Leverage based sampling for classification

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Abstract

We validate the results of leverage based sampling for Least-Squares regression, shown by Ma et al. [1]. We explore the possibility of using the sampling distribution from LS-regression on 2-class classification, and introduce a new leverage distribution based on a generalization of the idea.

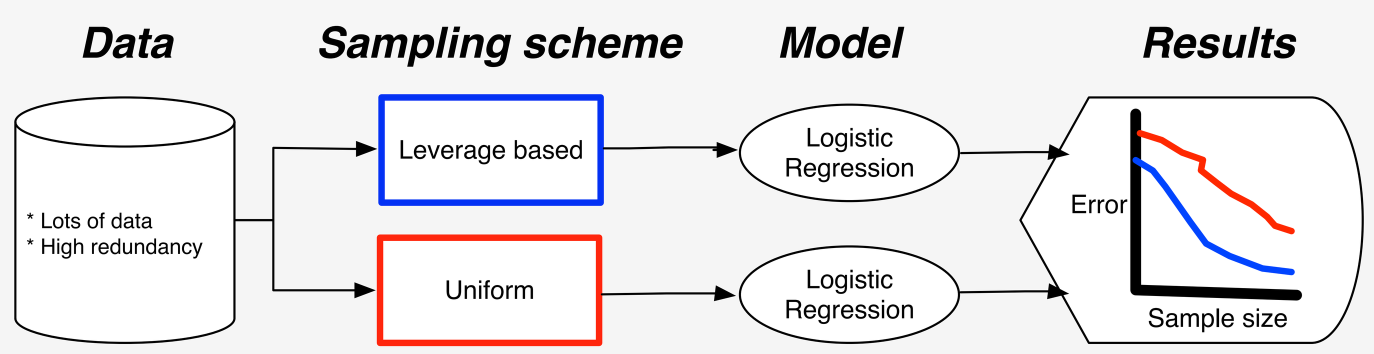
Motivation

For video the importance of sampling methods is exemplified by very large and high-dimensional datasets where

- It is not feasible to use all of the available data at once.
- There is a high redundancy between datapoints (25 fps).
- Computational cost is rarely linear to the input size.

We therefore want to explore alternative sampling methods, and try to identify datapoints which are important when fitting a model.

Concept



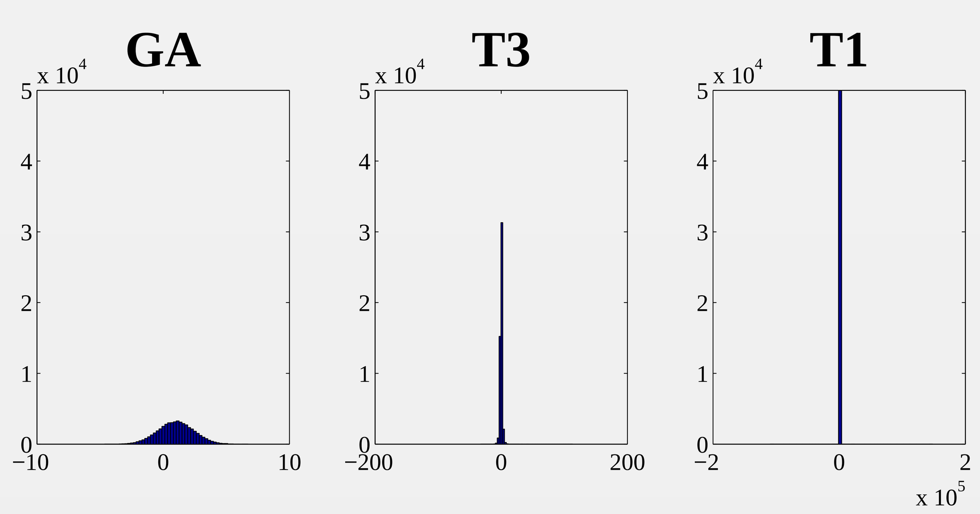
Research Questions

- Can we validate the results for least-squares regression shown by Ma et al. ?
- Will a linear regression based sampling distribution improve our performance in classification?
- Can leverage based sampling be generalized and used for classification?

Datasets

These datasets are drawn from distributions defined in Ma et al. [1] and characterised by

- GA: Nearly uniform leverage-scores
- T3: Mildly non-uniform leverage-scores
- T1: Very non-uniform leverage-scores



Figur 2: The three distributions considered.

Leveraging for least-squares regression

When fitting a model, we know that some datapoints are more important that others, leveraging is based on the idea that we can determine the importance of these point beforehand.

1. A leverage-score is calculated for each datapoint (its importance).
2. These scores are normalized into a distribution π to sample from.

