IIIT Hyderabad - Lazy Three

Contents	template <typename t=""></typename>
1. Contest	<pre>using minHeap = priority_queue<t, vector<t="">,</t,></pre>
2. Data Structures	<pre>greater<t>>; template <typename t=""> using maxHeap = priority_queue<t>;</t></typename></t></pre>
3. Graph 10	
4. Number Theory 18	
5. Strings	#define int long long
6. Numerical	<pre>#define all(s) s.begin(), s.end()</pre>
7. Geometry	<pre>#define sz(s) (int)s.size()</pre>
1. Contest	<pre>using longer =int128_t;</pre>
i. Contest	typedef vector <int> vi;</int>
1.1. template.h	typedef pair <int, int=""> pii;</int,>
<pre>#include <bits stdc++.h=""></bits></pre>	<pre>const int INF = numeric_limits<int>::max(); const int M = 1e9 + 7;</int></pre>
<pre>#include <ext assoc_container.hpp="" pb_ds=""></ext></pre>	<pre>void solve() {}</pre>
//#pragma GCC target("bmi,bmi2,lzcnt,popcnt")	<pre>int32_t main() {</pre>
//#pragma GCC optimize("02,unroll-loops")	<pre>ios_base::sync_with_stdio(0);</pre>
//#pragma GCC target("avx2")	<pre>cin.tie(NULL);</pre>
,,p. a.ga ooo tan got(a.m. ,	<pre>cout.tie(NULL);</pre>
//#pragma GCC optimize("02")	<pre>int tt;</pre>
//#pragma GCC optimize("Ofast")	cin >> tt;
//#pragma GCC target("avx,avx2,fma")	<pre>while (tt) solve();</pre>
using namespace std;	}
<pre>using namespacegnu_pbds;</pre>	1.2. mint.h
<pre>typedef tree<int, less<int="" null_type,="">,</int,></pre>	template <const i32="" mod=""></const>
rb_tree_tag, tree_order_statistics_node_update>	struct mint {
o_set;	constexpr mint(i32 $x = 0$) : val($x \% \mod + (x < 0)$
<pre>// order_of_key (val): returns the no. of values</pre>	<pre>* mod) {} mint &operator+=(const mint &b) { val += b.val;</pre>
less than val	
<pre>// find_by_order (k): returns the kth largest</pre>	
element.(0-based)	· · · · · · · · · · · · · · · · · · ·

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val -= mod * (val ≥ mod);
    return *this;
  }
  mint &operator-=(const mint &b) {
    val -= b.val;
    val += mod * (val < 0);</pre>
    return *this;
  mint &operator*=(const mint &b) {
    val = 111 * val * b.val % mod;
    return *this;
  mint &operator (const mint &b) { return *this *=
b.inv(); }
 mint inv() const {
    i32 \times = 1, y = 0, t;
    for (i32 a = val, b = mod; b; swap(a, b),
swap(x, y))
      t = a / b, a -= t * b, x -= t * y;
    return mint(x);
 mint pow(int b) const {
    mint a = *this, res(1);
    for (; b; a *= a, b \not= 2)
      if (b & 1) res *= a;
    return res;
  friend mint operator+(const mint &a, const mint
&b) { return mint(a) += b; }
  friend mint operator-(const mint &a, const mint
```

```
&b) { return mint(a) -= b; }
  friend mint operator*(const mint &a, const mint
&b) { return mint(a) *= b; }
  friend mint operator/(const mint &a, const mint
&b) { return mint(a) /= b; }
  friend bool operator=(const mint &a, const mint
&b) { return a.val = b.val; }
  friend bool operator*(const mint &a, const mint
&b) { return a.val *= b.val; }
  friend bool operator<(const mint &a, const mint
&b) { return a.val < b.val; }
  friend ostream &operator<<(ostream &os, const
mint &a) { return os << a.val; }
  i32 val;
};</pre>
```

2

2. Data Structures

2.1. SegTree.h

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```
= a[i];
                                                                                                                                                                                                                                                                                 if (t[1] < k) return n;
                  for (int i = n - 1; i \ge 1; i--) t[i] = f(t[2 *
                                                                                                                                                                                                                                                                                T rem = id;
i], t[2 * i + 1]);
                                                                                                                                                                                                                                                                                int v = 1;
        }
                                                                                                                                                                                                                                                                                while (v < n) {
         int i2leaf(int i) { return i + off - (i < 2 *</pre>
                                                                                                                                                                                                                                                                                         T resl = f(rem, t[2 * v]);
ct ? 0 : n); }
                                                                                                                                                                                                                                                                                         if (resl \geq k) {
          int leaf2i(int l) { return l - off + (l < off ?</pre>
                                                                                                                                                                                                                                                                                                   v = 2 * v;
n : 0); }
                                                                                                                                                                                                                                                                                          } else {
        T query(int l, int r) {
                                                                                                                                                                                                                                                                                                   rem = resl;
                  l = (l < 2 * ct) ? (l + off) : 2 * (l + off - off) : 2 * (l + off) : 2 * (l 
                                                                                                                                                                                                                                                                                                   v = 2 * v + 1;
n);
                                                                                                                                                                                                                                                                                         }
                   r = (r < 2 * ct) ? (r + off) : 2 * (r + off - off) : 2 * (r + off) : 2 * (r 
                                                                                                                                                                                                                                                                                 }
                                                                                                                                                                                                                                                                                return leaf2i(v);
n);
                                                                                                                                                                                                                                                                      }
                   r += (r \ge 2 * n);
                  T resl(id), resr(id);
                                                                                                                                                                                                                                                               };
                  for (; l \leq r; l \gg 1, r \gg 1) {
                                                                                                                                                                                                                                                               2.2. LazySegTree.h
                           if (l = r) {
                                                                                                                                                                                                                                                               template <typename T, typename U>
                                    resl = f(resl, t[l]);
                                                                                                                                                                                                                                                               struct seq_tree_lazy {
                                    break;
                                                                                                                                                                                                                                                                        int S, H;
                                                                                                                                                                                                                                                                        T zero;
                           if (l & 1) resl = f(resl, t[l++]);
                                                                                                                                                                                                                                                                        vector<T> value;
                            if (!(r \& 1)) resr = f(t[r--], resr);
                                                                                                                                                                                                                                                                        U noop;
                   }
                                                                                                                                                                                                                                                                        vector<bool> dirty;
                   return f(resl, resr);
                                                                                                                                                                                                                                                                        vector<U> prop;
                                                                                                                                                                                                                                                                        seq_tree_lazy(int _S, T _zero = T(), U _noop =
          void update(int v, T value) {
                                                                                                                                                                                                                                                              U()) {
                  for (t[v = i2leaf(v)] = value; v \gg 1;)
                                                                                                                                                                                                                                                                                 zero = _zero, noop = _noop;
                           t[v] = f(t[2 * v], t[2 * v + 1]);
                                                                                                                                                                                                                                                                                 for (S = 1, H = 1; S < _S;) S *= 2, H++;
                                                                                                                                                                                                                                                                                 value.resize(2 * S, zero);
          int lower_bound(int k) {
```

