Homework #4 STA 360

Jerry Xin STA 360: Homework 4

Due Friday September 17th, 5 PM EDT

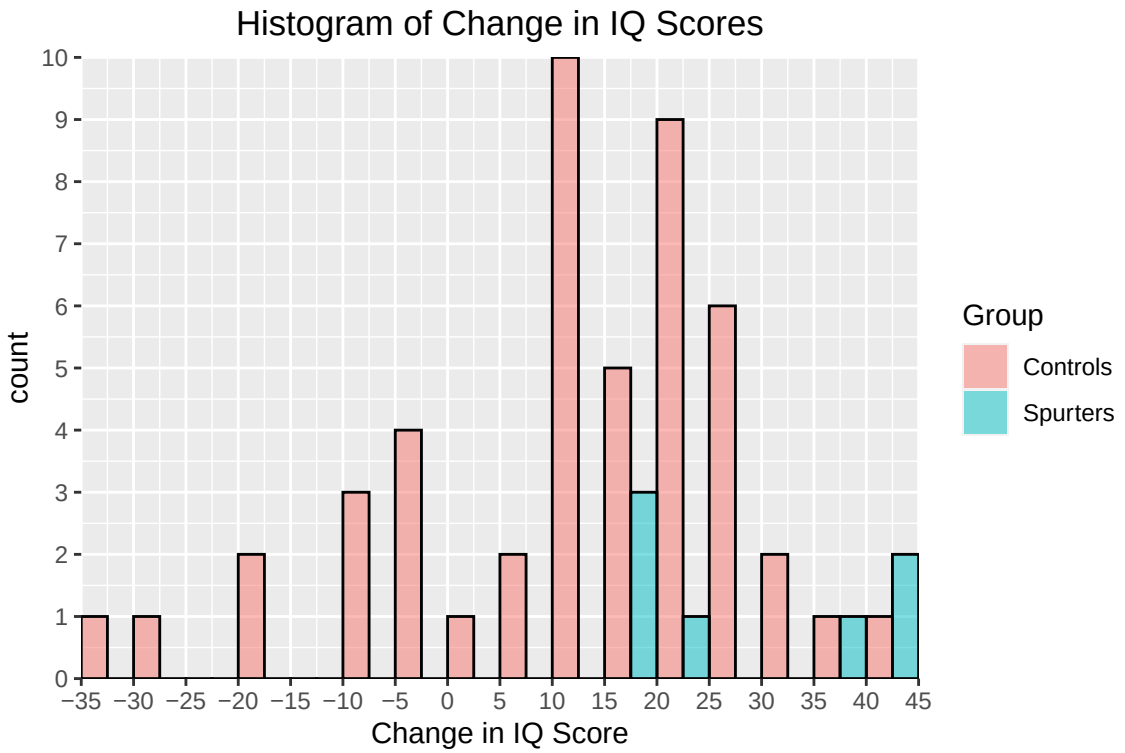
```
library(plyr)
library(ggplot2)
library(dplyr)
library(xtable)
library(reshape)
library(tidyverse)
```

1. (10 points, 5 points each) Hoff, 3.10 (Change of variables).
2. Lab component (25 points total) Please refer to lab 4 and complete tasks 4—5.

