

EE3980 Algorithms

Homework 5. Trading Stock II

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1. Introduction

In this homework, we improve our previous homework, trading stocks with finding the maximum contiguous subarray by both brute-force and divide-and-conquer method. As for the brute-force approach, we reduce the time complexity from $O(n^3)$ to $O(n^2)$, which might deviate from using the maximum subarray approach; and for the divide-and-conquer with time complexity of $O(n * \log n)$, we adopt a method called Kadane's algorithm to make it reach just $O(n)$. In the end, we compare the time consumed of both methods with what we got in the last homework, and analyze the cause of the differences.

2. Implementation

In the program, we use the main structure of the previous homework, and only revise two parts of the code. First, we revise the **MaxSubArrayBF** function to make it more efficient. Second, we replace the divide-and conquer method by Kadane's algorithm, which would be explained later, in the **MaxSubArray** function.

2.1. Brute-force approach

In our previous homework, we implement the brute-force approach with the time complexity of $O(n^3)$, since it has to runs through the array with the start and end points of the subarray, which contributes to $O(n^2)$, and then sum up all the items in the subarray, which contributes to $O(n)$.

```
1. Algorithm MaxSubArrayBF(A, n, low, high)
2. {
3.     max := 0; low := 1; high := n;           // Initialize
4.     for j := 1 to n do {                     // Try all possible ranges: A[j:k]
5.         for k := j to n do {
6.             sum := 0;
7.             for i := j to k do {             // Summation for A[j:k]
8.                 sum := sum + A[i];
9.             }
10.            if (sum > max) then {              // Record the max value and range
11.                max := sum; low := j; high := k;
12.            }
13.        }
14.    }
15.    return max;
16. }
```

We have to implement this way since we treat it as the maximum contiguous subarray sum problem. However, we had known the actual price of stocks at each moment, and the contiguous subarray sum stands for the price difference between the start point and the end point. So, we could get the sum just by calculating the difference of the prices between the buy date and sell date, instead of summing up the

price differences through the array. Thus, we replace line 6 ~ 9 by only one assign statement.

```
1. Algorithm MaxSubArrayBF_revised(A,n,low,high)
2. {
3.     max := 0; low := 1; high := n;    // Initialize
4.     for j := 1 to n do {              // Try all possible ranges: A[j:k]
5.         for k := j to n do {
6.             sum = A[k] - A[j];        // price difference between k and j
7.             if (sum > max) then {      // Record the maximum value and range
8.                 max := sum; low := j; high := k;
9.             }
10.        }
11.    }
12.    return max;
13. }
```

For the time complexity, the outer and inner loop still contribute $O(n^2)$, where the iterations may go through the array. And at line 6 ~ 9, there are at most only one comparison and four assignment, which contribute a time complexity of constant time. Thus, the overall time complexity of the brute-force approach becomes $O(n^2)$.

For the space complexity, we also need extra parameters: i, j, k, max, low, high, and the initial array with size N like the previous homework. So, the space complexity is $N + 6$ which is $O(n)$.

Time complexity: $O(n^2)$

Space complexity: $O(n)$

2.2. Kadane's algorithm

In our previous homework, we implemented the divide-and-conquer method to solve the maximum contiguous sum problem, with the time complexity of $O(n * \log n)$. Yet, we try to use the Kadane's algorithm, contributed by an American professor of computer science Joseph B. Kadane, in the **MaxSubArray** function to reduce the time complexity of the problem.

```
1. Algorithm Kadane(A, n, start, end)
2. {
3.     start := 2; end := 1;           // initialize two ends
4.     start_tmp := 2;                // temporary start index
5.     max := 0; now := 0;            // initialize values
6.     for i := 2 to n do {           // go through the array
7.         now := now + A[i];          // subarray value until i
8.         if (now < 0) then {          // if value < 0, reset
9.             now := 0;
10.            start_tmp := i + 1;
11.        }
12.        if (now > max) then {        // record if value is largest
13.            max := now;
14.            start := start_tmp;
15.            end := i;
16.        }
17.    }
18.    return max;
19. }
```

The feature of Kadane's algorithm is that the iteration only goes through the array one time, and it doesn't need any recursion. In the algorithm, we use two variables

max and *now* to record the value of the maximum contiguous sum so far and the contiguous sum with $A[i]$, respectively.

