	2011	1	22.28	0.82	246.05	1.38	215.12
	2	38.47		126.92		323.19		2	49.47		891.90		514.69
	3	109.89		568.93		1175.07		3	123.80		3048.15		1281.89
1999	1	17.39	0.15	48.04	0.53	95.04	2012	1	28.27	0.76	254.37	1.31	245.38
	2	43.43		165.40		477.36		2	64.96		797.94		680.17
	3	109.53		626.78		2278.61		3	498.84		2779.50		2144.59
2000	1	15.66	0.25	29.98	0.85	39.78							
	2	42.57		203.66		192.66							
	3	117.48		915.58		908.20							

stocks with $\beta \leq 0.18$ and $MktCap \leq 4.57$. The portfolio $P_{1,2}$ contains stocks with $\beta \leq 0.18$ and $4.57 \leq MktCap \leq 12.36$. Portfolio $P_{1,3}$ holds stocks with $\beta \leq 0.18$ and $12.36 \leq MktCap \leq 35.38$. Finally, the low-$\beta$ and high-*MktCap* portfolio ($P_{1,4}$) holds stocks with $\beta \leq 0.18$ and $MktCap \geq 35.38$.

Turning to the second β-based group of stocks, we see that this group contains stocks with $0.18 \leq \beta \leq 0.66$. Within this group, the *MktCap* breakpoints are 12.49, 39.76, and 142.60. Thus, for example, the portfolio $P_{2,3}$, which corresponds to stocks with βs between the 30th and 70th β percentiles and *MktCap*s between the 50th and 75th *MktCap* percentiles, holds stocks with $0.18 \leq \beta \leq 0.66$ and $39.76 \leq MktCap \leq 142.60$. Notice that the *MktCap* breakpoints are very different in the second β group than in the first β group. This is the ramification of performing a dependent sort. Finally, in the third β group, the *MktCap* breakpoints are 25.31, 126.79, and 796.57. The increasing *MktCap* breakpoints across the different groups of β are a manifestation of the positive correlation between β and *MktCap*.

5.3.2 Portfolio Formation

Portfolio formation for the dependent-sort analysis proceeds as one would expect given the procedure for calculating the breakpoints. Each time period t, all entities in the sample are first sorted into groups based on the breakpoints calculated using the first sort variable $X1$. Each of those groups is then sorted into portfolios based on the conditional breakpoints of the second sort variable, $X2$. In general, we can describe the portfolio holding stocks in group j of the first sort variable $X1$ and group k of the second sort variable $X2$ as

$$P_{j,k,t} = \{i|B1_{j-1,t} \leq X1_{i,t} < B1_{j,t}\} \cap \{i|B2_{j,k-1,t} \leq X2_{i,t} < B2_{j,k,t}\} \tag{5.21}$$

for $j \in \{1, 2, \ldots, n_{P1}\}$, $k \in \{1, 2, \ldots, n_{P2}\}$. As with the independent-sort analysis, the result is that all entities in the sample for each period t are placed into one of $n_{P1} \times n_{P2}$ portfolios. When the sample used to calculate the breakpoints is the same as the sample that is sorted into portfolios, the percentage of entities in any given portfolio is easily calculated from the percentiles used to calculate the breakpoints. This does not hold when the breakpoints sample is not identical to the sample that is used to form the portfolios.

Table 5.21 presents the number of stocks in each of the portfolio for each year t during our sample period. Notice that for each year t and within each β group, each of the *MktCap* portfolios has approximately 25% of the stocks. This is because we chose to use quartile breakpoints. There are two reasons that the number of stocks in each portfolio is not exactly 25% of the number of stocks in the given β group. The first is that the number of stocks in the given β group may not be divisible by four. The second is that when a stock has a value of a certain variable that is exactly equal to one of the breakpoints, it gets put in more than one portfolio. These issues are very minor, however. If these issues have a substantial impact on the conclusions drawn from the portfolio analysis, it means that the number of stocks (or entities in the general sense) in each portfolio is too small and that either the breakpoints must be adjusted or there are simply too few stocks (or entities) in the sample to effectively conduct a bivariate portfolio analysis.

TABLE 5.21 Bivariate Dependent-Sort Number of Stocks per Portfolio

This table presents the number of stocks in each of the 12 portfolios formed by sorting dependently into three β groups and then into four *MktCap* groups. The columns labeled t indicate the year of portfolio formation. The columns labeled β 1, β 2, and β 3 indicate the β group. The rows labeled *MktCap* 1, *MktCap* 2, *MktCap* 3, and *MktCap* 4 indicate the *MktCap* groups.

