Helicopter Lab

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Abstract

In this report, we present the results of experiments exploring four control methods for controlling a three-axis helicopter system. Our findings emphasize the relationship between theoretical assumptions and observed behaviour. We evaluated the performance of each control strategy in controlling elevation and pitch.

Contents

1 Introduction

In this report, we present a series of experiments where we controlled a helicopter using four different control methods. Each day, a new control method was tested, allowing us to compare the helicopter's behaviour across the different approaches. The experiments were structured into four lab sessions, covering the following methods: Monovariable Control, Multivariable Control, Luenberger Observer and Kalman Filter. This report is organized chronologically, following the sequence of the lab sessions.

Our focus is not on detailed derivations of the equations calculated during the preparatory work but we briefly outline what we did in preparation and how this informed the lab experiments. For each lab, we developed a test plan and formulated hypothesis about the expected behaviour of the system. The focus has been on analysing how the system behaved in relation to our assumptions, as well as identifying differences between theory and practice.

The helicopter system used in these experiments consists of two motors and a counterweight on the opposite side of the beam. In our preliminary assumptions, we considered the motors to be identical in weight and anticipated that identical input would yield identical output for both. The helicopter is mounted with three axes of rotation, which allow for pitch (p), elevation (e), and travel (λ), shown in figure 1. However, our focus throughout the experiments was primarily on controlling elevation and pitch.

Each control model was created in Simulink and MATLAB. After programming and simulating the models, they were uploaded to the helicopter, where we observed the system's response to each control method.

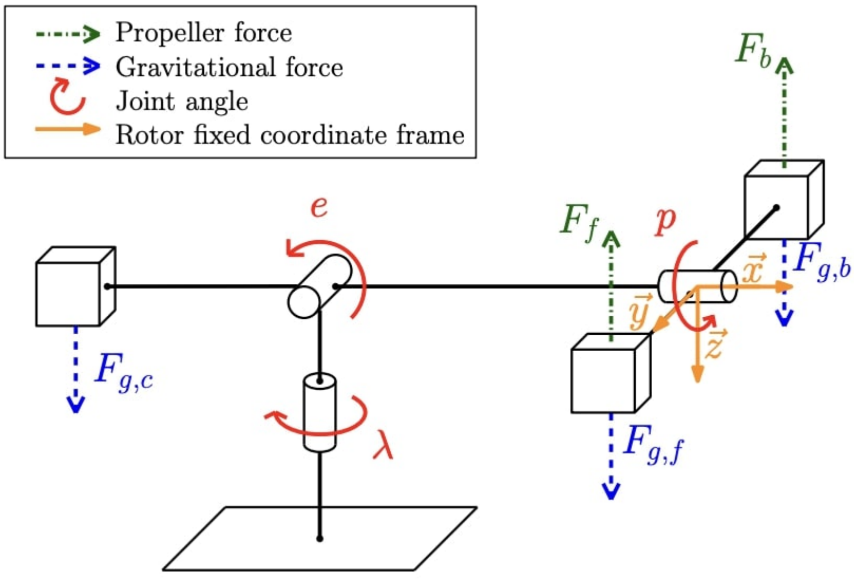

Figure 1: Forces and angles of the helicopter, source [2]

2 Part I - Monovariable control

2.1 Motivation and lab-preparation

In this part of the lab we wanted to control the helicopter by using a PD controller and pole placement. Through the lab-preparation we made the linearized equations of motion for the system, and applied a PD-controller to control the pitch. The system can be described through the following linearized equations:

$$\dot{p} = K_1 V_d \tag{1}$$

$$\ddot{e} = K_2 \tilde{V}_s \tag{2}$$

$$\ddot{\lambda} = K_3 p \tag{3}$$

where

$$K_1 = \frac{K_f}{2 m_p l_p} \tag{4}$$

$$K_2 = \frac{L_3}{J_e} \tag{5}$$

$$K_3 = \frac{L_4}{J_\lambda} V_{s,0} \tag{6}$$

We implemented a controller with state-feedback and made a test-plan with different pole placements. This setup was designed to allow control of the helicopter's pitch to both sides and enable it to hover stably in the lab environment. During the lab, we implemented the setup from our preparation phase, and set a constant control input, $V_{s,0}$, in order to move the helicopter to the linearization point for elevation. This approach allowed us to focus exclusively on controlling the pitch, which became the primary objective throughout the lab session.

When plotting the results, we chose to introduce a reference impulse with a square pulse to make the plots comparable. The key focus was on the system's response to this impulse. As shown in figure 2, we observed how the pitch followed the impulse and how it stabilized back to zero once the reference impulse changed.

2.2 Test-plan and hypothesis

From the first task in [1] , we wanted to experiment with different poles. In our test plan, we selected four different poles, each with distinct theoretical stability properties: one stable, one marginally stable and two unstable poles. Based on the theory, the marginally stable poles should produce oscillations with a constant amplitude over time, without converging to a steady-state value. The stable poles should make the system follow the impulse. In contrast, the unstable poles are expected to make the system unstable. In practice, this will cause the actuators to saturate in both directions.

2.3 Results and observations

When we first tested the stable set of poles, we observed behaviour that aligned with our expectations. As illustrated in figure 2, this result closely matches our hypothesis.

