Simulation of Aortic Stenosis and How the Heart Compensates

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1. Introduction:

Aortic stenosis a common valve disease, which involves a narrowing of the aortic valve. The narrowed valve restricts the blood flow from the left ventricle to the aorta. Such insufficient stroke volume might cause the following symptoms: chest pain, fluttering heartbeat, and feeling dizzy, light-headed, or short of breath. In order to maintain a relatively normal stroke volume, the heart will usually compensate by thickening the wall of the left ventricle or increase the left atrial pressure[1]. Such compensation is restricted by upper limit for left atrial pressure. There will be excessive fluid in the lung (pulmonary edema) if the left atrial pressure is higher than 25 mmHg[2]. In this project, I will try to explore the relationship between change in RAo, change in CLVD, change in CLVS, and change in PLA. In particular, how do change in CLVD, CLVS, and PLA affect the high systolic blood pressure caused by aortic stenosis.

In the mathematical modeling of the disease[3], the narrowing of aortic valve will be simulated by increasing RAo (the resistance of the aortic valve). The PV Diagram (pressure-volume diagram), and the PSa & PLV vs. Time Diagram will be plotted and analyzed, and VS, the stroke volume, of each case will be recorded. If the stroke volume is below the normal value, then CLVD, CLVS, and PLA will be changed in order to bring the stroke volume back to normal.

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[1] American Heart Association, Oct 26, 2020

[2] T. Brett Reece MD, David A. Fullerton MD, Mitral Stenosis, Abernathy's Surgical Secrets (Sixth Edition), 2009

[3] Frank C. Hoppensteadt, Charles S. Peskin, Modeling and Simulation in Medicine and the Life Sciences (Second Edition)

2. Equations[4]

The left ventricle is regarded as a compliance vessel whose compliance is a function of time. Thus, the left ventricle is described by $V(t) = V_d + C(t)P(t)$ (2.1) where $C(t)$ is a function of time with the qualitative character shown in Figure 2.1. The systolic compliance is small as the ventricle contracts, while the diastolic compliance is larger as the ventricle relaxes.

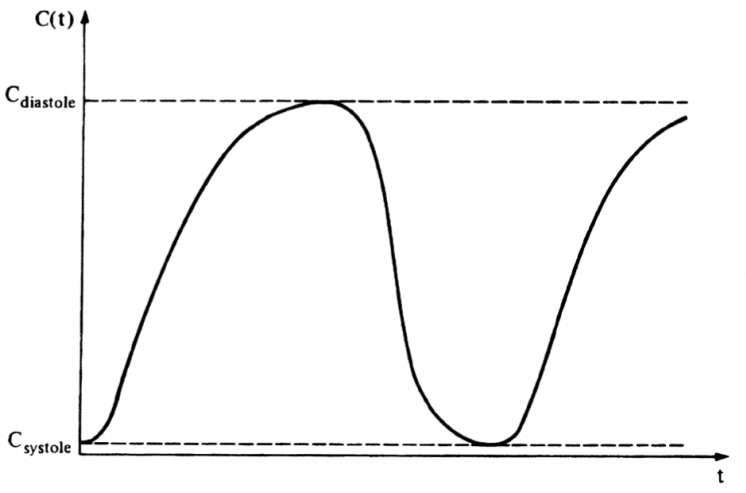

Figure 2.1.[5] Qualitative Character of $C(t)$

Using (2.1), a pressure-volume diagram of the cardiac cycle (Figure 1.5) can be constructed. The maximum volume of the ventricle at end-diastole is given by $V_{ED} = V_d + C_{diastole}P_v$ (2.2), while the minimum volume at end-systole is given by $V_{ES} = V_d + C_{systole}P_a$ (2.3), where P_a is the pressure in the arteries supplied by the ventricle and P_v is the pressure in the veins that fill it. Thus, the stroke volume is given by $V_{stroke} = V_{ED} - V_{ES} = C_{diastole}P_v - C_{systole}P_a$ (2.4)

In the result section, the model presented by Figure 1.5 is plotted with relatively realistic data. With aortic stenosis, B-C appears to be a parabola instead of a straight line because the increase in aortic valve resistance results in higher systolic blood pressure.

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[4] Frank C. Hoppensteadt, Charles S. Peskin, Chapter 1.4, Equation 1.4.1 - 1.4.4, Modeling and Simulation in Medicine and the Life Sciences (Second Edition)

[5] Frank C. Hoppensteadt, Charles S. Peskin, Chapter 1.4, Figure 1.4, Modeling and Simulation in Medicine and the Life Sciences (Second Edition)