Ma. et al. [1] use the leverage-scores for least-square regression defined as the diagonal elements of

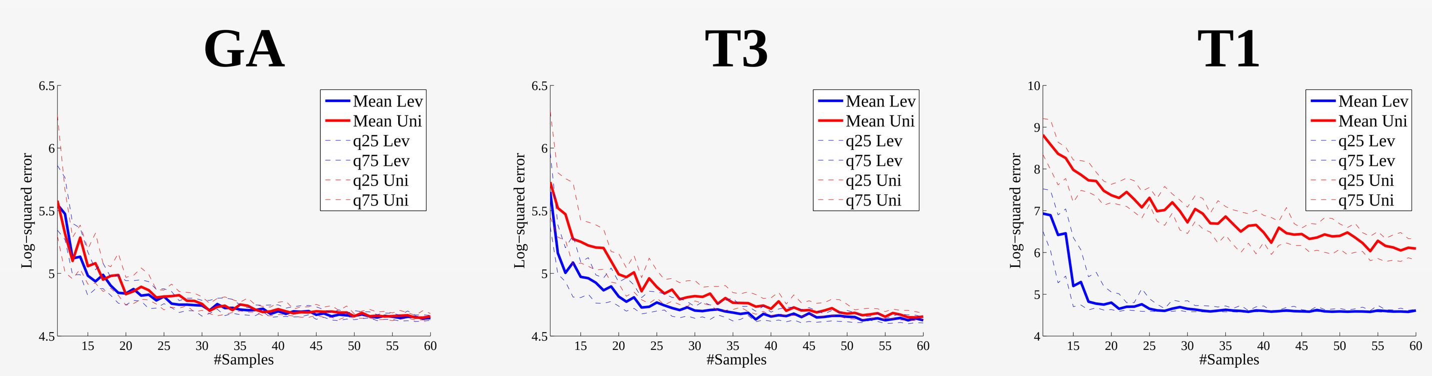
$$\mathbf{H} = \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T \tag{1}$$

This comes from the closed form expression for predictions which is linear in y

$$\hat{\mathbf{y}}_n = \mathbf{X}_n * \hat{\beta} \quad \text{where} \quad \hat{\beta} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}$$

Validation of the results Ma et al.

We have empirically tested and validated the results shown by Ma et al. [1].



Figur 3: Comparison of uniform **(red)** vs. leverage **(blue)** based sampling schemes for least-squares regression. $N = 1000$, $d = 10$.

- GA: The leverage score are approximately uniform, and thus there is no significant difference between the two sampling schemes.
- T3: Leveraging consistently provides slightly better results compared to uniform sampling.
- T1: With *very non-uniform* leverage-scores, leveraging clearly outperforms uniform sampling.

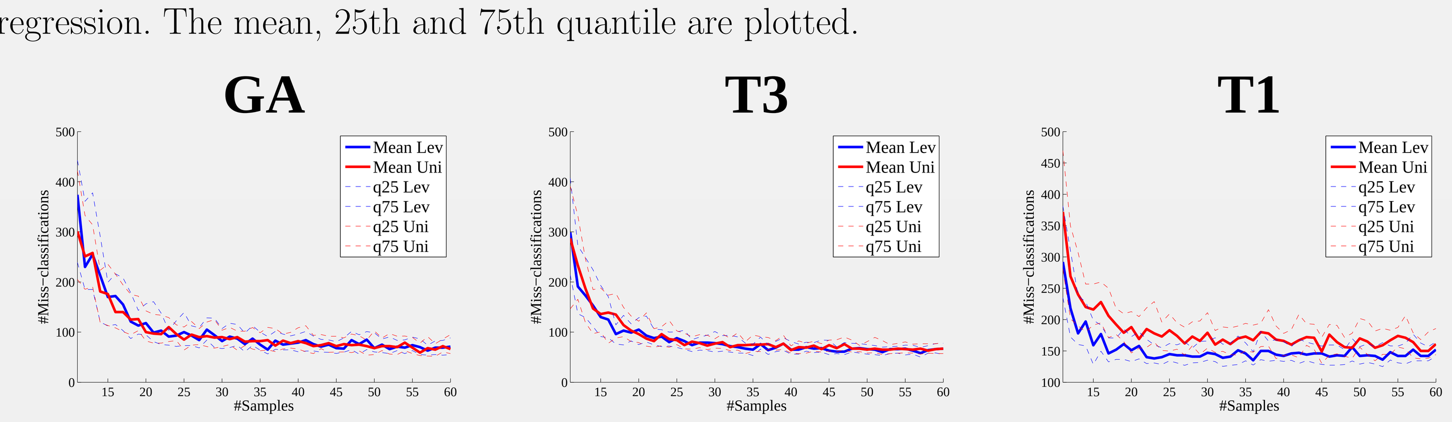
There results are consistent when varying N and d, although the level of improvement varies.

LS-based Distribution for Classification

We sample from the same distribution (1) as for least-squares regression. We use these samples to train a logistic regression model for 2 class classification, with equal class size.

Test Results

We compared the LS-distribution **(blue)** to a uniform-distribution **(red)** in sampling for a logistic regression. The mean, 25th and 75th quantile are plotted.



- Sampling from the LS-distribution is no better that uniform on datasets of type GA and T3.
- With very non-uniform leverage scores, T1, the LS-distribution slightly outperforms uniform sampling.

