Neoscholar 2024 – Robot Dynamics and Control – HW3

$$H_{4\times4} = \begin{bmatrix} R_{3\times3} & d_{3\times1} \\ 0_{3\times1} & 1 \end{bmatrix}$$

1. Given the homogeneous matrix H= $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 3 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and the coordinate P(2,1,0). What is the new coordinate after the transformation?

A. $(\sqrt{3}+\frac{5}{2}, \frac{\sqrt{3}}{2}-1, 0)$

B. $(\sqrt{3}-\frac{5}{2}, \frac{\sqrt{3}}{2}-1, 0)$

C. $(\sqrt{3}+\frac{5}{2}, \frac{\sqrt{3}}{2}+1, 0)$

D. $(\sqrt{3}-\frac{5}{2}, \frac{\sqrt{3}}{2}+1, 0)$

① $H \cdot P = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 3 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

=

② $\theta = 30°$ $\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{bmatrix} \sqrt{3}-\frac{1}{2} \\ 1+\frac{\sqrt{3}}{2} \end{bmatrix}$

$\begin{bmatrix} \sqrt{3}-\frac{1}{2} \\ 1+\frac{\sqrt{3}}{2} \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} \sqrt{3}+\frac{5}{2} \\ \frac{\sqrt{3}}{2}-1 \end{bmatrix}$.

2. Assume that the initial positions of the moving coordinate {O':u,v,w} system and the fixed coordinate system {O:x,y,z} coincide with each other, and undergo the following coordinate transformation:a

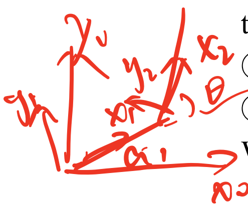
① Rotate counterclockwise 90 degrees around the z-axis

② Translation position vector $4\boldsymbol{i}$-$3\boldsymbol{j}$+$7\boldsymbol{k}$ relative to the fixed coordinate system

(4,-3,7)

current coordinate = $H_1 \times H_2$.

What is the synthetic homogeneous coordinate transformation matrix?



$\{1\} \to \{2\}$.

$\begin{bmatrix} a_1 \\ 0 \end{bmatrix}$

A. $\begin{bmatrix} 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 0 & -1 & 0 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 0 & 1 & -4 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

① $H = \begin{bmatrix} \cos 90° & -\sin 90° & 0 & 4 \\ \sin 90° & \cos 90° & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

② $H_1 = \begin{bmatrix} \cos 90° & -\sin 90° & 0 & 0 \\ \sin 90° & \cos 90° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$H_2 = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

local coordinate $H_2 \times H_1$.

3. In Q2, if a vector point $7\boldsymbol{i}$+$3\boldsymbol{j}$+$2\boldsymbol{k}$ is fixed on the moving coordinate system, that is, the expression of this vector in the moving coordinate system is constant at any time. After the above transformation, the moving coordinate system is expressed as a vector X in the fixed coordinate system, then X=

(7,3,2) in moving frame

A. (1,4,9)

B. (1,2,8)

C. (5,4,6)

D. (4,9,1)

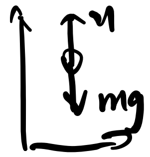
$X_0 = (7,3,2)$

rotation of a vector:

$V_1 = H \cdot V_0$

$X = HP = \begin{bmatrix} 0 & -1 & 0 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 9 \\ 1 \end{bmatrix}$

falling ball : $L = E_k - E_p = \frac{1}{2}m\dot{y}^2 - mgy$.



$\frac{\partial L}{\partial \dot{y}}$ $\frac{\partial L}{\partial y}$

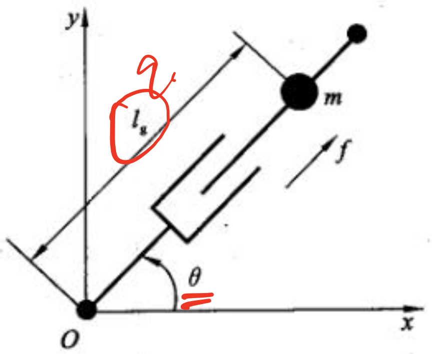
4. Given L=f(y, $\dot{y}$). How to calculate dL_ddy = $\frac{\partial L}{\partial \dot{y}}$ using MATLAB?