During each iteration step, we first add the current datum $A[i]$ to the variable *now*, which then stands for the contiguous subarray sum including $A[i]$. And if *now* is less than zero, it means that the contiguous sum is negative. That is, the price is lower at this point than where the sum take start. Thus, we reset the sum by assign *now* to zero, and take the current index to be the new beginning of the contiguous sum.

Then, we check whether the contiguous sum *now* is larger than the contiguous sum so far. If yes, we refresh the value of *max* to record the maximum sum since we want to get the maximum value until now. At the end of each iteration step, we can get the maximum contiguous sum from 1 to the iteration index i , which is either the maximum contiguous sum we recorded before or the contiguous sum including $A[i]$. Therefore, when the iteration goes to end, we could find the maximum contiguous sum from 1 to n .

For the time complexity, the loop at line 6 goes through the whole array with size n , which contribute the time complexity of $O(n)$. In the loop, there are only comparisons, addition, and assignment operations, which would take constant time.

Thus, the overall time complexity is $O(n)$, and that means we can solve the maximum contiguous sum problem by using Kadane's algorithm with linear time.

As for the space complexity, we need extra parameters: i , $start$, $start_tmp$, end , max , now and the initial array with size N . So, the space complexity is $N + 6$ which is $O(n)$.

Time complexity: $O(n)$

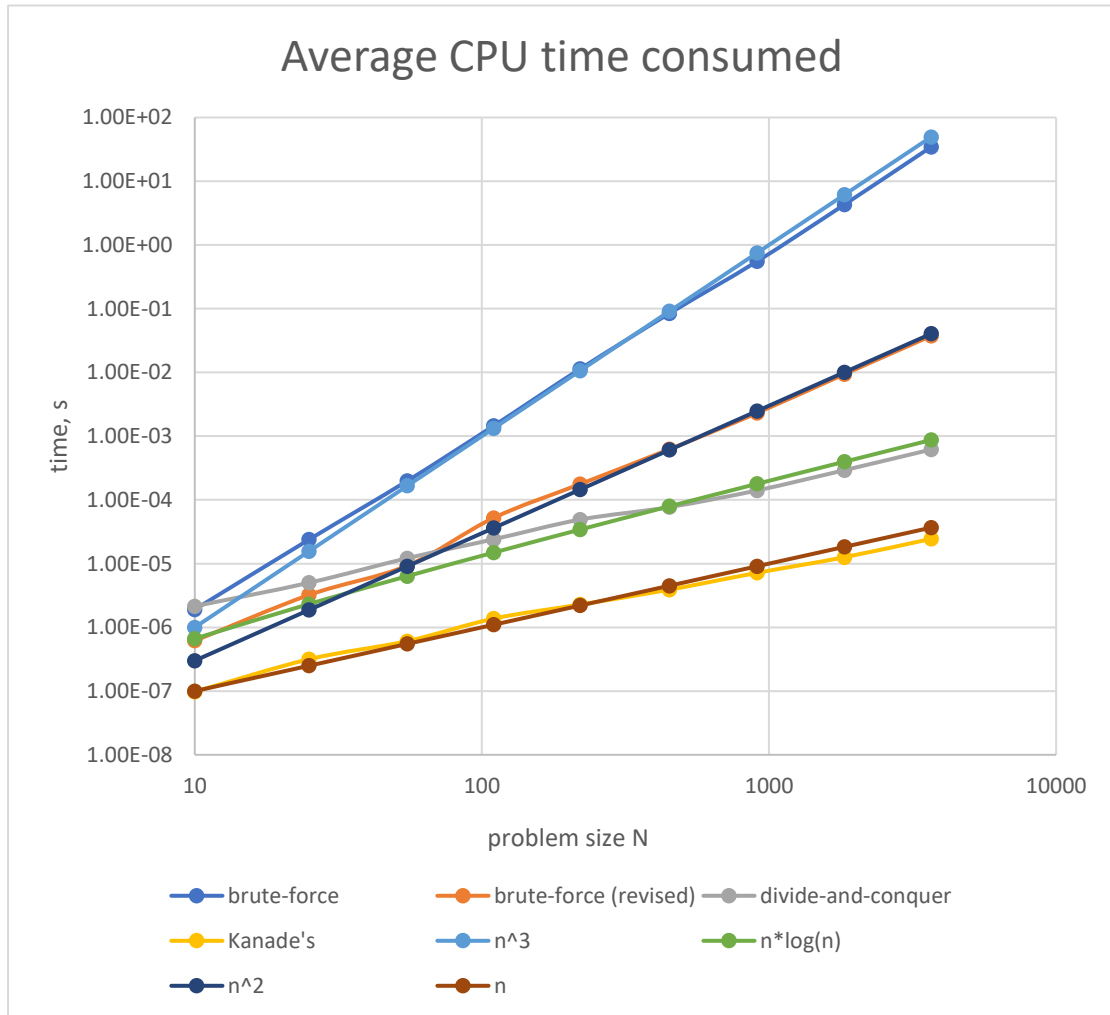
Space complexity: $O(n)$

3. Executing results

We run the testing data from $s1.dat$ to $s9.dat$ with different input data size by revised brute-force approach and Kadane's algorithm for 500 times, record the average CPU time used, and compare to the results we got in the previous homework.