The results shown are for dimension $p = 10$ and $N = 1000$ datapoints, but it is consistent when varying p and N.

Sensitivity Based Distribution

We generalize the leverage scores to other models by seeing that they can be described as:

$$\frac{\delta \hat{\mathbf{y}}_n}{\delta \mathbf{y}_n} = Diag(H) \tag{2}$$

Which we call the sensitivity of the model to a specific datapoint. For a general probabilistic discriminative model this requires the following:

$$\hat{\mathbf{y}}_n = p(y|\bar{\mathbf{x}}_n, \bar{\mathbf{w}}) \quad \bar{\mathbf{w}} \text{ s.t. } \frac{\delta L}{\delta \bar{\mathbf{w}}} = 0 \tag{3}$$

Since 3 depends both directly and indirectly on y we see that

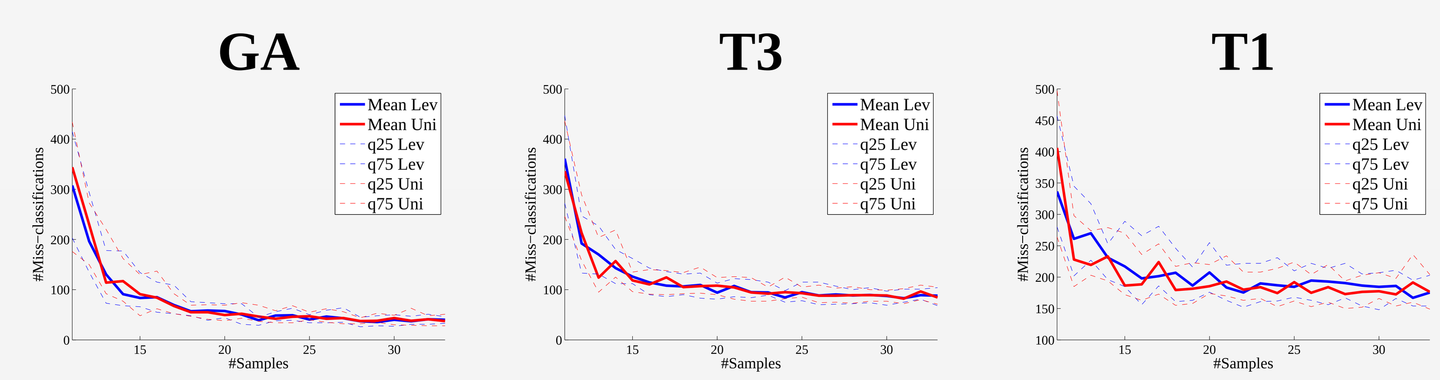
$$\frac{\delta}{\delta \mathbf{y}}\frac{\delta \mathcal{L}}{\delta \bar{\mathbf{w}}} = 0$$
$$\Downarrow$$
$$\frac{\delta^2 \mathcal{L}}{\delta \mathbf{y} \delta \bar{\mathbf{w}}} + \frac{\delta^2 \mathcal{L}}{\delta \bar{\mathbf{w}} \delta \bar{\mathbf{w}}^T}\frac{\delta \bar{\mathbf{w}}}{\delta \mathbf{y}} = 0$$

and from this we can get our leverage-score (2)

$$\frac{\delta \hat{\mathbf{y}}_n}{\delta \mathbf{y}_n} = \frac{\delta p(y|\bar{\mathbf{x}}_n, \bar{\mathbf{w}})}{\delta \bar{\mathbf{w}}^T}\frac{\delta \bar{\mathbf{w}}}{\delta \mathbf{y}} = -\frac{\delta p(y|\bar{\mathbf{x}}_n, \bar{\mathbf{w}})}{\delta \bar{\mathbf{w}}^T}\left[\frac{\delta^2 \mathcal{L}}{\delta \bar{\mathbf{w}} \delta \bar{\mathbf{w}}^T}\right]^{-1}\frac{\delta^2 \mathcal{L}}{\delta \mathbf{y} \delta \bar{\mathbf{w}}}$$

When evaluating this model, weights trained on a small initial training-set is used. This is expected to be better than LS-based sampling since it introduces dependence on class information.

Test results



Figur 4: Comparison of sensitivity vs. uniform -based sampling for logistic regression.

We see that the *sensitivity based sampling* gives us a performance equivalently to that of uniform sampling.

Future work

From our work several new question arise.

- How large show the initial sampling size be for sensitivity-based sampling?
- How should the non-linear sensitivity based leverage scores be normalised?
- Should all points be sampled from the initial weights found, or should the process be iterative?

Conclusion

In the case of linear regression, leverage-based sampling provides a improvement over uniform sampling when the leverage-scores are mildly or very non-uniform.
Using the LS-based sampling for classification is slightly better with very non-uniform leverage-scores, T1 data.
We have generalized the concept of leverage-based scores to classification with logistic regression and it has shown no improvements. However further analysis and tweaking might improved this approach.

References

Litteratur

[1] Ma et al. A statistical perspective on algorithmic leveraging. *arXiv:1306.5362v1 [stat.ME]*, June 2013.