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dirty[l] = false;
    dirty.resize(2 * S, false);
    prop.resize(2 * S, noop);
                                                             }
                                                            }
  void set_leaves(vector<T> &leaves) {
    copy(leaves.begin(), leaves.end(),
                                                            void upd(int i, int j, U update) {
value.begin() + S);
                                                              i += S, j += S;
    for (int i = S - 1; i > 0; i--)
                                                              propagate(i), propagate(j);
      value[i] = value[2 * i] + value[2 * i + 1];
                                                              for (int l = i, r = j; l \leq r; l \not= 2, r \not= 2)
  }
  void apply(int i, U &update) {
                                                                if ((l \& 1) = 1) apply(l \leftrightarrow , update);
                                                                if ((r \& 1) = 0) apply(r--, update);
    value[i] = update(value[i]);
   if (i < S) {
      prop[i] = prop[i] + update;
                                                              rebuild(i), rebuild(j);
      dirty[i] = true;
   }
                                                            T query(int i, int j) {
  }
                                                              i += S, j += S;
  void rebuild(int i) {
                                                              propagate(i), propagate(j);
    for (int l = i / 2; l; l \neq 2) {
                                                              T res_left = zero, res_right = zero;
      T combined = value[2 * l] + value[2 * l + 1];
                                                              for (; i \le j; i \not= 2, j \not= 2) \{
      value[l] = prop[l](combined);
                                                                if ((i \& 1) = 1) res_left = res_left +
                                                          value[i++];
                                                                if ((j \& 1) = 0) res_right = value[j--] +
  void propagate(int i) {
                                                          res_right;
    for (int h = H; h > 0; h--) {
      int l = i >> h;
                                                              return res_left + res_right;
      if (dirty[l]) {
        apply(2 * l, prop[l]);
                                                          };
        apply(2 * l + 1, prop[l]);
                                                          struct node {
                                                            int sum, width;
                                                            node operator+(const node &n) {
        prop[l] = noop;
```

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```
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    // Change 1
    return {sum + n.sum, width + n.width};
};
struct update {
  bool type; // 0 for add, 1 for reset
  int value;
  node operator()(const node &n) { // apply update
on n
    // Change 2
    if (type)
      return {n.width * value, n.width};
    else
      return {n.sum + n.width * value, n.width};
  }
  update operator+(const update &u) { // u is the
recent update, *this is the older update
    // Change 3
    if (u.type) return u;
    return {type, value + u.value};
};
2.3. RMQ.h
template <class T>
struct RMO {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) : jmp(1, V) {
    for (int pw = 1, k = 1; pw * 2 \leq sz(V); pw *=
```

2, ++k) {

```
jmp.emplace_back(sz(V) - pw * 2 + 1);
      for (int j = 0; j < sz(jmp[k]); j++)</pre>
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j]
+ pw]);
    }
  T query(int a, int b) {
    assert(a \leq b); // tie(a, b) = minimax(a, b)
    int dep = 63 - __builtin_clzll(b - a + 1);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)</pre>
+ 1]);
  }
};
2.4. Fenwick.h
template <typename T>
struct Fenwick {
  vector<T> bit;
  vector<T>& original;
  Fenwick(vector<T>& _arr) : bit(_arr.size(), OLL),
original(_arr) {
    int n = sz(\_arr);
    for (int i = 0; i < n; i++) {
      bit[i] = bit[i] + _arr[i];
      if ((i | (i + 1)) < n) bit[(i | (i + 1))] =
bit[(i | (i + 1))] + bit[i];
    }
 // returns smallest index i, st. sum[0..i] \ge x,
returns -1 if no such i exists
```

```
// returns n if x \ge sum of array
  // ASSUMES NON NEGATIVE ENTRIES IN TREE
  int lower_bound(int x) {
   if (x < 0) return -1;
   if (x = 0) return 0;
    int pos = 0;
    for (int pw = 1LL \ll 20; pw; pw \gg 1)
      if (pw + pos \leq sz(bit)) and bit[pos + pw - 1]
< x)
        pos += pw, x -= bit[pos - 1];
    return pos;
  }
  T query(int r) {
    assert(r < sz(bit));</pre>
    int ret = 0;
    for (r++; r > 0; r \&= r - 1) ret += bit[r - 1];
    return ret;
  T query(int l, int r) {
   T ret = query(r);
    if (l \neq 0) ret -= query(l - 1);
    return ret;
  void update(int i, int x) {
    int n = bit.size();
    T diff = x - original[i];
    original[i] = x;
    for (; i < n; i = i | i + 1) bit[i] += diff;
};
```