t		β 1	β 2	β 3	t		β 1	β 2	β 3
1988	*MktCap* 1	426	568	426	1998	*MktCap* 1	496	661	495
	MktCap 2	426	569	426		*MktCap* 2	496	660	496
	MktCap 3	427	568	426		*MktCap* 3	496	660	495
	MktCap 4	426	569	427		*MktCap* 4	496	661	495
1989	*MktCap* 1	414	551	414	1999	*MktCap* 1	457	609	457
	MktCap 2	413	551	413		*MktCap* 2	457	608	456
	MktCap 3	413	551	413		*MktCap* 3	456	608	457
	MktCap 4	414	552	414		*MktCap* 4	457	609	457
1990	*MktCap* 1	406	540	406	2000	*MktCap* 1	442	589	442
	MktCap 2	405	540	406		*MktCap* 2	441	589	442
	MktCap 3	406	540	405		*MktCap* 3	442	589	441
	MktCap 4	406	540	405		*MktCap* 4	442	589	442
1991	*MktCap* 1	398	530	398	2001	*MktCap* 1	413	551	413
	MktCap 2	397	530	397		*MktCap* 2	413	550	412
	MktCap 3	397	530	397		*MktCap* 3	413	550	413
	MktCap 4	398	530	398		*MktCap* 4	413	551	413
1992	*MktCap* 1	404	538	404	2002	*MktCap* 1	382	509	382
	MktCap 2	404	538	404		*MktCap* 2	381	508	381
	MktCap 3	404	538	403		*MktCap* 3	381	508	381
	MktCap 4	404	539	404		*MktCap* 4	382	509	382
1993	*MktCap* 1	425	567	425	2003	*MktCap* 1	355	473	355
	MktCap 2	425	567	425		*MktCap* 2	355	472	355
	MktCap 3	425	565	425		*MktCap* 3	355	473	355
	MktCap 4	425	567	425		*MktCap* 4	355	474	354
1994	*MktCap* 1	461	615	462	2004	*MktCap* 1	343	457	343
	MktCap 2	462	614	462		*MktCap* 2	343	457	343
	MktCap 3	462	614	460		*MktCap* 3	343	457	343
	MktCap 4	462	615	462		*MktCap* 4	343	457	343
1995	*MktCap* 1	472	629	472	2005	*MktCap* 1	337	450	337
	MktCap 2	471	628	471		*MktCap* 2	337	449	337
	MktCap 3	471	628	471		*MktCap* 3	337	448	337
	MktCap 4	472	629	472		*MktCap* 4	337	450	337
1996	*MktCap* 1	494	658	494	2006	*MktCap* 1	334	444	334
	MktCap 2	494	658	493		*MktCap* 2	334	445	333
	MktCap 3	494	658	494		*MktCap* 3	334	446	334
	MktCap 4	494	658	494		*MktCap* 4	334	446	334
1997	*MktCap* 1	514	686	515	2007	*MktCap* 1	325	432	325
	MktCap 2	515	687	515		*MktCap* 2	325	434	325
	MktCap 3	515	686	514		*MktCap* 3	325	432	325
	MktCap 4	515	685	515		*MktCap* 4	325	434	325

(*continued*)

TABLE 5.21 (*Continued*)

t		β 1	β 2	β 3	t		β 1	β 2	β 3
2008	*MktCap* 1	320	426	320	2011	*MktCap* 1	276	368	277
	MktCap 2	320	426	319		*MktCap* 2	277	368	276
	MktCap 3	319	426	320		*MktCap* 3	277	368	275
	MktCap 4	320	426	320		*MktCap* 4	276	368	277
2009	*MktCap* 1	298	398	298	2012	*MktCap* 1	266	354	266
	MktCap 2	299	396	298		*MktCap* 2	266	355	266
	MktCap 3	298	398	298		*MktCap* 3	266	355	266
	MktCap 4	299	397	299		*MktCap* 4	266	354	266
2010	*MktCap* 1	285	381	286					
	MktCap 2	285	380	285					
	MktCap 3	286	380	284					
	MktCap 4	286	381	286					

5.3.3 Average Portfolio Values

After the portfolios have been created, the next step is to calculate, for each time period t, the average value of the dependent variable Y for each of the portfolios. For each of the $n_{P1} \times n_{P2}$ portfolios, the procedure for calculating the average dependent variable value is identical to the procedure in the independent-sort analysis (see equation (5.13)). Similarly, calculation of the difference in averages between group n_{P2} and group one of the second sort variable, for each of the groups of the first sort variable ($\overline{Y}_{j,\,Diff,\,t}$, equation (5.14)), as well as the calculation of the average portfolio value for each $X2$ group across all $X1$ groups ($\overline{Y}_{Avg,k,t}$, equation (5.17)), for $k \in \{1, 2, \ldots, n_{P2}, Diff\}$, remain unchanged.

The only difference between the dependent-sort analysis and the independent-sort analysis is that, in dependent-sort analysis, we do not calculate the differences in mean values between groups n_{P1} and group one of the first sort variable $X1$ (equation (5.15)). The reason for this is that the dependent-sort analysis is only designed to assess the relation between the second sort variable $X2$ and the outcome variable Y. The conditional nature of the portfolio formation procedure leads to uncertain interpretation of the difference in average Y values between portfolio n_{P1} and portfolio one of the first sort variable $X1$ for a given group of $X2$. The one exception to this rule is the difference in differences portfolio, which can be used to detect differences in the relation between $X2$ and Y for entities with different levels of $X1$. We are also not interested in the average portfolio within each of the groups of the first sort variable (equation (5.16)).

The average one-year-ahead future excess returns for each of the portfolios in our example bivariate dependent-sort analysis are presented in Table 5.22. While the numbers are different from those of the independent-sort analysis, the discussion of these results would be similar to the previous discussion in Section 5.2.3. Further discussion is not necessary. We present these results so that the reader attempting to replicate the analysis has a reference point.