A. dL_ddy = jacobian(L,ddy)
B. dL_ddy = jacobian(L,y)
C. dL_ddy = jacobian(dL,dy)
D. dL_ddy = jacobian(L,dy)

5. As shown in the figure is a straight-forward (1P) robot with joints, the mass of the connecting rod is m, and the instantaneous distance from the center of gravity to the far point is l_g. Use the Lagrange equation to establish the dynamic equation of this system. What is the Lagrangian L in this system?

① q
② $L = E_k - E_p$
③ $\frac{\partial L}{\partial \dot{q}}$ $\frac{\partial L}{\partial q}$
④ 代入 Equation

$\frac{dl_g}{dt} = V_{lg}$



$L = E_k - E_p$

$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m \cdot \dot{l}_g^2$

$E_p = mgy = mg\, l_g \sin\theta$.

$\Rightarrow L = \frac{1}{2}m\dot{l}_g^2 - mg\, l_g \sin\theta$.

A. $L = \frac{1}{2}m\dot{l}_g^2 + mgl_g\sin(\theta)$

B. $L = \frac{1}{2}m\dot{l}_g^2 - mgl_g\sin(\theta)$

C. $L = \frac{1}{2}m\dot{l}_g^2 - mgl_g\cos(\theta)$

D. $L = \frac{1}{2}m\dot{l}_g^2 + mgl_g\cos(\theta)$

6. Consider the following simple pendulum system. What is the Lagrangian L in this system? (Considering Potential Energy = 0 at fix base)

$L = E_k - E_p$

$E_k = \frac{1}{2}mv^2 = \frac{1}{2}I w^2$

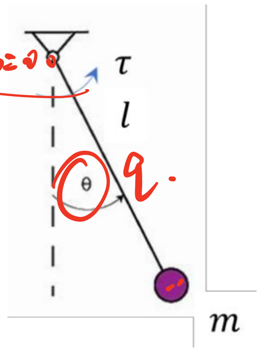
$= \frac{1}{2} \cdot ml^2 \cdot \dot{\theta}^2$

$E_p = -mgy = mg\,l - l\cos\theta)$

$= -mgl\cos\theta$

$L = \frac{1}{2}ml^2 \cdot \dot{\theta}^2 + mgl\cos\theta.$

$E_p = 0$



A. $L = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta$

B. $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta$

C. $L = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\sin\theta$

D. $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl\sin\theta$

7. Consider the same simple pendulum system as Q6. Which equations show the correct of the dynamics of the system:

A. $ml^2\ddot{\theta} + mgl\sin(\theta) = \tau$

B. $ml^2\ddot{\theta} - mgl\sin(\theta) = \tau$

C. $ml^2\ddot{\theta} + mgl\cos(\theta) = \tau$

D. $ml^2\ddot{\theta} - mgl\cos(\theta) = \tau$

$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2}ml^2 \cdot 2\dot{\theta} = ml^2 \cdot \dot{\theta}$

$\frac{\partial L}{\partial \theta} = -mgl\sin\theta$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta}$

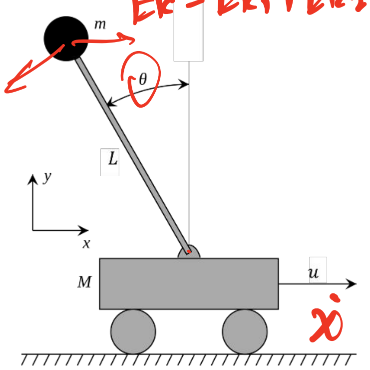
$\Rightarrow ml^2\ddot{\theta} + mgl\sin\theta = \tau$

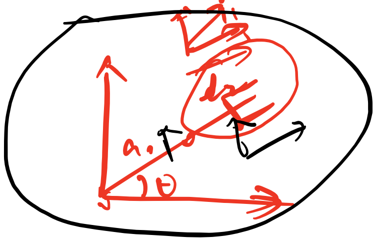
8. Consider the cart-pole system as shown in the figure. Which equations show the correct of the dynamics of the system.

x, y, θ

$E_k = E_{k1} + E_{k2}$ $q : x, \theta$

$L = E_k - E_p.$





θ, l_2

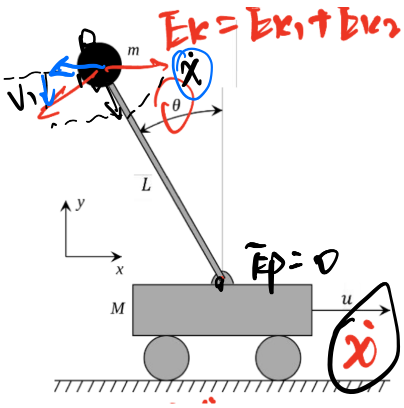
$E_k = E_{k1} + E_{k2}.$

A. $(M+m)\ddot{x} + mL\sin(\theta)\dot{\theta}^2 - mL\cos(\theta)\ddot{\theta} = u$
$mL^2\ddot{\theta} - mL\cos(\theta)\ddot{x} - mgL\sin(\theta) = 0$