2.5. cht.h

```
using i64 = int64_t;
struct Line {
  mutable i64 m, c, p;
  bool operator<(const Line& o) const { return m <</pre>
o.m: }
  bool operator<(i64 x) const { return p < x; }</pre>
};
struct LineContainer : multiset<Line, less<>>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const i64 inf = LONG_LONG_MAX;
  i64 div(i64 a, i64 b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b);
  bool isect(iterator x, iterator y) {
     if (y = end()) return x \rightarrow p = inf, 0;
    if (x \rightarrow m = y \rightarrow m)
       x \rightarrow p = x \rightarrow c > y \rightarrow c ? inf : -inf;
     else
       x \rightarrow p = div(y \rightarrow c - x \rightarrow c, x \rightarrow m - y \rightarrow m);
     return x \rightarrow p \geqslant y \rightarrow p;
  void add(i64 m, i64 c) {
     auto z = insert(\{m, c, 0\}), y = z++, x = y;
     while (isect(y, z)) z = erase(z);
     if (x \neq begin() \&\& isect(--x, y)) isect(x, y =
erase(y));
     while ((y = x) \neq begin() \&\& (--x) \rightarrow p \geqslant y \rightarrow p)
```

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```
isect(x, erase(y));
                                                             } else {
  }
                                                               parent[x] = y;
  i64 query(i64 x) {
                                                               size[y] += size[x];
    assert(!empty());
                                                             }
                                                           }
    auto l = *lower_bound(x);
                                                         };
    return l.m * x + l.c;
                                                         2.7. Fenwick2D.h
};
                                                         const int mxn = 1000;
2.6. DSU.h
                                                         int grid[mxn + 1][mxn + 1];
                                                         int bit[mxn + 1][mxn + 1];
struct DSU {
  int n;
                                                         void update(int row, int col, int d) {
  vector<int> parent;
                                                           grid[row][col] += d;
                                                           for (int i = row; i \leq mxn; i += (i \& -i))
  vector<int> size;
  DSU(int _n) : n(_n), parent(n), size(n, 1)
                                                             for (int j = col; j \leq mxn; j += (j & -j))
{ iota(parent.begin(), parent.end(), 0); }
                                                               bit[i][j] += d;
  int find_set(int x) {
    if (parent[x] = x) return x;
                                                         int sum(int row, int col) {
    return parent[x] = find_set(parent[x]);
                                                           // calculates sum from [1,1] till [row,col]
                                                           int res = 0:
                                                           for (int i = row; i > 0; i -= (i \& -i))
  int getSize(int x) { return
size[find_set(x)]; } // returns size of component
                                                             for (int j = col; j > 0; j -= (j & -j))
of x
                                                               res += bit[i][j];
  void union_sets(int x, int y) {
                                                           return res;
    x = find_set(x);
    y = find_set(y);
                                                         2.8. Mos.h
    if (x = y) return;
                                                         int BLOCK = DO_NOT_FORGET_TO_CHANGE_THIS;
    if (size[x] > size[y]) {
                                                         struct Query {
      parent[y] = x;
                                                           int l, r, id;
      size[x] += size[y];
                                                           Query(int _l, int _r, int _id) : l(_l), r(_r),
```

```
id(_id) {}
  bool operator<(Query &o) {</pre>
    int mblock = l / BLOCK, oblock = o.l / BLOCK;
    return (mblock < oblock) or</pre>
           (mblock = oblock and mblock \% 2 = 0
and r < o.r) or
           (mblock = oblock and mblock \% 2 = 1
and r > o.r);
 };
};
// Solve
void solve() {
  vector<Query> queries;
  queries.reserve(q);
  for (int i = 0; i < q; i++) {
    int l, r;
    cin >> l >> r;
   l--, r--;
    queries.emplace_back(l, r, i);
  sort(all(queries));
  int ans = 0;
  auto add = [&](int v) {};
  auto rem = [&](int v) {};
  vector<int> out(q); // Change out type if
necessarv
  int cur_l = 0, cur_r = -1;
  for (auto &[l, r, id] : queries) {
    while (cur_l > l) add(--cur_l);
    while (cur_l < l) rem(cur_l++);</pre>
```

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```
while (cur_r < r) add(++cur_r);</pre>
    while (cur_r > r) rem(cur_r--);
    out[id] = ans;
 }
2.9. Persistent.h
const int N = 5e5 + 10, LOGN = 18;
int L[N * LOGN], R[N * LOGN], ST[N * LOGN];
int nodeid = 0;
// usage newrootId = update(i, 0, n - 1, val,
oldrootId)
// [update index i to val]
int update(int pos, int l, int r, int val, int id)
  if (pos < l or pos > r) return id;
  int ID = ++nodeid, m = (l + r) / 2;
  if (l = r) return (ST[ID] = val, ID);
  L[ID] = update(pos, l, m, val, L[id]);
  R[ID] = update(pos, m + 1, r, val, R[id]);
  return (ST[ID] = ST[L[ID]] + ST[R[ID]], ID);
}
// usage query(l, r, 0, n - 1, rootId)
int query(int ql, int qr, int l, int r, int id) {
  if (ql > r or qr < l) return 0;
  if (ql \leq l \text{ and } r \leq qr) \text{ return ST[id]};
  int m = (l + r) / 2;
  return (query(ql, qr, l, m, L[id])) + query(ql,
qr, m + 1, r, R[id]);
```

```
// searches for upper bound of x, call as
                                                              void Node::recalc() { c = cnt(l) + cnt(r) + 1; }
descent(0, n - 1, x, rootId)
int descent(int l, int r, int x, int id) {
                                                              template <class F>
  if (l = r) return l;
                                                              void each(Node* n, F f) {
  int m = (l + r) / 2;
                                                                if (n) {
                                                                  each(n \rightarrow l, f);
  int leftCount = ST[L[id]];
  if (leftCount ≤ x) {
                                                                  f(n\rightarrow val);
    // is in right half
                                                                  each(n\rightarrow r, f);
    return descent(m + 1, r, x - leftCount, R[id]);
                                                               }
  } else {
                                                              }
    // is in left half
    return descent(l, m, x, L[id]);
                                                              pair<Node*, Node*> split(Node* n, int k) {
  }
                                                                if (!n) return {};
                                                                if (cnt(n\rightarrow l) \ge k) \{ // "n\rightarrow val \ge k" for
                                                              lower_bound(k)
2.10. Treap.h
                                                                  auto pa = split(n \rightarrow l, k);
/*A short self-balancing tree. It acts as a
                                                                  n \rightarrow l = pa.second;
sequential container with log-time splits/joins,
                                                                  n→recalc();
and
                                                                  return {pa.first, n};
is easy to augment with additional data.
                                                                } else {
Time: $0(\log N)$*/
                                                                   auto pa = split(n \rightarrow r, k - cnt(n \rightarrow l) - 1); //
                                                              and just "k"
struct Node {
                                                                  n \rightarrow r = pa.first;
  Node *l = 0, *r = 0;
                                                                  n→recalc();
  int val, y, c = 1;
                                                                  return {n, pa.second};
  Node(int val) : val(val), y(rand()) {}
  void recalc();
};
                                                              Node* merge(Node* l, Node* r) {
int cnt(Node* n) { return n ? n \rightarrow c : 0; }
                                                                if (!l) return r;
```