TABLE 5.22 Bivariate Dependent-Sort Mean Values

This table presents the equal-weighted excess returns for each of the 12 portfolios formed by sorting all stocks in the sample into three β groups and then, within each of the β groups, into four *MktCap* groups. The columns labeled $t/t+1$ indicate the year of portfolio formation (t) and the portfolio holding period ($t+1$). The columns labeled β 1, β 2, β 3, and β Avg indicate the β groups. The rows labeled *MktCap* 1, *MktCap* 2, *MktCap* 3, *MktCap* 4, and *MktCap* Diff indicate the *MktCap* groups.

$t/t+1$		β 1	β 2	β 3	β Avg
1988/1989	*MktCap* 1	−3.02	−0.24	−3.63	−2.29
	MktCap 2	3.22	−1.67	1.44	1.00
	MktCap 3	−0.91	3.96	12.38	5.14
	MktCap 4	5.75	13.61	19.98	13.11
	MktCap Diff	8.77	13.84	23.61	15.41
1989/1990	*MktCap* 1	−20.67	−32.48	−27.18	−26.78
	MktCap 2	−34.95	−38.05	−33.72	−35.58
	MktCap 3	−31.57	−30.87	−25.74	−29.39
	MktCap 4	−27.67	−19.88	−15.97	−21.17
	MktCap Diff	−7.00	12.60	11.21	5.60
1990/1991	*MktCap* 1	75.86	57.12	86.67	73.22
	MktCap 2	32.31	47.48	63.18	47.66
	MktCap 3	35.88	41.68	51.85	43.14
	MktCap 4	20.89	28.56	39.91	29.79
	MktCap Diff	−54.97	−28.57	−46.75	−43.43
1991/1992	*MktCap* 1	78.25	38.34	37.76	51.45
	MktCap 2	31.13	23.22	10.40	21.58
	MktCap 3	24.36	18.48	11.07	17.97
	MktCap 4	24.93	14.20	8.67	15.94
	MktCap Diff	−53.31	−24.14	−29.09	−35.51
1992/1993	*MktCap* 1	45.88	42.00	25.77	37.89
	MktCap 2	31.54	23.36	7.28	20.73
	MktCap 3	21.03	15.09	9.93	15.35
	MktCap 4	17.84	11.34	12.12	13.77
	MktCap Diff	−28.04	−30.67	−13.65	−24.12
1993/1994	*MktCap* 1	1.63	−1.48	−8.00	−2.62
	MktCap 2	−6.68	−4.60	−10.08	−7.12
	MktCap 3	−9.21	−4.70	−7.91	−7.28
	MktCap 4	−5.38	−6.10	−5.52	−5.66
	MktCap Diff	−7.00	−4.62	2.49	−3.05
1994/1995	*MktCap* 1	28.15	27.40	30.11	28.55
	MktCap 2	27.43	36.11	33.51	32.35
	MktCap 3	24.75	24.16	22.22	23.71
	MktCap 4	21.69	25.31	27.22	24.74
	MktCap Diff	−6.46	−2.09	−2.89	−3.82
1995/1996	*MktCap* 1	34.29	13.58	13.95	20.61
	MktCap 2	13.10	12.84	10.80	12.25
	MktCap 3	11.36	18.16	2.11	10.54
	MktCap 4	20.16	14.54	12.97	15.89
	MktCap Diff	−14.13	0.96	−0.98	−4.72

(*continued*)

TABLE 5.22 ***(Continued)***

$t/t+1$		β 1	β 2	β 3	β Avg
1996/1997	*MktCap* 1	26.75	16.86	2.99	15.53
	MktCap 2	31.27	21.71	−2.49	16.83
	MktCap 3	33.86	25.86	1.50	20.40
	MktCap 4	30.37	22.93	17.32	23.54
	MktCap Diff	3.63	6.08	14.33	8.01
1997/1998	*MktCap* 1	−2.22	−8.19	−16.45	−8.95
	MktCap 2	−12.88	−13.97	−6.83	−11.23
	MktCap 3	−11.64	−6.85	−10.62	−9.71
	MktCap 4	−4.84	−2.75	14.63	2.34
	MktCap Diff	−2.63	5.44	31.08	11.30
1998/1999	*MktCap* 1	43.73	59.50	93.85	65.69
	MktCap 2	18.83	42.90	70.65	44.13
	MktCap 3	4.90	11.46	53.39	23.25
	MktCap 4	−10.56	−4.72	37.46	7.39
	MktCap Diff	−54.29	−64.21	−56.39	−58.30
1999/2000	*MktCap* 1	−4.05	−13.45	−31.33	−16.28
	MktCap 2	−7.70	−8.89	−13.70	−10.10
	MktCap 3	−11.01	5.20	−10.31	−5.37
	MktCap 4	−0.19	13.59	−8.62	1.59
	MktCap Diff	3.86	27.04	22.71	17.87
2000/2001	*MktCap* 1	42.66	49.62	16.59	36.29
	MktCap 2	35.91	45.46	−3.51	25.95
	MktCap 3	31.95	12.97	3.05	15.99
	MktCap 4	14.93	2.80	−23.98	−2.08
	MktCap Diff	−27.73	−46.82	−40.57	−38.37
2001/2002	*MktCap* 1	7.86	−6.53	−34.32	−10.99
	MktCap 2	7.17	−9.37	−44.22	−15.47
	MktCap 3	16.93	−9.66	−42.12	−11.62
	MktCap 4	−0.45	−8.74	−37.27	−15.49
	MktCap Diff	−8.31	−2.21	−2.95	−4.49
2002/2003	*MktCap* 1	126.16	126.13	130.07	127.45
	MktCap 2	83.70	77.30	76.57	79.19
	MktCap 3	60.81	38.73	56.68	52.08
	MktCap 4	44.81	30.37	44.05	39.74
	MktCap Diff	−81.35	−95.76	−86.01	−87.71
2003/2004	*MktCap* 1	35.21	21.61	10.52	22.45
	MktCap 2	25.11	17.13	7.53	16.59
	MktCap 3	22.03	24.40	8.80	18.41
	MktCap 4	20.05	19.33	10.48	16.62
	MktCap Diff	−15.15	−2.28	−0.05	−5.83