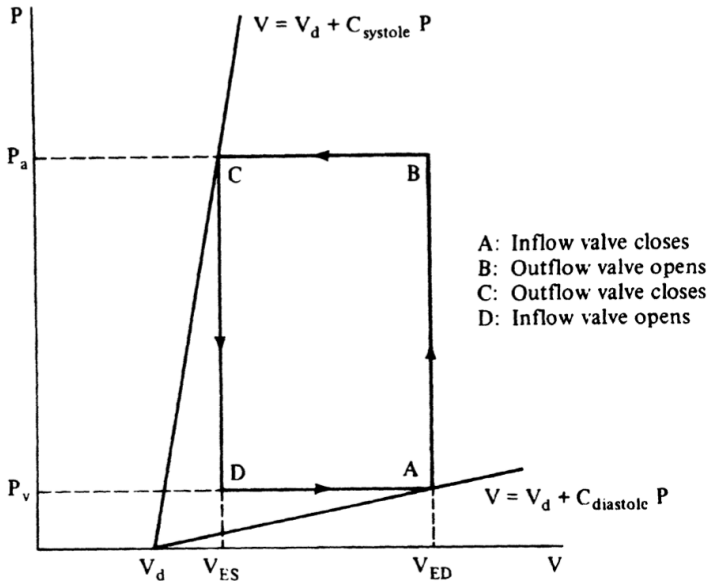

Figure 2.2.[6] Cardiac Cycle of the Left Ventricle. (A:Mitral valve closes; A-B: Isovolumetric contraction during which the left ventricular volume stays the same while the pressure increases; B: Aortic valve opens; B-C: Left ventricle sends out blood through aortic valve; C: Aortic valve closes; C-D: Isovolumetric relaxation during which the left ventricular volume stays the same while the pressure decreases; D: Mitral valve opens; D-A: Filling - blood flows from left atrium to left ventricle)

3. Methods:

The numerical method is Backward Euler Method. In the result section, only the steady state of each cardiac cycle is shown. When the systolic pressure appears to be too high (above 150 mmHg), parameters must be adjusted to bring down the systolic pressure while maintaining normal stroke volume (50-100 mL, or 0.0500 - 0.1000 L)[7].

[6] Frank C. Hoppensteadt, Charles S. Peskin, Chapter 1.4, Figure 1.5, Modeling and Simulation in Medicine and the Life Sciences (Second Edition)

[7] David Sidebotham, Ian J. Le Grice, Physiology and Pathophysiology, Cardiothoracic Critical Care, 2007

4. Results and Discussion

Normally, the resistance of aortic valve RAo = 0.01 mmHg/(liter/minute); The compliance of the left ventricle at systole CLVS = 0.00003 Min (systolic) value of CLV (liters/mmHg); The compliance of the left ventricle at diastole CLVD = 0.0146 %TBD %Max (diastolic) value of CLV (liters/mmHg); The pressure from the left atrium PLA = 5 mmHg. Under these conditions, the stroke volume is 0.0650 L, and normal cardiac cycle diagram and PLV & PSA vs. Time Diagram are plotted.

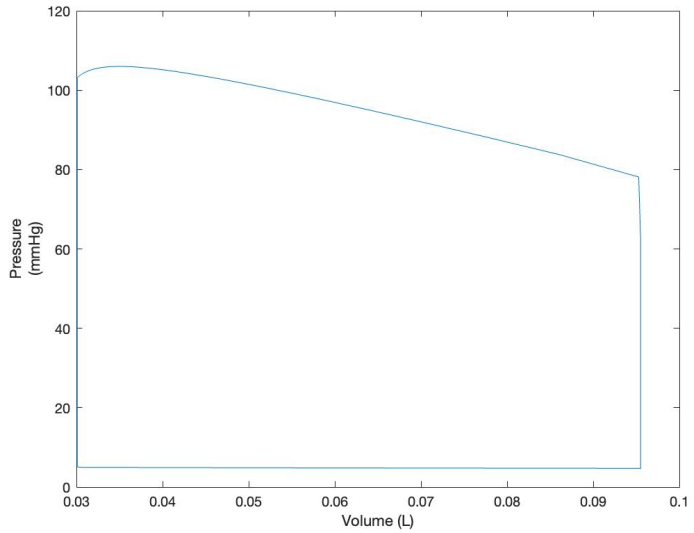

Figure 4.1. Normal Cardiac Cycle Diagram

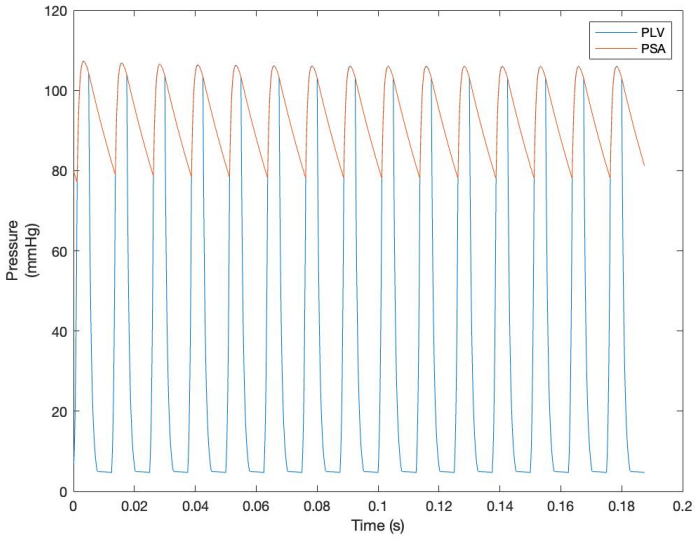

Figure 4.2. Normal PLV & PSA vs. Time Diagram

When RAo increases to hundreds of times its original value, the observed decrease in stroke volume is not significant, but the blood pressure at systolic increases drastically. In order to maintain blood pressure at a relatively realistic range (less then 300 mmHg) and in the mean time produce decrease in stroke volume (change in volume is higher than 0.0001 L), the highest RAo is set to be 6 mmHg (600 times the original value, which was 0.01 mmHg), which results in a systolic blood pressure of nearly 300 mmHg, and a 0.0004 L decrease in stroke volume. The general conclusion is that the decrease in stroke volume is negligible, but the increase in systolic blood pressure is very likely to cause a stroke.

