

METRICS

Introduction

Predictive Modeling works on constructive feedback principle. You build a model. Get feedback from metrics, make improvements and continue until you achieve a desirable accuracy. Evaluation metrics explain the performance of a model. An important aspect of evaluation metrics is their capability to discriminate among model results.

Simply, building a predictive model is not your motive. But, creating and selecting a model which gives high accuracy on out of sample data. Hence, it is crucial to check accuracy of the model prior to computing predicted values.

In our industry, we consider different kinds of metrics to evaluate our models. The choice of metric completely depends on the type of model and the implementation plan of the model. After you are finished building your model, these 7 metrics will help you in evaluating your model accuracy.

Types of Predictive models

When we talk about predictive models, we are talking either about a regression model (continuous output) or a classification model (nominal or binary output). The evaluation metrics used in each of these models are different.

In classification problems, we use two types of algorithms (dependent on the kind of output it creates):

- **Class output:** Algorithms like SVM and KNN create a class output. For instance, in a binary classification problem, the outputs will be either 0 or 1. However, today we have algorithms which can convert these class outputs to probability. But these algorithms are not well accepted by the statistics community.
- **Probability output:** Algorithms like Logistic Regression, Random Forest, Gradient Boosting, Adaboost etc. give probability outputs. Converting probability outputs to class output is just a matter of creating a threshold probability.

1. Confusion Matrix

A confusion matrix is an $N \times N$ matrix, where N is the number of classes being predicted. For the problem in hand, we have $N=2$, and hence we get a 2×2 matrix. Here are a few definitions; you need to remember for a confusion matrix:

Accuracy: the proportion of the total number of predictions that was correct.

Positive Predictive Value or Precision: the proportion of positive cases that were correctly identified.

Negative Predictive Value: the proportion of negative cases that were correctly identified.

Sensitivity or Recall: the proportion of actual positive cases which are correctly identified.

Specificity: the proportion of actual negative cases which are correctly identified.

Confusion Matrix		Target			
		Positive	Negative		
Model	Positive	a	b	Positive Predictive Value	$a/(a+b)$
	Negative	c	d	Negative Predictive Value	$d/(c+d)$
		Sensitivity	Specificity	Accuracy = $(a+d)/(a+b+c+d)$	
		$a/(a+c)$	$d/(b+d)$		

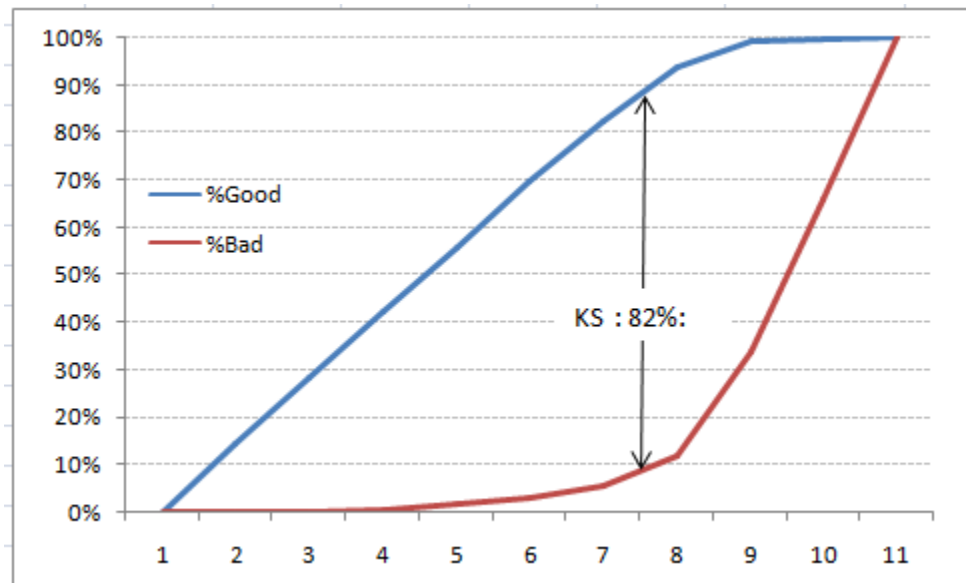
In general we are concerned with one of the above defined metric. For instance, in a pharmaceutical company, they will be more concerned with minimal wrong positive diagnosis. Hence, they will be more concerned about high Specificity. On the other hand an attrition model will be more concerned with Sensitivity. Confusion matrix is generally used only with class output models.

2. Kolmogorov Smirnov chart

K-S or Kolmogorov-Smirnov chart measures performance of classification models. More accurately, K-S is a measure of the degree of separation between the positive and negative distributions. The K-S is 100, if the scores partition the population into two separate groups in which one group contains all the positives and the other all the negatives.