TABLE 5.22 ***(Continued)***

$t/t+1$		β 1	β 2	β 3	β Avg
2004/2005	*MktCap* 1	3.31	0.26	−15.96	−4.13
	MktCap 2	1.76	1.13	−0.99	0.64
	MktCap 3	5.06	5.89	0.06	3.67
	MktCap 4	5.28	8.37	3.59	5.75
	MktCap Diff	1.97	8.12	19.55	9.88
2005/2006	*MktCap* 1	16.66	14.56	8.44	13.22
	MktCap 2	11.25	8.20	12.91	10.79
	MktCap 3	7.71	12.13	9.98	9.94
	MktCap 4	17.20	8.79	8.22	11.40
	MktCap Diff	0.53	−5.77	−0.22	−1.82
2006/2007	*MktCap* 1	−11.31	−11.33	−13.36	−12.00
	MktCap 2	−12.62	−13.25	−9.96	−11.94
	MktCap 3	−15.20	−4.46	−2.81	−7.49
	MktCap 4	−5.24	−0.24	8.37	0.96
	MktCap Diff	6.07	11.09	21.74	12.97
2007/2008	*MktCap* 1	−45.41	−51.23	−47.10	−47.91
	MktCap 2	−46.82	−37.86	−37.19	−40.62
	MktCap 3	−45.96	−33.62	−38.36	−39.31
	MktCap 4	−34.59	−35.08	−50.46	−40.05
	MktCap Diff	10.82	16.15	−3.36	7.87
2008/2009	*MktCap* 1	144.10	115.09	112.15	123.78
	MktCap 2	66.00	39.84	52.92	52.92
	MktCap 3	37.20	33.54	49.62	40.12
	MktCap 4	18.80	31.57	48.88	33.09
	MktCap Diff	−125.30	−83.52	−63.27	−90.69
2009/2010	*MktCap* 1	21.77	31.15	38.62	30.51
	MktCap 2	22.93	24.74	32.02	26.56
	MktCap 3	20.63	25.66	36.43	27.58
	MktCap 4	16.79	21.93	28.51	22.41
	MktCap Diff	−4.97	−9.22	−10.11	−8.10
2010/2011	*MktCap* 1	−10.16	−14.32	−19.46	−14.64
	MktCap 2	−7.71	−4.01	−12.26	−7.99
	MktCap 3	−5.92	0.35	−7.94	−4.50
	MktCap 4	8.45	2.10	−10.49	0.02
	MktCap Diff	18.61	16.42	8.97	14.67
2011/2012	*MktCap* 1	32.28	18.52	14.59	21.80
	MktCap 2	23.86	13.74	18.93	18.85
	MktCap 3	29.16	17.57	19.85	22.19
	MktCap 4	13.06	19.13	18.04	16.74
	MktCap Diff	−19.22	0.62	3.44	−5.05

5.3.4 Summarizing the Results

The procedure for summarizing the results for each of the time series of portfolio average values in a bivariate dependent-sort portfolio analysis is identical to that for univariate portfolio analysis and bivariate independent-sort portfolio analysis, described in Sections 5.1.4 and 5.2.4, respectively. For each portfolio, the time-series mean and inferential statistics are calculated. If the outcome variable Y is a return variable, then risk-adjustment may be performed. The Newey and West (1987) adjustment is usually employed.

The summarized results for our example are presented in Table 5.23. As with the bivariate independent-sort analysis, we present results for only the average one-year-ahead excess portfolio returns, risk-adjusted alphas using the FFC model, and the corresponding t-statistics, which are adjusted following Newey and West (1987) using six lags. For reasons discussed earlier, in a dependent-sort portfolio analysis, results for the average *MktCap* portfolio, as well as the β difference portfolios, are not calculated.

5.3.5 Interpreting the Results

The main difference in the interpretation of the bivariate dependent-sort portfolio analysis is that the only relation we are interested in understanding is the relation

TABLE 5.23 Bivariate Dependent-Sort Portfolio Results Risk-Adjusted Summary
This table presents the results of a bivariate dependent-sort portfolio analysis of the relation between *MktCap* and future stock returns after controlling for β.

