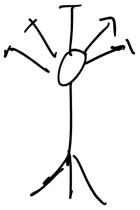
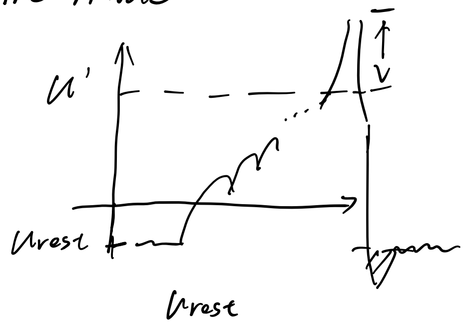
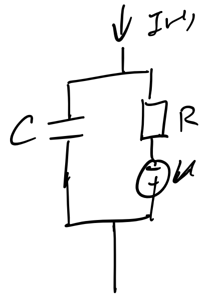
Ch 1 Foundations of Neuronal Dynamics

· Integral - Fire Model







Input : I(t)

输入: 持续的 i(t)

输出: spike u(t)

$\xrightarrow{\text{Model}}$

Information $\ni$ Time

I(t). u(t) 相近

$$I(t) = \frac{U - U_{rest}}{R} + C\frac{du}{dt}$$

$$du = \underline{\underline{I\,dt}} - \frac{U - U_r}{R}\,dt$$

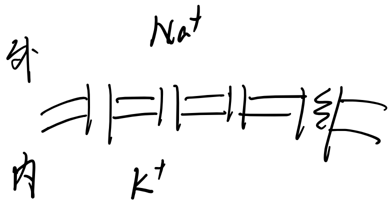
$\downarrow$

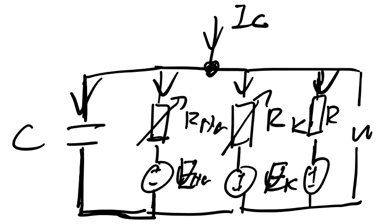
Q

$$U = U_r = \Delta u \exp\left(-\frac{t - t_0}{\tau_m}\right)$$

RI_0 $\quad$ $\underline{\underline{t > t_0}}$

Cha 2 HHM Hodgkin-Huxley Model





$$C \frac{du}{dt} = I_c - \sum I_{ion} \qquad m, n, h$$

$$I_{Na} = g_{Na} m^3 h (u - E_{Na}) \Rightarrow \dot{x} = -\frac{1}{\tau_x(u)} (x - x_0(u))$$

$$I_K = g_K n^4 (u - E_K)$$

$$I_L = g_L (u - E_L)$$

Cha 3 Dendrite & Synapse

- $I_{syn}(t) = g_{syn}(t) \cdot (u(t) - E_{syn})$

  - $g_{syn}(t) = \sum \bar{g}_{syn}(t) \cdot e^{-\frac{(t-t_f)}{\tau}} \cdot \theta(t-t_f)$
  - 不同神经递质 有不同的 τ、$\bar{g}_{syn}$、E_{syn}

- E_{syn}

  兴奋性 : 0 mV

  抑制性 : −75 mV

- Neuro Transmitter
  - 兴奋性神经递质

    NMDA (慢)　AMDA (快)
  - 抑制性

    GABA A (快)　GABA B (慢)

- contact point ∞ learning / memory
- synaptic facilitation/depression ∞ short-term plasticity
  - P_{rel} : Probability of channel releasing transmitter

$$\frac{dP_{rel}}{dt} = -\frac{P_{rel} - P_0}{\tau_P} + f_F (1 - P_{rel}) \sum \delta(t - t_f)$$

