1. Problem Background and Challenges

- **Market Impact**: Large trades may trigger adverse price movements.
- **Price Risk**: Slow execution may miss favorable price windows.

Traditional methods (e.g., the Almgren-Chriss model) rely on strict market assumptions (e.g., prices following Brownian motion), but real-world markets are complex and dynamic, limiting their practicality. **Reinforcement Learning methods** bypass market assumptions but face challenges:

- **Noisy Data**: Leads to low sample efficiency.
- **Imperfect Information**: Only historical data is available; future information is unknown.

Objective Function: Maximize the $$\arg\max_{q_1,q_2,\ldots,q_T}\sum_{t=0}^{T-1}(q_{t+1}\cdot p_{t+1}),s.t.\sum_{t=0}^{T-1}q_{t+1}=Q.$$

- q_t: Quantity traded at timestep t.
- p_t: Market price at timestep t.
- Q: Total quantity to be traded.

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2. MDP Formulation for Order Execution

Order execution is modeled as a **Markov Decision Process (MDP)**:

- **State**:
  - private variables (remaining inventory, elapsed time)
  - public variables (historical market data, e.g., open prices, trading volumes).
- **Action**:
  - The proportion of assets to trade at each step a_t,
  - constrained by$\sum_{t=0}^{T-1} a_t = 1$.
- **Reward**:
  - **Price Advantage**: Measures trading gains relative to the market average price(across the trading period, no information leakage).
  - **Market Impact Penalty**: Penalizes adverse price effects from large trades.

Reward Function:
$$\hat{R}t^+(\boldsymbol{s}t,a_t)=\frac{q{t+1}}{Q}\cdot\overbrace{\left(\frac{p{t+1}-\tilde{p}}{\tilde{p}}\right)}^{\text{price normalization}}=a_t\left(\frac{p_{t+1}}{\tilde{p}}-1\right)$$

$$\hat{R}_t^-=-\alpha(a_t)^2$$

$$ R_t = \hat{R}_t^+ + \hat{R}_t^-$$

- p_{t+1}: Market price at the next timestep.
- $\tilde{p}$: Average market price over the execution window (contains future information, since reward fucntion is not in state, no information leakage).
- α: Penalty coefficient controlling trade size impact (only temporary, market is resilient, price bounce bakct at next timestep).

Objective Function: $$\arg\max_\pi\mathbb{E}|*pi|left[|sum*{t=0}^{T-1}\gamma^tR_t(\boldsymbol{s}_t,a_t)\right].$$

- π: Policy function. Decides the action of trading volume at each timesteps.

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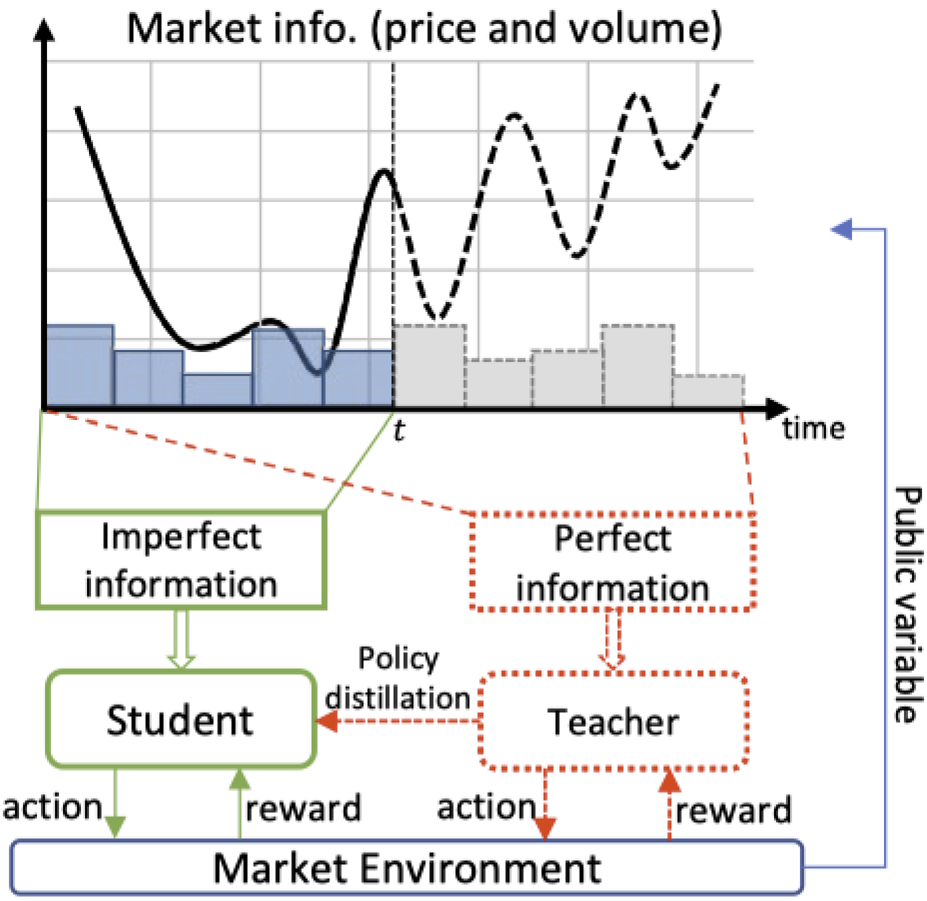
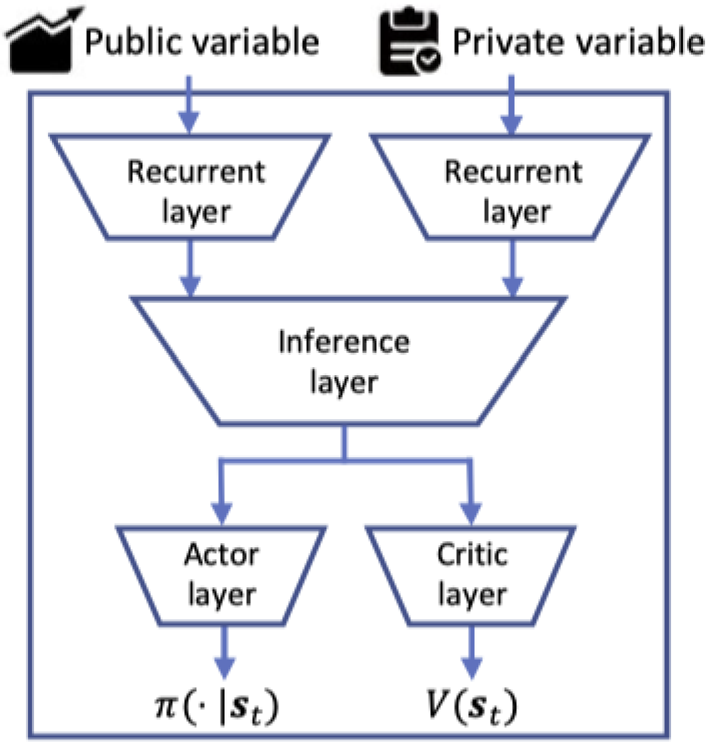
3. Policy Distillation Framework