B. $(M+m)\ddot{x} + mL\sin(\theta)\dot{\theta}^2 + mL\cos(\theta)\ddot{\theta} = u$
$mL^2\ddot{\theta} - mL\cos(\theta)\ddot{x} - mgL\sin(\theta) = 0$

C. $(M+m)\ddot{x} + mL\sin(\theta)\dot{\theta}^2 - mL\cos(\theta)\ddot{\theta} = u$
$mL^2\ddot{\theta} - mL\cos(\theta)\ddot{x} + mgL\sin(\theta) = 0$

D. $(M+m)\ddot{x} + mL\sin(\theta)\dot{\theta}^2 - mL\cos(\theta)\ddot{\theta} = u$
$mL^2\ddot{\theta} + mL\cos(\theta)\ddot{x} - mgL\sin(\theta) = 0$



$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} I \cdot w^2$$

$$E_k = E_{k1} + E_{k2}$$

$$E_{k1} = \frac{1}{2} M \cdot \dot{x}^2$$

$$E_{k2} = \frac{1}{2} m v^2 = \frac{1}{2} m \overbrace{\left[(\dot{x} - \dot{\theta} L\cos\theta)^2 + (\dot{\theta} L \cdot \sin\theta)^2\right]}^{v^2}$$

$$E_k = E_{k1} + E_{k2} = \frac{1}{2} M\dot{x}^2 + \frac{1}{2} m (\dot{x}^2 + \dot{\theta}^2 L^2 - 2\dot{\theta} L \dot{x}\cos\theta)$$

$$= \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2} m \dot{\theta}^2 L^2 - m\dot{\theta} L \dot{x}\cos\theta$$

$$E_p = mg \cdot L\cos\theta$$

$$L = E_k - E_p = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2} m\dot{\theta}^2 L^2 - m\dot{\theta} L\dot{x}\cos\theta - mgL\cos\theta$$

$$q = \begin{pmatrix} x \\ \theta \end{pmatrix}$$

$$\frac{d\dot{\theta}}{dt} = \ddot{\theta} \qquad \frac{d\cos\theta}{dt} = -\sin\theta \cdot \dot{\theta}$$

对x: $\frac{\partial L}{\partial \dot{x}} = (M+m)\dot{x} - m\dot{\theta} L\cos\theta$

jacobian( )

$$\frac{\partial L}{\partial x} = 0$$

$$\frac{d\left(\frac{\partial L}{\partial \dot{x}}\right)}{dt} = (M+m)\ddot{x} - mL\left[\ddot{\theta}\cos\theta + \dot{\theta}^2(-\sin\theta)\right]$$

代入 Equation:

$$(M+m)\ddot{x} - mL\ddot{\theta}\cos\theta + mL\sin\theta \cdot \dot{\theta}^2 = u \quad ①$$

对θ:

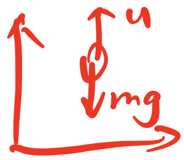
$\ddot{\theta}$ $\quad$ $\ddot{x}$ $\quad$ $-\sin\theta \cdot \dot{\theta}$

$$\frac{\partial L}{\partial \dot{\theta}} = mL^2\dot{\theta} - mL\dot{x}\cos\theta$$

$$\frac{\partial L}{\partial \theta} = m\dot{\theta} L\dot{x}\sin\theta + mgL\sin\theta$$

$$\frac{d\left(\frac{\partial L}{\partial \dot{\theta}}\right)}{dt} = mL^2\ddot{\theta} - mL\left(\ddot{x}\cos\theta + \dot{x}(-\sin\theta)\cdot\dot{\theta}\right)$$