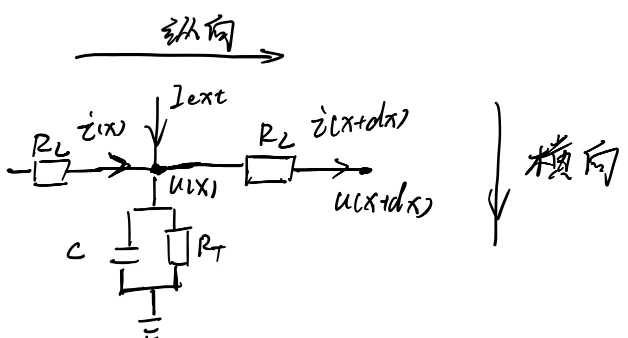
P_0: resting state　　f_F: degree of fasilitation/depression

τ_P: time constant　　$\in [0,1]$

$\bar{g}_{syn} = P_{rel} \cdot g_0$

- Cable Equation

  考虑"纵向"电流



Equation :

$$\frac{\partial u(x,t)}{\partial x} = r_L \cdot i(x)$$

$$\begin{cases} u(x) - u(x+dx) = R_L \cdot i(x+dx) \\ i(x) + I_{ext} = C\frac{du}{dt} = i(x+dx) + \sum I_C \end{cases}$$

$i(x)$

$$\Downarrow$$

$$i(x+dx) - i(x) = C\frac{du}{dt} + \sum I_{ion} - I_{ext}$$

$$\frac{\partial i(t,x)}{\partial x} = C \cdot \frac{\partial u(t,x)}{\partial t} + \sum i_{ion} - i_{ext}$$

$$\Rightarrow \frac{\partial u(x,t)}{\partial x^2} = r_L C \cdot \frac{\partial u(x,t)}{\partial t} + r_L \sum i_{ion} - r_L i_{ext}$$

- Passive Dendrite

$$\sum i_{ion} = \frac{u(x,t)}{r_m}$$

$$\lambda^2 = \frac{r_m}{r_L} \qquad \tau = C\, r_m$$

$$\lambda^2 \frac{\partial u(x,t)}{\partial x^2} = \underbrace{\tau \frac{\partial u(x,t)}{\partial t}} + u(x,t) - i_{ext}(x,t)$$

· Compartmental Dendrite
  - Different branches
  - Active circuit
    - Back Propagating Action Potential (BPAP)
    - Ca spikes

Cha 4 Dimensionality Reduction and Phase Plane Analysis

$$C\frac{du}{dt} = -g_{Na}[m(t)]^3 h(t)(u(t)-E_{Na}) - g_K[n(t)]^4(u(t)-E_K) - g_L(u(t)-E_L) + I(t)$$

Seperation of Time Scales → $m_0(u(t))$ → $1-w(t)$ → $\frac{w(t)}{a}$

Coupled Differention Equation

x ↑↓ C ↑ I

$\tau_1 \frac{dx}{dt} = x + c$

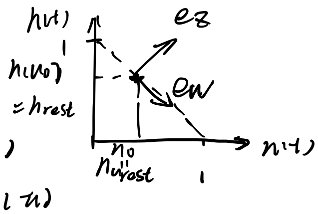
$\tau_2 \frac{dc}{dt} = c + f(x) + I(t)$

$\tau_1 \ll \tau_2$ $\quad x = c(t)$

Similarities of n-h

$1-h(t) = a\,n(t) = w(t)$

$w(t)$: $\frac{dw}{dt} = \frac{w - w_0(u)}{\tau_{eff}(u)}$



4D → 2D

$$\Rightarrow \begin{cases} C\frac{du}{dt} = f(u(t), w(t)) + I(t) \\ \frac{dw}{dt} = g(u(t), w(t)) \end{cases}$$

