DIFFERENTIAL CALCULUS

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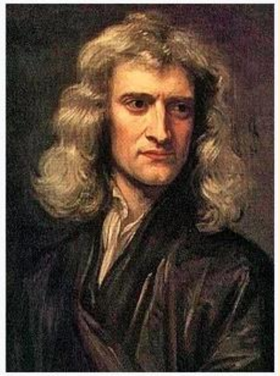
CALCULUS


Gottfried Wilhelm Leibniz (1646-1716)


Isaac Newton (1642-1726)

Branches:

Differential Calculus cuts something into small pieces to find how it changes

Integral Calculus joins the small pieces together to find how much there is.

FUNCTION

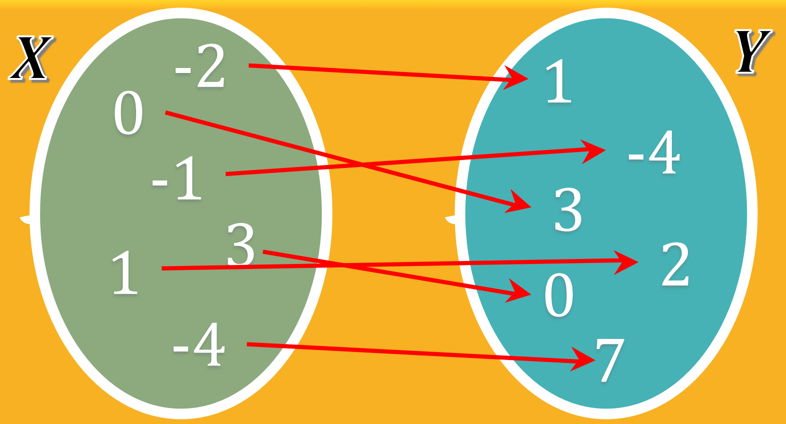
is a set of ordered pairs, (x, y) such that for **every x**, there corresponds a **unique value of y**.

Domain of the function is the set of all allowable values for x.

Range of the function is the set of all resulting $y - values$ after substituting the x − values.

FUNCTION

Ex. $Let\ f: X \to Y$



$\{(-2,1),(0,3),(-1,-4),(3,0),(1,2),(-4,7)\}$

Domain = $\{-4,-2,-1,0,1,3\}$

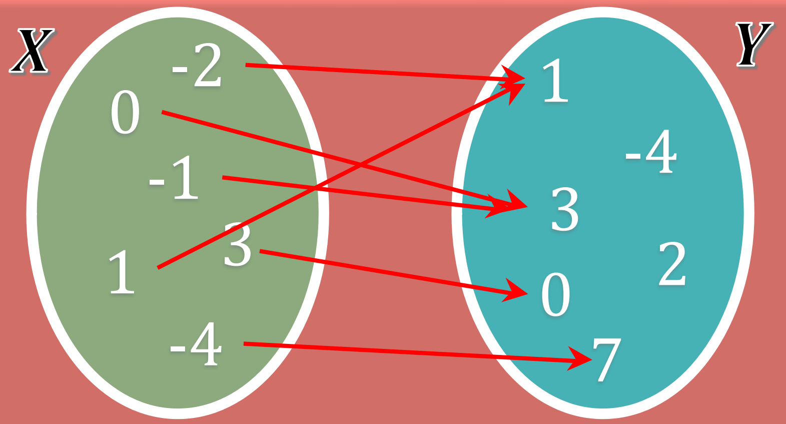
Domain = X

Co-domain = $\{-4,0,1,2,3,7\}$

Range = Y

***f* is a function**

Ex. $Let\ g: X \to Y$



$\{(-2,1),(0,3),(-1,3),(3,0),(1,1),(-4,7)\}$

Domain = $\{-4,-2,-1,0,1,3\}$

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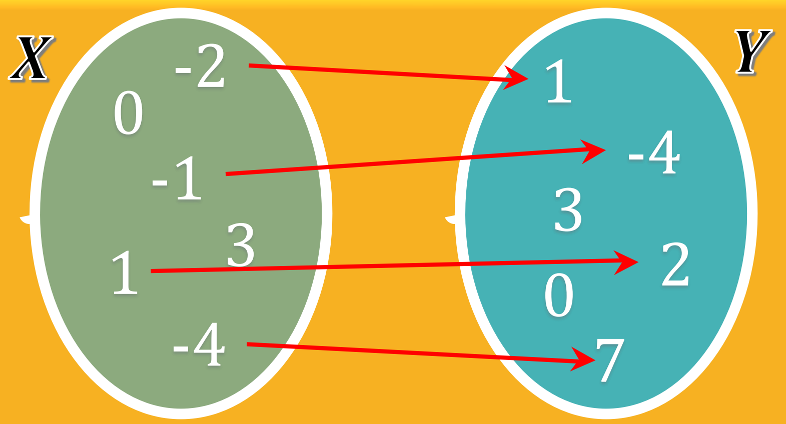
Co-Domain = $\{-4,0,1,2,3,7\}$