	Coefficient	β 1	β 2	β 3	β Avg
MktCap 1	Excess return	27.82	20.52	16.89	21.74
		(7.29)	(6.18)	(5.56)	(6.95)
	FFC α	21.91	12.30	6.83	13.68
		(7.49)	(5.44)	(2.09)	(5.08)
MktCap 2	Excess return	14.05	12.65	9.30	12.00
		(4.82)	(4.05)	(5.06)	(5.34)
	FFC α	4.39	0.73	−1.84	1.09
		(2.45)	(0.29)	(−1.35)	(0.91)
MktCap 3	Excess return	10.67	10.21	8.46	9.78
		(3.62)	(6.21)	(4.87)	(6.20)
	FFC α	0.24	1.10	−0.54	0.27
		(0.09)	(1.75)	(−0.37)	(0.42)
MktCap 4	Excess return	8.84	8.79	8.67	8.77
		(4.17)	(6.55)	(3.46)	(5.62)
	FFC α	0.31	1.51	−0.45	0.46
		(0.17)	(2.35)	(−0.33)	(1.33)
MktCap Diff	Excess return	−18.98	−11.73	−8.22	−12.98
		(−6.08)	(−3.74)	(−2.66)	(−4.50)
	FFC α	−21.60	−10.79	−7.27	−13.22
		(−5.74)	(−4.26)	(−3.00)	(−4.87)

between $X2$ and Y after controlling for $X1$. Interpretation of the results therefore will focus on the difference portfolios for the second sort variable. Statistically significant differences indicate a cross-sectional relation between $X2$ and Y after controlling for $X1$. Otherwise, interpretation of the results of the bivariate dependent-sort portfolio analysis is similar to that for the bivariate independent-sort portfolio analysis.

Examination of the results in Table 5.23 indicates that there is a strong negative cross-sectional relation between *MktCap* and future portfolio returns, as within each β group, the difference in returns between the portfolio comprised high-*MktCap* stocks and that of low-*MktCap* stocks is negative, economically large, and highly statistically significant. These results persist after adjusting for risk using the FFC risk model. The abnormal returns relative to the FFC model are driven by the low-*MktCap* portfolios, as the alphas of low-*MktCap* portfolio (*MktCap* 1) are statistically significant, but this is not the case for the second, third, and fourth *MktCap* portfolios (with two exceptions). Thus, it can be seen that, in this case, the results of the bivariate dependent-sort portfolio analysis are qualitatively the same as those of the independent-sort analysis discussed in Section 5.2.4. While it is usually the case that both types of bivariate-sort analyses produce similar results, this is not necessarily the case. Thus, it is standard to check the robustness of any results using both sorting methodologies.

5.3.6 Presenting the Results

The presentation of the results of bivariate dependent-sort portfolio analysis is basically identical to that of the independent-sort analysis, except that results for the $X1$ difference portfolios and $X2$ average portfolios are not presented. The reason for this is that, as discussed earlier, the objective of the dependent-sort analysis is to examine the relation between $X2$ and Y after controlling for $X1$. The excluded portfolios are of no use in assessing this relation. Due to the similarities between the presentation of the results for bivariate independent-sort portfolio analyses and bivariate dependent-sort analyses, we describe only briefly the different presentation styles for dependent-sort results. For completeness, however, we present the results of analyses analogous to all results presented in Section 5.2.6. Of course, in this section, the results are different because they are generated by dependent-sort, not independent-sort, analyses.

In Table 5.24, we present the average return for each of the 12 portfolios formed by sorting on β and then *MktCap*, as well as for the average β portfolio within each *MktCap* group. At the bottom of the table, we present average excess returns and FFC alphas for the *MktCap* difference portfolios, along with the associated Newey and West (1987) adjusted t-statistics. The results indicate that the negative relation between *MktCap* and future stock returns persists after controlling for β. The relation is strong within each of the β groups.

Notice that in Table 5.24, for *MktCap* groups one through four, we presented only the average excess returns, not the FFC alphas, as was done in Table 5.15. Presenting the excess returns for all portfolios and FFC alphas for only the difference portfolios is a common approach. The reason for this is that, if the excess returns do not exhibit a pattern across the different groups of $X2$ (*MktCap* in this case), then there is usually

TABLE 5.24 Bivariate Dependent-Sort Portfolio Results

This table presents the average abnormal returns relative to the FFC model for portfolios sorted dependently into three β groups and then, within each of the β groups, into four *MktCap* groups. The breakpoints for the β portfolios are the 30th and 70th percentiles. The breakpoints for the *MktCap* portfolios are the 25th, 50th, and 75th percentiles. Table values indicate the alpha relative to the FFC model with the corresponding t-statistics in parentheses.

	β 1	β 2	β 3	β Avg
MktCap 1	27.82	20.52	16.89	21.74
MktCap 2	14.05	12.65	9.30	12.00
MktCap 3	10.67	10.21	8.46	9.78
MktCap 4	8.84	8.79	8.67	8.77
MktCap 4-1	−18.98	−11.73	−8.22	−12.98
	(−6.08)	(−3.74)	(−2.66)	(−4.50)
FFC α	−21.60	−10.79	−7.27	−13.22
	(−5.74)	(−4.26)	(−3.00)	(−4.87)

little need to risk-adjust the returns. While it is possible that there is no significant pattern in returns but a significant pattern emerges after adjusting for risk, this sort of result is rare.