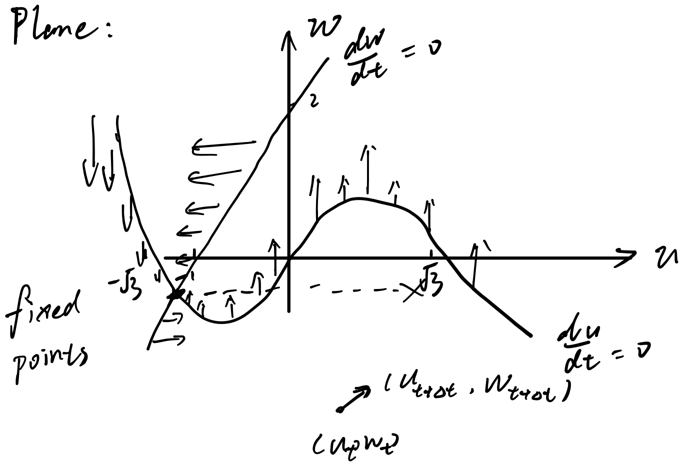
· Phase Plane Analysis

· 对神经元受刺激后的振荡解释 FitzHugh-Nagumo Model

Nullcline:
$$\begin{cases} \frac{du}{dt} = 0 & \tau\frac{du}{dt} = u - \frac{1}{3}u^3 - w + \underline{RI(t)} \\ \frac{dw}{dt} = 0 & \tau\frac{dw}{dt} = b_0 + b_1 u - w \end{cases}$$

u nullcline 的上下 shift

Plane:



不同的轨迹

Action potential

limit cycle ⇒ 与Arrows 垂直的平衡形成的区域

(repetitive firing; constant input)

⇒ 产生 fixed points 但不稳定 形成 limit cycle

Stability Math:

对 fixed points 周围的区域进行分析

$$\begin{cases} C\frac{du}{dt} = F(u,w) + RI \\ C\frac{dw}{dt} = G(u,w) \end{cases} \rightarrow \begin{cases} x = u - u_0 \\ y = w - w_0 \end{cases} \quad (u_0, w_0)$$

$$\frac{du}{dt} = \frac{\partial F}{\partial u}\Big|_{(u_0,w_0)} \cdot x + \frac{\partial F}{\partial w}\Big|_{(u_0,w_0)} \cdot y$$

$$\frac{dw}{dt} = \frac{\partial G}{\partial u}\Big|_{(u_0,w_0)} \cdot x + \frac{\partial G}{\partial w}\Big|_{(u_0,w_0)} \cdot y$$

$x = \binom{x}{y}$ $\quad \Rightarrow \frac{d}{dt}x = \begin{pmatrix} F_u & F_w \\ G_u & G_w \end{pmatrix} x$

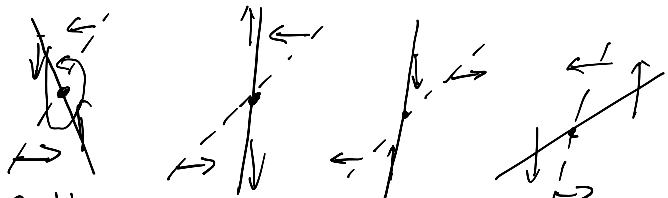
特征值分解: $x = e\exp(\lambda t) \Rightarrow \lambda_+, \lambda_-$

$Re\{\lambda\} > 0$ unstable

$Re\{\lambda_+\} > 0$, $Re\{\lambda_-\} < 0$ 两个 $Re\{\lambda\}$ saddle

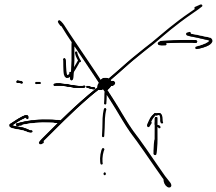
Fixed Points

$Re\{\lambda\} < 0$ stable



Stable Saddle unstabe

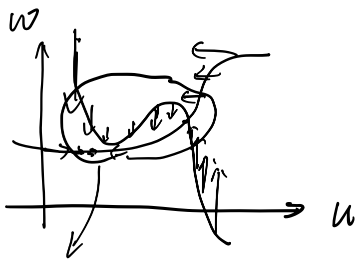
$$\begin{cases} \frac{du}{dt} = 0 \\ \frac{dw}{dt} = 0 \end{cases}$$