However, when we tested the marginally stable poles, the observed behaviour differed from theoretical predictions. In practice, the system initially displayed large oscillations following the impulse, then gradually decayed towards our reference. This behaviour typically characterizes complex poles in the left half-plane, not poles at the imaginary axis. This discrepancy became even more pronounced when testing one of the unstable poles, (1,1).

Testing the pole (1,1), we anticipated needing to stop quickly. But, as shown in figure 2, the system behaved as if the pole were stable. It followed the reference well, and although it exhibited some undershoot, it stabilized around a point. This was not the case when we selected poles located further into the right half-plane, where we had to stop testing early to avoid equipment damage. As shown in figure 2, we see that the system oscillated with the maximum amplitude possible over time.

The behaviour likely stems from multiple factors, primarily due to linearization and various assumptions in the physical model. Linearization simplifies the problem, meaning it is never a perfect representation of the actual system. This can result in a model that is too simplistic, creating discrepancies between theory and practice. Our physical assumptions in the model could also be inaccurate. For instance, we assumed that the motors were of equal weight and produced equal thrust for the same input. These assumptions may be wrong, especially considering the motors' age and possible replacements over the years. These factors may explain why a theoretically unstable pole behaved as if it were stable. Similarly, they might account for the system's tendency to stabilize below zero, instead of around the theoretically predicted point.

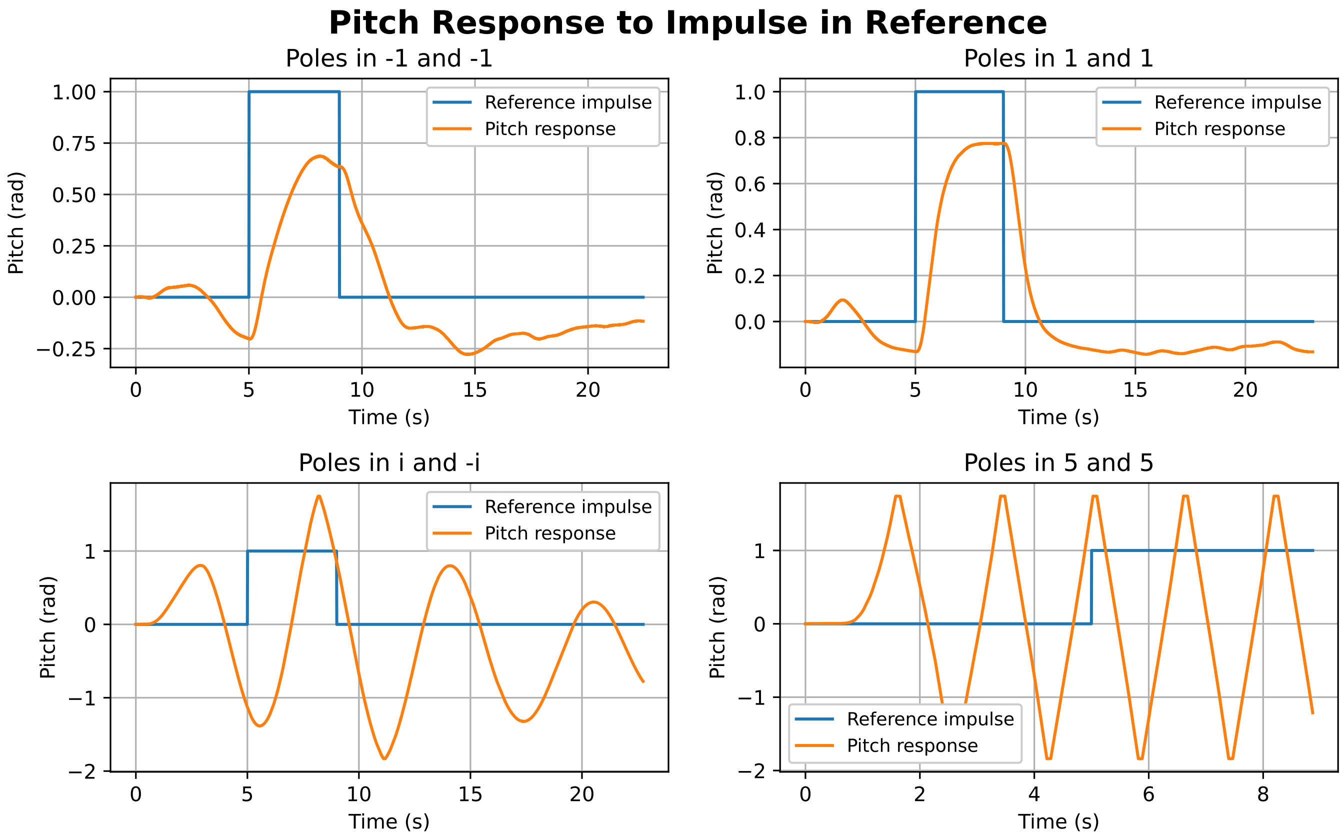

Figure 2: Plot of the pitch response with different poles

2.4 Conclusion

To summarize, the experimental results revealed unexpected behaviours, particularly with the marginally stable and unstable poles. While the stable poles matched theoretical expectations, the marginally stable and unstable poles displayed stability not predicted by the theoretical model. These discrepancies are likely due to limitations in the model, including simplifications from linearization and assumptions about motor performance.