代入: $mL^2\ddot{\theta} - mL\ddot{x}\cos\theta - mgL\sin\theta = 0 \quad ②$



9. Considering a falling ball problem. The ball only experiences gravity mg and vertical upward force u in the vertical direction. Given $m = 0.5\ (kg)$, $g = 9.81\ (m/s^2)$, $u = 5(N)$, $y(0) = 0.5$, $\dot{y}(0) = 0.2$. X_0
Use MATLAB ode solver (https://www.mathworks.com/help/matlab/ref/ode45.html) to simulate the dynamics of the system in 3 seconds. Which figure correctly reflects the changing relationship between displacement y and t?

$$\ddot{y} = \frac{u}{m} - g$$

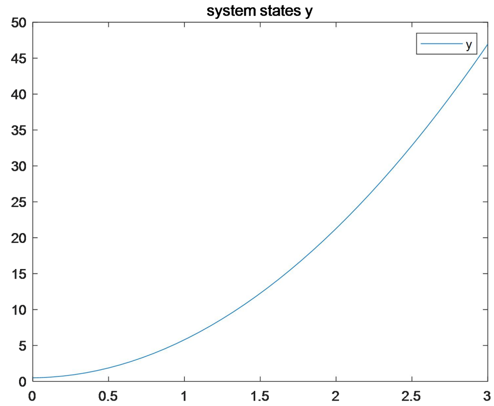
$y(0) = 0.5 \quad \dot{y}(0) = 0.2$

ode45 $\Rightarrow$ y(t)

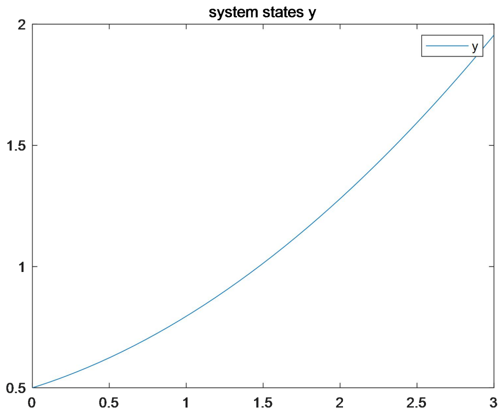
simulation t

$\Rightarrow$ [t, x] = ode45 (equation, tspan, x_0)

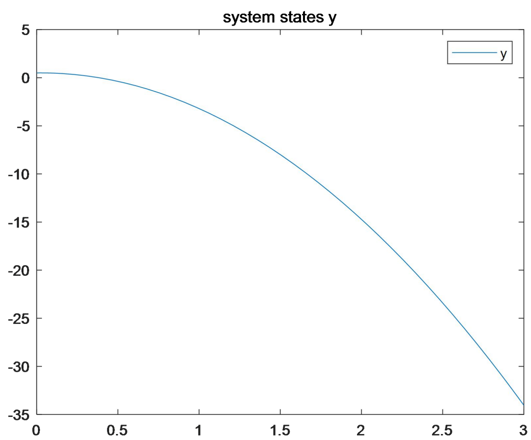
initial condition



A.



B.



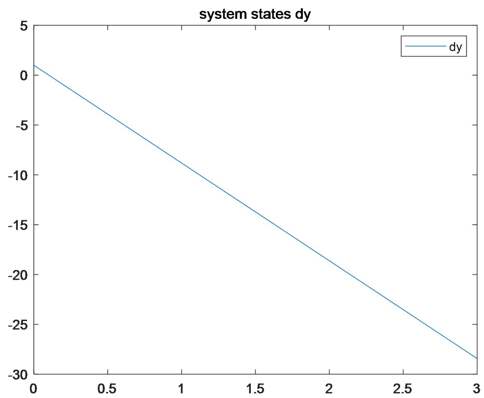
C.

D. None of the above is correct

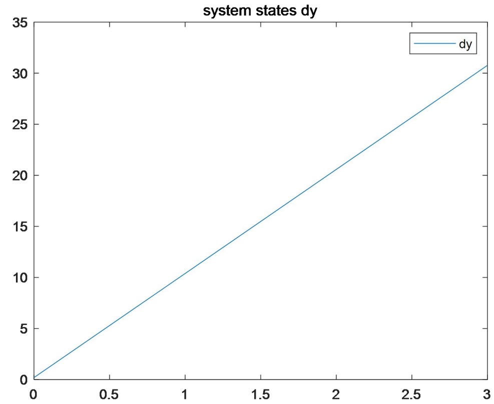
10. Considering a falling ball problem. The ball only experiences gravity mg and vertical upward force u in the vertical direction. Given $m = 0.5\ (kg)$, $g = 9.81\ (m/s^2)$, $u = 1(N)$, $y(0) = -0.5$, $\dot{y}(0) = 0$.