· Type I & II Neuron Model

· Frequency & Current

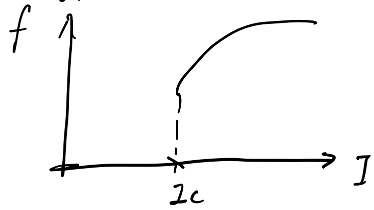
Type I



$\frac{du}{dt}$ 在这里非常小，

需要的时间更多，频率非常小

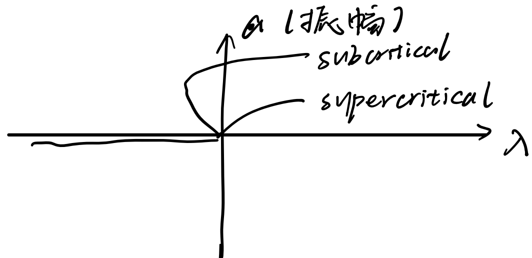
Type II : Bifurcation Model



Ic 的改变

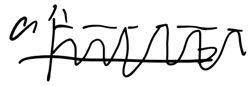
$Re\{\lambda\}$ 从 0^- 到 0^+

从 stable 到 unstable



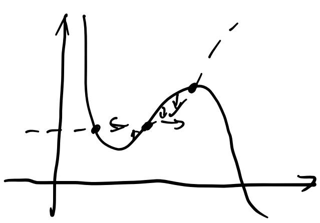
subcritical：振幅非常大

supercritical：振幅小后稳定



· Firing Threshold & Delayed Spike Initiation

↳ saddle point.



轨迹穿过 saddle point

唯一，Threshold

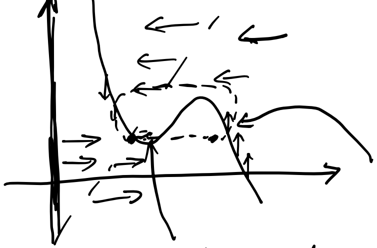
两侧轨迹均排斥后回 stable point

速度慢 ⇒ 延迟

Hopf :

$I_w \ll I_n$

几乎平行于 u



非常慢 : delayed initiation spike

Threshold

轨迹直接回 Stable

(Shortcut)

· Nonlinear Integral and Fire

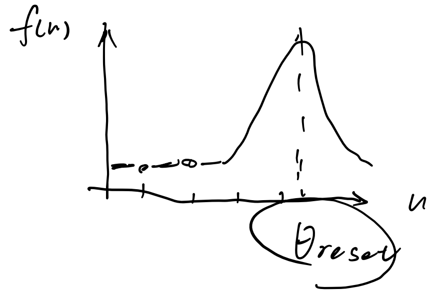
 · one dimension :

 w 的特征 : $\begin{cases} \text{改变慢} \\ \text{改变不大.} \quad \Delta w < 0.5 \end{cases}$

 $\Rightarrow$ 故将 $w \approx w_{rest}$

 $$\tau \frac{du}{dt} = \underline{\underline{f(u)}} + RI$$

 ↳ exponential



· aim : predict spikes & subthreshold voltage

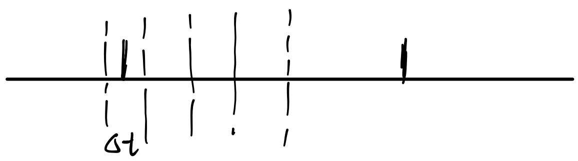
Cha 5

- Rate Code

$PSTH = \frac{n}{K \cdot \Delta t}$ K 次重复 Δt 的时间间隔

- Poisson Model

Homogeneous Poisson Process



$\underline{P_F} = \rho_0 \cdot \Delta t$

Probability firing

$\rho_0 = \lim_{\Delta t \to 0} \frac{P_F}{\Delta t}$

- Survival Function

$S(t) = e^{-\rho_0 t}$

- Inhomogeneous Process ρ_0 改变，时间上连续

$P_F = \rho_K \cdot \Delta t$

↳ $PSTH = \frac{n}{K \cdot \Delta t} \xrightarrow[\Delta t \to 0]{K \to 1000} \rho(t)$

