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Due to climate change, many communities across the world are concerned about its environmental effect on humans. In particular, this paper will be focusing on the effect of depleting water supply at the Colorado River System on the states of Arizona, California, Wyoming, New Mexico, Colorado, and other involved parties. Negotiators are asking for a mathematical model that can respond to these changes and allocate the water and hydroelectric supply from the river system to the involved parties. The mathematical solution for this paper focuses on two variables: supply and demand of water and electricity. Our supply model takes into account many factors like the Glen Canyon Dam, Hoover Dam, the inflow of water into the system, and more. This model gives us a measure of how much water and electricity the river system can supply annually. On the other hand, the demand model quantifies the demand of the respective parties that rely on the water and electricity supply from the system. The demand model accounts for the criteria delineated from the guidelines and requirements of the problem. This includes the agriculture, residential, and industrial sectors of each state or party. Taking into account these factors, we can assign each state with a new water and electricity demand value that will fit the annual supply. This model was testing used historical demand data from each state and involved parties. Since the current supply does fit the water and electricity demand for each state, we tested our demand model using theoretical supply values. These tests have shown that our model does make realistic adjustments to the demands for water and electricity for each state.

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Contents

1 Abstract

The impact of the modern climate change has had negative consequences on the availability of resources throughout the world. These consequences are now reflected on American communities like the states of California, Wyoming, Arizona, Colorado, and New Mexico. Climate changes has brought upon droughts and reduced rainfall around the Colorado river system. Part of the Colorado river flows from the Glen Canyon Dam to the Hoover Dam, and then finally enters the Gulf of California and Mexico. The communities of California, Wyoming, Arizona, Colorado, and New Mexico depend on the water and electricity supply brought from the reservoirs and hydroelectricity generators of both dams. As a result of the recent droughts and reduced rainfalls, a new water reallocation plan is needed to address the change in supply of water and electricity. This paper will present a mathematical model that reallocates these resources according to changes in supply and demand to the five states. The model will also account for the residual water from the river system that lands in the Gulf of California and communities of Mexico.

2 Assumptions and Variables

2.1 Assumptions

To eliminate complicated and unnecessary variables from the model, seven basic assumptions were made.
Assumption one : The total water and electricity demand for each states is constant over years. Change in demands caused by environmental factors are negligible.
Assumption two : Time and cost of transporting/distributing water and electricity is negligible.
Assumption three : The allocation of water and electricity depends solely on demand. Proximity is not considered.
Assumption four : Water flow is always consistent. Water supply in each cycle are the same. Electricity generated in each cycle are the same.
Assumption five : The influence from outer environment (e.g. evaporation) is negligible.
Assumption six : There is no causality between demand of water and electricity. They are independent.
Assumption seven : We'll change from The water level M and P to the water volume of Lake Mead and Lake Powell.

2.2 Notations

We summarize frequent-used notations in Table 1. Other non-frequent-used notations will be introduced once they are used.

Table 1: Notations used in this literature

Symbol	Definition
a	Agricultural coefficient in water demand model
i	Industrial coefficient in water demand model
r	Residential coefficient in water demand model
A	Agricultural water demand
I	Industrial water demand
R	Residential water demand
t	Test count

3 Model

3.1 Model Introduction

To create a mathematical model that will assist negotiators to respond to a fixed set of water supply and demand conditions, two separate models for the supply and demand were deemed to be necessary.

3.2 Supply-Side Model

Basic assumptions of the supply-side model:

1. Lake Mead is at the downstream while Lake Powell is at the upstream. Assume that all water in Lake Powell that discharged by Glen Canyon Dam will eventually enter Lake Mead.

2. Assume that there are pipes and cables connecting the outflow of Hoover Dam to five states. Recall water and electricity can be transported to fives states immediately without fees (Section 2.1). We decide not to take water from the two lakes. Instead, we decide to collect part of the outflow of Hoover Dam and transport water to the five states through the pipes and cables. The leftover outflow will flow in Gulf of California, and eventually will flow to Mexico since Mexico claims the residual of the water.

