NMF

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```
# load the packages
library(NMF)
library(tidyverse)
library(gridExtra)
library(readxl)
```

Procedure

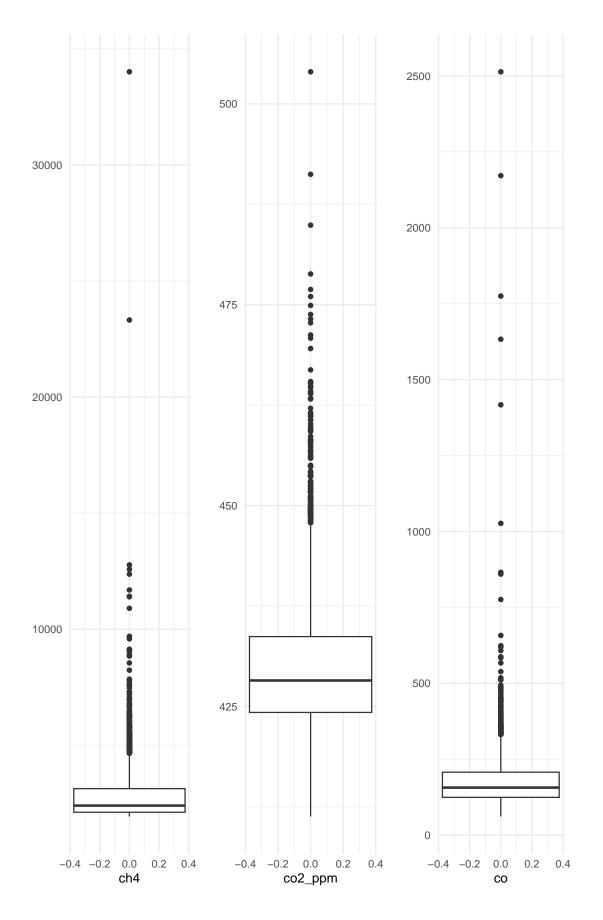
- 1. Remove hourly observation with missing observation for any chemical
- 2. Remove background noise level using min values (except for chemicals with minimum value < 2*LOD and maximum value > 100*LOD)
- 3. Zero values are converted to a random value between 0 and 0.5*LOD
- 4. Normalize using 0th & 99th quantile
- 5. Compute weight matrix according to Guha's paper, without LOQ

Reading the data

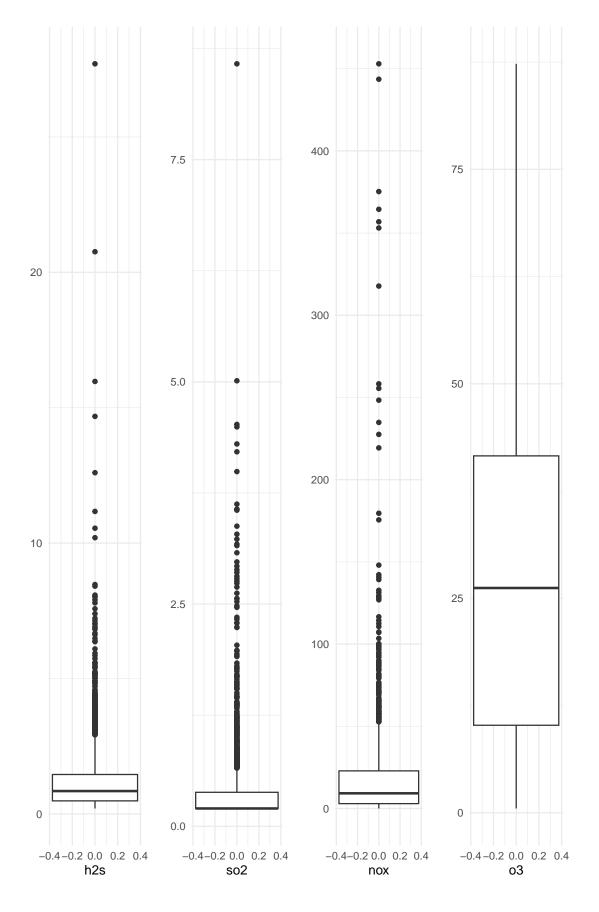
```
# read the radon data
# Old:
# hourly_radon <- readRDS("hourly_radon.rds")</pre>
hourly_data <- readRDS("../DataProcessing/Trailer_hourly_merge_20240905.rds")
# remove NAs
hourly_nona <- hourly_data %>% select(-c(temp_bb,rhi, esf_bb, distToLovi,inv_dist,
                                          distToLovi_wells, monthly_oil, monthly_gas)) %>% na.omit()
vocs <- c("ethane", "ethene", "propane", "propene",</pre>
                                         "1_3-butadiene", "i-butane", "n-butane",
                                         "acetylene", "cyclopentane", "i-pentane",
                                         "n-pentane", "n-hexane", "isoprene", "n-heptane",
                                         "benzene", "n-octane", "toluene", "ethyl-benzene",
                                         "m&p-xylene", "o-xylene")
# retrieving the vocs, removing everything else except the vocs
hourly_vocs <- hourly_nona %>% select(any_of(vocs))
# retrieving the non-vocs: co2_ppm, nox, ch4, h2s, so2, o3
# double check this
non_vocs <- c('ch4', 'co2_ppm', 'co', 'h2s', 'so2', 'nox', 'o3')</pre>
hourly_non_vocs <- hourly_nona %>% select(all_of(non_vocs))
hourly_full_nona <- cbind(hourly_non_vocs, hourly_vocs)
```