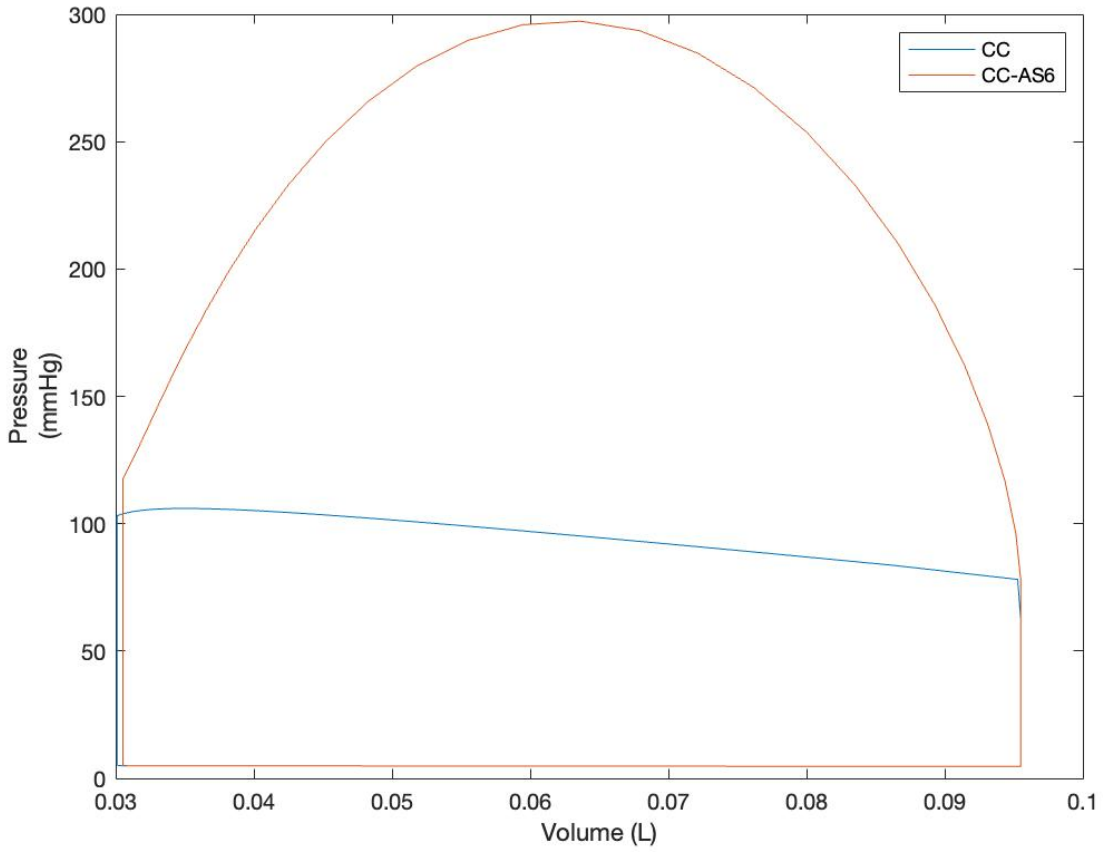

Figure 4.3. CC-AS6: Cardiac cycle diagram when RAo is increased to 6mmHg, which is 600 times its original value. The decrease in stroke volume is nearly neligible, while the systolic blood pressure (nearly 300 mmHg) is tremendous. The observed area of the aortic stenosis cardiac cycle is larger than the normal one which indicates that the heart does a lot more work when the resistance of the aortic valve is high.