Data size	Brute-force approach	Brute-force approach (revised)	Divide-and-conquer	Kadane's algorithm	Earning per share
10	$1.907 \mu s$	616.1ns	$2.146 \mu s$	97.75ns	9.065
25	$23.84 \mu s$	$3.222 \mu s$	$5.007 \mu s$	315.7ns	20.81
55	$198.8 \mu s$	$9.306 \mu s$	$12.16 \mu s$	608.0ns	96.02
110	1.456ms	$52.25 \mu s$	$24.08 \mu s$	$1.372 \mu s$	103.9
220	11.35ms	$177.0 \mu s$	$49.11 \mu s$	$2.280 \mu s$	204.1
450	84.34ms	$622.69 \mu s$	$77.01 \mu s$	$3.902 \mu s$	371.6

910	555.1ms	2.315ms	140.9 μ s	7.176 μ s	641.8
1830	4.295s	9.386ms	292.1 μ s	12.58 μ s	641.8
3671	34.56s	37.41ms	617.0 μ s	24.46 μ s	1185



4. Result analysis and conclusion

From the graph, we could observe that the advised brute-force approach has a trend of n^2 , and the Kadane's algorithm has a trend of n , which are same as our estimation.

Compared to the previous homework, the time complexity of the brute-force approach successfully reduces from $O(n^3)$ to $O(n^2)$, and we had found an algorithm whose time complexity is lower than $O(n * \log n)$. There are large scales of improvements on both the methods.

The Kadane's algorithm is the fastest on all input data. It might be the fact that it runs through the array for only one time, and it doesn't need any recursive function calls. Thus, the algorithm could be used on a wide range of input data size, making it faster than all other methods we mentioned above. Furthermore, if we just want to know the contiguous sum instead of the indexes, we could take off the start, start_tmp, end variables in the algorithm, which could be much faster.

The least time complexity of solving maximum contiguous sum we got so far is $O(n)$, and it might not be less. Since we must know the content of each index in the array with size n to solve the problem, we have to run through the array to get the values, which contributes the time complexity with $O(n)$. Thus, $O(n)$ must be the least time complexity of the maximum contiguous sum problem.