$S(t) = \exp\left(-\int_0^t \rho_0 \, dt\right)$

$\frac{n}{N \cdot \Delta t} \xrightarrow[\Delta t \to 0]{N \to 10^3} \rho(t)$

$\Rightarrow$ Interval Distribution 在 $\hat{t}$ 时产生动作电位

$$P(t|\hat{t}) = \underbrace{\rho(t)}_{\text{在}t\text{产生动作电位}} \cdot \underbrace{S(t|\hat{t})}_{t\text{与}\hat{t}\text{间不产生动作电位}}$$

$$\boxed{\langle S(t) \rangle = \langle \sum_{k} \sum_{t} \delta(t - t_k^f) \rangle = K\rho_0}$$

$$\langle u(t) \rangle = \int \langle \sum_f (f(t-t') \cdot \underline{\delta(t' - t_k^f)}) \rangle \cdot dt'$$

$$= \int f(t-t') \cdot \rho_0 \cdot dt'$$

Cha 6

- Spike Arrival

- Noise Models
  - Diffusion Noise
  - Escape Noise

Cha 7

AdEx.

→ limitations of LIF : Adaptations. Firing Patterns. Noise. ×

Exponential LIF : $\tau \frac{du}{dt} = -(u-u_{rest}) + \Delta \exp(\frac{u-\delta}{\Delta}) + RI$

AdEx : $\begin{cases} \tau \frac{du}{dt} = -(u-u_{rest}) + \underline{\Delta \exp(\frac{u-\delta}{\Delta})} - Rw + RI \\ \tau_w \frac{dw}{dt} = a(u-u_{rest}) - w + b\cdot\tau \underbrace{\sum_f \delta(t-t_f)}_{\text{spikes counter}} \end{cases}$

? 为什么可以解决上述 limit ? : phase plane analysis

vs LIF : AdEx vs $\tau \frac{du}{dt} = u - u_{rest} + I(t)\cdot R$, reset

↳ Δ 很小 $\Delta = 2mV$ 则 AdEx ≈ LIF

指数项相当于 reset

SRM : Spike Response Model

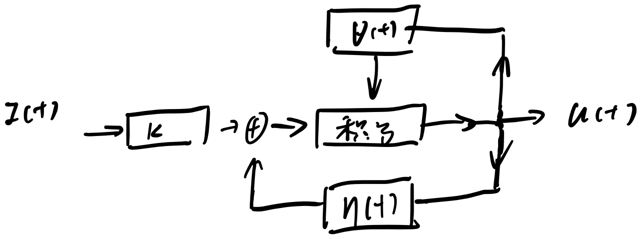
LIF : $\begin{cases} \tau \frac{du}{dt} = -(u-u_{rest}) - R\sum_k w_k + RI(t) \\ \tau_k \frac{dw_k}{dt} = a_k(u-u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t-t_f) \end{cases}$

改写成积分式：

$$u(t) = \sum_f \underbrace{\eta(t - t^f)}_{\text{spike after potential}} + \int_0^\infty ds\, \underbrace{\kappa(s)}_{\text{对spike的核函数}} I(t-s) + u_{rest}$$

Threshold: $\theta(t) = \theta_0 + \sum_f \theta_1(t - t^f)$

— 每一个spike后，Threshold增加



选择合适的参数

则可模拟spike。

GLM: SPM + noise

· 参数计算

Cha 8 Memory

Memory ⇒ neurons extension、connection

· Associations - auto-associative memory

· Pattern Recognition (Brain Style) Magnetic Materials
  - Similarity

· Hopfield Model

$$\begin{cases} w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu} \\ S_i(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j(t)\right] \\ \quad = \mathrm{sgn}\left[\sum_{\mu} p_i^{\mu} m^{\mu}(t)\right] \end{cases}$$