The results in Table 5.25 show the results of bivariate dependent-sort analyses of the relation between *MktCap* and future stock returns after controlling for each of β and *BM*. The table presents the average excess returns and FFC alphas for the *MktCap* difference portfolio within each group of the control variable, which is indicated in the first column of the table. The results indicate that the relation between *MktCap* and future stock returns is strong after controlling for each of β and *BM*. The results of the dependent-sort analyses are therefore similar, and lead to the same conclusions, as the results of the corresponding independent-sort analyses (see Table 5.16). The only exception to this is for the difference in differences portfolios (column 3-1). The results indicate that the negative relation between *MktCap* and future stock excess returns is much stronger for low-β stocks than for high-β stocks as the average excess return of the difference in differences portfolio is 10.77% per year with a corresponding t-statistic of 6.60. The FFC alpha for this portfolio of 14.32% per year (t-statistic = 5.99) is even larger. On the other hand, the negative relation between *MktCap* and future stock excess returns is stronger in high-*BM* stocks than in low-*BM* stocks because the difference in differences portfolio generates an average return of −15.18% per year (t-statistic = −8.17) and alpha of −23.76% per year (t-statistic = −6.86).

In Table 5.26, we present the excess return and FFC alpha for average portfolio across all groups of the control variable within each group of *MktCap*. These results help show that the strong negative relation between *MktCap* and future stock returns is driven mostly by stocks with low values of *MktCap*. The results are qualitatively similar to those from the independent-sort analyses (Table 5.17).

TABLE 5.25 Bivariate Dependent-Sort Portfolio Results—Differences
This table presents the average abnormal returns relative to the FFC model for long–short zero-cost portfolios that are long stocks in the highest quartile of *MktCap* and short stocks in the lowest quartile of *MktCap*. The portfolios are formed by sorting all stocks independently into groups based on β and *MktCap*. The breakpoints used to form the β groups are the 30th and 70th percentiles of β. Table values indicate the alpha relative to the FFC model with the corresponding t-statistics in parentheses.

Control	Coefficient	1	2	3	Avg	3-1
β	Excess return	−18.98	−11.73	−8.22	−12.98	10.77
		(−6.08)	(−3.74)	(−2.66)	(−4.50)	(6.60)
	FFC α	−21.60	−10.79	−7.27	−13.22	14.32
		(−5.74)	(−4.26)	(−3.00)	(−4.87)	(5.99)
BM	Excess return	−6.79	−10.31	−21.98	−13.03	−15.18
		(−2.01)	(−3.10)	(−7.04)	(−4.19)	(−8.17)
	FFC α	−4.26	−9.05	−28.02	−13.77	−23.76
		(−1.64)	(−2.81)	(−8.06)	(−5.59)	(−6.86)

TABLE 5.26 Bivariate Dependent-Sort Portfolio Results—Averages
This table presents the average abnormal returns relative to the FFC model for portfolios formed by sorting independently on β and *MktCap*. The table shows the portfolio FFC alphas and the associated Newey and West (1987)-adjusted t-statistics calculated using six lags (in parentheses) for the average β group within each group of *MktCap*.

Control	Coefficient	*MktCap* 1	*MktCap* 2	*MktCap* 3	*MktCap* 4	*MktCap* 4-1
β	Excess return	21.74	12.00	9.78	8.77	−12.98
		(6.95)	(5.34)	(6.20)	(5.62)	(−4.50)
	FFC α	13.68	1.09	0.27	0.46	−13.22
		(5.08)	(0.91)	(0.42)	(1.33)	(−4.87)
BM	Excess return	22.01	12.78	10.06	8.99	−13.03
		(6.84)	(6.27)	(6.66)	(6.62)	(−4.19)
	FFC α	14.64	3.48	0.54	0.87	−13.77
		(5.45)	(2.47)	(1.29)	(2.00)	(−5.59)

In Table 5.27, we present results of bivariate dependent-sort portfolio analyses of the relation between future stock returns and each of *MktCap* and *BM* after controlling for β. The table shows the average excess returns and FFC alphas for the *MktCap* and *BM* difference portfolios within each β group. Consistent with the independent-sort analyses (Table 5.18), the negative relation between *MktCap* and future stock returns and the positive relation between *BM* and future stock returns are both strong when using dependent-sort analyses. The strength of the positive relation between *BM* and future stock returns appears to be quite similar for stocks with high and low βs because the difference in differences portfolio (β 3-1 column) generates statistically insignificant average excess returns and alpha.

TABLE 5.27 Bivariate Dependent-Sort Portfolio Results—Differences

This table presents the average excess returns and FFC alphas for portfolios formed by sorting independently on β and a second sort variable, which is either *MktCap* or *BM*. The table shows the average excess returns and FFC alphas, along with the associated Newey and West (1987)-adjusted t-statistics calculated using six lags (in parentheses), for the difference between the portfolios with high and low values of the second sort variable (*MktCap* or *BM*). The first column indicates the second sort variable. The remaining columns correspond to different β groups, as indicated in the header.