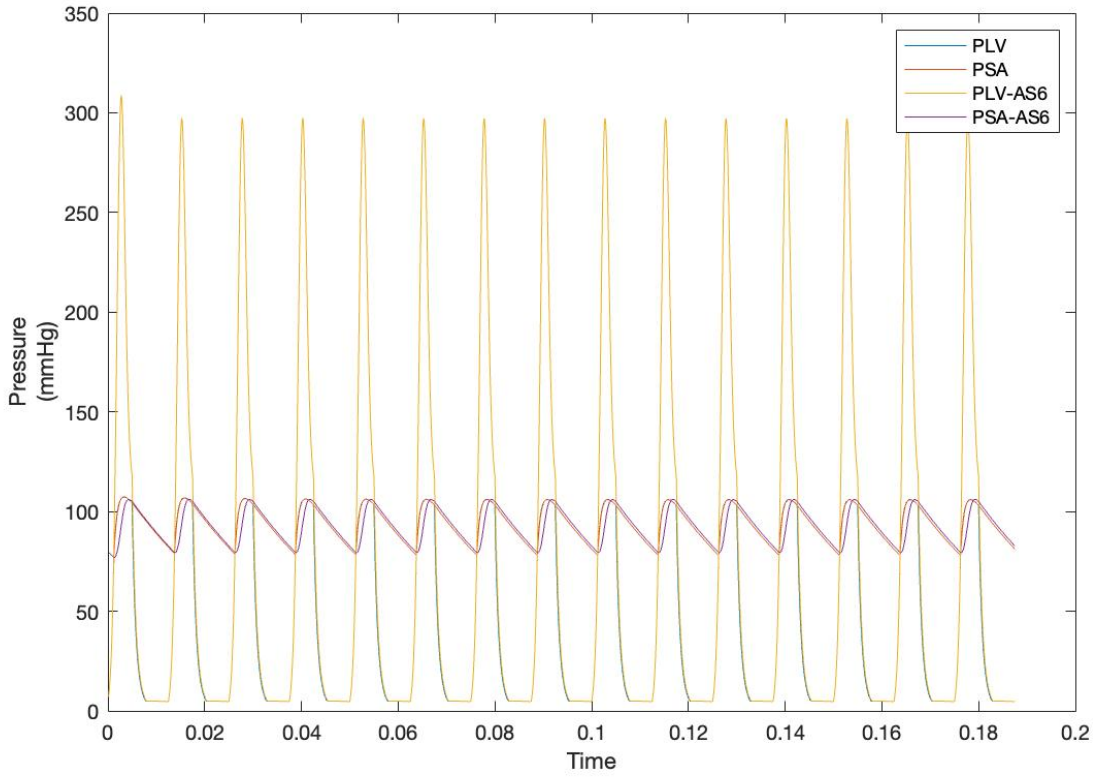

Figure 4.4. P-AS6: Observe that new PLV peaks (shown by PLV-AS6) are much higher than the normal ones. PSA-AS6 shifts to the right of the normal one, and, initially, the peak of the new curve is slightly below that of the normal curve probably because as the resistance of aortic valve increases, the cardiac output is decreased. Therefore, the systemic arterial pressure is decreased. But over time the difference in peaks between the two curves are almost negligible.

Usually the heart compensates aortic stenosis by increasing the left atrial pressure and the compliance of the left ventricle. As the left atrial pressure and the left ventricular muscle stiffens, the left ventricle is able to increase stroke volume. Based on the case when RAo is 6, if we increase CLVD from 0.0146 to 0.0200, CLVS from 0.00003 to 0.00004, and PLA from 5 to 5.5, The stroke volume is increased from 0.0650 to 0.0937. However, as shown in Figure 4.5 and Figure 4.6, such compensation is very likely to cause a stroke because the blood pressure. Observe the purple curve (CLVD, CLVS and PLA are all increased) that the systolic blood pressure is increased to 416.33 mmHg. Therefore, in the case when aortic stenosis increases systolic blood pressure significantly but the decrease in stroke volume is nearly negligible, the proper treatment should not be increasing PLA or CLVS & CLVD. These values should be decreased to avoid a stroke.

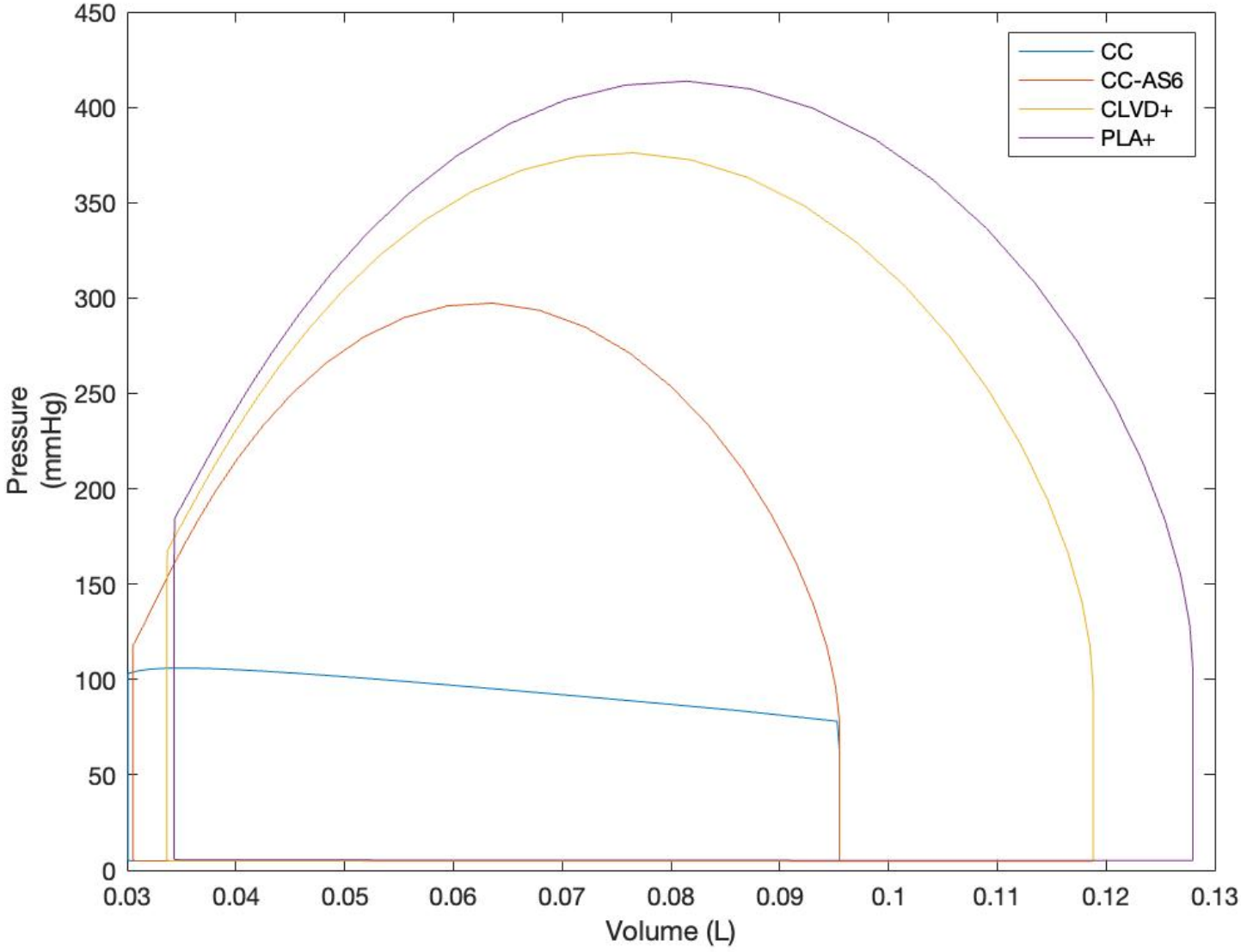

Figure 4.5: The yellow curve is the case when CLVS & CLVD are increased. When CLVS & CLVD and PLA are all increased, observe the purple curve that the increase of these three values have basically the same effect. The loop is enlarged, and systolic blood pressure keeps increasing.