```
# retrive a vector of yearmonth
hourly_dates <- hourly_nona %>%
  mutate(yearmonth = substring(day, 0, 7)) %>%
  pull(yearmonth)
```

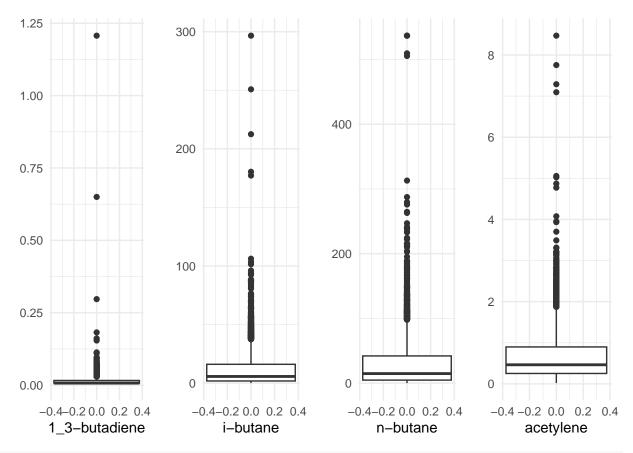
Data visualisation

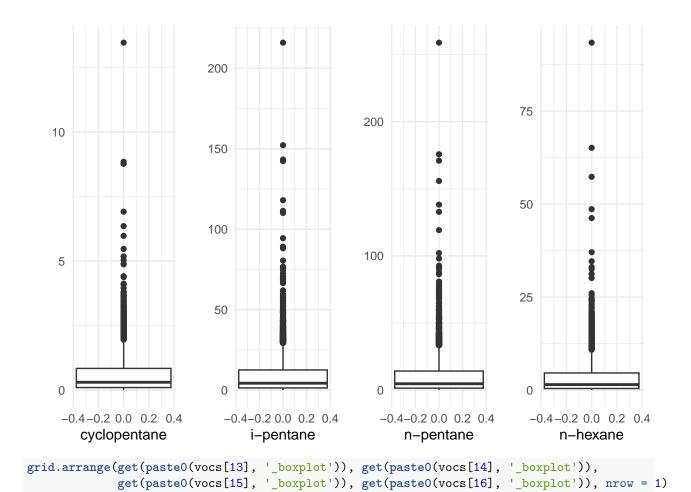


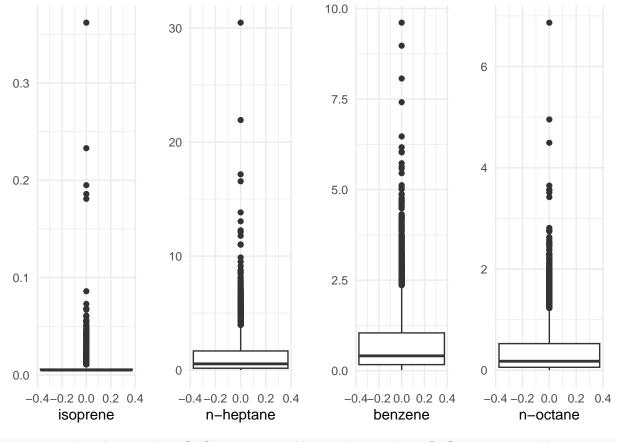
grid.arrange(h2s_boxplot, so2_boxplot, nox_boxplot, o3_boxplot, nrow = 1)

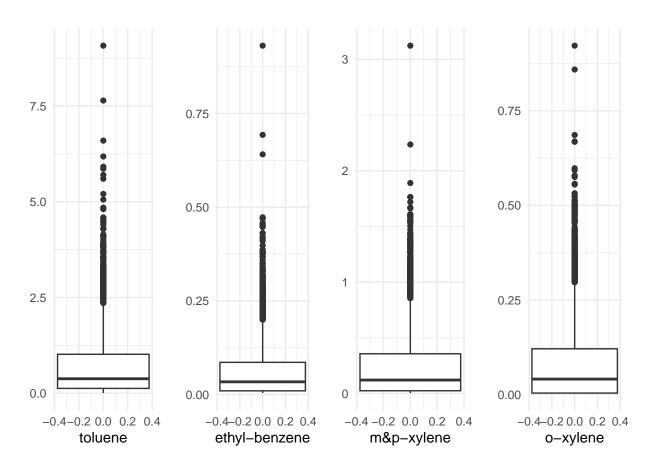


```
for (compound in vocs) {
  assign(pasteO(compound, '_boxplot'),
         ggplot(hourly_vocs) +
           geom_boxplot(aes(y = .data[[compound]])) +
           labs(x = compound, y = '') +
           theme_minimal())
}
grid.arrange(get(paste0(vocs[1], '_boxplot')), get(paste0(vocs[2], '_boxplot')),
             get(paste0(vocs[3], '_boxplot')), get(paste0(vocs[4], '_boxplot')), nrow = 1)
                                                   1250
   2000
                           15
                                                   1000
   1500
                                                                           4
                                                    750
                           10
   1000
                                                    500
                                                                           2
                            5
    500
                                                    250
                                                                           0
                                                      -0.4-0.20.0 0.2 0.4
       -0.4-0.20.0 0.2 0.4
                             -0.4-0.2 0.0 0.2 0.4
                                                                           -0.4 - 0.2 \ 0.0 \ 0.2 \ 0.4
            ethane
                                   ethene
                                                           propane
                                                                                 propene
grid.arrange(get(paste0(vocs[5], '_boxplot')), get(paste0(vocs[6], '_boxplot')),
```









Data preprocessing

```
# Define LOD for each chemical
LOD_non_voc \leftarrow c('ch4' = 0.9,
             co2 ppm' = 0.0433,
             'co' = 40,
             'h2s' = 0.4,
              'so2' = 0.4,
             'nox' = 0.05,
             '03' = 1)
LOD_voc_monthly <- read_csv('../data/LNM_VOC_LOD_Rounded.csv') %>% select(-1)
# extract the yearmonth from date variables
LOD_voc_monthly <- LOD_voc_monthly %>%
  mutate(yearmonth = strftime(as.POSIXct(start_date, format = '%Y-%m-%d %H:%M:%S', tz = 'UTC'), '%Y-%m'
LOD_voc_monthly <- LOD_voc_monthly %>%
  select(-c(start_date, end_date)) %>%
  select(!any_of(ends_with('half_ldl')))
colnames(LOD_voc_monthly) <- str_replace_all(names(LOD_voc_monthly), '_ldl', '')</pre>
LOD_voc_avg <- read_xlsx('../data/LNM_VOC_Uncertainties.xlsx', skip = 1)</pre>
LOD_voc_avg <- LOD_voc_avg %>%
  select(1, 4) %>%
```

```
rename('LOD' = 2, 'chemical' = 1) %>%
  head(20)
# find the min for background-levels
background_levels <- sapply(hourly_full_nona, min)</pre>
background_levels
##
              ch4
                        co2_ppm
                                             СО
                                                          h2s
                                                                         so2
##
        1928.000
                        411.300
                                        61.630
                                                        0.200
                                                                       0.200
##
             nox
                             о3
                                        ethane
                                                       ethene
                                                                     propane
##
           0.025
                          0.500
                                         0.916
                                                        0.011
                                                                       0.224
##
         propene 1_3-butadiene
                                      i-butane
                                                     n-butane
                                                                   acetylene
##
           0.009
                          0.007
                                         0.035
                                                        0.090
                                                                       0.019
##
    cyclopentane
                      i-pentane
                                     n-pentane
                                                     n-hexane
                                                                    isoprene
##
           0.005
                          0.038
                                                        0.021
                                                                       0.005
                                         0.042
##
       n-heptane
                        benzene
                                      n-octane
                                                      toluene ethyl-benzene
##
           0.004
                          0.017
                                         0.004
                                                        0.004
                                                                       0.004
##
      m&p-xylene
                       o-xylene
##
           0.004
                          0.004
get_info <- function(column) {</pre>
  N <- length(column)
  background <- quantile(column, 0)</pre>
  quantile1 <- quantile(column, 0.01)</pre>
  quantile99 <- quantile(column, 0.99)</pre>
  n_background <- sum(column == background)</pre>
  max <- max(column)</pre>
  return(c(N, quantile1, quantile99, max, background, n_background))
}
info_table <- hourly_full_nona %>%
  reframe(across(everything(), ~ get_info(.x)))
info_table <- info_table %>%
  mutate(rownames = c('N', '1st percentile', '99th percentile', 'Max', 'Background', '# Background')) %
  pivot_longer(-rownames) %>%
  pivot_wider(names_from = rownames, values_from = value)
knitr::kable(info_table)
```

						#
name	N	1st percentile	99th percentile	Max	Background	Background
ch4	4497	1963.40000	6318.81200	34010.900	1928.000	1
$co2$ _ppm	4497	417.09000	457.87120	503.990	411.300	1
co	4497	84.90720	444.04320	2513.440	61.630	1
h2s	4497	0.20000	5.18084	27.700	0.200	777
so2	4497	0.20000	1.83896	8.578	0.200	3065
nox	4497	0.22700	92.01080	452.959	0.025	2
o3	4497	0.50000	72.11200	87.300	0.500	255
ethane	4497	1.80852	536.67200	2060.000	0.916	1
ethene	4497	0.01100	3.52212	16.970	0.011	163
propane	4497	0.81700	305.54000	1211.000	0.224	1
propene	4497	0.00900	0.70228	5.528	0.009	401
1_3-butadiene	4497	0.00700	0.05904	1.207	0.007	3126

name	N	1st percentile	99th percentile	Max	Background	# Background
i-butane	4497	0.14496	63.53760	296.600	0.035	1
n-butane	4497	0.34792	171.37600	536.900	0.090	1
acetylene	4497	0.04900	2.66204	8.471	0.019	1
cyclopentane	4497	0.00500	3.12356	13.460	0.005	96
i-pentane	4497	0.10396	51.02080	215.900	0.038	1
n-pentane	4497	0.10300	58.10280	258.800	0.042	1
n-hexane	4497	0.04196	18.32640	93.360	0.021	2
isoprene	4497	0.00500	0.03204	0.362	0.005	2815
n-heptane	4497	0.01500	6.58924	30.470	0.004	5
benzene	4497	0.02700	3.87512	9.610	0.017	3
n-octane	4497	0.00400	2.01452	6.867	0.004	100
toluene	4497	0.01296	3.53640	9.077	0.004	11
ethyl-benzene	4497	0.00400	0.31604	0.931	0.004	898
m&p-xylene	4497	0.00400	1.31824	3.123	0.004	814
o-xylene	4497	0.00400	0.45912	0.922	0.004	1266