On the other hand, If the model cannot differentiate between positives and negatives, then it is as if the model selects cases randomly from the population. The K-S would be 0. In most classification models the K-S will fall between 0 and 100, and that the higher the value the better the model is at separating the positive from negative cases.

						Cumulative		
Lift/Gain	Column			%Rights	%Wrongs	Cum %Rig	Cum %Wrong	K-S
Row La	0	1	Grand Tot	0%	0%	0%	0%	0%
1		543	543	14%	0%	14%	0%	14%
2	2	542	544	14%	0%	28%	0%	28%
3	7	537	544	14%	0%	42%	1%	42%
4	15	529	544	14%	1%	56%	2%	54%
5	20	524	544	14%	1%	69%	3%	67%
6	42	502	544	13%	3%	83%	5%	77%
7	104	440	544	11%	7%	94%	12%	82% K-S
8	345	199	544	5%	22%	99%	34%	65%
9	515	29	544	1%	32%	100%	66%	34%
10	540	5	545	0%	34%	100%	100%	0%
Grand Tot	1590	3850	5440					



3. Gain and Lift charts

Gain and Lift chart are mainly concerned to check the rank ordering of the probabilities. Here are the steps to build a Lift/Gain chart:

Step 1: Calculate probability for each observation

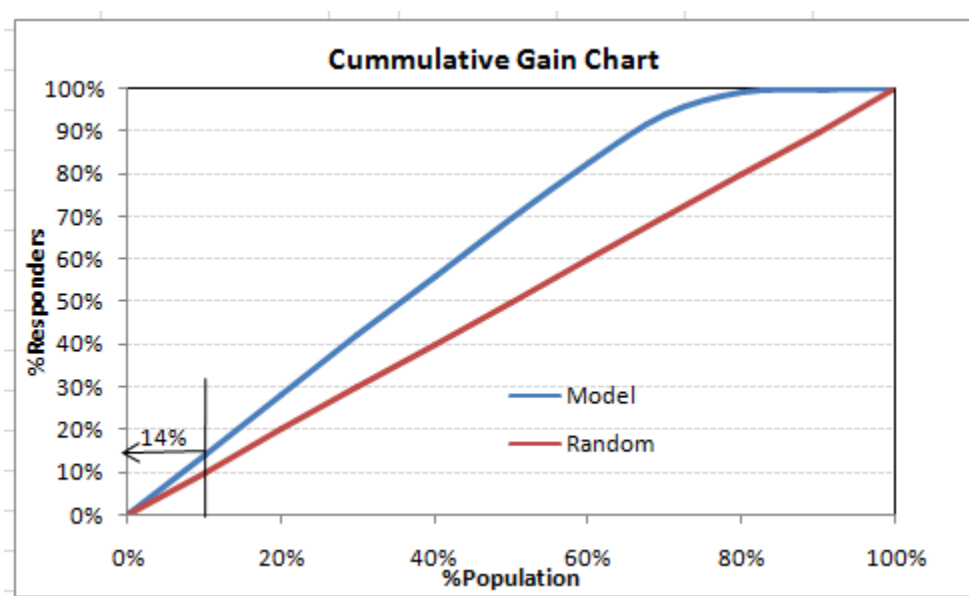
Step 2: Rank these probabilities in decreasing order.

Step 3: Build deciles with each group having almost 10% of the observations.

Step 4: Calculate the response rate at each deciles for Good (Responders) ,Bad (Non-responders) and total.

You will get following table from which you need to plot Gain/Lift charts:

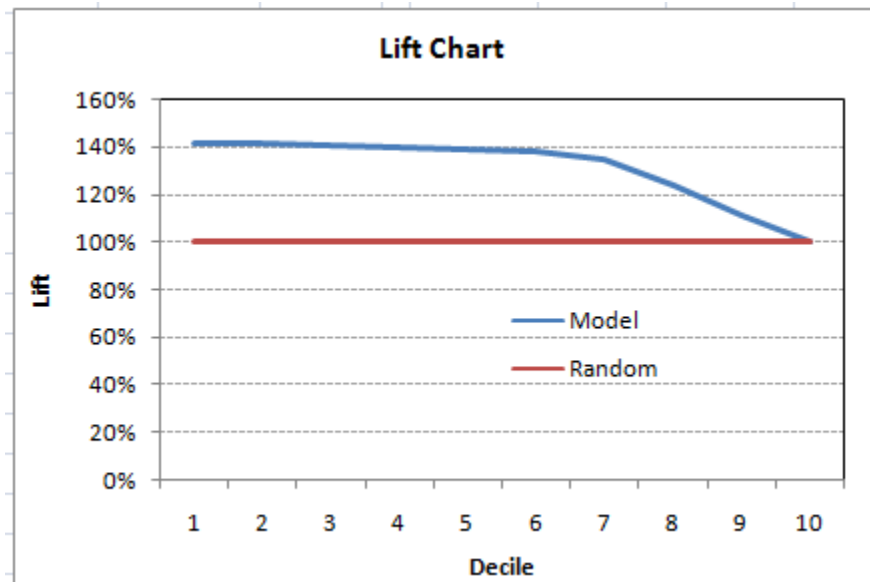
Lift/Gain	Column Labels			%Rights	%Wrongs	%Population	Cum %Right	Cum %Pop	Lift @decile	Total Lift
Row Labels	0	1	Grand Total							
1	543		543	14%	0%	10%	14%	10%	141%	141%
2	2	542	544	14%	0%	10%	28%	20%	141%	141%
3	7	537	544	14%	0%	10%	42%	30%	139%	141%
4	15	529	544	14%	1%	10%	56%	40%	137%	140%
5	20	524	544	14%	1%	10%	69%	50%	136%	139%
6	42	502	544	13%	3%	10%	83%	60%	130%	138%
7	104	440	544	11%	7%	10%	94%	70%	114%	134%
8	345	199	544	5%	22%	10%	99%	80%	52%	124%
9	515	29	544	1%	32%	10%	100%	90%	8%	111%
10	540	5	545	0%	34%	10%	100%	100%	1%	100%
Grand Total	1590	3850	5440							



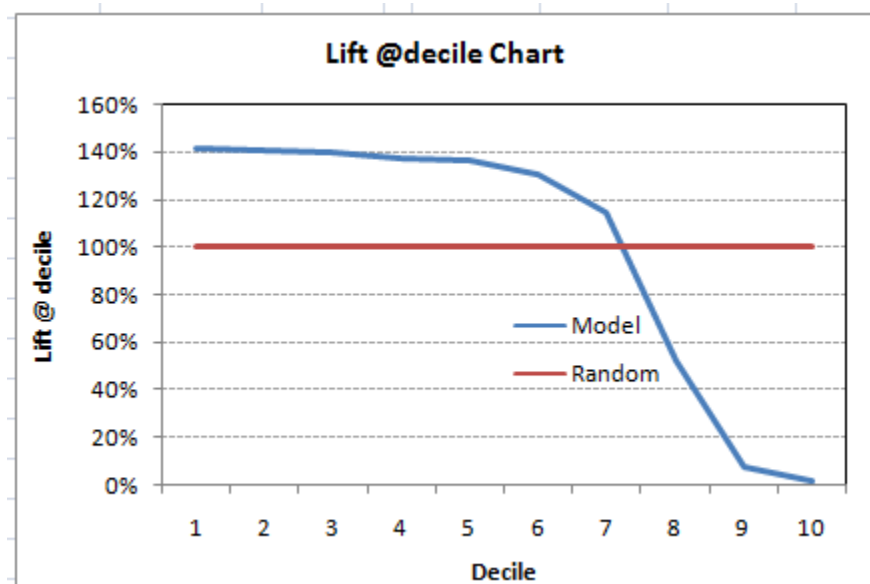
This graph tells you how well is your model segregating responders from non-responders. For example, the first decile however has 10% of the population, has 14% of responders. This means we have a 140% lift at first decile.

What is the maximum lift we could have reached in first decile? From the first table of this article, we know that the total number of responders are 3850. Also the first decile will contains 543 observations. Hence, the maximum lift at first decile could have been $543/3850 \sim 14.1\%$. Hence, we are quite close to perfection with this model.

Let's now plot the lift curve. Lift curve is the plot between total lift and %population. Note that for a random model, this always stays flat at 100%. Here is the plot for the case in hand:



You can also plot decile wise lift with decile number :



What does this graph tell you? It tells you that our model does well till the 7th decile. Post which every decile will be skewed towards non-responders. Any model with lift @ decile above 100% till minimum 3rd decile and maximum 7th decile is a good model. Else you might consider over sampling first.

Lift / Gain charts are widely used in campaign targeting problems. This tells us till which decile can we target customers for an specific campaign. Also, it tells you how much response you expect from the new target base.

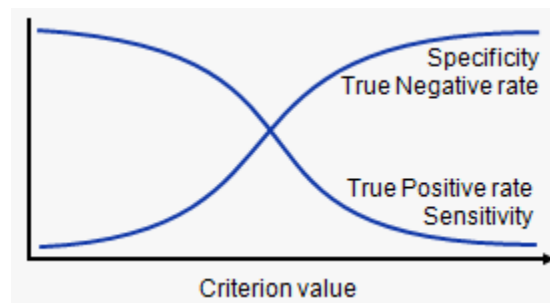
4. Area Under the ROC curve (AUC – ROC)

The biggest advantage of using ROC curve is that it is independent of the change in proportion of responders.