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```
if (!r) return l;
  if (l \rightarrow y > r \rightarrow y) {
    l \rightarrow r = merge(l \rightarrow r, r);
    l→recalc();
    return l:
  } else {
    r \rightarrow l = merge(l, r \rightarrow l);
    r→recalc();
    return r;
  }
}
Node* ins(Node* t, Node* n, int pos) {
  auto [l, r] = split(t, pos);
  return merge(merge(l, n), r);
}
// Example application: move the range [l, r) to
index k
void move(Node*& t, int l, int r, int k) {
  Node *a, *b, *c;
  tie(a, b) = split(t, l);
  tie(b, c) = split(b, r - l);
  if (k \leq 1)
    t = merge(ins(a, b, k), c);
  else
    t = merge(a, ins(c, b, k - r));
}
```

3. Graph

```
3.1. SCC.h
struct SCC {
  int n;
  vi val, cc, z;
  vvi comps;
  SCC(vvi\& adj) : n(sz(adj)), val(n), cc(n, -1) {
    int timer = 0:
    function<int(int)> dfs = [&](int x) {
      int low = val[x] = ++timer, b;
      z.push_back(x);
      for (auto y : adj[x])
        if (cc[v] < 0)
          low = min(low, val[y] ?: dfs(y));
      if (low = val[x]) {
        comps.push_back(vi());
        do {
          b = z.back();
          z.pop_back();
          comps.back().push_back(b);
          cc[b] = sz(comps) - 1;
        } while (x \neq b);
      }
      return val[x] = low;
    };
    for (int i = 0; i < n; i++)
      if (cc[i] < 0) dfs(i);</pre>
  int operator[](int i) { return cc[i]; }
```

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```
int size(int i) { return sz(comps[cc[i]]); }
                                                         d[lca(a, b)]; }
};
                                                         };
                                                         3.3. 2sat.h
3.2. LCA.h
struct LCA {
                                                         /*
  int T = 0:
                                                         ts.either(x, y);
                                                         ts.either(~x, ~y); these two do x xor y
  vi st, path, ret;
  vi en, d;
  RMO<int> rmg;
                                                         ts.setValue(x, x); assert x is true
  LCA(vector < vi > \& C) : st(sz(C)), en(sz(C)),
d(sz(C)), rmq((dfs(C, 0, -1), ret)) {}
                                                        use ~x to denote not x
  void dfs(vvi& adj, int v, int par) {
    st[v] = T++;
                                                         call ts.solve() to run the solver, returns if a
    for (auto to : adj[v])
                                                         solution exists
      if (to \neq par) {
                                                         if exists: ts.values[i] contains the assignments
        path.pb(v), ret.pb(st[v]);
                                                         */
        d[to] = d[v] + 1;
        dfs(adj, to, v);
                                                         struct TwoSat {
                                                           int N;
    en[v] = T - 1;
                                                           vector<vi> qr;
                                                          vi values; // 0 = false, 1 = true
  bool anc(int p, int c) { return st[p] ≤ st[c]
and en[p] \ge en[c];
                                                           TwoSat(int n = 0) : N(n), gr(2 * n) {}
  int lca(int a, int b) {
    if (a = b) return a;
                                                           int addVar() { // (optional)
    tie(a, b) = minmax(st[a], st[b]);
                                                             gr.emplace_back();
    return path[rmq.query(a, b - 1)];
                                                             gr.emplace_back();
  }
                                                            return N++;
  int dist(int a, int b) { return d[a] + d[b] - 2 *
```

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```
void either(int f, int j) {
  f = max(2 * f, -1 - 2 * f);
  j = max(2 * j, -1 - 2 * j);
  gr[f].push_back(j ^ 1);
  gr[i].push_back(f ^ 1);
void setValue(int x) { either(x, x); }
                                                         }
void atMostOne(const vi& li) { // (optional)
  if (sz(li) ≤ 1) return;
                                                         bool solve() {
  int cur = ~li[0];
  for (int i = 2; i < sz(li); i++) {</pre>
    int next = addVar();
                                                           comp = val;
    either(cur, ~li[i]);
    either(cur, next);
    either(~li[i], next);
    cur = ~next;
                                                           return 1;
  either(cur, ~li[1]);
                                                       };
                                                       3.4. Dinic.h
vi val, comp, z;
int time = 0;
int dfs(int i) {
  int low = val[i] = ++time, x;
  z.push_back(i);
  for (int e : gr[i])
    if (!comp[e])
                                                       struct Dinic {
      low = min(low, val[e] ?: dfs(e));
                                                         struct Edge {
  if (low = val[i]) do {
```

```
x = z.back();
        z.pop_back();
        comp[x] = low;
        if (values[x \gg 1] = -1)
          values[x \gg 1] = x & 1;
      } while (x \neq i);
    return val[i] = low;
    values.assign(N, -1);
    val.assign(2 * N, 0);
    for (int i = 0; i < 2 * N; i++)
      if (!comp[i]) dfs(i);
    for (int i = 0; i < N; i++)
      if (comp[2 * i] = comp[2 * i + 1]) return 0;
// Flow algorithm with complexity $0(VE\log U)$
where U = \max |\text{text}\{cap\}|.
// $0(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$;
$0(\sqrt{V}E)$ for bipartite matching.
using ll = long long;
#define rep(i, j, k) for (int i = j; i < k; i \leftrightarrow j
```