3 Part II - Multivariable control

3.1 Motivation and lab-preparation

In this part of the lab we want to control the helicopter by using an LQR-controller. In the preparatory work we started by creating a state space formulation, and examining the controllability of the state space system. By computing rank($\mathcal{C}$) where $\mathcal{C}$ is the controllability matrix, we concluded that the system was controllable, and LQR control could be applied. The computations of the gain matrix $\mathbf{K}$ was computed using the matlab function `LQR(A,B,Q,R)`. `Q` and `R` is chosen as diagonal matrices based on a compromise between punishing the deviation of a state from the desired response, against minimizing large control signals. In this part of the lab, the measurements from the encoder was used. For the second part of the lab exercise, integral action was added to the controller. This was done by adding the states: $\dot{\gamma} = p_c - p$ and $\dot{\zeta} = \dot{e_c} - \dot{e}$ and augmenting the system matrices to reflect the changes in the state vector $\mathbf{x}$.

3.2 Test-plan and hypothesis

We wanted to experiment with different ratios $\mathbf{Q}/\mathbf{R}$ in order to observe how the response was affected. In order to get comparative results, we gave a square pulse as input for all tests. For a high $\mathbf{Q}/\mathbf{R}$ ratio, we thought the helicopter would have a faster response to changes in reference, but at the cost of a large control input. We separated the controller before and after applying integral action. In the case of giving a square pulse as reference for the elevation rate $\dot{e}$, we assumed that with no integral action, the controller would be unable to maintain the elevation, as the reference $\dot{e}$ dropped to zero. We think that adding integral effect to the controller, will be helpful for reducing some of the stationary deviation. Especially for counteracting the effect of the gravity on the elevation e. The values $\mathbf{Q}$ and $\mathbf{R}$ for a selection of the conducted tests, are written over the corresponding plots of responses in the subsequent figures. The entries represents the values along the diagonal of the matrices.

3.3 Results and observations

3.3.1 LQR controller without integral action

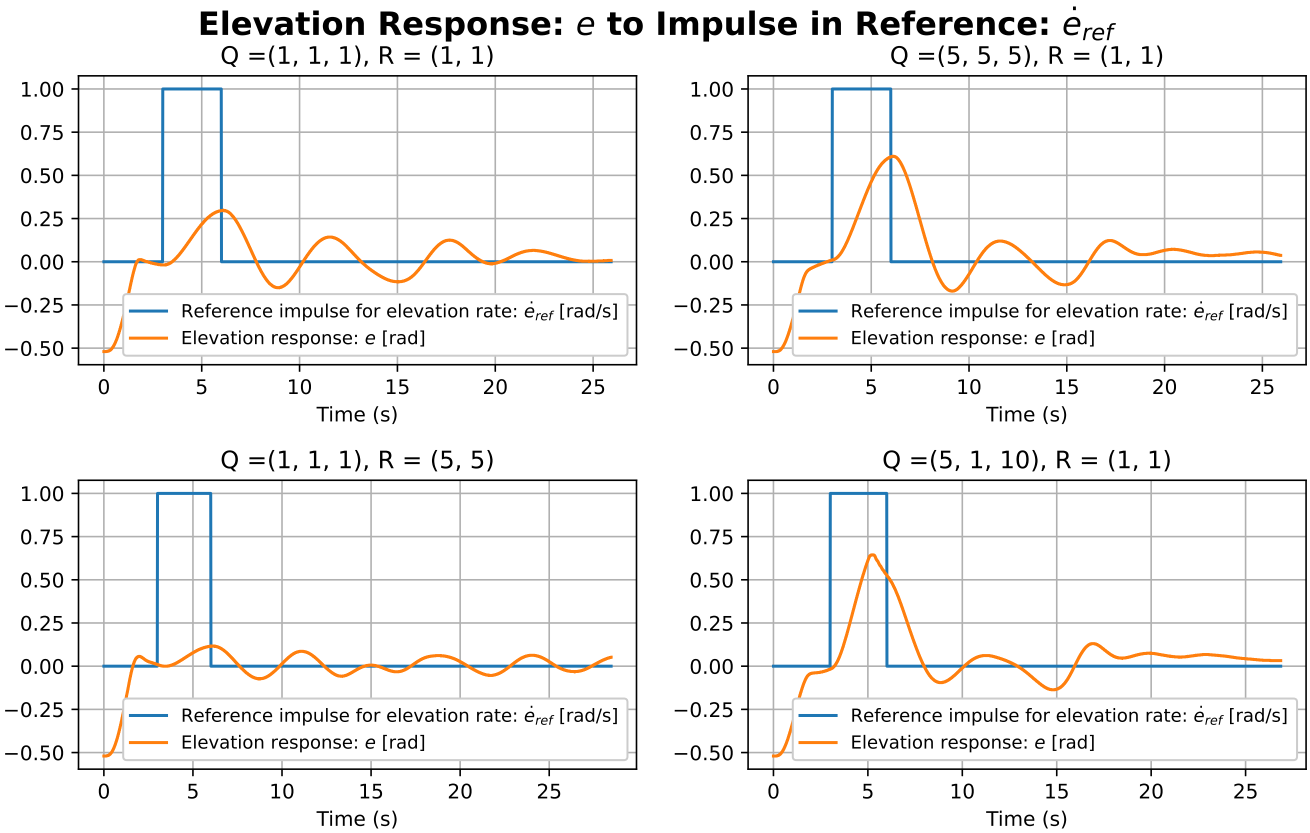

Figure 3: Plot of the elevation against reference in elevation rate, with no integral action