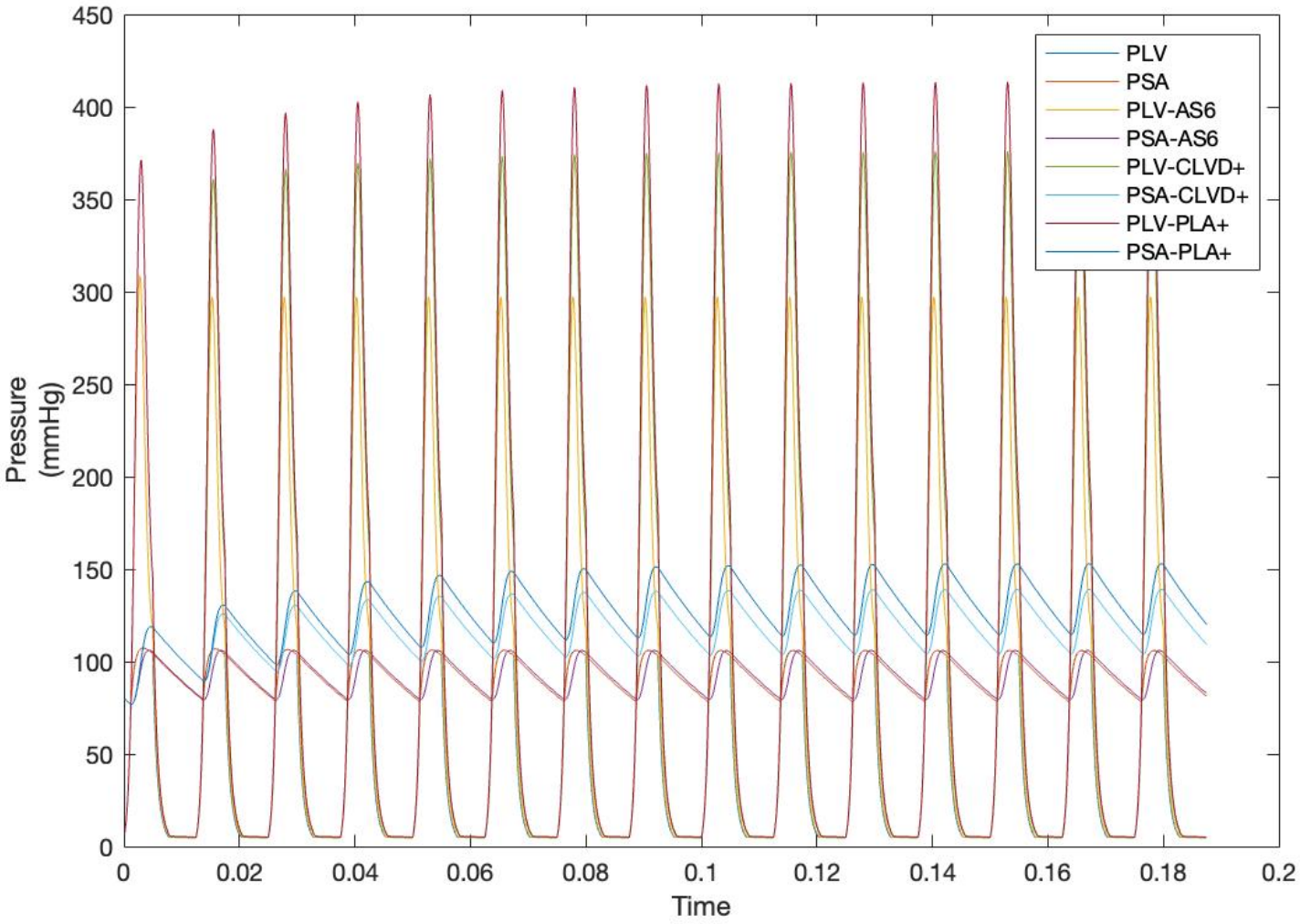

Figure 4.6. Both PLV and PSA curves have increasing peak values. It can be inferred that if the time axis is lengthened, then the peak values will decrease and arrive at steady state. At steady state the peaks should still be higher than the original ones.

The regular stroke volume should be between 0.0500 mL to 0.1000 mL[8]. In the case when aortic resistance is increased from 0.01 to 6, the stroke volume decreases from 0.0650 L to 0.0649 L. The decrease is negligible and the new value is still within the normal range. When CLVS, CLVD, and PLA are all increased, as shown by Figure 4.5, the stroke volume increased to 0.0937 L. In such case when the main problem caused by aortic stenosis is high blood pressure, the goal is to explore how the heart can compensate by maintaining the stroke volume while bringing the systolic blood pressure back to normal (lower than 150 mmHg). It can be inferred that if CLVS, CLVD and PLA are all decreased, the loop will be much smaller, which decreases both systolic blood pressure and the stroke volume.