Range = $\{0,1,3,7\}$

***g* is a function**

FUNCTION

Ex. *Let* $h: X \to Y$



$\{(-2,1),(-1,-4),(1,2),(-4,7)\}$

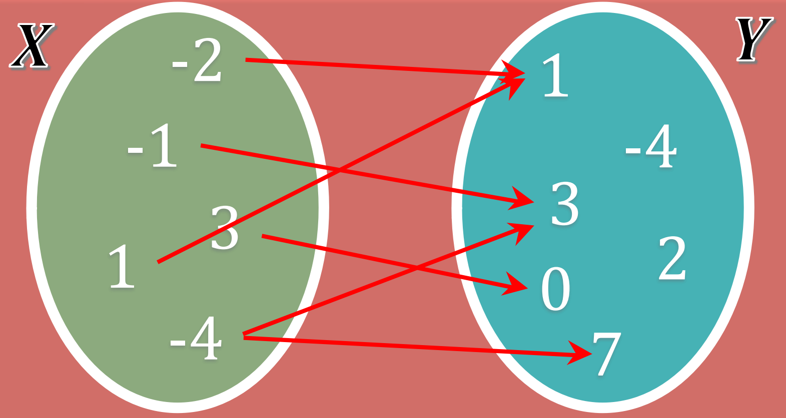
Domain $= \{-4, -2, -1, 0, 1, 3\}$

Co-Domain $= \{-4, 0, 1, 2, 3, 7\}$

Range $= \{-4, 1, 2, 7\}$

***h* is a NOT function**

Ex. *Let* $F: X \to Y$



$\{(-2,1),(-1,3),(3,0),(1,1),(-4,7),(-4,3)\}$

Domain $= \{-4, -2, -1, 1, 3\}$

Co-Domain $= \{-4, 0, 1, 2, 3, 7\}$

Range $= \{0, 1, 3, 7\}$

***F* is a NOT function**

FUNCTION

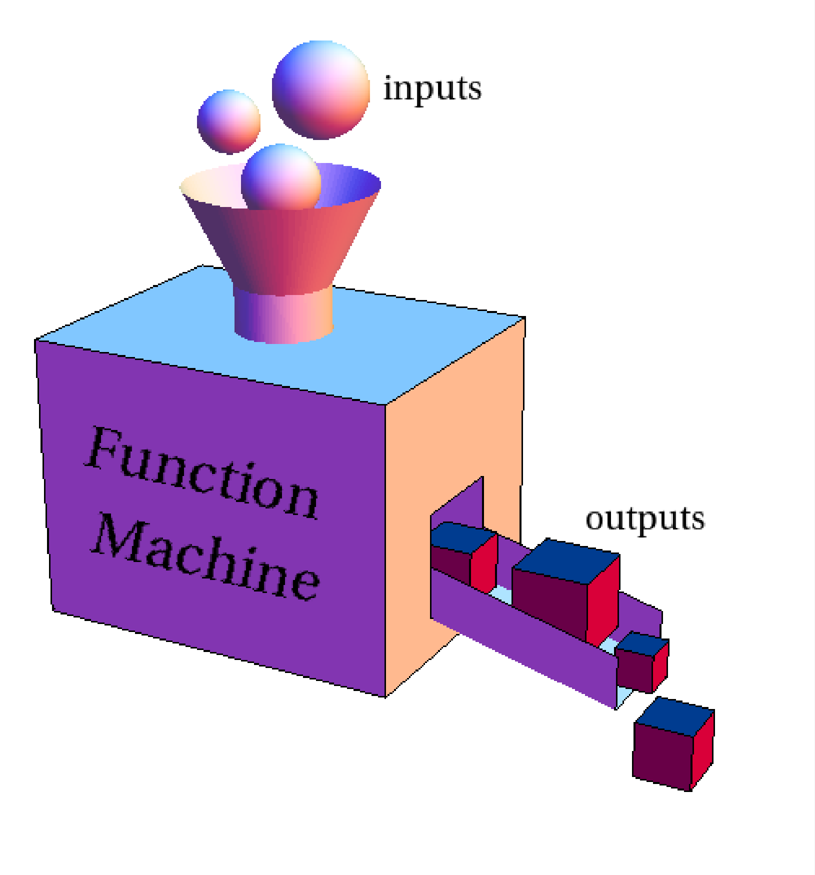
The quantity y is a *function* of x if every value of x in the domain corresponds to a unique value of y in the co-domain.

"y is a function of x"

$$y = f(x)$$

dependent variable

independent variable



FUNCTION

The given function is $f(x) = x^2 - 3$

$x = \cdots, -2, -1, 0, 1, 2, \ldots$

INPUT

function

$f(x) = x^2 - 3$

OUTPUT

$y = -3, -2, -1, 0, 1, 2, \ldots$

Domain = set of all real numbers

$= \mathbb{R}$ or $(-\infty, \infty)$

$= \{x | x \in \mathbb{R}\}$

$= \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Range = set of all real numbers greater than or equal to -3

$= [-3, \infty)$

$= \{y | y \in \mathbb{R} \geq -3\}$

$= \{-3, -2, -1, 0, 1, 2, \ldots\}$

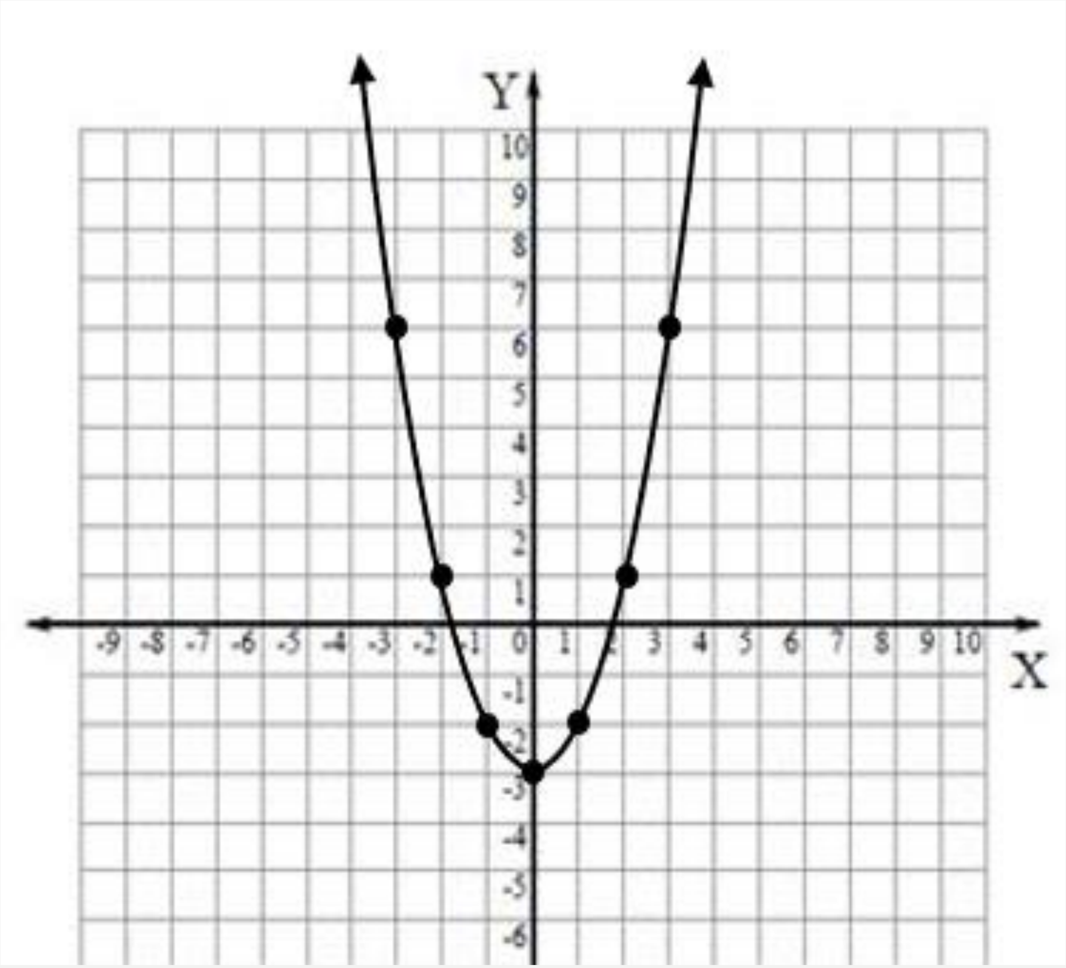
FUNCTION

The function $f(x) = x^2 - 3$ can also be defined by an equation like $y = x^2 - 3$.

$$y = f(x)$$

Plotting some points from the set of ordered pairs

{..., (-4, 13), (-3, 6), (-2, 1), (-1, -2), (0, -3), (1, -2), (2, 1), (3, 6), ...}



FUNCTION

TYPES:

A. *Algebraic Functions* are functions formed by a finite combination of algebraic expression using algebraic operations such as addition, subtraction, multiplication, division, raising to powers and root extractions.