3. According to our research, Glen Canyon Dam has higher electricity generating efficiency. Therefore, all water in Lake Powell should go through Glen Canyon Dam to maximize electricity generation. Therefore, electricity generation is divided into three parts: water in Lake Powell going through the turbine of Glen Canyon Dam; water in Lake Mead going through the turbine of Hoover Dam; water in Lake Powell going through the turbine of Hoover Dam.

To begin with our supply-side model, first, we assume that there are only two available sources of water - Lake Powell and Lake Mead. Other sources of water including rainfall and the Colorado River are ignored in the first stage. The volume is the largest water storing capacity of a lake, and the discharge rate of a dam is how fast water can flow through the dam. Below are the setups of variables

$$\text{Volume of Lake Powell: } vLP = 30.001 km^3$$
$$\text{Volume of Lake Mead: } vLM = 32.236 km^3$$
$$\text{Discharge rate of Glen Canyon Dam: } dGCD = 1.13267386 \times 10^{-7} km^3/s$$
$$\text{Discharge rate of Hoover Dam: } dHD = 3.3413879 \times 10^{-6} km^3/s$$

Since $dHD > dGCD$, as Hoover Dam is discharging water in Lake Mead, Glen Canyon Dam can discharge water in Lake Powell at the same time, and water discharged from Glen Canyon Dam will flow into Lake Mead (which is in the lower stream area) without causing the total volume of water in Lake Mead to exceed the water storing capacity of Lake Mead. Therefore, we have the following equations:

Hoover Dam is discharging water from Lake Mead. Lake Mead is being replenished by the water from Lake Powell. The net decreasing rate of water in Lake Mead is denoted by dNet:
$dNet = dHD - dGCD = 3.2281 \times 10^{-6} km^3/s$ - [Equation 3.2.1]

When Lake Mead is drained up, the two lakes run out of water completely (recall that Lake Powell is replenishing Lake Mead, so Lake Mead being drained up means Lake Powell also runs out of water). The the total time for two lakes to be drained up is denoted by "tCycle." It can be calculated by dividing the total volume of Lake Mead by the net decreasing rate of water in Lake Mead:
$tCycle = vLM/dNet = 9.9860 \times 10^6 s$ - [Equation 3.2.2]

We assumed that the two lakes were the only available sources of water. The system will eventually run out of water when the two lakes are drained up.And the process of two lakes from being fully replenished to drained up is called one "cycle." Now we introduce Colorado River to the system to replenish the two lakes in order to create more cycles. Assume that the Colorado River has infinite amount of water storage, and can replenish the two lakes at a constant rate. To keep the cycles running, we need vLM+vLP amount of water to be replenished into the two lakes for every tCycle (recall that tCycle is the total time for one cycle to complete).The rate of replenishment "repRate" should be:

$repRate = (vLP + vLM)/tCycle = 6.2324 \times 10^{-6} km^3/s$ - [Equation 3.2.3]

To get precise estimates, the unit of tCycle is in second. So how long is tCycle roughly (denoted by tCycleR)? How many "cycles" happen in annually (denoted by numCycle1Y)?

$$secMon = howmanysecondsarethereinonemonth \approx 2592000s$$
$$secYear = howmanysecondsarethereinoneyear \approx 31536000s$$
$$tCycleR = tCycle/secMon \approx 3.8526months - [Equation3.2.4]$$
$$numCycle1Y = secYear/tCycle \approx 3.1580cycles - [Equation3.2.5]$$

For every cycle, the total amount of water is the combined volume of the two lakes. So for every cycle there is $vLP + vLMkm^3$ water available. The total amount of water supply per year is denoted by wSup1Y:

$wSup1Y = numCycle1Y * (vLP + vLM) = 196.5458km^3$ - [Equation 3.2.6]

The water supply model is finished. Now we need to consider the electricity supply model:
Electricity generated by Hoover Dam in one year = $eHD1Y = 4 \times 10^9 kWh$
Electricity generated by Glen Canyon Dam in one year = $eGCD1Y = 5 \times 10^9 kWh$
Since $eGCD1Y > eHD1Y$, we concluded that Glen Canyon Dam has higher efficiency generating electricity. After considering the annual electricity generation, how much electricity can each dam generate per cycle?
Hoover Dam: $eHD1C = eHD1Y/numCycle1Y = 1266624445.85kWh/cycle$ - [Equation 3.2.7]
Glen Canyon Dam: $eGCD1C = eGCD1Y/numCycle1Y = 1583280557.31kWh/cycle$ - [Equation 3.2.8]