(a) (10) Task 4 (Finish for Homework)

```
set.seed(123)
# input data
# spurters
x = c(18, 40, 15, 17, 20, 44, 38)
# control group
y = c(-4, 0, -19, 24, 19, 10, 5, 10,
      29, 13, -9, -8, 20, -1, 12, 21,
      -7, 14, 13, 20, 11, 16, 15, 27,
      23, 36, -33, 34, 13, 11, -19, 21,
      6, 25, 30,22, -28, 15, 26, -1, -2,
      43, 23, 22, 25, 16, 10, 29)
# store data in data frame
iqData = data.frame(Treatment = c(rep("Spurters", length(x)),
                                  rep("Controls", length(y))),
                                  Gain = c(x, y))
```

```
xLimits = seq(min(iqData$Gain) - (min(iqData$Gain) %% 5),
              max(iqData$Gain) + (max(iqData$Gain) %% 5),
              by = 5)

ggplot(data = iqData, aes(x = Gain, fill = Treatment, colour = I("black"))) +
  geom_histogram(position = "dodge", alpha = 0.5, breaks = xLimits, closed = "left")+
  scale_x_continuous(breaks = xLimits,
                     expand = c(0,0))+
  scale_y_continuous(expand = c(0,0),
breaks = seq(0, 10, by = 1))+
  ggtitle("Histogram of Change in IQ Scores") + labs(x = "Change in IQ Score",
                                                     fill = "Group") +
                                                theme(plot.title = element_text(hjust = 0.5))
```




```
prior = data.frame(m = 0, c = 1, a = 0.5, b = 50)
findParam = function(prior, data){
  postParam = NULL
  c = prior$c
  m = prior$m
  a = prior$a
  b = prior$b
  n = length(data)
  postParam = data.frame(m = (c*m + n*mean(data))/(c + n),
                c = c + n,
                a = a + n/2,
                b =  b + 0.5*(sum((data - mean(data))^2)) +
                  (n*c *(mean(data)- m)^2)/(2*(c+n)))
  return(postParam)
}
postS = findParam(prior, x)
postC = findParam(prior, y)
```

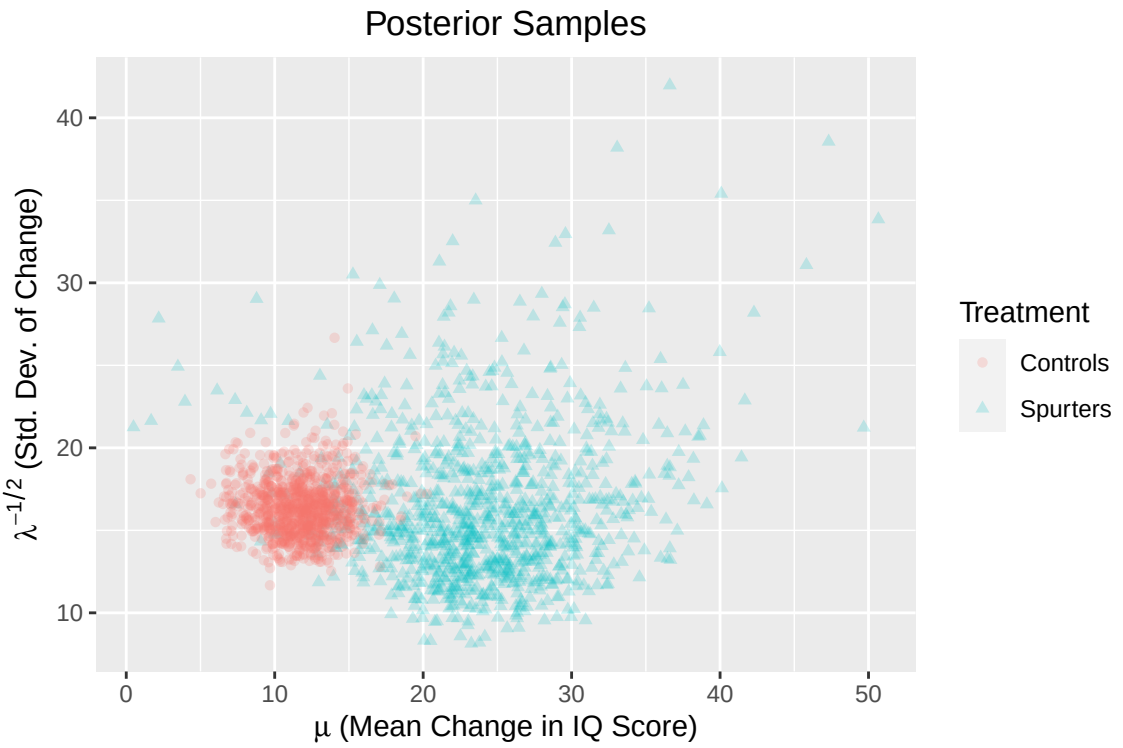
% latex table generated in R 4.0.2 by xtable 1.8-4 package % Fri Sep 10 18:43:35 2021

	m	c	a	b
prior	0.00	1.00	0.50	50.00
Spurters Posterior	24.00	8.00	4.00	855.00
Controls Posterior	11.80	49.00	24.50	6343.98