- Constant Function
- Identity Function
- Linear Function
- Quadratic Functions
- Other Polynomial Function
- Rational Function
- Radical Function
- Piecewise-defined Functions

B. *Transcendental Functions* are function that cannot be expressed as a finite sequence of algebraic operations.

- Trigonometric Function
- Inverse Trigonometric Function
- Exponential Function
- Logarithmic Functions

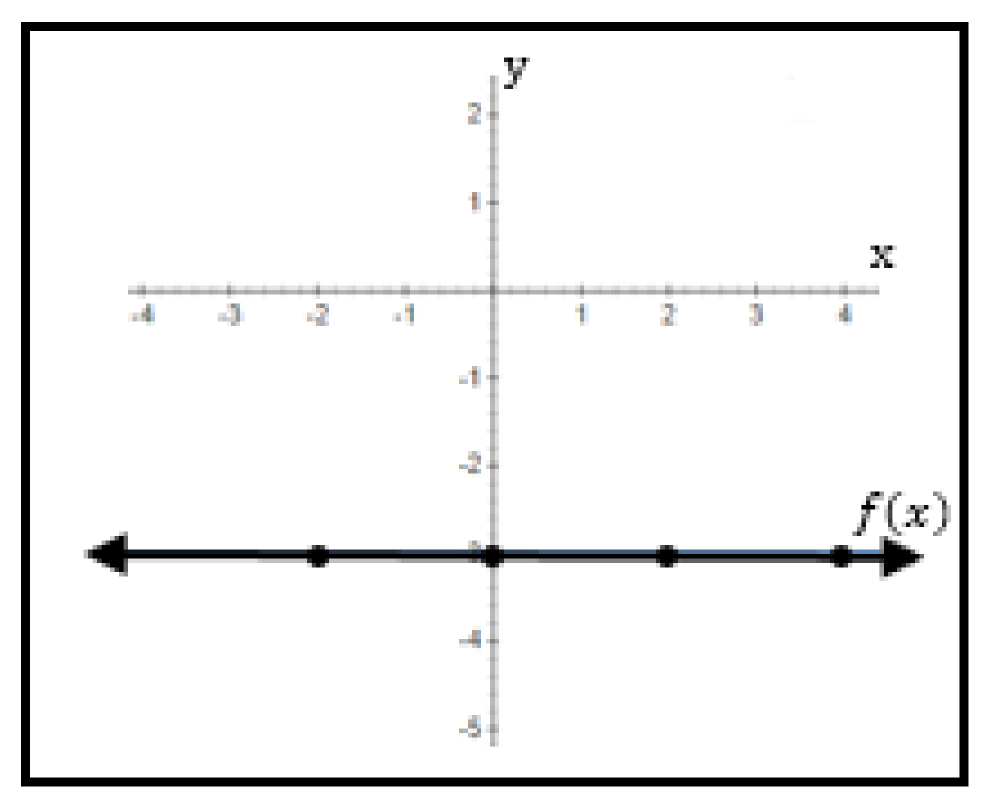
ALGEBRAIC FUNCTION

A. Algebraic Functions

a. Constant function: $f(x) = c$ or $y = c$, where c is constant

Example: $f(x) = -3$

x	-4	-2	0	2	4
$f(x)$	-3	-3	-3	-3	-3



Domain $= \mathbb{R}\ or\ (-\infty, \infty)$

Range = {-3}

ALGEBRAIC FUNCTION

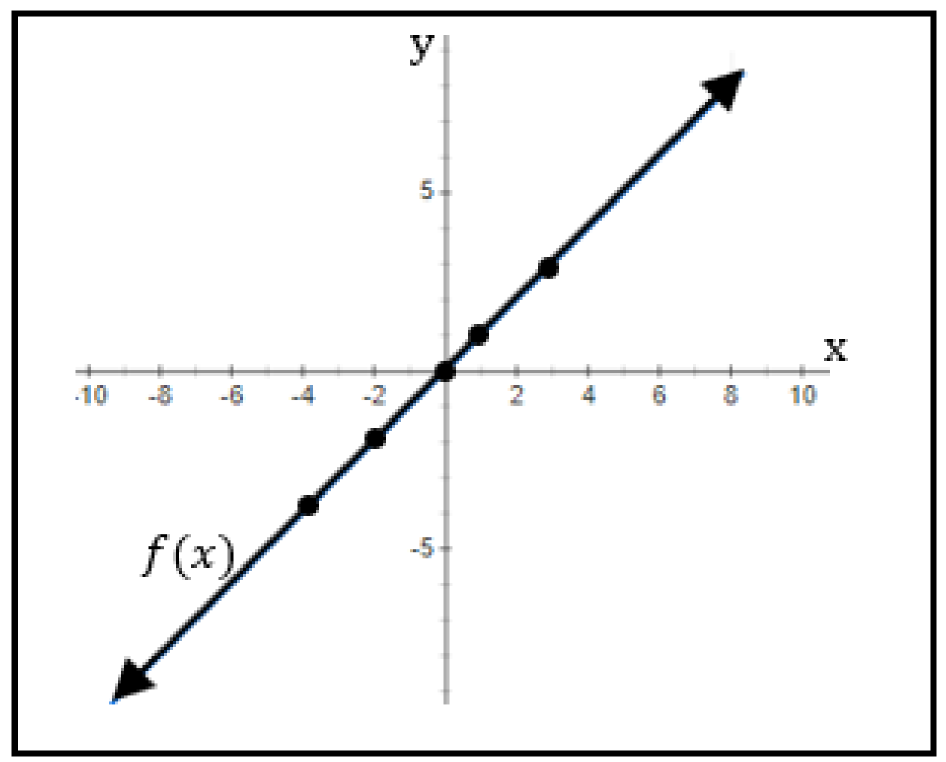
b. Identity Function: $f(x) = x$ or $y = x$