```
int to, rev;
    ll c, oc;
    ll flow() { return max(oc - c, OLL); } // if
you need flows
  };
  vi lvl, ptr, q;
  vector<vector<Edge>> adj;
  Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
  void addEdge(int a, int b, ll c, ll rcap = 0) {
    adj[a].push_back({b, sz(adj[b]), c, c});
    adj[b].push_back({a, sz(adj[a]) - 1, rcap,
rcap});
  }
  ll dfs(int v, int t, ll f) {
    if (v = t || !f) return f;
    for (int& i = ptr[v]; i < sz(adj[v]); i++) {</pre>
      Edge\& e = adj[v][i];
      if (lvl[e.to] = lvl[v] + 1)
        if (ll p = dfs(e.to, t, min(f, e.c))) {
          e.c -= p, adj[e.to][e.rev].c += p;
          return p;
        }
    }
    return 0;
  ll calc(int s, int t) {
   Il flow = 0;
    q[0] = s;
    rep(L, 0, 31) do { // 'int L=30' maybe faster
for random data
```

```
lvl = ptr = vi(sz(q));
      int qi = 0, qe = lvl[s] = 1;
      while (qi < qe && !lvl[t]) {</pre>
        int v = q[qi+];
        for (Edge e : adj[v])
          if (!lvl[e.to] && e.c >> (30 - L))
            q[qe ++] = e.to, lvl[e.to] = lvl[v] + 1;
      while (ll p = dfs(s, t, LLONG_MAX)) flow +=
p;
    while (lvl[t])
    return flow;
  bool leftOfMinCut(int a) { return lvl[a] ≠ 0; }
};
3.5. HLD.h
struct HLD {
  int n, timer = 0;
  vi top, tin, p, sub;
  HLD(vvi \&adj) : n(sz(adj)), top(n), tin(n), p(n,
-1), sub(n, 1) {
   vi ord(n + 1);
   for (int i = 0, t = 0, v = ord[i]; i < n; v =
ord[#i])
      for (auto &to : adj[v])
        if (to \neq p[v]) p[to] = v, ord[++t] = to;
    for (int i = n - 1, v = ord[i]; i > 0; v =
```

```
ord[--i]) sub[p[v]] += sub[v];
    for (int v = 0; v < n; v +++)
      if (sz(adj[v])) iter_swap(begin(adj[v]),
max_element(all(adj[v]), [&](int a, int b) { return
make_pair(a \neq p[v], sub[a]) < make_pair(b \neq p[v],
sub[b]); }));
    function<void(int)> dfs = [&](int v) {
      tin[v] = timer++;
      for (auto &to : adj[v])
        if (to \neq p[v]) {
          top[to] = (to = adj[v][0] ? top[v] :
to);
          dfs(to);
        }
   };
    dfs(0);
  int lca(int u, int v) {
    return process(u, v, [](...) {});
  template <class B>
  int process(int a, int b, B op, bool ignore_lca =
false) {
    for (int v;; op(tin[v], tin[b]), b = p[v]) {
      if (tin[a] > tin[b]) swap(a, b);
      if ((v = top[b]) = top[a]) break;
    if (int l = tin[a] + ignore_lca, r = tin[b]; l
< r) op(l, r);</pre>
    return a;
```

```
}
  template <class B>
  void subtree(int v, B op, bool ignore_lca =
false) {
    if (sub[v] > 1 or !ignore_lca) op(tin[v] +
ignore_lca, tin[v] + sub[v] - 1);
};
3.6. KthAnc.h
struct LCA {
  int n;
  vvi& adjLists;
  int lg;
  vvi up;
  vi depth;
  LCA(vvi& _adjLists, int root = 0) :
n(sz(_adjLists)), adjLists(_adjLists) {
    lq = 1;
    int pw = 1;
    while (pw \leq n) pw \ll 1, lq++;
    // lq = 20
    up = vvi(n, vi(lg));
    depth.assign(n, -1);
    function<void(int, int)> parentDFS = [&](int
from, int parent) {
      depth[from] = depth[parent] + 1;
      up[from][0] = parent;
      for (auto to : adjLists[from]) {
        if (to = parent) continue;
```

```
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```

```
parentDFS(to, from);
   }
  };
  parentDFS(root, root);
  for (int j = 1; j < lq; j++) {
    for (int i = 0; i < n; i++) {
      up[i][j] = up[up[i][j - 1]][j - 1];
}
int kthAnc(int v, int k) {
  int ret = v;
  int pw = 0;
  while (k) {
    if (k & 1) ret = up[ret][pw];
    k \gg 1;
    pw++;
  return ret;
int lca(int u, int v) {
  if (depth[u] > depth[v]) swap(u, v);
  v = kthAnc(v, depth[v] - depth[u]);
  if (u = v) return v;
  while (\upsilon p[\upsilon][0] \neq \upsilon p[\upsilon][0]) {
    int i = 0;
    for (; i < lg - 1; i++) {
      if (up[u][i + 1] = up[v][i + 1]) break;
    u = up[u][i], v = up[v][i];
```

```
}
   return up[u][0];
  };
  int dist(int u, int v) {
   return depth[v] + depth[v] - 2 * depth[lca(v,
v)];
 }
};
3.7. MinCostMaxFlow.h
template <const int MAX_N, typename flow_t,</pre>
          typename cost_t, flow_t FLOW_INF,
          cost_t COST_INF, const int SCALE = 16>
struct CostScalingMCMF {
#define sz(a) a.size()
#define zero_stl(v, sz) fill(v.begin(), v.begin() +
(sz), 0
  struct Edge {
   int v;
   flow_t c;
   cost_t d;
   int r;
   Edge() = default;
    Edge(int v, flow_t c, cost_t d, int r) : v(v),
c(c), d(d), r(r) {}
 };
  vector<Edge> g[MAX_N];
  cost_t negativeSelfLoop;
  array<cost_t, MAX_N> pi, excess;
```