3.3 Demand-Side Model

3.3.1 Cases

After projecting the estimated water and electricity supply from the Hoover and Glen Canyon Dam, a demand model was needed to address the following scenarios: water and electric demand exceeds supply, water demand exceeds supply and electric demand does not exceed supply, electric demand exceeds supply and water demand does not exceed supply, and lastly water and electricity demand do no exceed supply. The last case was ignored because the problem expects there to be unfulfilled water and electricity demands due to climate change.

3.3.2 What is demand?

To address the relationship between demand and supply, a model for demand is needed. Demand for water and electricity is based on the following 3 criteria: agriculture, industrial, and residential demand. Thus, demand is modelled using the following equation.

$$\text{Total Water/Electric Demand} = aA + iI + rR \text{ - [Equation 3.3.2.1]}$$
$$a = 1 - 0.25t, i = 1 - 0.25t, r = 1 - 0.1t$$

(a- agricultural coefficient, i- industrial coefficient, r- residential coefficient, t- test count)
(A- agricultural demand, I- industrial demand, R- residential demand) unit: km^3

This equation models the demand for water/hydroelectricity with respect to the concerned criteria: agriculture, industrial, and residence. The variables a, i, r are used to adjust the demand when the supply of water or hydroelectricity is less than demand. Each variable is initially set to 1 for $t = 0$ because each sector's demands for water/electricity should be met if possible. If the supply fails to meet the demand, we adjust the variables by incrementing t such that the total demand will fall. Each variable also decreases at different rates when t is incremented. This is because the model assumes that residential demands are more important than the equally important industrial and agricultural demands. However, negotiators should can have the free will to adjust the rate which each variable decreases according to their preferences. The following section summarizes the methodology for operating the model in the context of this problem.

3.3.3 How does the model work?

1. Set total demand from the five states equal to the projected water/electric supply from Colorado River System beginning with $a = i = r = 1$.
2. If the demand is less than or equal to the supply. No further action is needed.
3. If the demand is greater than the supply. Repeat step 1 but for $a = 1 - 0.25t, i = 1 - 0.25t, r = 1 - 0.1t$ where t is the number of times step 1 has been performed.
4. If either a, i, or r reaches 0. Move to step 5.
5. Given the new values of a, i, and r, calculate the adjusted total demand for each state by inputting the new values and the STATE sector demands A, I, R into the demand equation.
6. The adjusted total demand for the given state is the water/electric supply that should be allocated to that state.

4 Analysis

4.1 Introduction

To deal with the situation when drought happens that influences the water supply, we need to create a solution that water supply matches with water demand. Since our water demand equation contained linear factors, we need to adjust factors allocation limited water based on the importance of demand. Due to the COVID pandemic, our historical data had a deviation from the future situation. Therefore, we utilize hypothetical data to test our models and verify their validity.
Some constant historical data is used in our testing:(in our testing zero, t is equal to zero when we do not need to change the demand since all demand can be satisfied by supply

$$\text{Total Water Demand for a year}$$
$$= 16.38 + 5.7 + 6.216 + 0.516 + 1.728 + 3.947 + 0.0135 + 0.407 + 16.32 + 1.26 + 6.312 + 0.444$$
$$= 59.2435km^3 \text{ - [Equation 5.1.1]}$$
$$\text{Total Electricity Demand for a year}$$
$$= 250,174,672 + 81,960,074 + 24,777,155 + 56,050,264 + 42,010,989 = 454973154kWh \text{ - [Equation 5.1.2]}$$

4.2 Demand-Side Model testing

4.2.1 normal case

In normal case, annual water supply and annual electricity supply will be greater than annual water demand and annual electricity demand due to our supply model.
To test data in test 0, we get the value:

$$\text{Annual Total Water Supply - Annual Total Water Demand =}$$
$$196.5458km^3 - 59.2435km^3$$
$$= 137.3023 \text{ km}^3$$

$$\text{Annual Total electricity Supply - Annual Total electricity Demand =}$$
$$910^9kWh = 454973154kWh$$
$$= 4.45*10^8kWh$$

Since Mexico consumed shares after the five states, normal the excess water supply and excess electricity supply can get into Mexico, which is $137.3023km^3$ and 910^9kWh annually.