Example: $f(x) = x$ or $y = x$

x	-4	-2	0	1	2	3
$f(x)$	-4	-2	0	1	2	3

Domain $= \mathbb{R}$ *or* $(-\infty, \infty)$

Range $= \mathbb{R}$ *or* $(-\infty, \infty)$



ALGEBRAIC FUNCTION

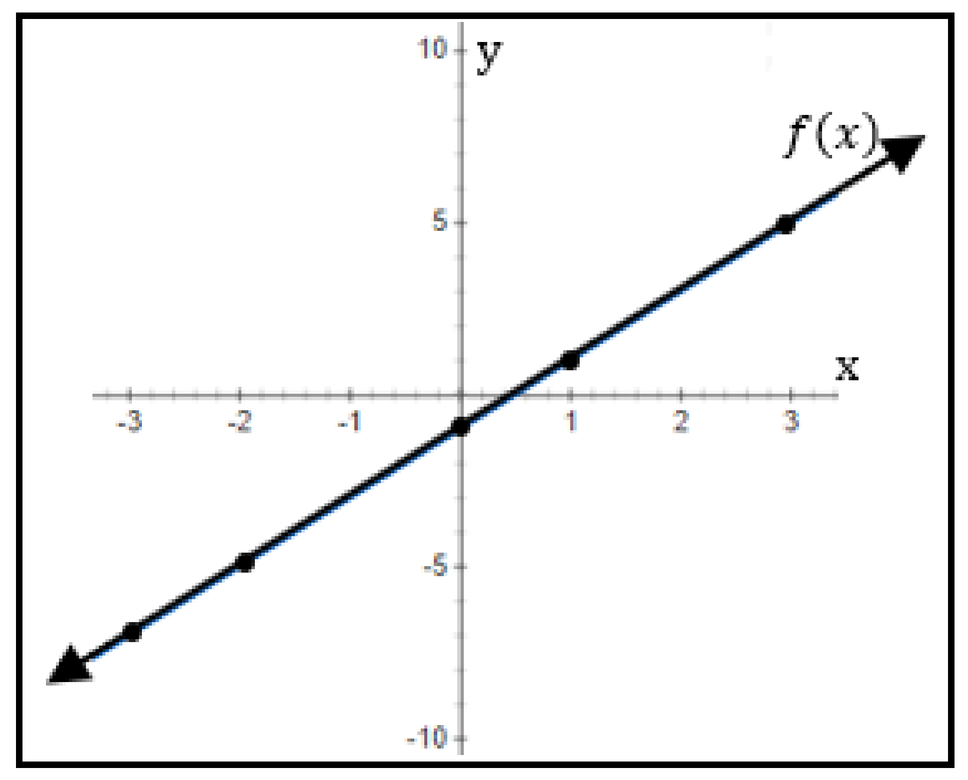
c. Linear Function: $f(x) = mx + b$ where $m \neq 0$ and m and b are constants

Example: $f(x) = 2x - 1$

x	-3	-2	0	1	3
$f(x)$	-7	-5	-1	1	5

$\text{Domain} = \mathbb{R}\ or\ (-\infty, \infty)$

$\text{Range} = \mathbb{R}\ or\ (-\infty, \infty)$



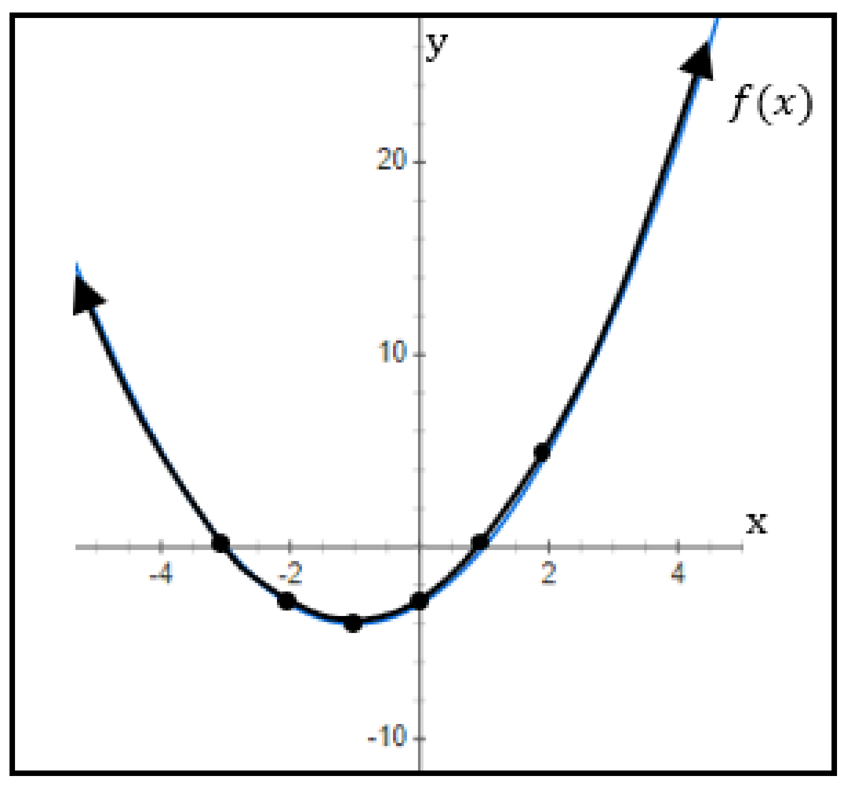
d. Quadratic Function: $f(x) = ax^2 + bx + c$ where a, b and c are constants

Example: $f(x) = x^2 + 2x - 3$

x	-3	-2	-1	0	2	3
$f(x)$	0	-3	-4	-3	5	12

Domain $= \mathbb{R}$ or $(-\infty, \infty)$

Range $= \mathbb{R} \geq -4$ or $[-4, \infty)$



e. Other *Polynomial Functions*:

$$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$$
$$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$$

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$$f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + +a_2x^2 + a_1x + a_0$$