Note that in figure 3, the reference is not for elevation e, but for the elevation rate $\dot{e}_{ref}$. A reference for elevation rate $\dot{e}_{ref} = 0$, will result in the helicopter attempting to stop moving, not for the helicopter to move towards position 0. We can observe that without the integral action, the helicopter is unable to remain up, after $\dot{e}_{ref}$ drops to 0. This is in accordance with our theory. Furthermore we can note that for a higher ratio $\mathbf{Q}/\mathbf{R}$ the elevation e converges faster to a stationary condition, as the reference for elevation rate drops to zero. For our values this still takes some time. In the case of $\mathbf{R}$ being greater than $\mathbf{Q}$ the helicopter appears to have a minimal response to the pulse in reference, and besides some minor oscillations remains at its stationary point. This could align with the theory, as the controller attempts to limit large control signals and this possibly results in the controller not being able to respond in fast changes in reference from the step impulse. In retrospect trying even higher ratios $\mathbf{Q}/\mathbf{R}$ could have been of interest.

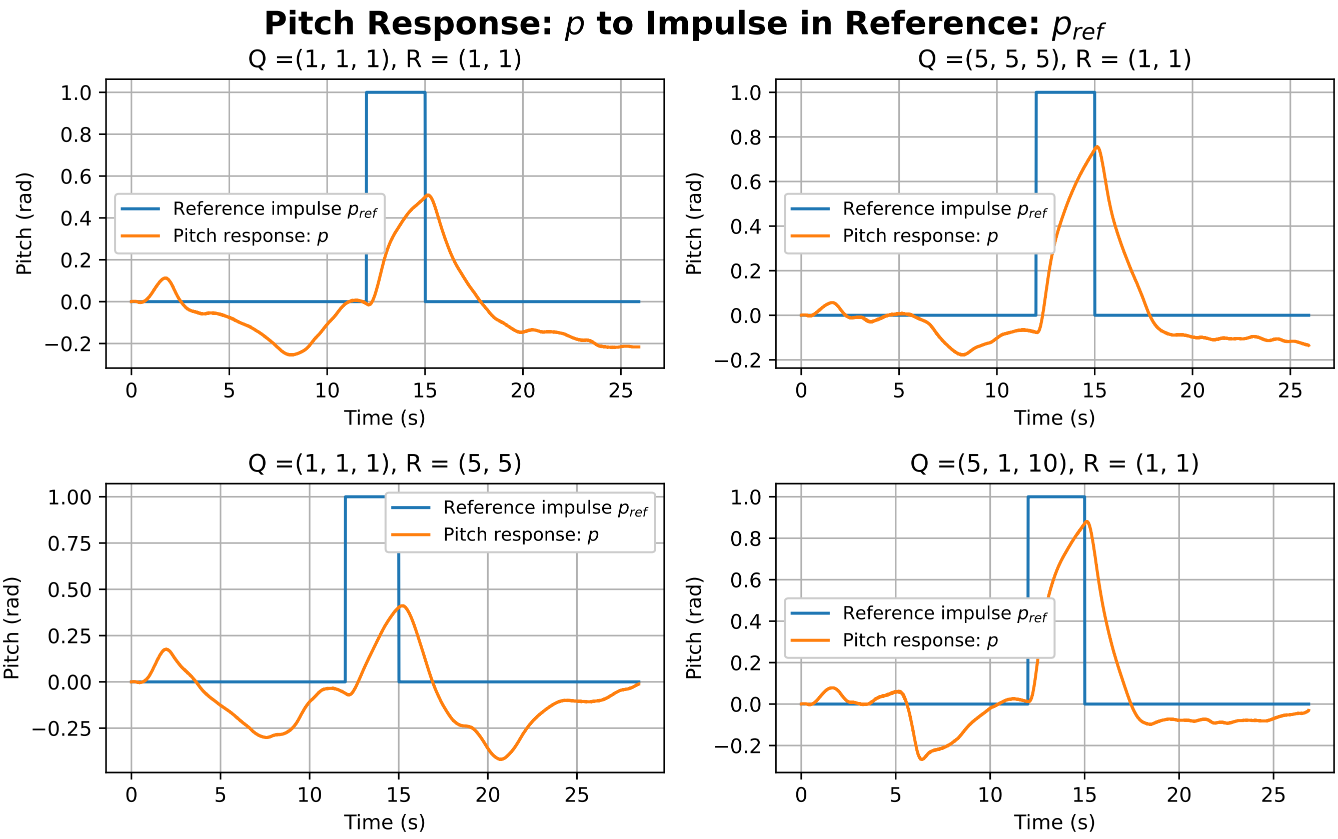

Figure 4: Plot of the pitch against reference in pitch, with no integral action

In contrast to the previous plot where the reference was given in terms of change, the reference for the pitch is given directly as p_{ref}. We can make similar observations for the different $\mathbf{Q}/\mathbf{R}$ ratios. A high ratio $\mathbf{Q}/\mathbf{R}$ results in a quicker response. In the opposite case when $\mathbf{R}$ is greater, the deviations from reference increases. This is reasonable as a high $\mathbf{Q}/\mathbf{R}$ means that deviation from desired response is punished more. We can note that some stationary deviation is still present, something we believe integral action will improve upon.