Table 1: Parameters

```
# sampling from two posteriors

# Number of posterior simulations
sim = 1000
```

```r
# initialize vectors to store samples
mus = NULL
lambdas = NULL
muc = NULL
lambdac = NULL

# Following formula from the NormalGamma with
# the update paramaters accounted accounted for below

lambdas = rgamma(sim, shape = postS$a, rate = postS$b)
lambdac = rgamma(sim, shape = postC$a, rate = postC$b)


mus = sapply(sqrt(1/(postS$c*lambdas)),rnorm, n = 1, mean = postS$m)
muc = sapply(sqrt(1/(postC$c*lambdac)),rnorm, n = 1, mean = postC$m)

# Store simulations
simDF = data.frame(lambda = c(lambdas, lambdac),
                   mu = c(mus, muc),
                   Treatment = rep(c("Spurters", "Controls"),
                                   each = sim))

simDF$lambda = simDF$lambda^{-0.5}

# Plot the simulations
ggplot(data = simDF, aes(x = mu, y = lambda, colour = Treatment, shape = Treatment)) +
  geom_point(alpha = 0.2) +
  labs(x = expression(paste(mu, " (Mean Change in IQ Score)")),
       y = expression(paste(lambda^{-1/2}, " (Std. Dev. of Change)")))  +
  ggtitle("Posterior Samples")+
  theme(plot.title = element_text(hjust = 0.5))
```



Now, we can answer our original question: "What is the posterior probability that $\mu_S > \mu_C$?"

The easiest way to do this is to take a bunch of samples from each of the posteriors, and see what fraction of times we have $\mu_S > \mu_C$. This is an example of a Monte Carlo approximation (much more to come on this in the future).

To do this, we draw $N = 10^6$ samples from each posterior:

```
mp <- 24
cp <- 8
ap <- 4
bp <- 855

ms <- 11.8
cs <- 49
as <- 24.5
bs <- 6344.0

#Sampling from posterior
S <- 1000000


# Number of posterior simulations
sim = 1000000

# initialize vectors to store samples
mu_s = NULL
lambda_s = NULL
mu_c = NULL
lambda_c = NULL

# Following formula from the NormalGamma with
```

```r
# the update paramaters accounted accounted for below

lambda_c = rgamma(S, as, bs)
lambda_s = rgamma(S, ap, bp)

mu_c = rnorm(S, ms, sd = sqrt(1/(cs*lambda_c)))
mu_s = rnorm(S, mp, sd = sqrt(1/(cp*lambda_s)))

# Store simulations
dataF = data.frame(lambda = c(lambda_s, lambda_c),
                   mu = c(mu_s, mu_c),
                   Treatment = rep(c("Spurters", "Controls"),
                                   each = sim))

dataF$lambda = dataF$lambda^{-0.5}
mean(mu_s > mu_c)
```

```
## [1] 0.970602
```

Interpret the posterior probability that you compute above.

After sampling 10^6 times, we get 0.970831. 0.970831 means that 97.0831% of people who were told were spurters performed better than the control group.

(b) (15) Task 5 (Finish for Homework)

Let's return back to the prior assumptions. There are a few ways that you can check that the prior conforms with our prior beliefs. Let's go back and checks these. Draw some samples from the prior and look at them—this is probably the best general strategy. See Figure pygmalion-prior. It's also a good idea to look at sample hypothetical datasets $X_{1:n}$ drawn using these sampled parameter values.

Please replicate a plot similar to Figure pygmalion-prior and report your findings.