```
#adjustments that were made according to paper
#William: I'm guessing this refers to Gunnar's paper section 2.2 and Guha 3.3
adjusting_neg_bg_from_lod <- function(chemical, LOD, background, hourly_data){
    # get min and max
   min_value <- min(hourly_data[chemical], na.rm = TRUE)</pre>
   max_value <- max(hourly_data[chemical], na.rm = TRUE)</pre>
    # if min less than double LOD or max > 100 times LOD
    # adjust to -100 (for entire column???)
    if (min_value < 2 * LOD & max_value > 100 * LOD ){
      return (0)
 return (background)
# Check if background is negligible for non voc
# merge background and LOD
background_lod_non_voc <- tibble(chemical = non_vocs,</pre>
                                 LOD = LOD_non_voc,
                                 background = unname(background_levels[non_vocs]))
adjusted background non voc <- background lod non voc %>%
 rowwise() %>%
  mutate(min = min(hourly_data[chemical], na.rm = TRUE),
         LODx2 = 2 * LOD,
         criterion1 = min(hourly_data[chemical], na.rm = TRUE) < 2 * LOD,</pre>
         max = max(hourly_data[chemical], na.rm = TRUE),
         LODx100 = 100 * LOD,
         criterion2 = max(hourly_data[chemical], na.rm = TRUE) > 100 * LOD,
         adjusted_background = adjusting_neg_bg_from_lod(chemical, LOD, background, hourly_data))
# Check if background is negligible for voc
# merge background and LOD
background_lod_voc <- LOD_voc_avg %>%
  left_join(tibble(chemical = setdiff(names(background_levels), non_vocs),
                   background = background_levels[setdiff(names(background_levels), non_vocs)]))
## Joining with `by = join_by(chemical)`
```

```
adjusted_background_voc <- background_lod_voc %>%
  rowwise() %>%
  mutate(min = min(hourly_data[chemical], na.rm = TRUE),
         LODx2 = 2 * LOD,
         criterion1 = min(hourly_data[chemical], na.rm = TRUE) < 2 * LOD,</pre>
         max = max(hourly_data[chemical], na.rm = TRUE),
         LODx100 = 100 * LOD,
         criterion2 = max(hourly_data[chemical], na.rm = TRUE) > 100 * LOD,
         adjusted_background = adjusting_neg_bg_from_lod(chemical, LOD, background, hourly_data))
# So now we have the adjusted background concentrations
subtract_adj_bg <- function(column, chemical) {</pre>
  print(chemical)
  result <-
  return (result)
hourly_nona_bgrm <- hourly_full_nona %>%
  mutate(across(adjusted_background_non_voc$chemical, ~ .x - adjusted_background_non_voc$adjusted_back
hourly_nona_bgrm <- hourly_nona_bgrm %>%
  mutate(across(adjusted_background_voc$chemical, ~ .x - adjusted_background_voc$adjusted_background[a
# look at zero values
colSums(hourly_nona_bgrm == 0)
##
             ch4
                                                         h2s
                                                                        so2
                        co2_ppm
                                            СО
##
                                             1
                                                         777
                                                                       3065
               1
                              1
##
             nox
                             о3
                                        ethane
                                                      ethene
                                                                    propane
##
               0
                              0
                                                           0
                                             1
                                                                          1
##
         propene 1_3-butadiene
                                     i-butane
                                                    n-butane
                                                                  acetylene
##
                           3126
                                             1
##
    cyclopentane
                      i-pentane
                                    n-pentane
                                                    n-hexane
                                                                   isoprene
##
               0
                              1
                                             1
                                                           0
                                                                       2815
##
       n-heptane
                        benzene
                                     n-octane
                                                     toluene ethyl-benzene
##
                                                           0
                              0
                                             0
                       o-xylene
##
      m&p-xylene
##
               0
                              0
# replace negative values with random values between 0 and 0.5*LOD
replace_zero_with_random <- function(column, name, LOD_df){</pre>
  LOD <- LOD_df$LOD[LOD_df$chemical == name]
  column <- if_else(column == 0, round(runif(length(column), 0, 0.5 * LOD), 3), column)</pre>
  return (column)
}
hourly_nona_bgrm_zerorepl <- hourly_nona_bgrm %>%
  mutate(across(adjusted_background_non_voc$chemical, ~ replace_zero_with_random(.x, cur_column(), adju
hourly_nona_bgrm_zerorepl <- hourly_nona_bgrm_zerorepl %>%
```

Normalize the non-vocs

mutate(across(adjusted_background_voc\$chemical, ~ replace_zero_with_random(.x, cur_column(), adjusted

```
#normalizing function
normalize_column <- function(column){
  background <- quantile(column, 0)
  max <- quantile(column, 1) # this could be adjusted
  return ((column - background)/(max - background))
}</pre>
```

normalize all

hourly_nona_bgrm_zerorepl_norm <- as_tibble(sapply(as.list(hourly_nona_bgrm_zerorepl), normalize_column summary(hourly_nona_bgrm_zerorepl_norm)