[8] David Sidebotham, Ian J. Le Grice, Physiology and Pathophysiology, Cardiothoracic Critical Care, 2007

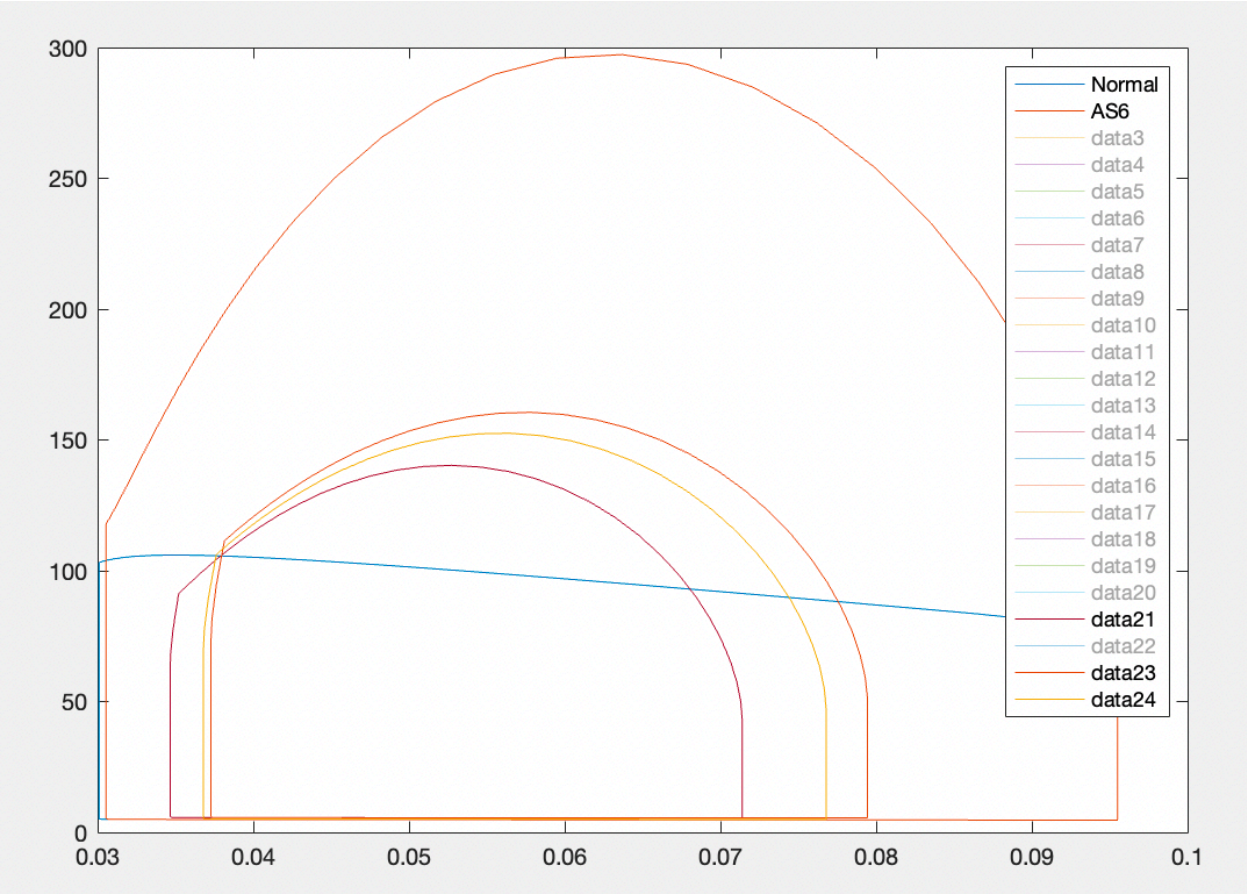

Figure 4.7. Attempts to fix systolic blood pressure with aortic valve resistance = 6. The displayed attempts (data 21, 23, 24) are close to desired situation, in which the systolic blood pressure is below 150 mmHg while the stroke volume is between 50 and 100 mL. For these three attempts, CLVS values are around 0.0001, CLVD values are around 0.001, PLA values are around 5.2. As shown, the stroke volume and systolic blood pressure are positively correlated (either both increase or decrease). The problem with these three attempts is that the stroke volumes are all below 50 mL.

The aortic valve resistance is probably too high. Such high resistance of the aortic valve might exceed the compensation capability of the heart. In later simulation, I will decrease the aortic valve resistance, and explore how the heart can compensate a less severe aortic stenosis.