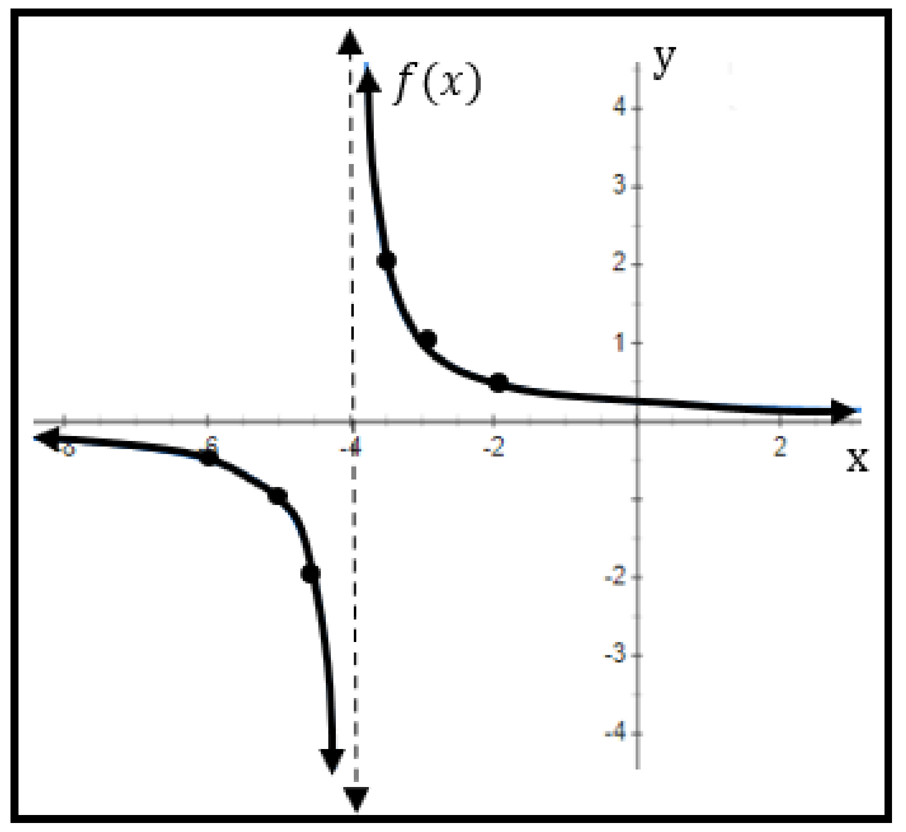
ALGEBRAIC FUNCTIONS

f. *Rational Function:*

$$f(x) = \frac{N(x)}{D(x)}$$ where $N(x)$ and D(x) are functions and $D(x) \neq 0$

Example: $f(x) = \frac{1}{x+4}$

x	-6	-5	-4.5	-4	-3.5	-3	-2
$f(x)$	-0.5	-1	-2	undef	2	1	0.5



Domain = $\mathbb{R}$`$\{-4\}$ or $(-\infty, -4) \cup (-4, \infty)$

Range = $\mathbb{R}$`$\{0\}$ or $(-\infty, 0) \cup (0, \infty)$

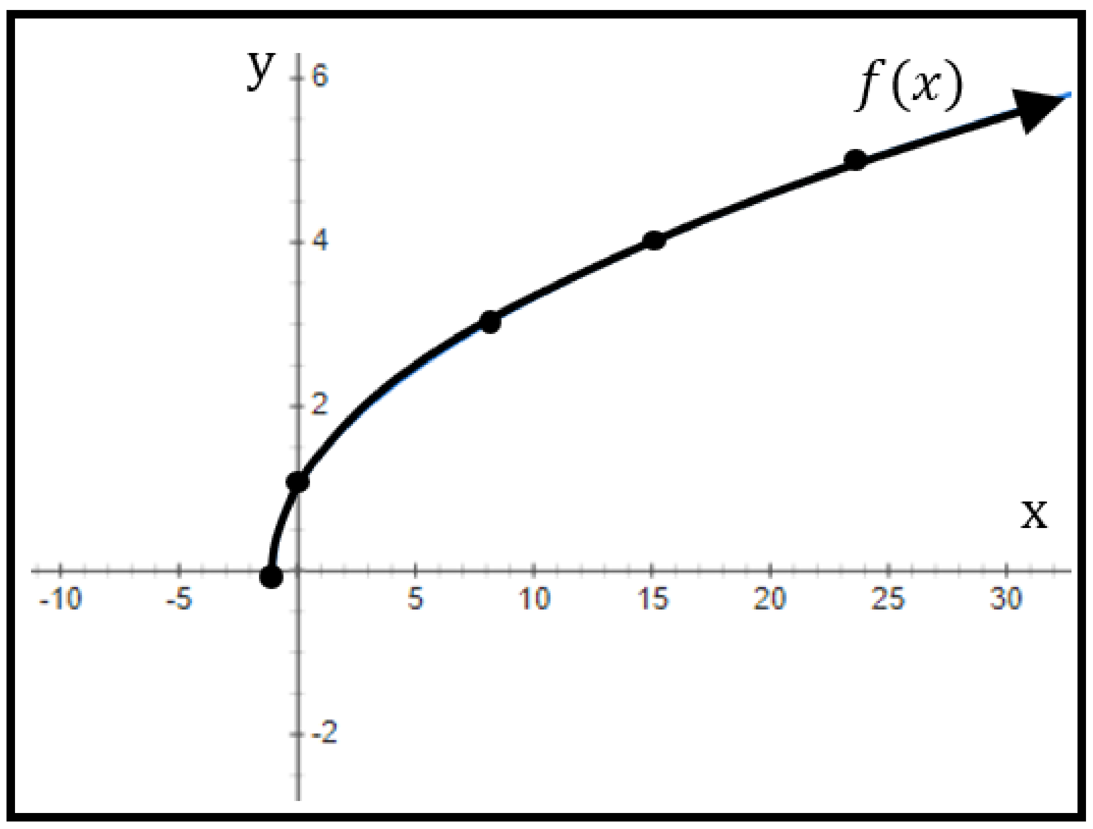
g. Radical Function:

Example: $f(x) = \sqrt{x+1}$

x	-1	0	8	15	24
$f(x)$	0	1	3	4	5

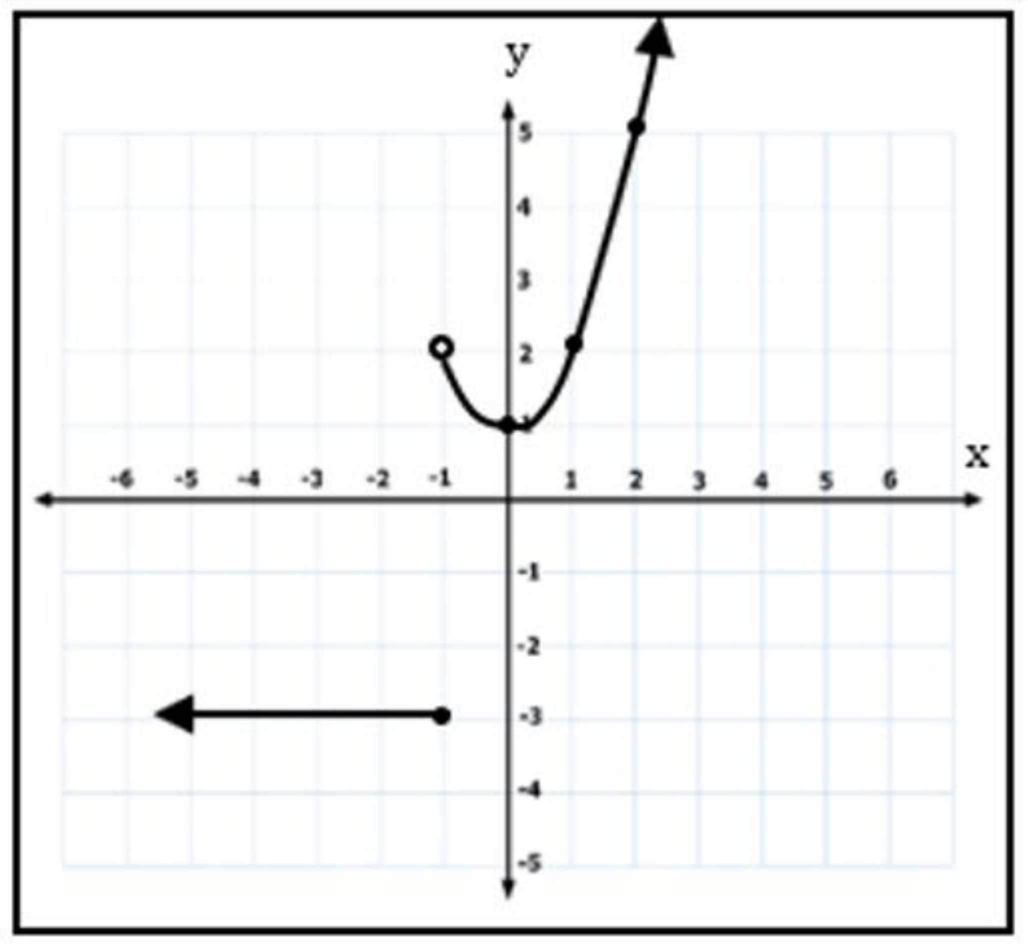
Domain: $\mathbb{R} \geq -1$ or $[-1, \infty)$

Range: $\mathbb{R} \geq 0$ or $[0, \infty)$



h. Piecewise-defined Function: is defined by multiple sub-functions and each sub-function is valid for some domain.

Example: $f(x) = \begin{cases} -3, & x \leq -1 \\ x^2 + 1, & x > -1 \end{cases}$



$f(x) = -3$

x	-1	-2	-3	-4
$f(x)$	-3	-3	-3	-3

$f(x) = x^2 + 1$

x	-1	0	1	2	3
$f(x)$	2	1	2	5	10

Domain: $\mathbb{R}$ or $(-\infty, \infty)$

Range: $\mathbb{R} \geq 1 \cup \{-3\}$ or $[1, \infty) \cup \{-3\}$

TRANSCENDENTAL FUNCTIONS

B. Transcendental Functions

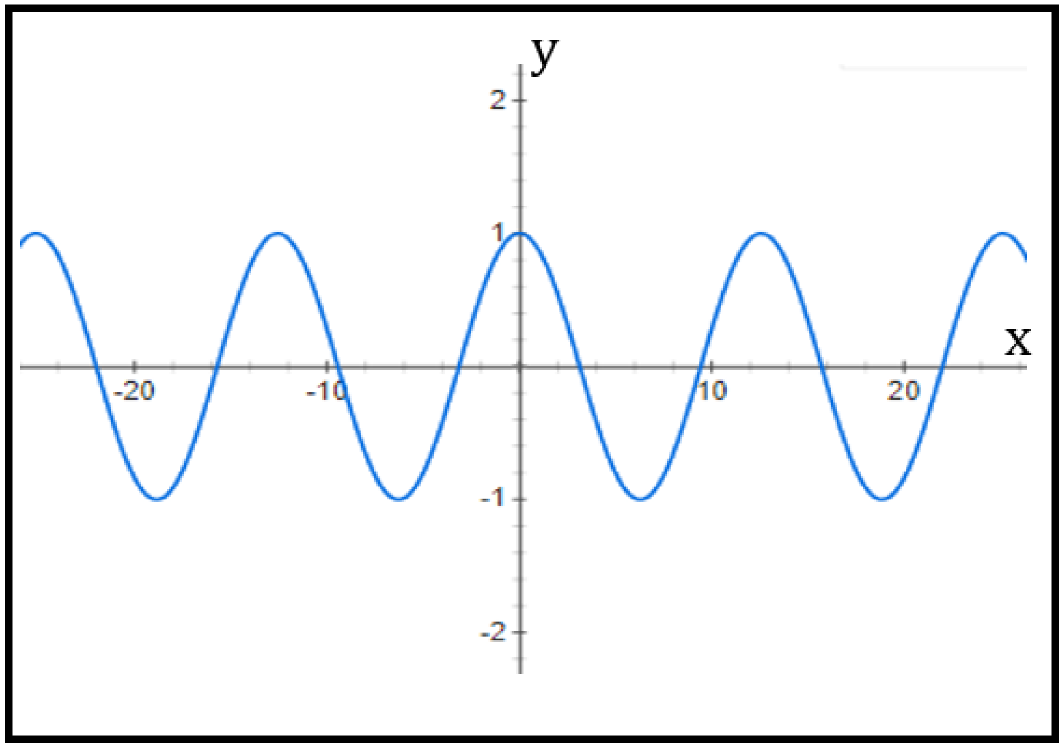
a. *Trigonometric Functions* are also called Circular Functions. Sine, Cosine, Tangent, Cotangent, Secant and Cosecant functions