Sort Variable	Coefficient	β 1	β 2	β 3	β Avg	β 3-1
MktCap	Excess return	−18.98	−11.73	−8.22	−12.98	10.77
		(−6.08)	(−3.74)	(−2.66)	(−4.50)	(6.60)
	FFC α	−21.60	−10.79	−7.27	−13.22	14.32
		(−5.74)	(−4.26)	(−3.00)	(−4.87)	(5.99)
BM	Excess return	12.33	9.29	9.84	10.48	−2.49
		(4.72)	(3.21)	(3.77)	(5.27)	(−1.09)
	FFC α	12.38	6.82	7.38	8.86	−5.00
		(5.54)	(2.07)	(2.25)	(3.45)	(−1.46)

TABLE 5.28 Bivariate Dependent-Sort Portfolio Results—Averages

This table presents the average excess returns and FFC alphas for portfolios formed by sorting independently on β and a second sort variable, which is either *MktCap* or *BM*. The table shows the average excess returns and FFC alphas, along with the associated Newey and West (1987)-adjusted t-statistics calculated using six lags (in parentheses), for the difference between the portfolios with high and low values of the second sort variable (*MktCap* or *BM*). The first column indicates the second sort variable. The remaining columns correspond to different β groups, as indicated in the header.

Sort Variable	Coefficient	1	2	3	4	4-1
MktCap	Excess return	21.74	12.00	9.78	8.77	−12.98
		(6.95)	(5.34)	(6.20)	(5.62)	(−4.50)
	FFC α	13.68	1.09	0.27	0.46	−13.22
		(5.08)	(0.91)	(0.42)	(1.33)	(−4.87)
BM	Excess return	8.63	11.93	13.89	19.11	10.48
		(4.98)	(7.38)	(8.09)	(7.68)	(5.27)
	FFC α	0.97	3.80	4.85	9.83	8.86
		(0.59)	(3.23)	(4.47)	(5.34)	(3.45)

Finally, in Table 5.28, we present the results of the same bivariate dependent-sort portfolio analyses whose results were presented in Table 5.27. In Table 5.28, however, we show the results for the average β portfolio within each of the different groups of *MktCap* and *BM*. The columns labeled 1, 2, 3, 4, and 4-1 indicate the *MktCap* or *BM* group for which the average β portfolio results are shown. The results in the column labeled 4-1 demonstrate that the negative relation between *MktCap* and future stock returns as well as the positive relation between *BM* and future stock returns, both persist after controlling for β. These results are qualitatively similar to those of the independent-sort analyses (Table 5.19).

In summary, our examples show that the results of bivariate independent-sort and dependent-sort portfolio analyses produce similar results. While this will usually be the case, it is not necessarily so. It is therefore a good idea to perform both sets of analyses. If the results of the independent-sort and dependent-sort analyses lead to substantially different conclusions, then further investigation is warranted.

5.4 INDEPENDENT VERSUS DEPENDENT SORT

Having separately presented the bivariate independent-sort and dependent-sort portfolio methodologies, we proceed to a comparison of the two sorting procedures. While in most cases both sorting procedures produce qualitatively similar results, this is not always the case, and when it is not the case, it is important to understand what may be driving the difference. As mentioned previously, the most salient difference between the two types of bivariate portfolio analyses is that dependent-sort analysis can only be used to examine the relation between the second sort variable, $X2$, and the outcome variable, Y, after controlling for $X1$. Independent-sort analysis permits examination of this relation, as well as the relation between $X1$ and Y, controlling for $X2$. Our discussion therefore focuses on differences in the examination of the relation between $X2$ and Y after controlling for $X1$, as investigation of this relation is common to both types of sorts. We exemplify the differences between the sorting procedures using the analyses that take β to be the first sort variable and *MktCap* to be the second sort variable.

When using the independent-sort procedure, each of the sorts is an unconditional sort, meaning that the sort on $X2$ (*MktCap* in our example) is performed on all entities regardless of the value of $X1$ (β in our example). As a result, any given portfolio $P_{j,k,t}$ contains the set of entities that fall into the jth group based on sort variable $X1$ and the kth *unconditional* group based on sort variable $X2$. For example, stocks in the low-β and low-*MktCap* portfolio represent stocks that have both unconditionally low values of β and, more importantly, unconditionally low values of *MktCap*. Similarly, stocks in the low-β and high-*MktCap* portfolio have unconditionally low values of β and, more importantly, unconditionally high values of *MktCap*. Thus, when taking the difference in returns between the high-*MktCap* portfolio and the low-*MktCap* portfolio, we are comparing average returns for stocks with unconditionally high values of *MktCap* and stocks with unconditionally low values of *MktCap*, among only stocks with low values of β. Similarly, when taking the difference in returns between the high-*MktCap* and low-*MktCap* portfolios in the high-β group, the comparison is once again between stocks with unconditionally high and low values of *MktCap*, this time among only stocks with high values of β.

To see this, in Table 5.29 we present the average values of *MktCap* for each of the 12 portfolios generated by the bivariate independent-sort analysis. The results show that, regardless of the level of β, the average values of *MktCap* within each *MktCap* quartile are similar (a small exception may be found in the high-*MktCap* group).

TABLE 5.29 Bivariate Independent-Sort Portfolio Average *MktCap*

This table presents the average *MktCap* for portfolios formed by sorting independently on β and *MktCap*.

