

Assignment 2: Introduction to Language Theory

Task 1

You will implement `mdc`: a program that interprets reverse-polish notation and applies mathematical operations on numbers. See slides from the second lecture for more information. The code from lecture 2 can supplement your understanding of `mdc`.

You also need to give a high level description of how your `mdc` works, i.e., how it takes the postfix expression and computes the result.

Code

[Link to code for task 1 here](#)

High level description

The `mdc` command is split by space character, turned into records corresponding to the `mdc` action, and finally run through a set of functions that return the updated stack given the command until every command has been processed.

a)

Split by space character (32 in ASCII)

```
declare fun {Lex Input} {String.tokens Input 32} end
```

b)

Simple `elseif` function for returning records corresponding to `lexem`. Use built in `Map` function for converting `Lexemes` array.

```
declare fun {Token Lexem}
  if Lexem == "+" then operator(type:plus)
  elseif Lexem == "-" then operator(type:minus)
  elseif Lexem == "*" then operator(type:multiply)
  elseif Lexem == "/" then operator(type:divide)
  elseif Lexem == "p" then command(print)
  elseif Lexem == "d" then command(duplicate)
  elseif Lexem == "i" then command(inverse)
  elseif Lexem == "c" then command(clear)
  else number({String.toInt Lexem}) end
end

declare fun {Tokenize Lexemes} {Map Lexemes Token} end
```

c)

1. Initial `interpret` function creates an empty stack and calls `InterpretRec`
2. `InterpretRec` goes through each token and completes it until there are no more tokens remaining.
3. `ParseToken` uses case pattern matching to figure out what type of token it is and runs their corresponding job.

4. `Calc` does the mathematical operation passed through its `Operator` parameter.

```
declare fun {Calc Operator S} N1=S.1 N2=S.2.1 in
  if      Operator == plus      then N1 + N2
  elseif Operator == minus      then N1 - N2
  elseif Operator == multiply   then N1 * N2
  elseif Operator == divide     then N1 / N2
  end
end

declare fun {Command Command S} ... end

declare fun {ParseToken T S}
  case T of
    number(N)      then N | S
  [] operator(type:0) then {Calc 0 S} | S.2.2
  [] command(C)     then {Command C S}
  end
end

declare fun {InterpretRec Token Stack} if Token==nil then Stack else
  {InterpretRec Token.2 {ParseToken Token.1 Stack}} end
end

declare fun {Interpret Tokens} {InterpretRec Tokens {List.make 0}} end
```

d) e) f) g)

The `Command` function performs the extra rules added in d, e, f, and g.

```
declare fun {Command Command S}
  if      Command == print      then {Show S} S
  elseif Command == duplicate   then S.1 | S
  elseif Command == inverse     then S.1 * ~1 | S.2
  elseif Command == clear      then {List.make 0}
  end
end
```

Task 2

You will implement `fun {ExpressionTree Tokens}`, where `Tokens` is a list of tokens as defined in Task 1. The function will return a record organized as a tree, representing the expression parsed from the postfix representation of the mdc-style input string.

In addition, you need to give a high level description of how you convert postfix notation to infix notation.

Code solution

```

\insert '/Users/jesperhustad/Documents/assignments/pl_4165/2/task1.oz'

declare fun {MakeExpression Operator S} N1=S.1 N2=S.2.1 in
  if      Operator == plus      then plus(N1 N2)
  elseif Operator == minus     then minus(N1 N2)
  elseif Operator == multiply  then multiply(N1 N2)
  elseif Operator == divide   then divide(N1 N2)
  end
end

declare fun {ExpressionTreeInternal Tokens ExpressionStack} T=Tokens S=ExpressionStack in

  % all tokens already processed
  if T==nil then S.1 else

    % add numbers to expression stack
    case T.1 of number(N) then {ExpressionTreeInternal T.2 N|S}

    % if operator: make corresponding expression and remove 2 from stack
    [] operator(type:0) then {ExpressionTreeInternal T.2 {MakeExpression 0 S}|S.2.2}

    end end
end

declare fun {ExpressionTree Tokens} {ExpressionTreeInternal Tokens {List.make 0}} end

```

High level description

The solution follows closely the algorithm given in the assignment:

- When you encounter a non-operator token in the input list, push it to the expression stack.
- When you encounter an operator token in the input list, remove two expressions from the expression stack and use them as operands, in a newly built expression. Push this expression to the stack.
- When all the input tokens have been processed, return the element at the top of the expression stack.

To make the function recursive the final check that the stack contains no more elements is performed first.

MakeExpression function simply returns an expression, inspired from the Calc function already implemented in Task1

Task 3: Theory

a) Formally describe the regular grammar of the lexemes in Task 1.

They are one character lexemes divided by single space characters. They can be divided into three types of tokens: numbers, operators and commands. Other than the order each lexem is context-free. It can therefore be described as:

```
v ::= γ
```

b) Describe the grammar of the records returned by the ExpressionTree function in Task 3, using (E)BNF.

The expressions tree in (Extended) Backus-Naur Form can be described as a set of grouping with parantheses. The token to the left of the grouping describes a function which is performed on the grouping.

***c) Which kind of grammar is the grammar you defined in step a)? Is it regular, context-free, context-sensitive,**

or unconstrained? What about the one from step b)?*

The grammar can be defined as context-sensitive. Because the order of the lexemes has a big effect on the output it can not be defined as context-free. The grammar in task 2 can be described as regular because (E)BNF ideas such as grouping and functions have been implemented in the grammar.