Example: $f(x) = \cos\frac{x}{2}$

Domain: $\mathbb{R}$

Range: $-1 \leq \mathbb{R} \leq 1$ or [-1, 1]

x (in rad)	*f(x)*
-3π	0
-2π	-1
-π	0
0	-1
Π	0
2π	-1



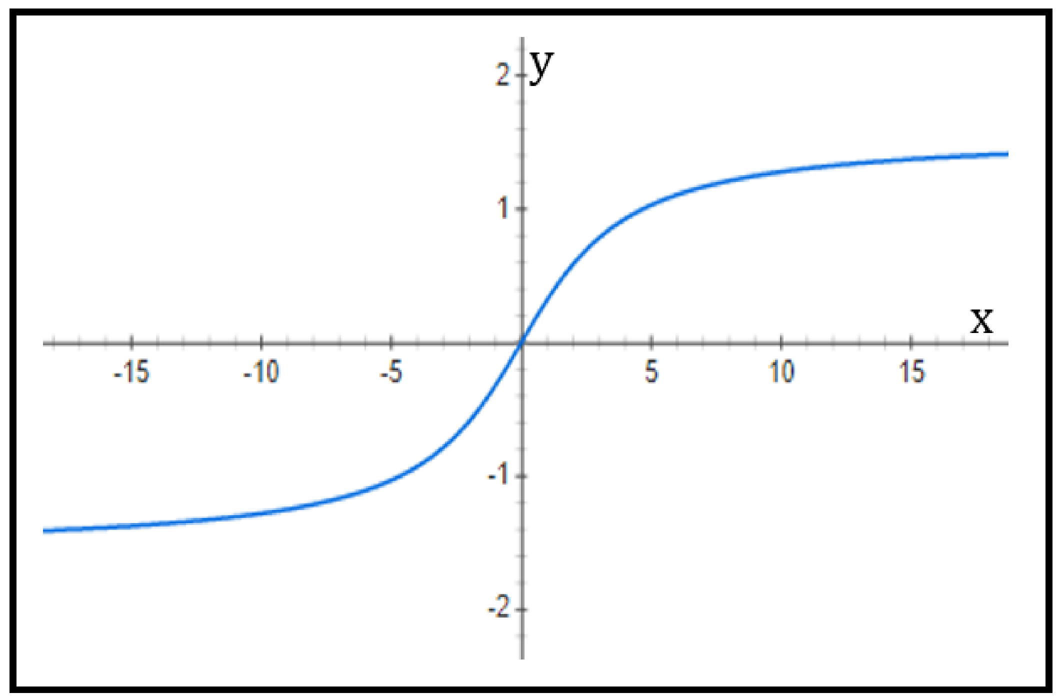
b. Inverse Trigonometric Functions are the inverse functions of the six trigonometric functions: arc-sine or sin^{-1}, arc-cosine or cos^{-1}, arc-tangent or tan^{-1}, arc-cotangent or cot^{-1}, arc-secant or sec^{-1}, arc-cosecant or csc^{-1}.

Example: $f(x) = \arctan \frac{x}{2}$

x	*f(x) (in rad)*
-3π	-1.36
-2π	-1.26
-π	-1
0	0
Π	1
2π	1.26
3π	1.36

Domain: $\mathbb{R}$

Range: $-\frac{\pi}{2} \leq \mathbb{R} \leq \frac{\pi}{2}$ or $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



TRANSCENDENTAL FUNCTIONS

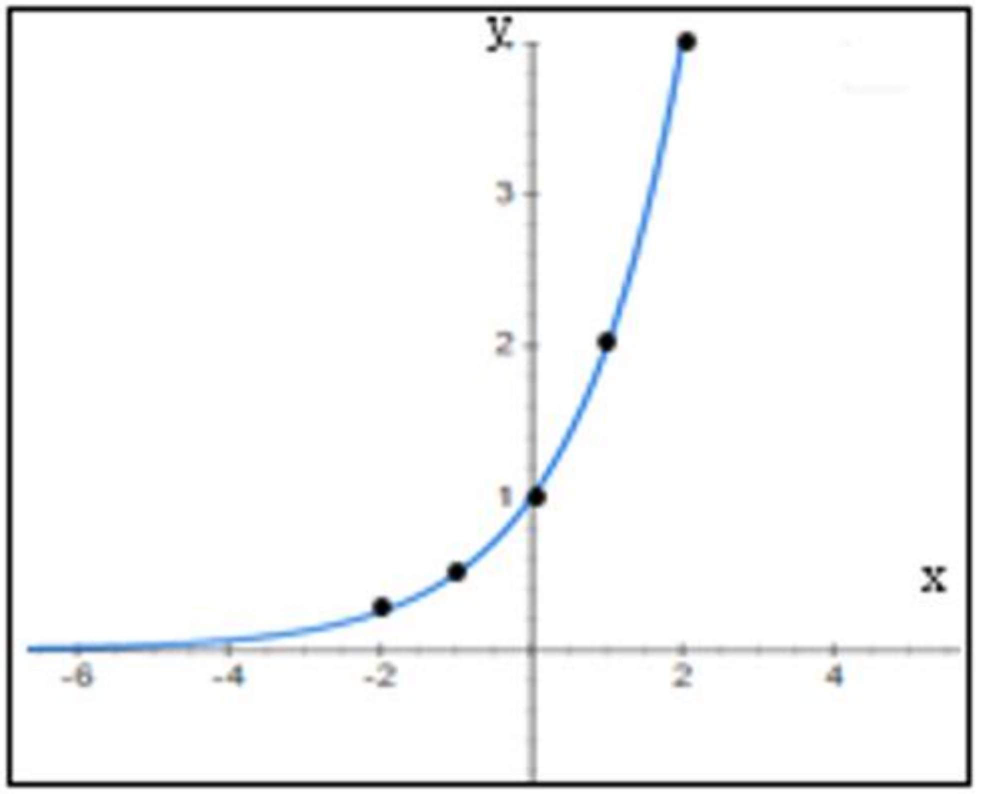
c. *Exponential Function* is defined by $f(x) = a^x$ where the base a is constant