Use MATLAB ode solver (https://www.mathworks.com/help/matlab/ref/ode45.html) to simulate the dynamics of the system in 3 seconds. Which figure correctly reflects the changing relationship between displacement dy and t? plot(t, x(:,2))

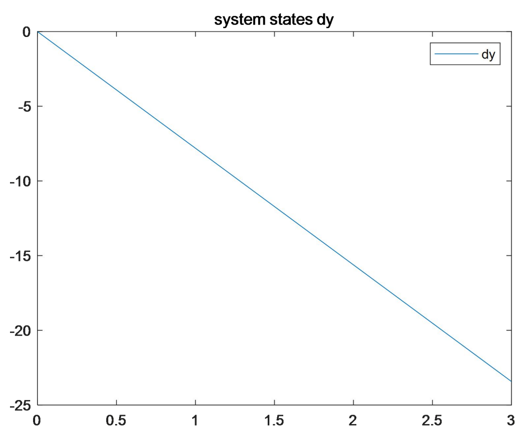
dy



A.



B.



C.

D. None of the above is correct

Main function

```matlab
% demo simulation of a 1D point mass moving along the vertical axis
% system states: X = [y;dy];
% control input: u = F;
clc; clear all; close all;
global params;

m = 0.5 ; g = 9.81 ; u=1;
params.m=m;
params.g =g;
params.u=u;
x0=[0.1;0.2]; % x0 is the intial state of the system
tspan=[0; 1]; % simulation time
[t, x]=ode45(@sys_dynamics, tspan, x0);
 % plot the simulation data
figure; plot(t, x); legend('y','dy'); title('system states');
 % plot the simulation data
%figure; plot(t, x(:, 1)); legend('y'); title('system states y');
%figure; plot(t, x(:, 2)); legend('dy'); title('system states dy');

function dx=sys_dynamics(t, x)
global params;
dy = x(2);
ddy = params.u/params.m-params.g;
dx = [dy;ddy];
end
```

$X = \begin{pmatrix} y \\ dy \end{pmatrix}$ $dx = \begin{pmatrix} \dot{y} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} \dot{y} \\ \frac{u}{m} - g \end{pmatrix}$

$dx \to \begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix}$

$\ddot{y} = \frac{u}{m} - g \to y' \to y.$

$= [$

ode45 : 一阶

$\to \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$

fun

1) 构建 main function

① parameters of the system
$[m, g, L, u]$

② x_0, tspan

③ $[t, x] = \text{ode45}(@\text{sys\_dynamics}, \text{tspan}, x_0)$
$[y(t), y'(t)]$

④ plot

2) dynamics function

$X = \begin{pmatrix} y \\ \dot{y} \end{pmatrix}$ $\dot{X} = dx = \begin{pmatrix} \dot{y} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} x(2) \\ \frac{u}{m} - g \end{pmatrix}$

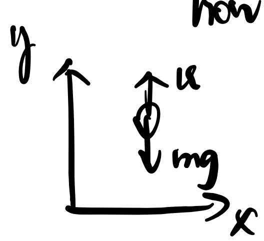
返回 $dx = \begin{bmatrix} x(2) \\ \frac{u}{m} - g \end{bmatrix}$

Dynamics simulation

kinematics : $\underline{\underline{q}} = (\theta_1; \theta_2; \cdots \theta_n) \to P_E^0$: draw the robot

$q(t)$

how to solve $q(t)$? → Dynamics



$q : y$

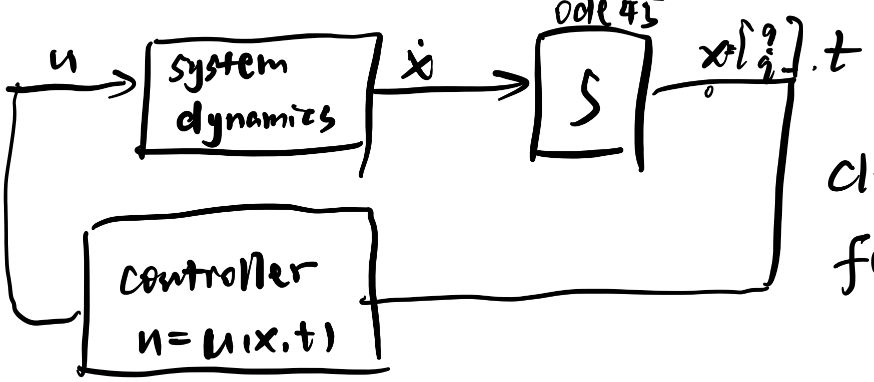
$u - mg = m\ddot{y} \Rightarrow \boxed{\ddot{y} = \frac{u}{m} - g} \Rightarrow y(t)$

ode45

$y(0)$ initial condition

$q(0)$

总结: system dynamics ⇒ solve $q(t)$ ⇒ simulate the system



close-loop system
feedback control

Build system dynamics.