Overlap : $m^{\mu}(t+1) = \frac{1}{N} \sum_j \underset{\text{pattern}}{\underline{P_j^{\mu}}} \underset{\text{Current state}}{\underline{S_j(t+1)}}$

· Stochastic Hopfield Model

Dymacs : $\begin{cases} Pr(S_i(t+1) = +1 \mid h_i\} = g\left[\sum_{\mu} p_i^{\mu} m^{\mu}(t)\right] \\ Pr(S_i(t+1) = -1 \mid h_i\} = g\left[\sum_{\mu} p_i^{\mu} m^{\mu}(t)\right] \end{cases}$

· Hebbian Learning of Associations

  association of memory prototype.

· Energy Space

$$E = -\frac{1}{2} \sum w_{ij} S_i S_j$$
$$= -\frac{1}{2} N \sum_{\mu} (m_{\mu})^2$$

· 当状态改变, Energy 减少

· Biology

· Mean activity of pattern.

  - $w_{ij} = c \sum_{\mu} (\xi_i^{\mu} - a)(\xi_j^{\mu} - b)$ $\quad \xi \in \{0, 1\}$ $\quad$ a, b 指平均值
  - overlap : $m^{\mu}(t) = c \sum_{j=1}^{N} (\xi_j^{\mu} - a) S_j(t)$
  - Dynamics :

$$S_i(t+1) = \mathrm{sgn}[h_i(t)] = \mathrm{sgn}\left[\sum_{j=1}^{N} w_{ij} S_j(t)\right]$$

$$= \mathrm{sgn}\left[\sum_{j=1}^{N} c \sum_{\mu} (\xi_i^{\mu} - a)(\xi_j^{\mu} - b)\, S_j(t)\right]$$

$$= \mathrm{sgn}\left[\sum_{\mu} m^{\mu}(t) \cdot (\xi_i^{\mu} - a)\right]$$

※ asymmetric weights. capacity calculation

- Binary neural data
  - $S(t) = \pm 1 \rightarrow \sigma(t) \in \{0,1\}$ $\quad S(t) = 2\sigma(t) - 1$
  - $h_i(t) = \sum w_{ij} S_j(t) = \sum w_{ij} (2\sigma_j(t) - 1)$

    $= \sum 2 w_{ij} \sigma(t) - \sum w_{ij}$
- Seperation of Excitation & Inhibition

- Low activity pattern
- IIF model

- Microstate EEG. Cognition

Cha 9. Population Activity

- Population Activity.

$$A(t) = \lim_{\Delta t \to 0} \frac{n}{N \cdot \Delta t} = \frac{1}{N} \sum_{t'} \sum_{n} \delta(t - t')$$

- Receptive field
  - sensitive stimulus features. locations
    → orientation. colors

- Stationary asynchronous activity
  - $\bar{A}(t)$. & Input $I(t)$ ⇒ filter function
    Identical. Independent of N
  - Independent of repititions M

- Mean field - fully connected network
  - All neurons receive the same total input current

$$I_t = \sum_f \sum_k w_{ik} \alpha(t - t_k^f)$$
$$= w_0 \int \alpha(s) A(t-s) ds = w_0 \bar{A}(t)$$

  - Spatial Average = Temperal Average

$$A_0 = \nu = g(I_0)$$

放电率 → ν　　gain function → g

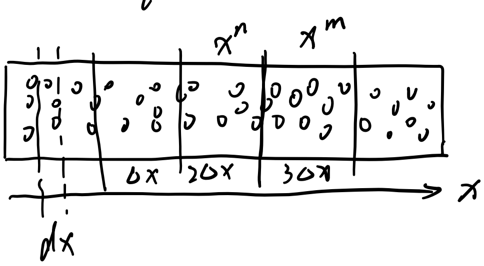
- Stationary Solution
  - balanced

Transient & Continuum

- Transients - uncoupled neurons
  - High noise
    - Slow Transient
    - $A(t) = F(h(t))$

      $$\tau \frac{d}{dt} h(t) = -h(t) + R I^{ext} + J_0 q F(h(t))$$

- Continuum
  - Discrete → Continue
  - Field Equation



$A(t) \rightarrow A(n\Delta x, t)$

$h(t) \rightarrow h(n\Delta x, t)$

Group size:

$N \rightarrow \underline{d} \cdot \Delta x$

density function

$$\tau \frac{d}{dt} h(t) = -h(t) + R I^{ext} + R \underbrace{\underline{I}^{int}}$$

$n-n$ within population　　$n-m$ outside population

↓

$$\tau \frac{d}{dt} h(x',t) = -h(x',t) + R I^{ext} + R \cdot \sum_m d\Delta x \cdot \underline{w(x',m)} \int \alpha(s) \cdot A(x', t-s)\, ds$$

↓

$$\tau \frac{d}{dt} h(x,t) = -h(x,t) + R I^{ext} + R \int d\, w(|x-x'|) \cdot \left(\int \alpha(s) A(x, t-s) ds\right) dx$$

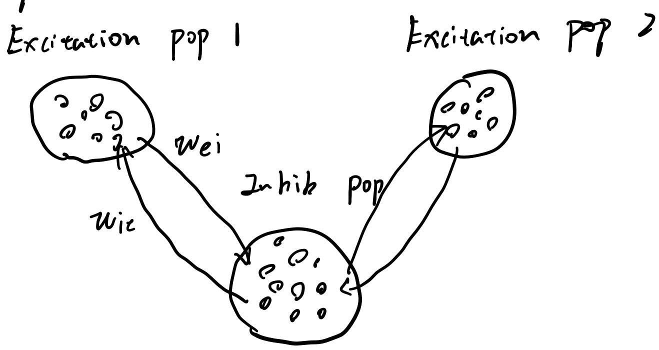
- Coupling : $w(|x-x'|)$
  - Mexican hat
    - local exitation, distant inhibition
- Solution type
  - Homogenous solution
  - Bump solution

Decision Making

- Model of neuron population

  - MT、LIP、FEF、'46'

  - Competition



Two assumptions:

- inhib: fast -> $\tau_{inh} \ll \tau$
- linear: $g(h(t)) = h(t)$

  - 两个 Excitation population 相互竞争

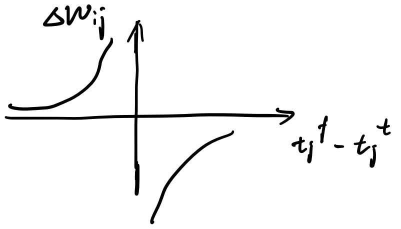
Neural Plasticity

- Short-term plasticity
  - induced 0.5 sec
  - recover 1 sec

- Long-term plasticity
  - change persist for a long time
  - LTP / LTD

    Long term potentiation

- Spike-timing Dependent Plasticity



$$\tau_+ \frac{d}{dt} z_j^+ = -z_j^+ + \sum \delta(t - t_j^{pre})$$

$$\tau_- \frac{d}{dt} z_j^- = -z_j^- + \sum \delta(t - t_j^{post})$$

$$\frac{d}{dt} w_{ij} = \underbrace{a(w_{ij}) z_j^+ \delta(t - t_j^{post})}_{\text{pre-post}} - \underbrace{b(w_{ij}) z_j^- \delta(t - t^{pre})}_{\text{post-pre}}$$

- Hebbian Model
  - unsupervised model

$$\frac{d}{dt} w_{ij} = F(w_{ij}, v_j^{pre}, v_j^{post}, \underbrace{Mod}_{\text{supervized learning}})$$

BCM:

$$\frac{d}{dt} w_{ij} = b\, v_j^{post} (v_j^{post} - \theta)\, v_j^{pre}$$

- Correlation & Competition