```
##
         ch4
                                                                   h2s
                            co2_ppm
                                                 co
    Min.
           :0.000000
                                :0.0000
                                                  :0.00000
                                                                      :0.00000
##
                        Min.
                                          Min.
                                                              Min.
##
    1st Qu.:0.005697
                        1st Qu.:0.1397
                                           1st Qu.:0.02347
                                                              1st Qu.:0.01022
    Median :0.014615
                        Median :0.1826
                                           Median :0.03663
                                                              Median: 0.02338
    Mean
            :0.027164
                        Mean
                                :0.1998
                                           Mean
                                                  :0.04550
                                                              Mean
                                                                      :0.03517
    3rd Qu.:0.037362
                        3rd Qu.:0.2415
                                           3rd Qu.:0.05734
                                                              3rd Qu.:0.04564
                                                  :1.00000
##
    Max.
            :1.000000
                        Max.
                                :1.0000
                                          Max.
                                                              Max.
                                                                      :1.00000
##
         so2
                                                   о3
                              nox
                                                                   ethane
##
    Min.
            :0.000000
                                :0.000000
                                             Min.
                                                    :0.0000
                                                               Min.
                                                                       :0.000000
    1st Qu.:0.007997
                        1st Qu.:0.006515
                                             1st Qu.:0.1118
                                                               1st Qu.:0.008171
                        Median :0.020376
    Median : 0.015756
                                             Median :0.2961
                                                               Median :0.026203
            :0.026295
                                :0.036923
                                                                       :0.051422
##
    Mean
                        Mean
                                             Mean
                                                    :0.3079
                                                               Mean
    3rd Qu.:0.023633
                        3rd Qu.:0.050500
                                             3rd Qu.:0.4735
                                                               3rd Qu.:0.075895
##
    Max.
            :1.000000
                        Max.
                                :1.000000
                                             Max.
                                                    :1.0000
                                                               Max.
                                                                       :1.000000
        ethene
                                                                1 3-butadiene
##
                          propane
                                               propene
            :0.00000
                                                                Min.
                                                                        :0.00000
##
    Min.
                               :0.000000
                                                   :0.000000
                       Min.
                                            Min.
    1st Qu.:0.01303
                       1st Qu.:0.009005
                                            1st Qu.:0.005798
                                                                1st Qu.:0.001667
##
    Median : 0.03615
                       Median: 0.028067
                                            Median :0.018663
                                                                Median : 0.004167
    Mean
                                            Mean
            :0.05116
                       Mean
                               :0.054283
                                                   :0.028932
                                                                Mean
                                                                        :0.007500
##
    3rd Qu.:0.07324
                       3rd Qu.:0.081075
                                            3rd Qu.:0.043305
                                                                3rd Qu.:0.007500
##
    Max.
            :1.00000
                       Max.
                               :1.000000
                                            Max.
                                                   :1.000000
                                                                Max.
                                                                        :1.000000
       i-butane
                           n-butane
##
                                               acetylene
                                                                 cyclopentane
##
    Min.
            :0.000000
                        Min.
                                :0.000000
                                             Min.
                                                    :0.00000
                                                                Min.
                                                                        :0.000000
    1st Qu.:0.005999
                        1st Qu.:0.008556
                                             1st Qu.:0.02769
                                                                1st Qu.:0.007284
                                                                Median :0.022445
    Median :0.019042
                        Median :0.027302
                                             Median :0.05265
##
    Mean
           :0.038815
                        Mean
                                :0.055538
                                             Mean
                                                    :0.07600
                                                                Mean
                                                                        :0.043917
##
    3rd Qu.:0.054019
                        3rd Qu.:0.078140
                                             3rd Qu.:0.10400
                                                                3rd Qu.:0.062356
##
    Max.
            :1.000000
                        Max.
                                :1.000000
                                             Max.
                                                    :1.00000
                                                                Max.
                                                                        :1.000000
                          n-pentane
##
      i-pentane
                                                n-hexane
                                                                   isoprene
##
    Min.
            :0.000000
                        Min.
                                :0.000000
                                             Min.
                                                    :0.00000
                                                                Min.
                                                                        :0.000000
##
    1st Qu.:0.006092
                        1st Qu.:0.005596
                                             1st Qu.:0.00466
                                                                1st Qu.:0.002801
    Median : 0.019717
                        Median: 0.018229
                                             Median :0.01579
                                                                Median :0.005602
                                                    :0.03512
##
    Mean
            :0.041448
                        Mean
                                :0.039184
                                             Mean
                                                                Mean
                                                                        :0.010377
    3rd Qu.:0.057853
                        3rd Qu.:0.054789
                                             3rd Qu.:0.04926
                                                                3rd Qu.:0.011204
    Max.
##
            :1.000000
                        Max.
                                :1.000000
                                             Max.
                                                    :1.00000
                                                                Max.
                                                                        :1.000000
      n-heptane
                           benzene
                                               n-octane
                                                                   toluene
##
            :0.000000
                                :0.00000
                                                   :0.000000
                                                                        :0.00000
    Min.
                        Min.
                                            Min.
                                                                Min.
    1st Qu.:0.005317
##
                        1st Qu.:0.01574
                                            1st Qu.:0.008014
                                                                1st Qu.:0.01345
    Median : 0.017889
                        Median: 0.04097
                                            Median: 0.025499
                                                                Median :0.04122
                                                                        :0.07760
    Mean
           :0.039271
                        Mean
                                :0.07624
                                            Mean
                                                   :0.054094
                                                                Mean
##
    3rd Qu.:0.055078
                        3rd Qu.:0.10706
                                            3rd Qu.:0.076206
                                                                3rd Qu.:0.11165
    Max.
            :1.000000
                        Max.
                                :1.00000
                                            Max.
                                                   :1.000000
                                                                Max.
                                                                        :1.00000
    ethyl-benzene
                          m&p-xylene
                                                o-xylene
```

```
## Min. :0.000000 Min. :0.000000 Min. :0.00000
## 1st Qu.:0.006472 1st Qu.:0.006733 1st Qu.:0.00000
## Median :0.032363 Median :0.037512 Median :0.04031
## Mean :0.061284 Mean :0.076834 Mean :0.08577
## 3rd Qu.:0.088457 3rd Qu.:0.113177 3rd Qu.:0.12745
## Max. :1.000000 Max. :1.000000 Max. :1.00000
```

Combine and Transpose

```
normalized_matrix <- as.matrix(hourly_nona_bgrm_zerorepl_norm) #important: using the normalized VOCs fo

# Transpose <- cbind(Normalized_Data, Merged_VOCs)

# rownames(Transpose) <- as.character(Transpose[,1]) # I'm not able to run this line, but it shouldn't

# transpose_normalized_matrix <- t(as.matrix(normalized_matrix))

number_row<- dim(normalized_matrix)[1] #store number of rows (used for checking)

number_column<- dim(normalized_matrix)[2] #store number of columns
```

NMF section

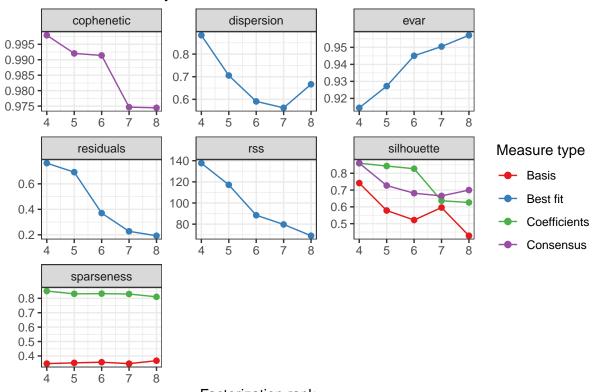
```
# creating uncertainty Matrix
for (i in 1:number_row) {
  for (j in 1:number_column) {
    xij <- normalized_matrix[i, j]</pre>
    LOD <- LOD_merged$LOD[[j]]</pre>
    # Get LOD value for this row
    if (j == 1) {
      # based on equation 6, we sqrt ch4 (at column = 1) and times by 1
      weight_matrix[i, j] <- sqrt(xij)</pre>
    } else if (j == 2) {
      # 0.25 for co2
      weight_matrix[i, j] <- 0.25 * sqrt(xij)</pre>
    } else if (j == 3) {
      # 0.5 for CO
      weight_matrix[i, j] <- 0.5 * sqrt(xij)</pre>
    } else if (xij <= LOD) {</pre>
      weight_matrix[i, j] <- 2 * LOD # equation 5a) in reference paper</pre>
      weight_matrix[i, j] \leftarrow sqrt(((0.1 * xij)**2 + LOD**2)) #equation 5c) in reference paper
    }
  }
```