array<int, MAX_N> level, ptr;

```
CostScalingMCMF() { negativeSelfLoop = 0; }
  void clear() {
    negativeSelfLoop = 0;
   for (int i = 0; i < MAX_N; i++) g[i].clear();</pre>
  void addEdge(int s, int e, flow_t cap, cost_t
cost) {
    if (s = e) {
      if (cost < 0) negativeSelfLoop += cap * cost;</pre>
      return;
    }
    g[s].push_back(Edge(e, cap, cost, sz(g[e])));
    g[e].push_back(Edge(s, 0, -cost, sz(q[s]) -
1));
  flow_t getMaxFlow(int V, int S, int T) {
    auto BFS = [&]() {
      zero_stl(level, V);
      queue<int> q;
      q.push(S);
      level[S] = 1;
      for (q.push(S); !q.empty(); q.pop()) {
        int v = q.front();
        for (const auto &e : q[v])
          if (!level[e.v] && e.c) q.push(e.v),
level[e.v] = level[v] + 1;
      return level[T];
    function<flow_t(int, flow_t)> DFS = [&](int v,
```

```
flow_t fl) {
      if (v = T || fl = 0) return fl;
      for (int &i = ptr[v]; i < (int)g[v].size();</pre>
i++) {
        Edge &e = q[v][i];
        if (level[e.v] \neq level[v] + 1 \mid | !e.c)
continue:
        flow_t delta = DFS(e.v, min(fl, e.c));
        if (delta) {
          e.c -= delta;
          q[e.v][e.r].c += delta;
          return delta:
        }
      return flow_t(0);
   };
    flow_t maxFlow = 0, tmp = 0;
    while (BFS()) {
      zero_stl(ptr, V);
      while ((tmp = DFS(S, FLOW_INF))) maxFlow +=
tmp;
    return maxFlow;
  }
  pair<flow_t, cost_t> maxflow(int N, int S, int T)
    flow_t maxFlow = 0;
    cost_t eps = 0, minCost = 0;
    stack<int, vector<int>> stk;
    auto c_pi = [&](int v, const Edge &edge)
```

```
{ return edge.d + pi[v] - pi[edge.v]; };
                                                                  Edge &e = q[v][i];
                                                                  if (e.c \le 0) continue;
    auto push = [\&](int v, Edge &edge, flow_t
delta, bool flaq) {
                                                                  cost_t cp = c_pi(v, e);
      delta = min(delta, edge.c);
                                                                  if (cp < 0) {
      edge.c -= delta;
                                                                    if (lookAhead(e.v)) {
      g[edge.v][edge.r].c += delta;
                                                                      i--;
      excess[v] -= delta;
                                                                      continue;
      excess[edge.v] += delta;
      if (flag && 0 < excess[edge.v] &&
                                                                    push(v, e, excess[v], true);
excess[edge.v] < delta) stk.push(edge.v);
                                                                    if (excess[v] = 0) return;
    };
                                                                  } else
    auto relabel = [&](int v, cost_t delta) { pi[v]
                                                                    delta = min(delta, cp);
-= delta + eps; };
    auto lookAhead = [&](int v) {
                                                                relabel(v, delta);
      if (excess[v]) return false;
                                                                stk.push(v);
                                                              };
      cost_t delta = COST_INF;
      for (auto &e : q[v]) {
                                                              zero_stl(pi, N);
        if (e.c \leq 0) continue;
                                                              zero_stl(excess, N);
        cost_t cp = c_pi(v, e);
                                                              for (int i = 0; i < N; i++)
        if (cp < 0)
                                                                for (auto &e : q[i]) minCost += e.c * e.d,
                                                          e.d \star= MAX_N + 1, eps = max(eps, e.d);
          return false;
        else
                                                              maxFlow = getMaxFlow(N, S, T);
          delta = min(delta, cp);
                                                              while (eps > 1) {
      }
                                                                eps \models SCALE;
                                                                if (eps < 1) eps = 1;
      relabel(v, delta);
                                                                stk = stack<int, vector<int>>();
      return true;
    };
                                                                for (int v = 0; v < N; v ++)
                                                                  for (auto &e : q[v])
    auto discharge = [&](int v) {
      cost_t delta = COST_INF;
                                                                    if (c_pi(v, e) < 0 \&\& e.c > 0) push(v, e, e)
      for (int i = 0; i < sz(q[v]); i \leftrightarrow) {
                                                         e.c, false);
```

```
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      for (int v = 0; v < N; v +++)
        if (excess[v] > 0) stk.push(v);
      while (stk.size()) {
        int top = stk.top();
        stk.pop();
        discharge(top);
      }
    }
    for (int v = 0; v < N; v \leftrightarrow)
      for (auto &e : g[v]) e.d \not\models MAX_N + 1,
minCost -= e.c * e.d;
    minCost = minCost / 2 + negativeSelfLoop;
    return {maxFlow, minCost};
 }
};
void solve() {
  CostScalingMCMF<102, int, int, 100, 100> flow;
  int n, m;
  cin >> n >> m;
  int start = 0;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j \leftrightarrow ) {
      int inp;
      cin >> inp;
      if (inp) {
        flow.addEdge(i + 1, n + 1 + j, 1, 0);
        start++;
      } else
        flow.addEdge(i + 1, n + 1 + j, 1, 1);
```

```
}
  }
 int counta = 0, countb = 0;
 for (int i = 0; i < n; i++) {
   int inp;
   cin >> inp;
    counta += inp;
   flow.addEdge(0, i + 1, inp, 0);
 for (int i = 0; i < m; i++) {
   int inp;
   cin >> inp;
    countb += inp;
   flow.addEdge(n + i + 1, n + m + 1, inp, 0);
 }
 if (counta ≠ countb) {
    cout << -1 << endl;
   return;
 pii t = flow.maxflow(102, 0, n + m + 1);
 if (t.first # counta) {
    cout << -1 << endl;
   return;
 }
 cout << t.second + start + t.second - counta <<</pre>
endl;
```