Example: $f(x) = 2^x$

x	-4	-2	-1	0	1	2
$f(x)$	.06	.25	0.5	1	2	4

Domain: $\mathbb{R}$ or $(-\infty, \infty)$

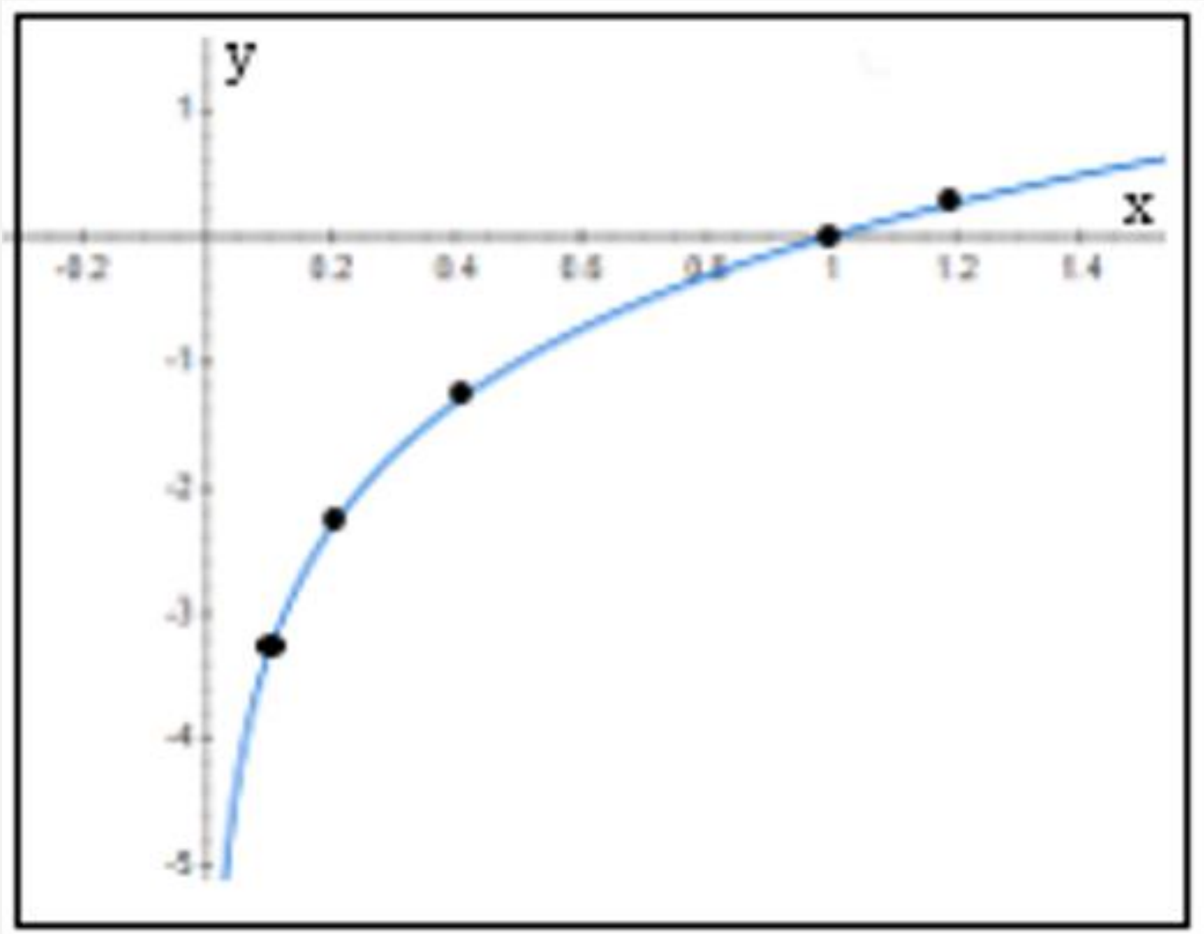
Range: $\mathbb{R} > 0$ or $(0, +\infty)$



d. Logarithmic Function is the inverse of exponential functions

Example: $f(x) = \log_2 x$

x	$f(x)$
0.01	-6.64
0.1	-3.32
0.2	-2.32
0.4	-1.32
1.0	0
1.2	0.26



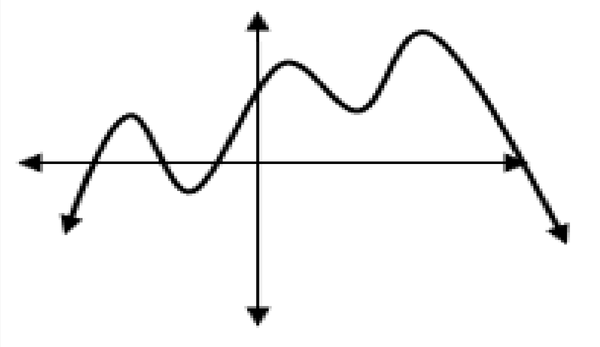
Domain: $\mathbb{R} > 0$ or $(0, +\infty)$

Range: $\mathbb{R}$ or $(-\infty, \infty)$

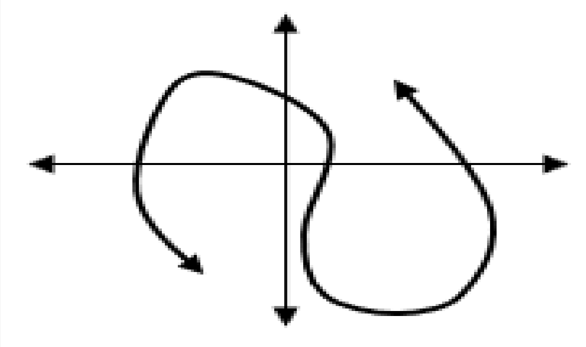
Note: *Vertical-line Test.* A set of points in the plane is the graph of a function if and only if the graph intersects every very vertical line in at most one point.

Example: Determine whether each graph is a graph of a function

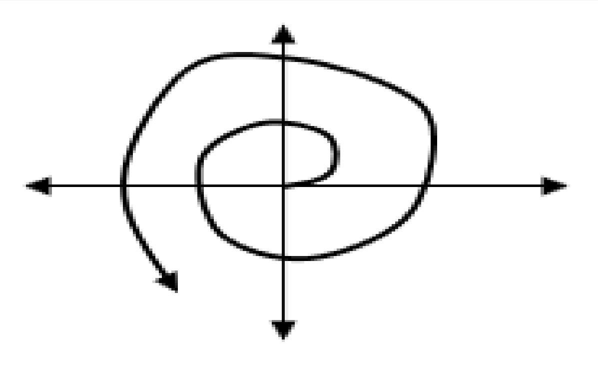
(a)



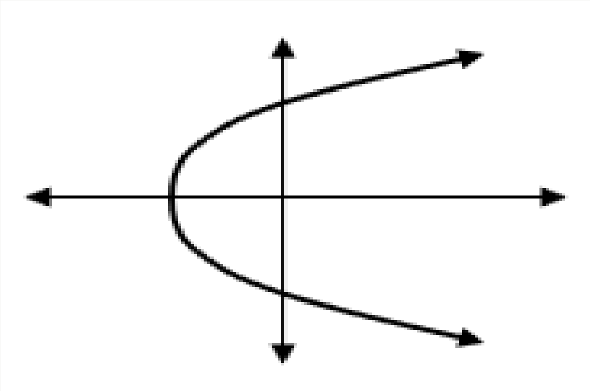
(c)



(b)



(d)



FUNCTIONS

Home Work #1