```
# set a seed for nmf
# set.seed(123)
# #function below used to estimate the optimal rank and will be used in the nmf() function.
# # takes around 20-30 mins to run
# estimate_rank <- nmfEstimateRank(normalized_matrix, 4:8, method = "ls-nmf", weight = weight_matrix, 3
# # # changing the range of rank to 2:20 from 4:20
# saveRDS(estimate_rank, 'estimate_rank.rds')

estimate_rank <- readRDS('estimate_rank.rds')
measures <- estimate_rank$measures
fit <- estimate_rank$fit
consensus <- estimate_rank$consensus

# plots the NMF rank survey
plot(estimate_rank)</pre>
```

NMF rank survey



Factorization rank

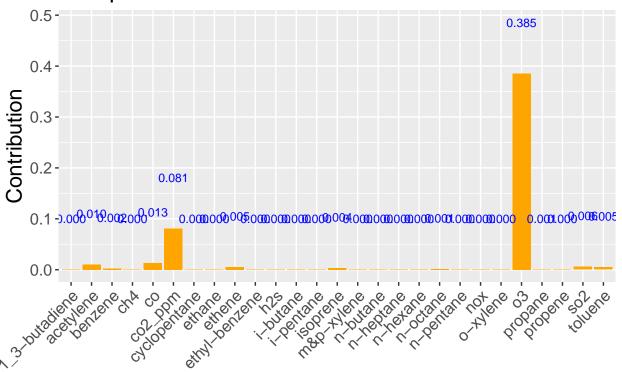
```
# fitting the optimal rank based on the above plots
# the choice of the optimal rank needs to be discussed
output <- nmf(normalized_matrix, rank = 4, weight = weight_matrix, method = "ls-nmf")
W <- basis(output)
H <- coef(output)</pre>
```

Source contributions

```
# Convert H to a data frame for ggplot
H_df <- as.data.frame(H)
```

```
# Add a column for component
H_df$Component <- names(as.data.frame(W))</pre>
# Reshape data to long format
H_long <- pivot_longer(H_df, cols = -Component, names_to = "Chemical", values_to = "Contribution")</pre>
get_component_plot <- function(data, component, title) {</pre>
  component_data <- subset(data, Component == component)</pre>
  plot <- ggplot(component_data, aes(x = Chemical, y = Contribution)) +</pre>
            geom_bar(stat = "identity", position = "dodge", fill = "orange") +
            theme(axis.text.x = element_text(angle = 45, hjust = 1)) +
            geom_text(aes(label = sprintf("%.3f", Contribution)), color = "blue", size = 3, nudge_y = 0
            labs(x = "Chemical", y = "Contribution", title = title)+
            theme(
            text = element_text(size = 14), # Base text size for all text elements
            axis.title = element_text(size = 16), # Size of axis titles
            axis.text = element_text(size = 12), # Size of axis text (tick labels)
            plot.title = element_text(size = 18) # Size of the plot title
 return(plot)
nmfplt_1_ls <- get_component_plot(H_long, 'V1', 'Component 1 ls-nmf Full')</pre>
nmfplt_1_ls
```

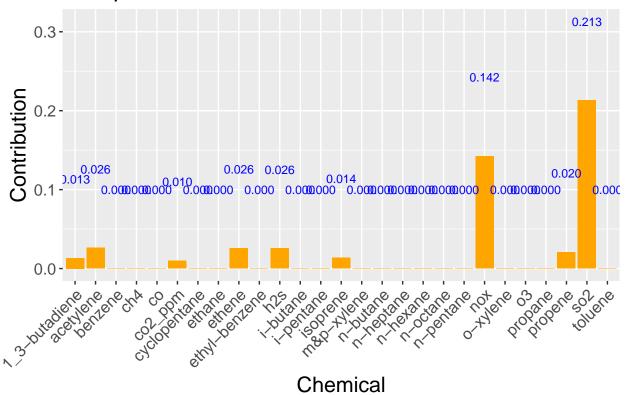
Component 1 Is-nmf Full



Chemical

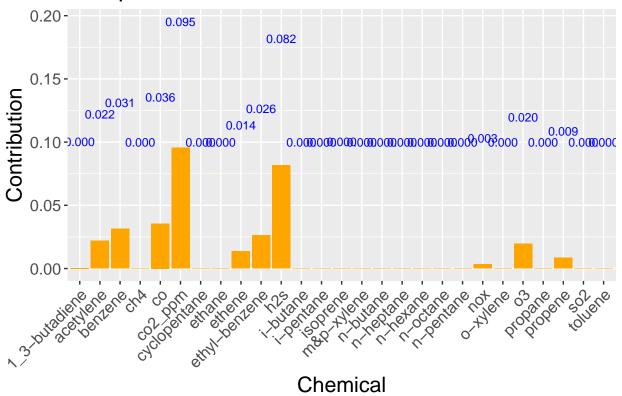
```
nmfplt_2_ls <- get_component_plot(H_long, 'V2', 'Component 2 ls-nmf Full')
nmfplt_2_ls</pre>
```

Component 2 Is-nmf Full



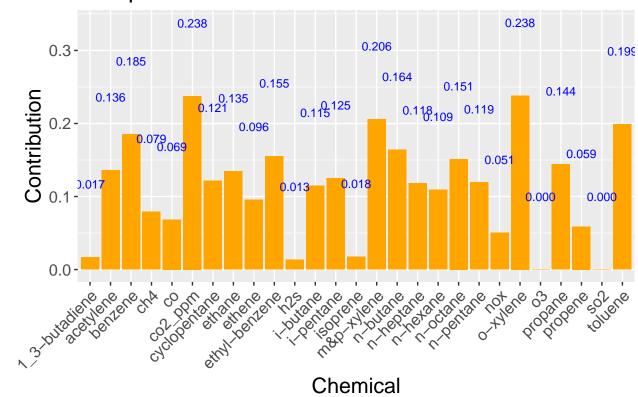
nmfplt_3_ls <- get_component_plot(H_long, 'V3', 'Component 3 ls-nmf Full')
nmfplt_3_ls</pre>

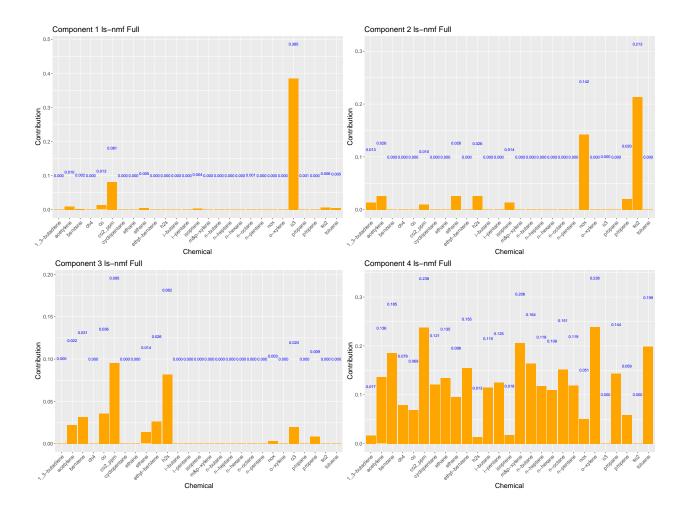
Component 3 Is-nmf Full



nmfplt_4_ls <- get_component_plot(H_long, 'V4', 'Component 4 ls-nmf Full')
nmfplt_4_ls</pre>

Component 4 Is-nmf Full

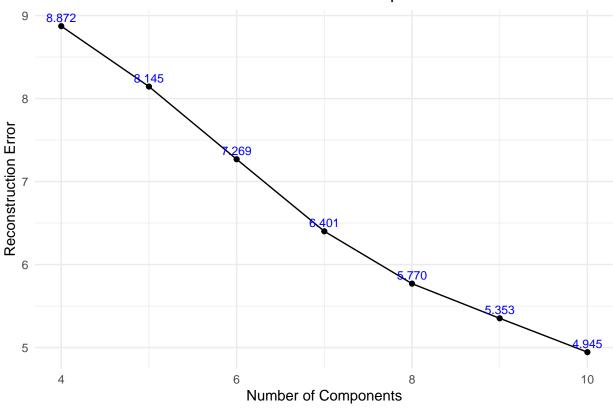




NMF - Eva