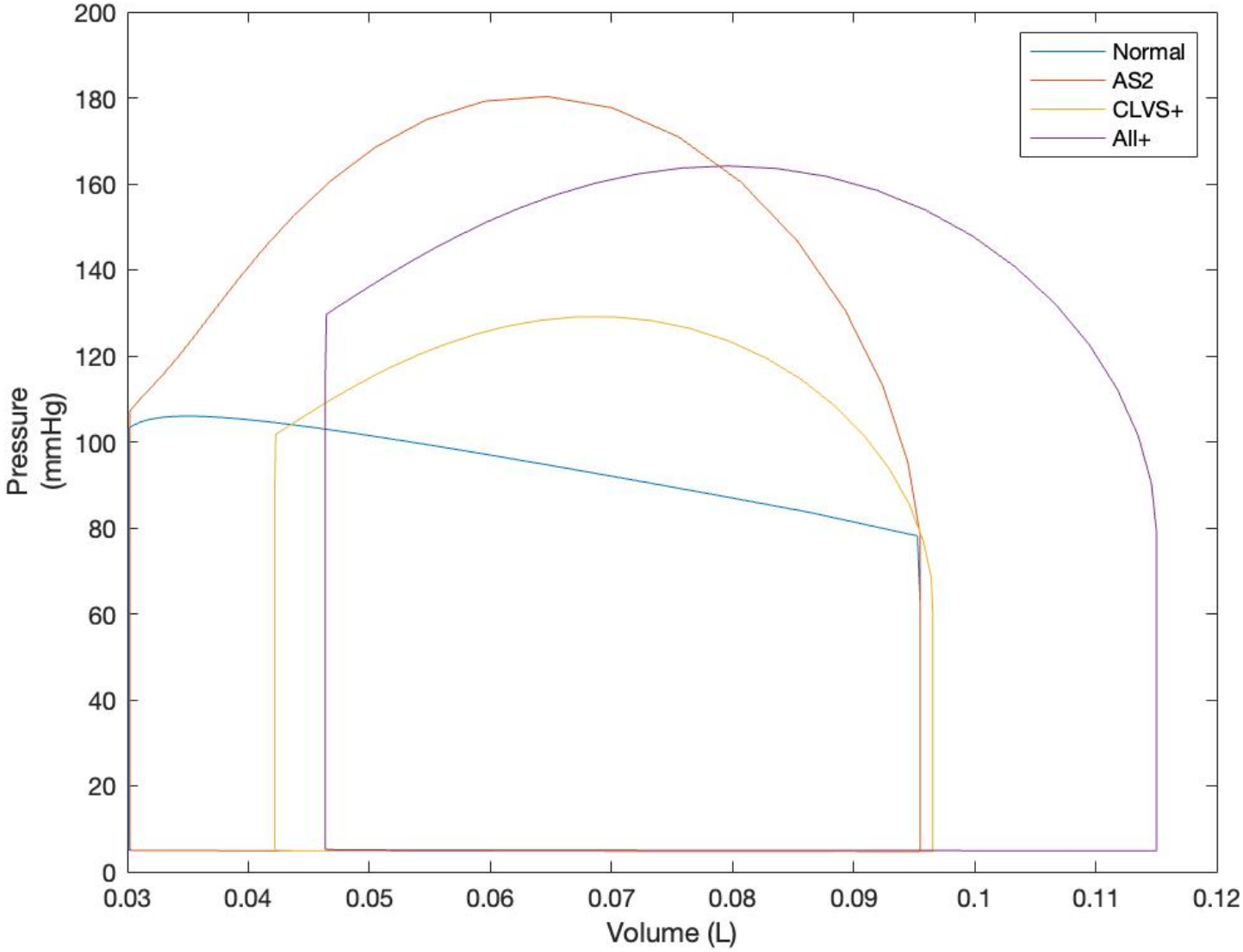

Figure 4.8. When aortic valve resistance is reduced to 2 (diagram AS2), there is a less severe aortic stenosis. The decrease of stroke volume is almost negligible, but the systolic blood pressure is very high (almost 180 mmHg). In this case, the heart can compensate by increasing CLVS only. When CLVS is increased from 0.00003 to 0.00015 (diagram CLVS+), the systolic blood pressure is decreased to nearly 130 mmHg, and the stroke volume is over 50 mL. If we further increase the values of CLVD from 0.0146 to 0.0180, and PLA from 5 to 5.2 (diagram All+), the stroke volume is increased significantly but the systolic blood pressure also increased.

Importantly, observe that the B-C curve (see Figure 2.2, B-C curve is where left ventricle sends out blood) of CLVS+ and All+ are changing relatively milder compared to that of AS2. It can be inferred that the ideal shape (shown by diagram Normal) might be achieved with right values of CLVD, CLVS, and PLA.