4.2.2 extreme drought case

Imagine the situation when drought happened, the period for cycling and water replenishment increased. The water supply diminished under $59km^3$ per year(smaller than water demand), which displayed a smaller value than water demand. To deal with the situation of water and electricity shortage, we need to modify coefficient (a, i, r) by matching supply with demand. Therefore, factors are alternated to reduce demand to a level lower than supply.

$$\text{Total water supply} > \text{Total Water Demand} = (1-0.25t)I + (1-0.25t)A + (1-0.1t)R$$
$$\text{California}: \ 16.38 * (1-0.25t) + 5.7 * (1-0.175t)$$
$$\text{Arizona}: \ 6.216 * (1-0.25t) + 0.516 * (1-0.25t) + 1.728 * (1-0.1t)$$
$$\text{New Mexico}: \ 3.947 * (1-0.25t) + 0.0135 * (1-0.25t) + 0.407 * (1-0.1t)$$
$$\text{Colorado}: \ 16.32 * (1-0.25t) + 1.26 * (1-0.175t)$$
$$\text{Wyoming}: \ 6.312 * (1-0.25t) + 0.444 * (1-0.1t)$$

Electricity case is similar, only that we can always keep a as 4 since the importance for Electricity on Agriculture is negligible.

$$\text{Total electricity supply} > \text{Total electricity Demand} = (1-0.25t)I + (1-0.1t)R$$
$$\text{California}: \ 58000000 * (1-0.25t) + 46750000 * (1-0.1t)$$
$$\text{Arizona}: \ 11890000 * (1-0.25t) + 21730000 * (1-0.1t)$$
$$\text{New Mexico}: \ 8275000 * (1-0.25t) + 4475000 * (1-0.1t)$$
$$\text{Colorado}: \ 15176000 * (1-0.25t) + 13552000 * (1-0.1t)$$
$$\text{Wyoming}: \ 24530000 * (1-0.25t) + 420000 * (1-0.1t)$$

in test 1, we assume the water supply goes to $50km^3$ per year, and electricity supply goes to $300000000kWh$ per year. Therefore,

$$Watersupply = 50km^3 > (1-0.25t)A + (1-0.25t)I + (1-0.1t)R$$

* since CaliforniaColoradoWyoming combined industrialresidential data together, we utilized (i + r) and calculate the factor as (1 - 0.175t). So the equation becomes :

$$50km^3 > (1-0.25t)A + (1-0.175t)(I+R)$$
$$t > 0.665$$
$$a = 0.834; i = 0.834; r = 0.934$$
$$\text{Electricity Supply} = 300000000 > (1-0.25t)I + (1-0.1t)R$$
$$t > 2.308$$
$$i = 0.423; r = 0.769$$

We calculate our data in test 2, where the water supply goes to $25km^3$ per year. Therefore,

$$25km^3 > (1-0.25t)A + (1-0.25t)I + (1-0.1t)R$$
$$25km^3 > (1-0.25t)A + (1-0.175t)(I+R)$$
$$t > 2.463$$
$$a = 0.616; i = 0.616; r = 0.754$$

4.3 Linear Progression Model

To testify the demand model we utilized when water demand is greater than water supply, we need to generate the solution for better allocation of water and electricity. Given that the coefficient factors before demands can be altered due to their importance under different situations, we need to allocate resources in proportions due to demand numbers, which are determined by the three coefficients. We

need to lower our demand; in the meantime, we have to satisfy as much demand as possible. We can satisfy their needs when demand gets a bit lower than the supply. Therefore, instead of using linear equations to figure out the coefficient and diminished coefficients at the same time, we could also utilize another method to calculate the maximum number of coefficients sum, which allowed us to test our model by satisfying maximum needs within limited resources.
As an alternative model, linear progression is used. Instead of using a singlet in linear equations, we utilize t1,t2, and t3 while calculating the agricultural coefficient, industrial coefficient, and residential coefficient. Since we assume that residential demands are more important than agricultural and industrial demand, we convert the importance level into coefficient digits, and use linear progression to find a maximum number for $0.25t1 + 0.25t2 + 0.1t3$ The equation follows several restrictions:

$$9.2435 < 12.294t1 + 0.763t2 + 0.84425t3 < 59.2435$$
$$0 < t1 < 4$$
$$0 < t2 < 4$$
$$0 < t3 < 10$$