4. Number Theory

4.1. MillerRabin.h

```
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```

```
u64 \text{ mult}(u64 \text{ a}, u64 \text{ b}, u64 \text{ m} = \text{M})  {
  i64 ret = a * b - m * (u64)(1.L / m * a * b);
  return ret + m * (ret < 0) - m * (ret ≥ (i64)m);
}
u64 pw(u64 b, u64 e, u64 m = M) {
  u64 \text{ ret} = 1;
  for (; e; b = mult(b, b, m), e \gg 1)
    if (e & 1) ret = mult(ret, b, m);
  return ret;
bool isPrime(u64 n) { // determistic upto 7e^18
  if (n < 2 | | n \% 6 \% 4 \neq 1) return (n | 1) = 3;
  u64 A[] = \{2, 325, 9375, 28178, 450775, 9780504,
1795265022},
      s = __builtin_ctzll(n - 1), d = n >> s;
  for (u64 a : A) {
    u64 p = pw(a \% n, d, n), i = s;
    while (p \neq 1 \&\& p \neq n - 1 \&\& a \% n \&\& i--)
      p = mult(p, p, n);
    if (p \neq n - 1 \&\& i \neq s) return 0;
  return 1;
}
4.2. gcdextended.h
int euclid(int a, int b, int &x, int &y) {
  if (!b) return x = 1, y = 0, a;
  int d = euclid(b, a \% b, y, x);
  return y -= a / b * x, d;
}
```

4.3. spf.h

```
#define SIEVE_TILL (int)1e6
vector<int> primes;
vector<int> spf;
void sieve() {
  spf = vector<int>(SIEVE_TILL + 1, 0);
 for (int i = 2; i ≤ SIEVE_TILL; i++) {
    if (spf[i] = 0) primes.push_back(i), spf[i] =
i;
    for (int j = 0; j < sz(primes) and i *</pre>
primes[j] ≤ SIEVE_TILL; j++) {
      spf[i * primes[j]] = primes[j];
     if (spf[i] = primes[j]) break;
   }
 }
bool isPrime(int n) {
  if (n \le 1) return false;
  return spf[n] = n;
}
```

5. Strings

5.1. Manacher.h

```
/* Description: p[0][i] = half length of longest
even palindrome behind pos i,
p[1][i] = longest odd with center at pos i(half
rounded down). */
array<vi, 2> manacher(const string& s) {
  int n = sz(s);
  array<vi, 2> p = {vi(n + 1), vi(n)};
```

```
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```

```
for (int z = 0; z < 2; z++)
                                                                  int pow2 = (111 << 3111);
    for (int i = 0, l = 0, r = 0; i < n; i \leftrightarrow ) {
                                                                  trieobject* temp = &base;
      int t = r - i + !z;
                                                                  while (pow2 > 0) {
      if (i < r) p[z][i] = min(t, p[z][l + t]);
                                                                    if (temp \rightarrow children[1 \&\& (x \& pow2)] = NULL)
      int L = i - p[z][i], R = i + p[z][i] - !z;
                                                             {
      while (L \ge 1 \&\& R + 1 < n \&\& s[L - 1] = s[R
                                                                      temp\rightarrowchildren[1 && (x & pow2)] = new
+ 1])
                                                             trieobject;
        p[z][i]++, L--, R++;
      if (R > r) l = L, r = R;
                                                                    temp\rightarrowchildren[1 && (x & pow2)]\rightarrownumelems\leftrightarrow;
                                                                    temp = temp\rightarrowchildren[1 && (x & pow2)];
                                                                    pow2 \neq 2;
  return p;
}
                                                                  }
                                                                }
5.2. Trie.h
                                                                // ADD FUNCTION BELOW
struct trieobject {
                                                             };
  trieobject() {
    children[0] = NULL;
                                                             5.3. SuffixArray.h
    children[1] = NULL;
                                                             /*Builds suffix array for a string.
                                                              \texttt{sa[i]} is the starting index of the suffix
    numelems = 0;
  };
                                                             which
                                                             is $i$'th in the sorted suffix array.
  struct trieobject* children[2];
                                                             The returned vector is of size $n+1$, and
  int numelems;
                                                             \text{texttt}\{sa[0] = n\}.
};
                                                             The \texttt{lcp} array contains longest common
struct trie {
                                                             prefixes for
  trieobject base;
                                                             neighbouring strings in the suffix array:
  trie() {
                                                             \text{texttt{lcp[i]} = lcp(sa[i], sa[i-1])},
                                                             \text{texttt{lcp[0] = 0}}.
    trieobject base;
                                                             The input string must not contain any zero bytes.
                                                             Time: O(n \loa n)*/
  void add(int x) {
```

```
#define rep(i, j, k) for (int i = j; i < k; i \leftrightarrow j
struct SuffixArray {
  vi sa, lcp;
  SuffixArray(string& s, int lim = 256) { // or
basic_string<int>
    int n = sz(s) + 1, k = 0, a, b;
    vi \times (all(s)), y(n), ws(max(n, lim));
    x.push_back(0), sa = lcp = y, iota(all(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j *
2), lim = p) {
      p = j, iota(all(y), n - j);
      rep(i, 0, n) if (sa[i] \geqslant j) y[p++] = sa[i] -
j;
      fill(all(ws), 0);
      rep(i, 0, n) ws[x[i]]++;
      rep(i, 1, lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] =
y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      rep(i, 1, n) = sa[i - 1], b = sa[i], x[b] =
(y[a] = y[b] \& y[a + j] = y[b + j]) ? p - 1 : p+
+;
    for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
      for (k \&\&k--, j = sa[x[i] - 1];
           s[i + k] = s[j + k]; k++)
        ï
};
```