```
# Try Eva's approach
components <- 4:10
errors <- numeric(length(components) - 4)</pre>
# Loop over the number of components
# for (n in components) {
    nmf\_result \leftarrow nmf(normalized\_matrix, rank = n, method = "KL", seed='nndsvd')
    reconstruction <- basis(nmf_result) %*% coef(nmf_result)</pre>
#
   error <- norm(normalized_matrix - reconstruction, type = "F")</pre>
#
    errors[n-3] <- error</pre>
#
    print(pasteO('Completed', n - 3, 'out of 7'))
# }
# saveRDS(errors, 'errors_norm.rds')
errors <- readRDS('errors_norm.rds')</pre>
```

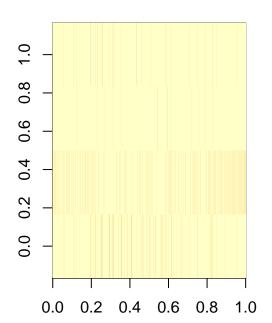
NMF Reconstruction Error vs. Number of Components

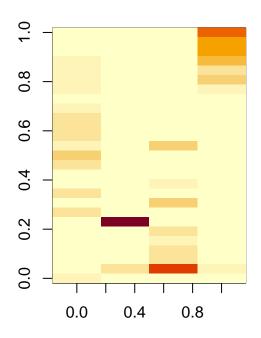


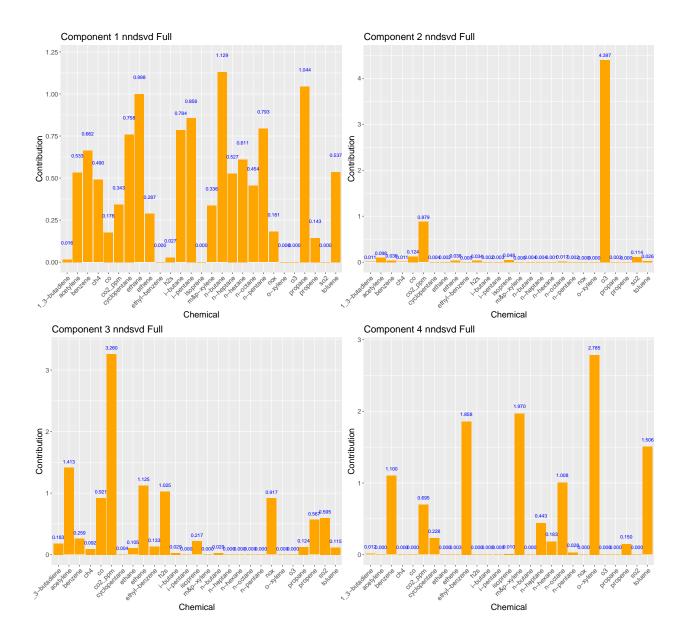
[1] 8.872266

- ## Warning in sqrt(S[i] * termp) * uup: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
- ## Warning in sqrt(S[i] * termp) * vvp: Recycling array of length 1 in array-vector arithmetic is depre
 ## Use c() or as.vector() instead.
- ## Warning in sqrt(S[i] * termp) * uup: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
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- ## Warning in sqrt(S[i] * termn) * uun: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
- ## Warning in sqrt(S[i] * termn) * vvn: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.

Basis Matrix (W)

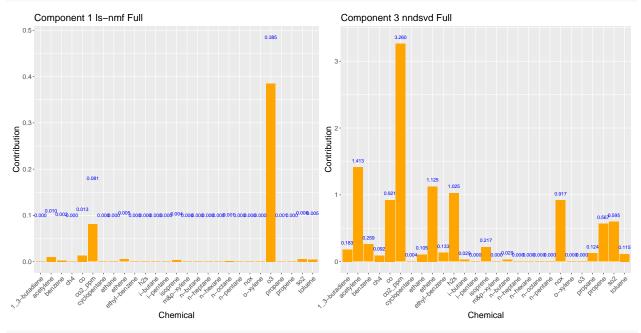




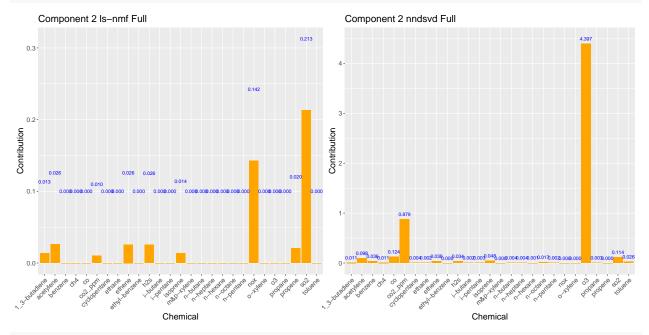


Method comparisons

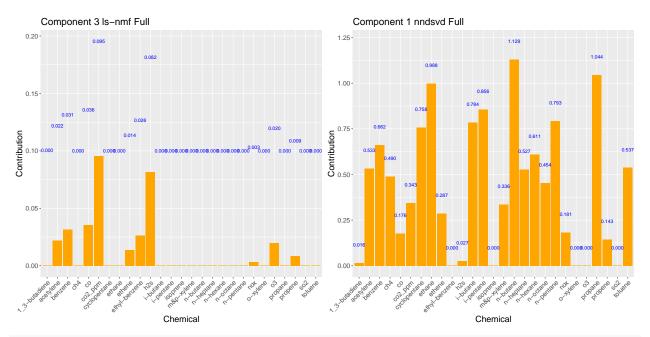
grid.arrange(nmfplt_1_ls, nmfplt_3_svd, ncol = 2)



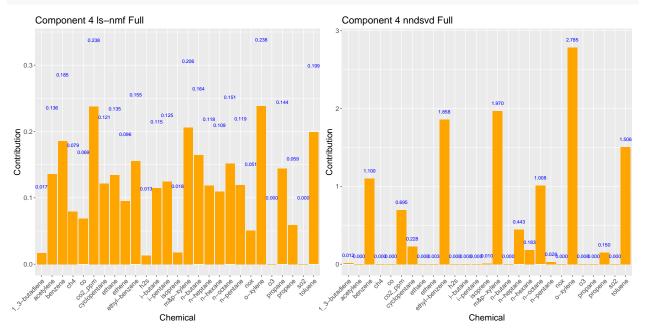
grid.arrange(nmfplt_2_ls, nmfplt_2_svd, ncol = 2)



grid.arrange(nmfplt_3_ls, nmfplt_1_svd, ncol = 2)



grid.arrange(nmfplt_4_ls, nmfplt_4_svd, ncol = 2)



Remove Ozone

4 Components