If we look at the confusion matrix below, we observe that for a probabilistic model, we get different value for each metric.

Confusion Matrix		Target			
		Positive	Negative		
Model	Positive	a	b	Positive Predictive Value	$a/(a+b)$
	Negative	c	d	Negative Predictive Value	$d/(c+d)$
		Sensitivity	Specificity	Accuracy = $(a+d)/(a+b+c+d)$	
		$a/(a+c)$	$d/(b+d)$		

Hence, for each sensitivity, we get a different specificity. The two vary as follows:



The ROC curve is the plot between sensitivity and (1- specificity). (1- specificity) is also known as false positive rate and sensitivity is also known as True Positive rate.

Note that the area of entire square is $1*1 = 1$. Hence AUC itself is the ratio under the curve and the total area. For the case in hand, we get AUC ROC as 96.4%. Following are a few thumb rules:

.90-1 = excellent (A)

.80-.90 = good (B)

.70-.80 = fair (C)

.60-.70 = poor (D)

.50-.60 = fail (F)

We see that we fall under the excellent band for the current model. But this might simply be over-fitting. In such cases it becomes very important to to in-time and out-of-time validations.

Points to Remember:

1. For a model which gives class as output, will be represented as a single point in ROC plot.
2. Such models cannot be compared with each other as the judgement needs to be taken on a single metric and not using multiple metrics. For instance, model with parameters (0.2,0.8) and model with parameter (0.8,0.2) can be coming out of the same model, hence these metrics should not be directly compared.
3. In case of probabilistic model, we were fortunate enough to get a single number which was AUC-ROC. But still, we need to look at the entire curve to make conclusive decisions. It is also possible that one model performs better in some region and other performs better in other.

5. Gini Coefficient

Gini coefficient is sometimes used in classification problems. Gini coefficient can be straight away derived from the AUC ROC number. Gini is nothing but ratio between area between the ROC curve and the diagonal line & the area of the above triangle. Following is the formulae used:

$$Gini = 2 * AUC - 1$$

6. Concordant – Discordant ratio

This is again one of the most important metric for any classification predictions problem. To understand this let's assume we have 3 students who have some likelihood to pass this year. Following are our predictions :

A – 0.9

B – 0.5

C – 0.3

Now picture this. if we were to fetch pairs of two from these three student, how many pairs will we have? We will have 3 pairs : AB , BC, CA. Now, after the year ends we saw that A and C passed this year while B failed. No, we choose all the pairs where we will find one responder and other non-responder. How many such pairs do we have?

We have two pairs AB and BC. Now for each of the 2 pairs, the concordant pair is where the probability of responder was higher than non-responder. Whereas discordant pair is where the vice-versa holds true. In case both the probabilities were equal, we say its a tie. Let's see what happens in our case :

AB – Concordant

BC – Discordant

Hence, we have 50% of concordant cases in this example. Concordant ratio of more than 60% is considered to be a good model. This metric generally is not used when deciding how many customer to target etc. It is primarily used to access the model's predictive power. For decisions like how many to target are again taken by KS / Lift charts.

7. Root Mean Squared Error (RMSE)

RMSE is the most popular evaluation metric used in regression problems. It follows an assumption that errors are unbiased and follow a normal distribution. Here are the key points to consider on RMSE:

1. The power of 'square root' empowers this metric to show large number deviations.
2. The 'squared' nature of this metric helps to deliver more robust results which prevents cancelling the positive and negative error values. In other words, this metric aptly displays the plausible magnitude of error term.
3. It avoids the use of absolute error values which is highly undesirable in mathematical calculations.
4. When we have more samples, reconstructing the error distribution using RMSE is considered to be more reliable.
5. RMSE is highly affected by outlier values. Hence, make sure you've removed outliers from your data set prior to using this metric.
6. As compared to mean absolute error, RMSE gives higher weightage and punishes large errors.

RMSE metric is given by:

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (Predicted_i - Actual_i)^2}{N}}$$

How do we choose k in k-fold Cross-Validation?

This is the tricky part. We have a tradeoff to choose k.

- For a small k, we have a higher selection bias but low variance in the performances.
- For a large k, we have a small selection bias but high variance in the performances.

Think of extreme cases :

$k = 2$: We have only 2 samples similar to our 50-50 example. Here we build model only on 50% of the population each time. But as the validation is a significant population, the variance of validation performance is minimal.

$k = \text{number of observations } (n)$: This is also known as “Leave one out”. We have n samples and modelling repeated n number of times leaving only one observation out for cross validation. Hence, the selection bias is minimal but the variance of validation performance is very large.

Generally a value of $k = 10$ is recommended for most purpose.

LINK:

[1.] <https://www.analyticsvidhya.com/blog/2016/02/7-important-model-evaluation-error-metrics/>