6. Numerical

6.1. NTT.h

```
/* Description: Can be used for convolutions modulo
specific nice primes of the form 2<sup>a</sup> b+1, where the
convolution result has size at most 2<sup>a</sup>
 * (125000001 << 3) + 1 = 1e9 + 7, therefore do not
use this for M = 1e9 + 7.
 * For p < 2^30 there is also e.g. (5 << 25, 3),
(7 \ll 26, 3),
 * For other primes/integers, use two different
primes and combine with CRT. (479 << 21, 3) and
(483 << 21, 5). The last two are > 10^9
* Inputs must be in [0, mod).
// Requires mod func
const int M = 998244353;
const int root = 3;
// (119 << 23) + 1, root = 3; // for M = 998244353
void ntt(int* x, int* temp, int* roots, int N, int
skip) {
 if (N = 1) return;
  int n2 = N / 2;
  ntt(x, temp, roots, n2, skip * 2);
  ntt(x + skip, temp, roots, n2, skip * 2);
  for (int i = 0; i < N; i + i + j) temp[i] = x[i *
skip];
  for (int i = 0; i < n2; i++) {
    int s = temp[2 * i], t = temp[2 * i + 1] *
roots[skip * i];
```

```
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```

```
x[skip * i] = (s + t) % M;
    x[skip * (i + n2)] = (s - t) % M;
  }
}
void ntt(vi& x, bool inv = false) {
  int e = pw(root, (M - 1) / sz(x));
  if (inv) e = pw(e, M - 2);
  vi roots(sz(x), 1), temp = roots;
  for (int i = 1; i < sz(x); i++) roots[i] =
roots[i - 1] * e % M;
  ntt(&x[0], &temp[0], &roots[0], sz(x), 1);
}
// Usage: just pass the two coefficients list to
qet a*b (modulo M)
vi conv(vi a, vi b) {
  int s = sz(a) + sz(b) - 1;
  if (s \leq 0) return \{\};
  int L = s > 1? 32 - \underline{builtin_clzll(s - 1)} : 0,
n = 1 << L;
  if (s ≤ 200) { // (factor 10 optimization for |
a|,|b| = 10)
    vi c(s);
    for (int i = 0; i < sz(a); i++)
     for (int j = 0; j < sz(b); j++)
        c[i + j] = (c[i + j] + a[i] * b[j]) % M;
    return c;
  a.resize(n);
  ntt(a);
  b.resize(n);
```

```
ntt(b);
vi c(n);
int d = pw(n, M - 2);
for (int i = 0; i < n; i++) c[i] = a[i] * b[i] %
M * d % M;
ntt(c, true);
c.resize(s);
return c;
}</pre>
```

6.2. FastFourierTransform.h

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
  int n = sz(a), L = 31 - \underline{builtin_clz(n)};
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1); // (^ 10% faster if
double)
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n);
    rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    rep(i, k, 2 * k) rt[i] = R[i] = i & 1 ? R[i / ]
2] * x : R[i / 2];
 }
  vi rev(n);
  rep(i, 0, n) rev[i] = (rev[i / 2] | (i & 1) <<
L) / 2;
  rep(i, 0, n) if (i < rev[i]) swap(a[i],
```

```
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a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j, 0, k)
{
        // C z = rt[j+k] * a[i+j+k]; // (25% faster
if hand-rolled) /// include-line
        auto x = (double*)&rt[j + k], y =
(double*)&a[i + j + k]; /// exclude-line
        C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1]
+ x[1] * y[0]); /// exclude-line
        a[i + j + k] = a[i + j] - z;
        a[i + j] += z;
      }
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {};
  vd res(sz(a) + sz(b) - 1);
  int L = 32 - \underline{builtin\_clz(sz(res))}, n = 1 \ll L;
  vector<C> in(n), out(n);
  copy(all(a), begin(in));
  rep(i, 0, sz(b)) in[i].imag(b[i]);
  fft(in);
  for (C\& x : in) x *= x;
  rep(i, 0, n) out[i] = in[-i & (n - 1)] -
conj(in[i]);
 fft(out);
  rep(i, 0, sz(res)) res[i] = imag(out[i]) / (4 *
n);
  return res;
```

}

7. Geometry

```
7.1. ConvexHull.h
```

```
// Needs point
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) ≤ 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts) + 1);
  int s = 0, t = 0;
 for (int it = 2; it--; s = --t,
reverse(all(pts)))
   for (P p : pts) {
     while (t \ge s + 2 \&\& h[t - 2].cross(h[t - 1],
p) \leq 0) t--;
      h[t++] = p;
  return \{h.begin(), h.begin() + t - (t = 2 \&\&
h[0] = h[1];
}
7.2. Point.h
```

```
template <class T>
int sgn(T x) { return (x > 0) - (x < 0); }
template <class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x = 0, T y = 0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x, y) <
  tie(p.x, p.y); }</pre>
```

```
bool operator=(P p) const { return tie(x, y) =
tie(p.x, p.y); }
  P operator+(P p) const { return P(x + p.x, y +
p.y); }
  P operator-(P p) const { return P(x - p.x, y -
p.y); }
  P operator*(T d) const { return P(x * d, y *
d); }
  P operator/(T d) const { return P(x / d, y /
d); }
 T dot(P p) const \{ return x * p.x + y * p.y; \}
  T cross(P p) const { return x * p.y - y * p.x; }
  T cross(P a, P b) const { return (a -
*this).cross(b - *this); }
  T dist2() const { return x * x + y * y; }
  double dist() const { return
sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this / dist(); } //
makes dist()=1
  P perp() const { return P(-y, x); }
                                             //
rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around
the origin
  P rotate(double a) const {
    return P(x * cos(a) - y * sin(a), x * sin(a) +
y * cos(a));
  }
```

```
return os << "(" << p.x << "," << p.y << ")";
 }
};
7.3. ClosestPair.h
// Requires point
typedef Point<int> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });</pre>
  pair<int, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (int)sqrtl(ret.first), 0};
    while (v[j].y \le p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi =
S.upper_bound(p + d);
    for (; lo \neq hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo,
p}});
    S.insert(p);
  }
 return ret.second;
```

friend ostream& operator<<(ostream& os, P p) {</pre>