	β 1	β 2	β 3
MktCap 1	21	22	25
MktCap 2	85	110	117
MktCap 3	421	432	421
MktCap 4	8767	7427	6983

Thus, regardless of which β group we are examining, the difference in average excess returns between the high-*MktCap* and low-*MktCap* portfolios represents a difference in average excess returns between stocks with unconditionally high values of *MktCap* and unconditionally low values of *MktCap*. Despite the fact that we are, in some way, controlling for the effect of β ($X1$) when examining the relation between *MktCap* ($X2$) and future stock returns (Y), the results of an independent-sort analysis must, therefore, be interpreted as indicative of the unconditional relation between *MktCap* ($X2$) and future excess stock returns (Y).

When using the dependent-sort methodology, the groupings on the second sort variable are conditional on the values of the first sort variable. Thus, any given portfolio $P_{j,k,t}$ contains entities that fall into the jth group based on sort variable $X1$ and the kth group based on sort variable $X2$ *conditional* on the entity having a value of $X1$ that places it in the jth $X1$ group. To exemplify this, we begin by recalling that the correlations presented in Table 3.3 and the portfolio analysis shown in Table 5.7 indicate a strong positive cross-sectional relation between β and *MktCap*. Thus, stocks in the low-β group are likely to have low values of *MktCap* relative to the entire sample. When performing the second sort within the low-β group, therefore, we are effectively stratifying a group of stocks that are mostly low-*MktCap* stocks into conditional levels of *MktCap*. Thus, the low-β and low-*MktCap* portfolio is likely to contain stocks with very low values of *MktCap*. The low-β and high-*MktCap* portfolio contains stocks with the highest values of *MktCap* among (conditional on) low-β stocks, but these values of *MktCap* may actually be quite low relative to the entire sample because the low-β group contains predominantly low *MktCap* stocks. Thus, when we calculate the difference in average excess returns between the high-*MktCap* and low-*MktCap* portfolios within the low-β group, we are effectively comparing stocks with conditionally high values of *MktCap* to stocks with conditionally low values of *MktCap*, with β being the conditioning variable. The same can be said for any of the groups of β ($X1$).

In Table 5.30, we present the average values of *MktCap* for each of the portfolios formed using the dependent-sort methodology. The results are exactly as would be expected. Stocks in the low-β and high-*MktCap* portfolio have high values of *MktCap* relative to other stocks with low values of β, but unconditionally, the average *MktCap* of these stocks is not as high as the average *MktCap* of stocks in the high-*MktCap* portfolios within groups two and three of β are much higher. Similarly,

TABLE 5.30 Bivariate Dependent-Sort Portfolio Average *MktCap*

This table presents the average *MktCap* for portfolios formed by sorting dependently on β and then on *MktCap*.

	β 1	β 2	β 3
MktCap 1	10	44	76
MktCap 2	29	202	237
MktCap 3	69	726	689
MktCap 4	3974	9523	9008

the high-β and low-*MktCap* portfolio contains stocks with conditionally low values of *MktCap*, but unconditionally, these stocks do not have low *MktCap*, especially compared to the low-*MktCap* portfolios for the first and second β groups. The results of a dependent-sort analysis are therefore indicative of the relation between *MktCap* ($X2$) and future stock excess returns (Y), conditional on β ($X1$).

In summary, while both independent-sort and dependent-sort analyses control for the effect of one sort variable ($X1$) while examining the relation between the other sort variable ($X2$) and the outcome variable (Y), the method of controlling for the effect of the first variable is different. Independent-sort analyses examine the unconditional relation between $X2$ and Y, while dependent sorts examine the relation between $X2$ and Y conditional on $X1$.

5.5 TRIVARIATE-SORT ANALYSIS

While univariate-sort and bivariate-sort portfolio analyses are most common, some researchers employ trivariate-sort portfolio analysis to assess the relations between three sort variables and an outcome variable. As with the bivariate-sort analysis, trivariate sorts can be independent or dependent in nature. In fact, it is possible to make the second sort dependent on the first sort, but the third sort independent of the second sort, or vice versa. The procedure for implementing a trivariate-sort portfolio analysis can easily be inferred from the above discussions of bivariate independent-sort and dependent-sort analyses, and thus will not be discussed in detail here. Perhaps the main drawback of trivariate portfolio analyses is that, unless the sample being used is very large or the number of breakpoints used in each sort is low, the number of entities in each portfolio is likely to be quite small. This is especially true when using independently sorted portfolios with sort variables that exhibit substantial cross-sectional correlation.

5.6 SUMMARY

In summary, in this section, we have discussed the procedure for implementing univariate-sort, bivariate dependent-sort, and bivariate independent-sort portfolio analyses. Univariate-sort analysis, as the name implies, examines the cross-sectional relation between two variables. Bivariate-sort analyses examine the relation between a given sort variable and the outcome variable after controlling for the effect of the other sort variable. While the interpretation of the results of the different types of bivariate-sort analyses differs slightly, in most cases, they lead to similar conclusions. When the outcome variable represents a security return, then the average portfolio values represent portfolio returns. In this case, it is usually appropriate to risk-adjust the excess returns using a factor model.

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