A **teacher-student framework** addresses imperfect information:

- **Teacher Policy**: Trained with perfect information (including future prices) via RL to generate optimal strategies.
- **Student Policy**: Accesses only historical data and learns generalized strategies by mimicking the teacher.

Training Process:



Loss Function: Combines policy loss L_p), value function loss L_v), and distillation loss L_d):
$$ L(\theta) = L_p + \lambda L_v + \mu L_d$$

- **Policy Loss (L_p)**: Maximizes expected rewards by adjusting policy parameters.
- **Value Function Loss (L_v)**: Minimizes the difference between predicted and actual state values. $$L_v(\boldsymbol{\theta})=\mathbb{E}*t|left[|V*{\boldsymbol{\theta}}(\boldsymbol{s}*t)-V_t|2|right]$$ $$V_t=|sum{t^{|prime}=t}^{T-1}|mathbb{E}|left[|gamma^{T-t^{|prime}-1}R*{t^{\prime}}(s_{t^{\prime}},a_{t^{\prime}})\right]$$ you can imagine this is the reward value of future timesteps using monte carlo simulation.
- **Distillation Loss (L_d)**: Aligns student and teacher policies to enhance generalization. $$L_d=-\mathbb{E}_t\left[\log\Pr(a_t=\tilde{a}*t|\pi*{\boldsymbol{\theta}},\boldsymbol{s}*t;|pi*{\boldsymbol{\phi}},\tilde{\boldsymbol{s}}_t)\right]$$ **Network Architecture**:
- **Recurrent Neural Network (RNN)** processes sequential market data.
- Shared-parameter networks output both action distributions (Actor) and state values (Critic).

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4. Experiments and Results

4.1 Experimental Setup

- **Dataset**: Minute-level trading data from China's A-share market (2017–2019), covering 3,569 assets.

4.2 Evaluation Metrics

1. **Reward**: Mean cumulative reward.

2. **Price Advantage (PA)**: Return improvement over market average (basis points, BP).
3. **Gain-Loss Ratio (GLR)**: Ratio of positive to negative returns.
$$\begin{aligned} & \text{price advantage (PA)}=\frac{10^4}{|\mathbb{D}|}\sum_{k=1}^{|\mathbb{D}|}(\frac{\bar{P}_{\mathrm{strategy}}^k}{\tilde{p}^k}-1) \ & \text{gain-loss ratio (GLR)}=\frac{\mathbb{E}[\mathrm{PA|PA>0}]}{\mathbb{E}[\mathrm{PA|PA<0}]} \end{aligned}$$

4.3 Results

Category	Strategy	Reward($\times 10^{-2}$)	PA	GLR
financial model-based	TWAP (Bertsimas et al. 1998)	-0.42	0	0
	AC (Almgren et al. 2001)	-1.45	2.33	0.89
	VWAP (Kakade et al. 2004)	-0.30	0.32	0.88
learning-based	DDQN (Ning et al. 2018)	2.91	4.13	1.07
	PPO (Lin et al. 2020)	1.32	2.52	0.62
	OPD^S (pure student)	3.24	5.19	1.19
	OPD (our proposed)	**3.36***	**6.17***	**1.35**