3.3.2 LQR controller with integral action

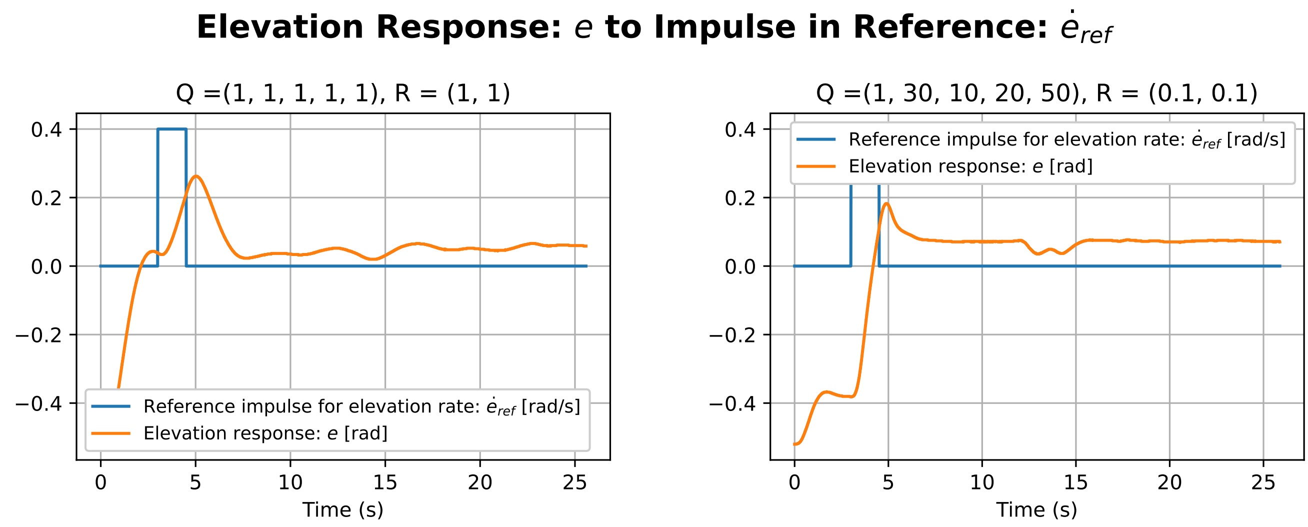

Figure 5: Plot of the elevation against reference in elevation rate, with integral action

From what we see in figure 5, the performance of the controller is greatly improved by adding integral action. By adding the integral effect to the controller, the helicopter is now able to remain up, after the reference impulse for elevation rate $\dot{e}_{ref}$ drops to zero. After some experimentations with different values of $\mathbf{Q}$ and $\mathbf{R}$ we obtained the response in the right plot in figure 5, which we deemed adequate for use in the subsequent lab tasks.

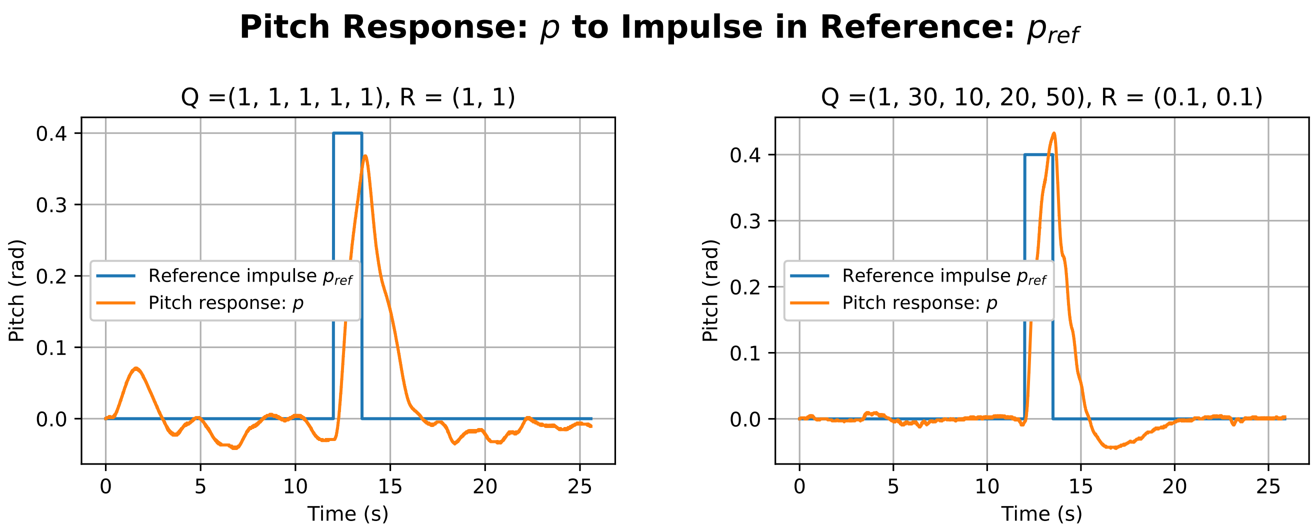

Figure 6: Plot of the pitch against reference in pitch, with integral action

For the pitch, we can observe that applying integral action, results in less deviation from reference, and a quicker response in comparison to figure 4. When using the same $\mathbf{Q}$ and $\mathbf{R}$ as the best performing controller from figure 5, we get some overshoot, but a quick response and overall a well performing system.