```
normalized_matrix_less_o3 <- normalized_matrix[ ,setdiff(colnames(normalized_matrix), "o3")]
nmf_result_4c_less_o3 <- nmf(normalized_matrix_less_o3, rank = 4, method = "KL", seed='nndsvd')</pre>
```

Warning in sqrt(S[i] * termn) * uun: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.

```
## Warning in sqrt(S[i] * termn) * vvn: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
```

Warning in sqrt(S[i] * termn) * uun: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.

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Use c() or as.vector() instead.

Warning in sqrt(S[i] * termn) * uun: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.

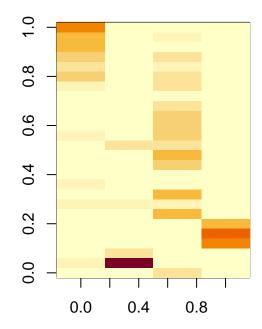
Warning in sqrt(S[i] * termn) * vvn: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.

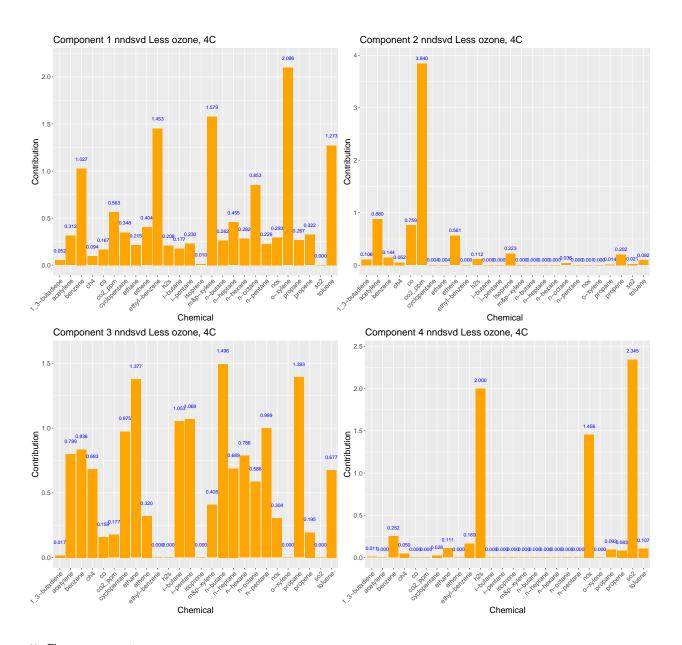
```
basis_matrix_4c_less_o3 <- basis(nmf_result_4c_less_o3)
coef_matrix_4c_less_o3 <- coef(nmf_result_4c_less_o3)

par(mfrow = c(1, 2))
image(basis_matrix_4c_less_o3, main = "Basis Matrix (W)")
image(coef_matrix_4c_less_o3, main = "Coefficient Matrix (H)")</pre>
```

Basis Matrix (W)

0.0 0.2 0.4 0.6 0.8 1.0





5 Components

```
nmf_result_5c_less_o3 <- nmf(normalized_matrix_less_o3, rank = 5, method = "KL", seed='nndsvd')
```

- ## Warning in sqrt(S[i] * termn) * uun: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
- ## Warning in sqrt(S[i] * termn) * vvn: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
- ## Warning in sqrt(S[i] * termn) * uun: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
- ## Warning in sqrt(S[i] * termn) * vvn: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
- ## Warning in sqrt(S[i] * termn) * uun: Recycling array of length 1 in array-vector arithmetic is depre
 ## Use c() or as.vector() instead.

```
## Warning in sqrt(S[i] * termn) * vvn: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
```

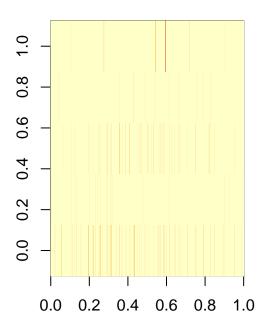
Warning in sqrt(S[i] * termp) * uup: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.

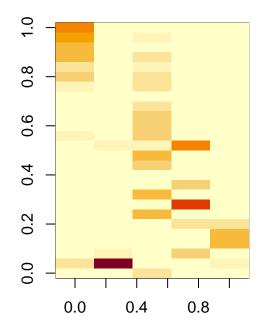
Warning in sqrt(S[i] * termp) * vvp: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.

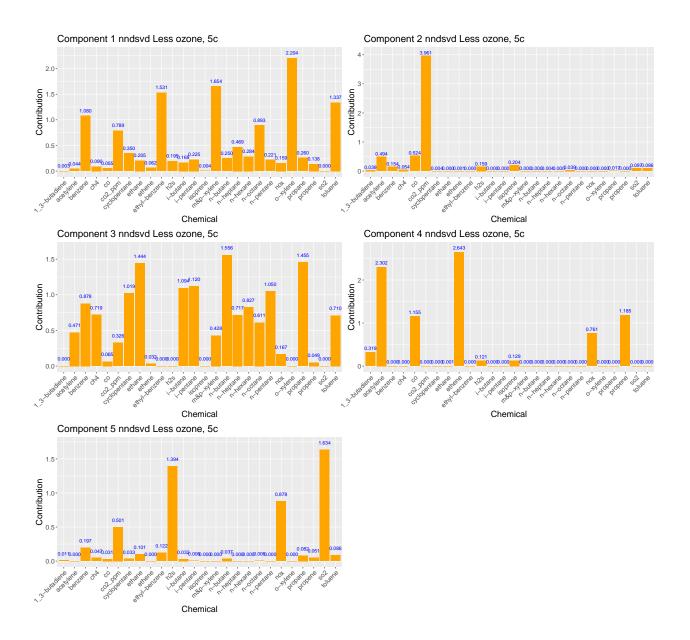
```
basis_matrix_5c_less_o3 <- basis(nmf_result_5c_less_o3)
coef_matrix_5c_less_o3 <- coef(nmf_result_5c_less_o3)