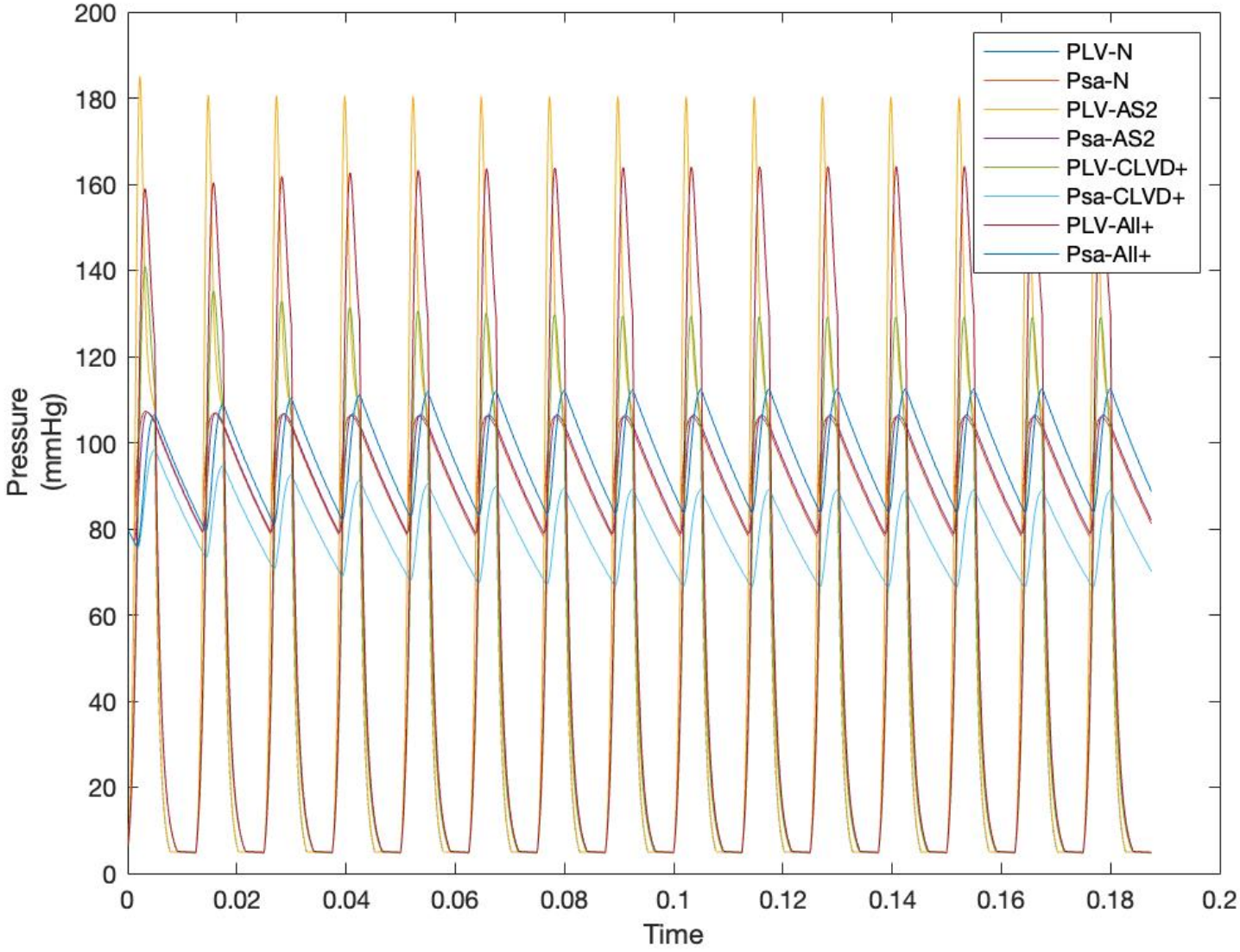

Figure 4.9. The PLV and Psa curves when the aortic valve resistance is 2

5. Conclusion

Unexpectedly, when RAo increases, the restriction of stroke volume is not significant. The most significant effect of increasing RAo is the significant increase of systolic blood pressure. When RAo is 6, the decrease in stroke volume is less than 1 mL, while the blood pressure is too high for the heart to compensate. Therefore, to consider a more realistic case, RAo should be no bigger than 2 (at which systolic blood pressure is 180 mmHg), or it is very likely for the patient to have a stroke.

Under this condition, it is still hard to conclude a numerical relationship between CLVS, CLVD, PLA, and RAo. Qualitatively, the stroke volume and systolic blood pressure are positively correlated. The increase in CLVD and PLA have roughly the same effect.

In the follow up projects, the following two topics will be explored:

1. When CLVD and PLA are increased, the systolic blood pressure and stroke volume will both increase. The systolic blood pressure always increase more rapidly. Whether CLVD or PLA results in the rapid growth of systolic blood pressure should be determined, and that parameter should be restricted.
2. The increase of CLVS has a good restricting effect on systolic blood pressure, but it also decreases stroke volume. Given a certain increase of CLVS, whether systolic blood pressure or stroke volume decreases more rapidly should be determined.

6. Appendix:

Two important MatLab programs, in_LV_sa.m and LV_sa.m, are borrowed from https://www.math.nyu.edu/~peskin/ModSimPrograms/ch1/.

For the input file in_LV_sa.m, the modified values are RAo, CLVS, CLVD, and PLA. And the following codes are modified or added:

```
nStepsPerBeat = 100; % The goal is to plot the steady state of cardiac cycle


dt=1/nStepsPerBeat*T    %Time step duration (minutes)
```

For the output file LV_sa.m, the following codes are modified or added:

```
figure(1)


plot(VLV_plot(end-nStepsPerBeat:end),PLV_plot(end-nStepsPerBeat:end))


hold on


% The goal is to plot the steady steady state of cardiac cycle diagram


figure(2)


plot(t_plot,PLV_plot,t_plot,Psa_plot)


hold on


% The goal is to plot PLV & Psa against time.
```

7. Reference (Alphabetic Order)

American Heart Association, https://www.heart.org/en/health-topics/heart-valve-problems-and-disease/heart-valve-problems-and-causes/problem-aortic-valve-stenosis

David Sidebotham, Ian J. Le Grice, Physiology and Pathophysiology, Cardiothoracic Critical Care, 2007, https://www.sciencedirect.com/topics/nursing-and-health-professions/heart-stroke-volume

Frank C. Hoppensteadt, Charles S. Peskin, Chapter 1.4, Equation 1.4.1 - 1.4.4, Figure 1.4, Figure 1.5, Modeling and Simulation in Medicine and the Life Sciences (Second Edition)

MatLab Programs, LV_sa, in_LV.sa, https://www.math.nyu.edu/~peskin/

T. Brett Reece MD, David A. Fullerton MD, Mitral Stenosis, Abernathy's Surgical Secrets (Sixth Edition), 2009, https://www.sciencedirect.com/topics/medicine-and-dentistry/heart-left-atrium-pressure