3.4 Conclusion

Through experimentation with different $\mathbf{Q}$ and $\mathbf{R}$ matrices we gained insight into how the theoretical effects of different $\mathbf{Q}/\mathbf{R}$ ratios translated into the physical behaviour of the system. In hindsight, plots of the control signal $\mathbf{u}(t)$ could have been an interesting addition, as this would have allowed more information about the effects of different $\mathbf{Q}$ and $\mathbf{R}$ matrices on the control signal $\mathbf{u}(t)$ itself. Using a LQR-controller with integral effect, we were able to achieve a controller for the helicopter, that works sufficiently well. The matrices we ended up using for the LQR-controller are

$$\mathbf{Q} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 50 \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}$$

4 Part III - Luenberger observer

4.1 Motivation and lab-preparation

Controlling a free-flying helicopter is impossible using the encoders employed in the previous tasks. Therefore, most modern drones receive their measurements from accelerometers and gyroscopes. The challenge when measuring the helicopter's states with these instruments is that the measurements are often very noisy. A common way to overcome these challenges is by using state estimation to estimate the states needed to control the helicopter. In the following lab session, we implemented state estimation by using a Luenberger observer given by:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}) \tag{7}$$

A prerequisite for using the Luenberger observer to estimate the full state vector of the system is that we are able to measure a minimal set of states that still makes the system observable. In the preparation tasks, we experimented the with observability of the system with different sets of states. The theoretical minimal set, that still made the system observable, was achieved by only measuring e and $\dot{\lambda}$.

The system was tested with different pole placements for the observer gain, $\mathbf{L}$, using the Matlab function `place(A,C,p)`. To control the helicopter, we kept the LQR-controller with integral effect developed in section 3. The measured states, needed in the LQR-controller, was replaced with the estimates given by the Luenberger observer. We compared the estimates against both the IMU and encoder measurements with different pole placements.

4.2 Test-plan and hypothesis

In the lab, we wanted to experiment with different pole placements for the observer gain, $\mathbf{L}$. Based on theory about the observer gain, we developed a hypothesis for the experiments. In general, we believed that higher observer gain would result in more noise on the estimated state. Since the different states have different behaviours given the same input, we also thought that they would respond different to equal observer gain.

4.3 Results and observations

The following figures shows the pitch and elevation rate response with a square wave change in reference. We have selected the plots for the observer poles in -10 and -100.

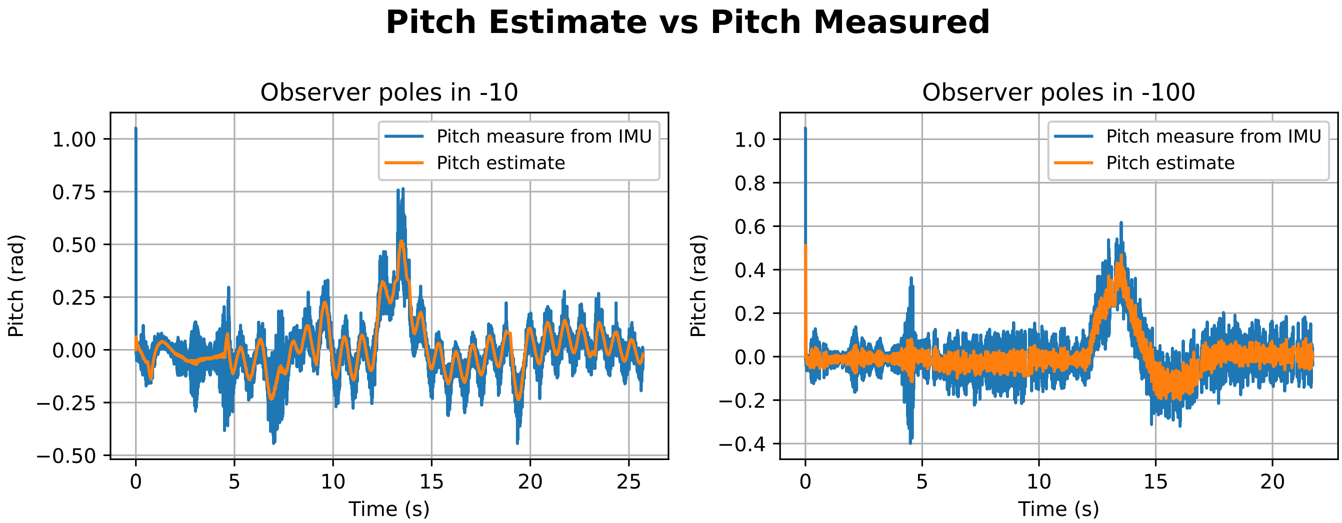

Figure 7: Plot of the pitch estimate against the measured pitch from the IMU.

