HW2 - MA232

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I pledge my honor that I have abided by the Stevens Honor System.

Problem 1

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 3 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 = R_2 + R_1$

$$A_{21} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 8 & 5 & 3 \end{bmatrix} \quad E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 = R_3 + 4R_1$

$$A_{31} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix} \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

$E = E_{21} \cdot E_{31} =$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

$E * A =$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

Problem 2

$$A = \left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 = R_1 + \frac{1}{5}R_4$
$R_2 = R_2 + \frac{1}{4}R_3$
$R_3 = R_3 + \frac{1}{3}R_2$
$R_4 = R_4 + \frac{1}{2}R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & \frac{1}{5} \\ 0 & 1 & 3 & 0 & 0 & 1 & \frac{1}{4} & 0 \\ 0 & 4 & 1 & 0 & 0 & \frac{1}{3} & 1 & 0 \\ 5 & 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & 1 \end{array}\right]$$

$R_4 = R_4 - 5R_1$
$R_3 = R3 - 4R_2$
$R_2 = R_2 - 3R_3$
$R_1 = R_1 - 2R_5$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{5} \\ 0 & 1 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & 0 \end{array}\right]$$

$A^{-1} =$

$$\begin{bmatrix} 0 & 0 & 0 & \frac{1}{5} \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{bmatrix}$$

$$B = \left[\begin{array}{cccc|cccc} 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 6 & 5 & 0 & 0 & 1 & 0 \\ 0 & 0 & 7 & 6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_4 = R_4 - R_3$

$$\left[\begin{array}{cccc|cccc} 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 6 & 5 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

$R_3 = R_3 - 5R_4$

$$\left[\begin{array}{cccc|cccc} 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 4 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 6 & -5 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

$R_4 = R_4 - R_3$
$R_2 = R_2 - R_1$

$$\left[\begin{array}{cccc|cccc} 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 6 & -5 \\ 0 & 0 & 0 & 1 & 0 & 0 & -7 & 6 \end{array}\right]$$

$R_1 = R_1 - 2R_2$
$R_2 = R_2 - R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 3 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -4 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 6 & -5 \\ 0 & 0 & 0 & 1 & 0 & 0 & -7 & 6 \end{array}\right]$$

$B^{-1} =$

$$\begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & -7 & 6 \end{bmatrix}$$

Problem 3

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = R_2 + \frac{1}{2}R_1$

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = R_3 + \frac{2}{3}R_2$

$$U = \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{4}{3} & 0 & \frac{2}{3} & 1 \end{array}\right]$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & -\frac{2}{3} & 1 \end{bmatrix}$$

$A = LDL^T =$

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & -\frac{2}{3} & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & \frac{4}{3} \end{bmatrix} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 4

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$b = \begin{bmatrix} x_1 + x_2 + x_3 \\ x_3 \\ x_3 \end{bmatrix}$$

Problem 5

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$R_2 - R_1$

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$R_3 - R_2$

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 - 2R_2$

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$$

$\frac{1}{2}R_1$

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$$

$R_3 - 2R_2$

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$\frac{1}{4}R_2$

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 - 2R_2$

$$B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$