par(mfrow = c(1, 2))
image(basis_matrix_5c_less_o3, main = "Basis Matrix (W)")
image(coef_matrix_5c_less_o3, main = "Coefficient Matrix (H)")</pre>
```

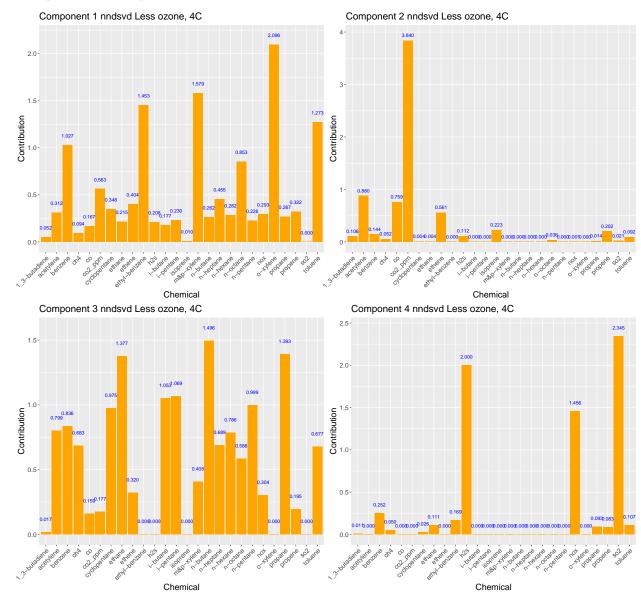
Basis Matrix (W)







Compare to 4 components



Remove Ozone + chemicals with more than 500+ background values

4 Components

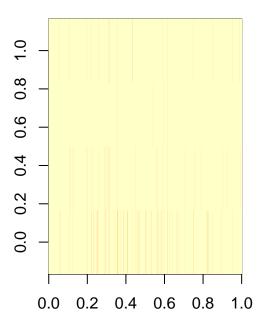
```
normalized_matrix_less_o3_lotbg <- normalized_matrix[ ,setdiff(colnames(normalized_matrix), c('o3', 'h2 nmf_result_4c_less_o3_lotbg <- nmf(normalized_matrix_less_o3_lotbg, rank = 4, method = "KL", seed='nnds ## Warning in sqrt(S[i] * termp) * uup: Recycling array of length 1 in array-vector arithmetic is depre
```

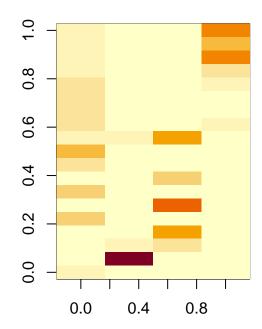
- ## Use c() or as.vector() instead.
 ## Warning in sqrt(S[i] * termp) * vvp: Recycling array of length 1 in array-vector arithmetic is depre
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- ## Warning in sqrt(S[i] * termp) * uup: Recycling array of length 1 in array-vector arithmetic is depre

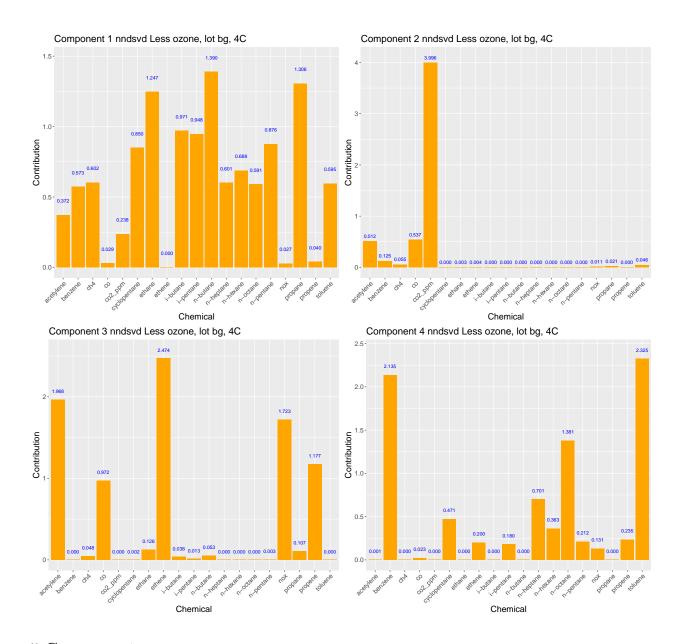
```
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## Use c() or as.vector() instead.
## Warning in sqrt(S[i] * termn) * uun: Recycling array of length 1 in array-vector arithmetic is depre
## Use c() or as.vector() instead.
## Warning in sqrt(S[i] * termn) * vvn: Recycling array of length 1 in array-vector arithmetic is depre
## Use c() or as.vector() instead.
basis_matrix_4c_less_o3_lotbg <- basis(nmf_result_4c_less_o3_lotbg)
coef_matrix_4c_less_o3_lotbg <- coef(nmf_result_4c_less_o3_lotbg)

par(mfrow = c(1, 2))
image(basis_matrix_4c_less_o3_lotbg, main = "Basis Matrix (W)")
image(coef_matrix_4c_less_o3_lotbg, main = "Coefficient Matrix (H)")</pre>
```

Basis Matrix (W)







5 Components

```
nmf_result_5c_less_o3_lotbg <- nmf(normalized_matrix_less_o3_lotbg, rank = 5, method = "KL", seed='nnds
```

- ## Warning in sqrt(S[i] * termp) * uup: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
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```
## Warning in sqrt(S[i] * termn) * vvn: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.
```

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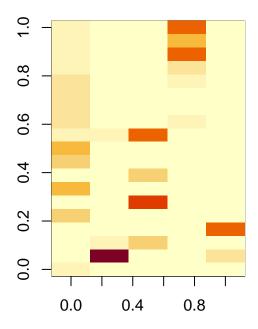
Warning in sqrt(S[i] * termp) * vvp: Recycling array of length 1 in array-vector arithmetic is depre ## Use c() or as.vector() instead.

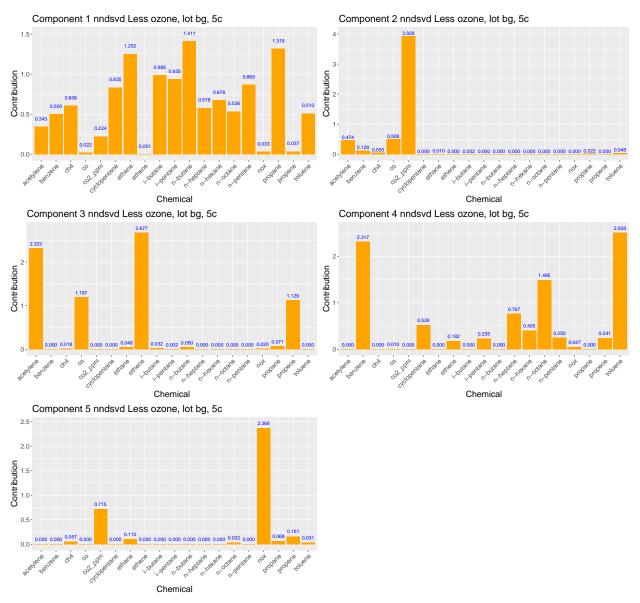
```
basis_matrix_5c_less_o3_lotbg <- basis(nmf_result_5c_less_o3_lotbg)
coef_matrix_5c_less_o3_lotbg <- coef(nmf_result_5c_less_o3_lotbg)

par(mfrow = c(1, 2))
image(basis_matrix_5c_less_o3_lotbg, main = "Basis Matrix (W)")
image(coef_matrix_5c_less_o3_lotbg, main = "Coefficient Matrix (H)")</pre>
```

Basis Matrix (W)

0.0 0.2 0.4 0.6 0.8 1.0





Compare to 4 components