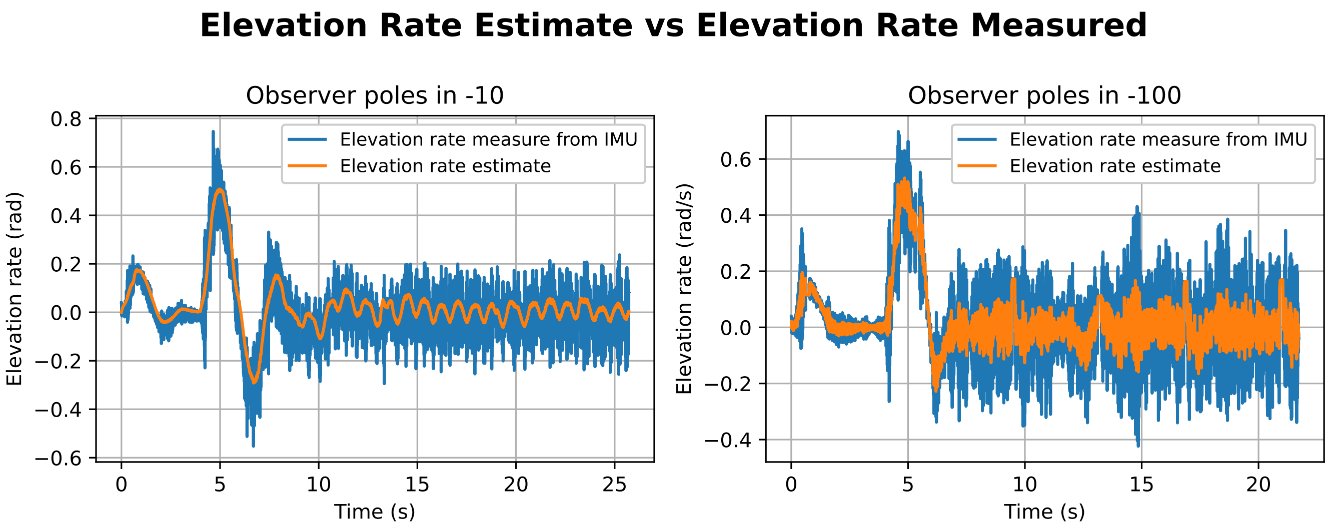

Figure 8: Plot of the elevation rate estimate against the measured elevation rate from the IMU.

From figure 7 and 8 we can see that our hypothesis about noise was correct. The state estimates became more noisy when placing the estimator poles further in the left half of the complex plane. Although the plots also show that even with observer poles in -100, the estimate is still a lot less noisy than the IMU measurement.

Compared to the IMU measurements, the encoder measurements was a lot more accurate. To evaluate the performance of the estimates, we therefore also plotted the estimates against the encoder measurements. These plots, with the same observer poles, can be seen in figure 9 and 10.

From the pitch estimate we can see that even with observer poles, only in -10, the estimates was very fast. We see that the estimate follows the encoder measurement very closely with almost no noise. As in the plots where we compared the estimate against the IMU measurements, the pitch estimate was still very noisy with poles in -100. We see that even though large observer gain is supposed to make the estimates follow the true measurement fast and close, in this case it comes with the consequence of too much noise on the estimate.

When looking at the plot for the elevation rate, we observe that in contrary to the pitch from the encoder, the elevation rate measurement from the encoder was full of noise. This is reasonable since very small disturbances to the helicopter would give noisy measurements even though the measurement instrument itself does not have a lot of measurement noise. We observe from this plot that with small observer poles, the state estimate may become useful even when using the encoder to measure states, since the estimate has less noise. When testing with observer poles in -100, we experience the same challenges discussed before. Compared to the encoder measurement, the estimate became full of noise.