Therefore, we write our codes as following to find optimized value for t1,t2 and t3:

$f = [0.25, 0.25, 1]$;
A = [-12.294,-0.763,-0.84425;12.294,0.763,0s.84425 ; -1,0,0 ; 0,-1,0 ; 0,0,-1; 1,0,0 ; 0,1,0 ; 0,0,1;];
B = [9.2435,59.2435,0,0,0,4,4,10];
x = linprog(f, A, B)

We get the value of t1,t2,t3

$$t1 = 0.2055$$
$$t2 = 0.0063$$
$$t3 = 0.0458$$

5 Drought and Thirst - Magazine Report

Water and electricity are essential. Our report addresses the issue of water allocation and electricity distribution among Arizona (AZ), California (CA), Wyoming (WY), New Mexico (NM), and Colorado (CO). They rely on Colorado River System for water resources, and Hoover Dam and Glen Canyon Dam provide electricity for them.

Three investigators from our group worked out a mathematical model for supply and demand of water and electricity. According to our research, after satisfying the demand of water and electricity of the five states, there is $137.3023km^3$ water and $4.45 \times 10^8 kWh$ electricity. Therefore, ideally, after satisfying the need of the five states, the leftover electricity can be distributed among other states in need. The leftover water will belong to Mexico. If the Mexico's demand of water is smaller than $137.3023km^3$, then final leftover will flow into the Gulf of California.

Once drought happens, water resources become deficient. To reconcile the tension between whether water or electricity should be prioritized, our team propose to purify sea water in Gulf of California after satisfying demand for electricity, or appeal to wind power plant to generate electricity after satisfying demand for water. We call for your action to save natural resources and energy.

6 Conclusion

In this section we will briefly conclude how the model satisfies the requirements. The water level is not considered because Lake Mead and Lake Powell do not occupy regular geometric spaces (like cuboid or cylinder). Their volume is not proportional to the water levels. We are aware that when the water level is too low, not enough water can go through the dams, and there might not be constant electricity supply. Therefore, we assumed an ideal situation: the Colorado river is continuously replenishing the two lakes at the rate of $repRate = 6.2324 \times 10^{-6} km^3/s$ (Equation 3.2.3). If the replenishing rate = repRate, the two lakes are always at their maximum capacity, meaning the two lakes are always "full of water." In another word, the two lakes always have the highest possible water levels.

We only allow the Colorado River to replenish the two lakes. Therefore, no additional water sources (such as precipitation) are available. If there is no replenishment at all, two lakes, starting at full capacity, will be drained up within $tCycle = 9.9860 \times 10^6 s$ (Equation 3.2.2).

According to our research the total demand of water of the five states is $dw1Y = 59.2435 km^3/year$. And the total demand of electricity of the five states is assumed to be $de1Y(kWh)$. Suppose every year the water supply is larger than dw1Y, and electricity supply is larger than de1Y, then the leftover supply of water can be transported to Mexico. If there is still leftover after Mexico's demand is satisfied, the rest of the water will flow in the the Gulf of California. The The leftover supply of electricity will be distributed to states in need.

The purpose of the magazine report is to use a qualitative way to call for people's attention. By laying out demands and supplies, we hope people to make good use of natural resources and energy. When the supply is greater than demand, we should not waste resources or energy. Rather, we should be grateful, and use them wisely. When the supply is lower than demand, we should not panic. Rather, we should work out a better way to allocate the resources. The importance of residents, agriculture and industry will be weighed according to the Demand-Side model, and hopefully advanced technology can also help. Sea water purifying techniques and new ways to generate electricity (including solar photovoltaics and wind power plant) should be popularized.

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