A Differentiable POGLM with Forward-Backward Message Passing

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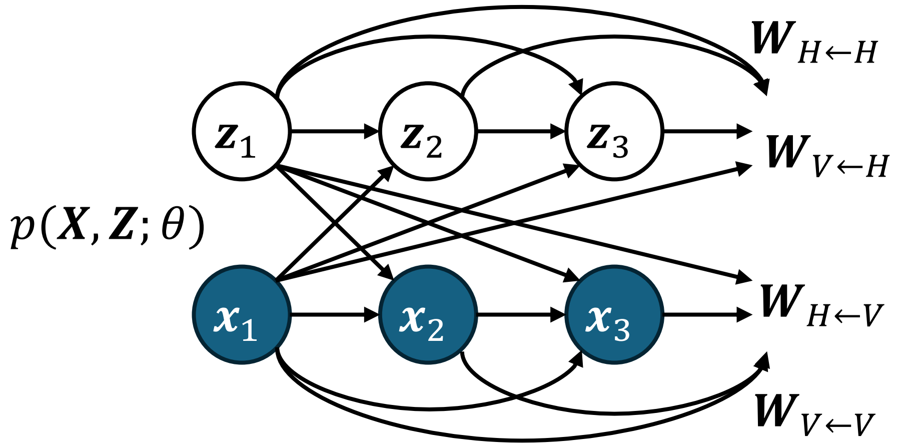
1 Partially observable generalized linear model (POGLM)

Introduction

- The partially observable generalized linear model (POGLM) is a powerful tool for understanding neural connectivity under the assumption of existing hidden neurons.
- POGLM itself is a difficult problem.
- Two main issues and our contributions:
  - The gradient estimator used in variational inference (VI).
  - The sampling scheme of the variational model.
- Comprehensive experiments on one synthetic and two real-world datasets.

POGLM

- V visible neurons and $H = N - V$ hidden neurons.



$$\underbrace{\begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{z}_t \end{bmatrix}}_{\text{visible/hidden spike counts}} \sim \text{Pois}\left(\underbrace{\begin{bmatrix} \boldsymbol{f}_t \\ \boldsymbol{g}_t \end{bmatrix}}_{\text{visible/hidden firing rates}}\right)$$

$$\begin{bmatrix} \boldsymbol{f}_t \\ \boldsymbol{g}_t \end{bmatrix} = \sigma\left(\underbrace{\begin{bmatrix} \boldsymbol{b}_V \\ \boldsymbol{b}_H \end{bmatrix}}_{\text{bias}} + \underbrace{\begin{bmatrix} \boldsymbol{W}_{V \leftarrow V} & \boldsymbol{W}_{H \leftarrow V} \\ \boldsymbol{W}_{V \leftarrow H} & \boldsymbol{W}_{H \leftarrow H} \end{bmatrix}}_{\text{weight}} \underbrace{\left(\sum_{l=1}^{L} \psi_l \begin{bmatrix} \boldsymbol{x}_{t-1} \\ \boldsymbol{z}_{t-1} \end{bmatrix}\right)}_{\text{convolved history}}\right)$$

- Observed variable: visible spike train $\boldsymbol{X} = [\boldsymbol{x}_1, \dots, \boldsymbol{x}_T]^{\mathrm{T}} \in \mathbb{N}^{T \times V}$.
- Latent variable: hidden spike train $\boldsymbol{Z} = [\boldsymbol{z}_1, \dots, \boldsymbol{z}_T]^{\mathrm{T}} \in \mathbb{N}^{T \times H}$.
- Generative parameter set: $\theta = \left\{\boldsymbol{b} \in \mathbb{R}^{V+H}, \boldsymbol{W} \in \mathbb{R}^{(V+H) \times (V+H)}\right\}$.

2 Variational inference (VI)

Variational inference (VI)

- Variational model $q(\boldsymbol{Z}|\boldsymbol{X};\phi)$ parameterized by ϕ.
- **Inference methods**: $\boldsymbol{z}_t \sim \mathrm{Pois}(\boldsymbol{g}_t)$.
- **Sampling schemes**: $\boldsymbol{g}_t = \mathrm{function}(\boldsymbol{X};\phi)$.
- Evidence lower bound:

$$\mathrm{ELBO}(\boldsymbol{X};\theta,\phi) = \mathbb{E}_q[\ln p(\boldsymbol{X},\boldsymbol{Z};\theta) - \ln q(\boldsymbol{Z}|\boldsymbol{X};\phi)]$$

Inference methods: Gradient estimators

	score function	pathwise
distribution	any distribution	continuous distribution with reparameterization trick $\boldsymbol{Z}\vert\boldsymbol{X};\phi = r(\boldsymbol{\epsilon}\vert\boldsymbol{X};\phi)$
samples	$\{\boldsymbol{Z}^{(k)}\}_{k=1}^{K} \sim q(\boldsymbol{Z}\vert\boldsymbol{X};\phi)$	$\{\boldsymbol{\epsilon}^{(k)}\}_{k=1}^{K} \sim R(\boldsymbol{\epsilon})$
$\frac{\partial \mathrm{ELBO}(\boldsymbol{X};\theta,\phi)}{\partial\phi} \approx$	$\frac{\partial}{\partial\phi}\frac{-1}{2K}\sum_{k=1}^{K}\left[\ln p\left(\boldsymbol{X},\boldsymbol{Z}^{(k)};\theta\right) - \ln q\left(\boldsymbol{Z}^{(k)}\vert\boldsymbol{X};\phi\right)\right]$	$\frac{\partial}{\partial\phi}\frac{1}{K}\sum_{k=1}^{K}\left[\ln p\left(\boldsymbol{X},r\left(\boldsymbol{\epsilon}^{(k)}\vert\boldsymbol{X};\phi\right);\theta\right) - \ln q\left(r\left(\boldsymbol{\epsilon}^{(k)}\vert\boldsymbol{X};\phi\right)\vert\boldsymbol{X};\phi\right)\right]$

Inference methods: Relaxation

Inference methods: Candidate distributions

- Replace the distribution $\mathbb{P}[z; g]$ in the generative and variational model with the following distributions governed by $\mathbb{E}[z] = g$.

Sampling schemes

3 Experiments

Synthetic

Retinal ganglion neurons (Pillow & Scott, 2012)

Primary visual cortex (cncrs: PVC-5)

Summary

Thanks for listening!