∘ Euler-lagrange Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L}{\partial q} = \tau$$

⇒ Equation of motion (EOM)

$$\underbrace{D(q)}_{n\times n}\cdot \ddot q + \overbrace{\underbrace{C(q,\dot q)\cdot \dot q}_{n\times 1} + \underbrace{g(q)}_{n\times 1}}^{N(q,\dot q)} = \tau .$$

惯性矩阵　　离心力　　重力

$$D(q)\,\ddot q + N(q,\dot q) = \tau$$

eg. HW3 Q8:

$$\begin{cases} (M+m)\ddot x - mL\ddot\theta\cos\theta + mL\sin\theta\cdot\dot\theta^2 = u \\ mL^2\ddot\theta - mL\ddot x\cos\theta - mgl\sin\theta = 0 \end{cases}$$

$\ddot x =$

$\ddot\theta =$

$$q = \begin{pmatrix} x \\ \theta \end{pmatrix} \quad D(q) = \begin{pmatrix} M+m & -ml\cos\theta \\ -ml\cos\theta & ml^2 \end{pmatrix}$$

$$N(q,\dot q) = \begin{pmatrix} ml\sin\theta\cdot\dot\theta^2 \\ -mgl\sin\theta \end{pmatrix}$$

$q = \begin{pmatrix} x \\ \theta \end{pmatrix}$ $\quad \ddot x =$, $\ddot\theta =$

$\ddot q = \begin{pmatrix} \ddot x \\ \ddot\theta \end{pmatrix}$

$$D(q)\,\ddot q + N(q,\dot q) = \tau$$

$$\ddot q = D(q)^{-1}\cdot(\tau - N)$$

$[x(t)\ \theta(t)\ \dot{x}(t)\ \dot{\theta}(t)]$

$ddq = inv(D)\,(2-N)$

plot(t, x) , plot(t, x(:,3))

$x = \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}$ $dx = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix}$

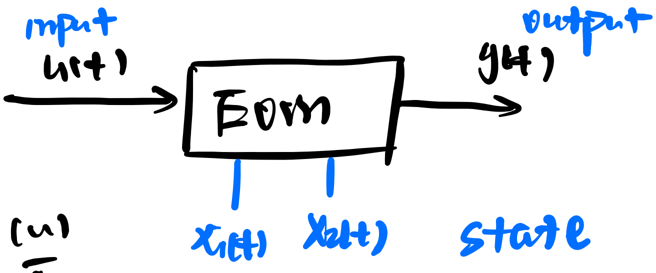
ode45

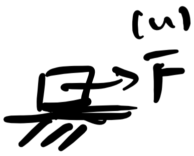
$dx = \begin{bmatrix} x(3) \\ x(4) \\ ddq(1) \\ ddq(2) \end{bmatrix}$

EOM → System dynamics $\dot{x} = f(x,u)$

– state space model

for linear system: $\dot{x} = Ax + Bu$





$F = ma \Rightarrow u = m\ddot{x}$

$X = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$ $\dot{x} = \begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} x(2) \\ \frac{u}{m} \end{pmatrix}$

$\dot{x} = \begin{pmatrix} x(2) \\ \frac{u}{m} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} u$

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix}$

$Y = Cx + Du$

假设 $y(t) = \dot{x}(t)$

$= (0\ 1)\begin{pmatrix} x \\ \dot{x} \end{pmatrix} + [0]\,u$

$C = [0\ 1]$ $D = 0$

Linear System:

形如 $a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt}$

$= b_m \frac{d^m x(t)}{dt^m} + \cdots + b_1 \frac{dx_1(t)}{dt} + b_0 x(t)$

$x(t) \to y(t)$

① 叠加原理. $x_1(t) \to y_1(t) \quad x_2(t) \to y_2(t)$

$x_1(t) + x_2(t) \to y_1(t) + y_2(t)$.

② 齐次原理. $a x_1(t) \to a y_1(t)$