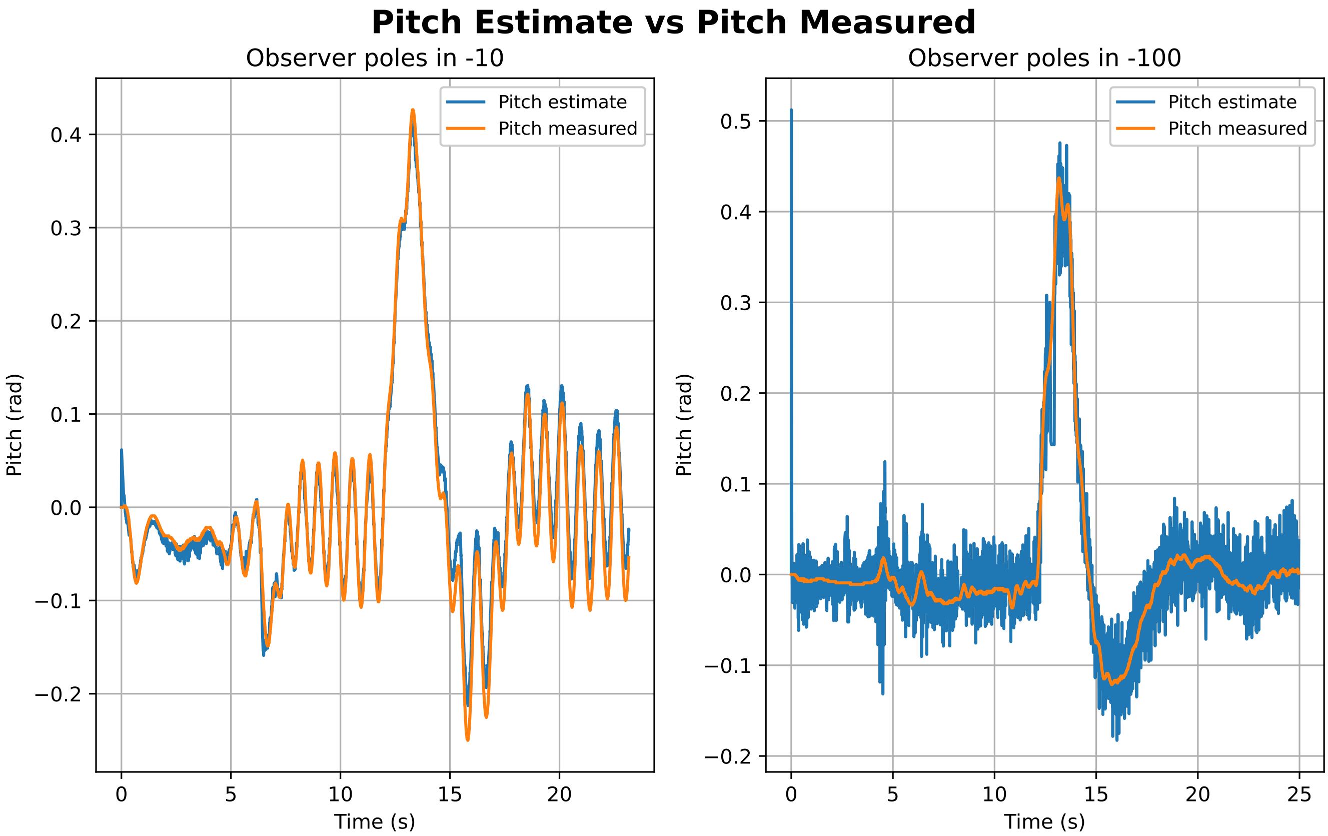

Figure 9: Plot of the pitch estimate against the measured pitch from the encoder.

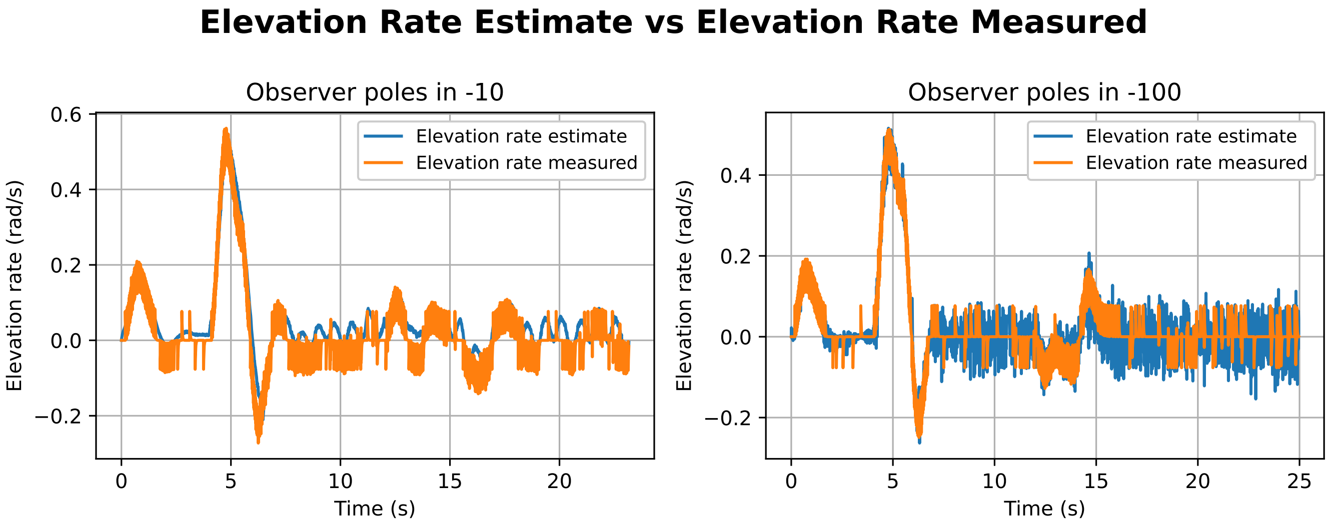

Figure 10: Plot of the elevation rate estimate against the measured elevation rate from the encoder.

4.4 Conclusion

To summarize, the experiences in the lab followed the theory. When increasing the the observer gain, the estimates became more noisy. If we were to work further with the implementation of the Luenberger observer, it would be interesting to see how well the helicopter would behave if we manually tuned each state estimation. This would enable us to have as fast estimates as possible for all states with minimal noise.

5 Part IV - Kalman filter

5.1 Motivation and lab-preparation

In the final part of the lab we wanted to estimate the states by implementing a discrete time Kalman filter. The filter incorporates our assumptions about the model uncertainty as well as the measurement uncertainty into the state estimates. In the preparatory tasks, the state vector $\mathbf{x}$ was augmented to include the travel rate $\dot{\lambda}$, and the system matrices was augmented to comply with this change. The continuous time system was discretized through the matlab function `c2d(sysc,Ts)`, where `sysc` is the continuous time state space formulation, and `Ts` is the discrete time step. In order to implement the filter in the lab, information about the measurement noise was needed. By disconnecting the control signal from the helicopter, and keeping it stationary, we took out measurements. This was used to compute the covariance of the measurement noise $\mathbf{R_d}$. The computation of the prediction step, as well as the correction step, was implemented by matlab function blocks in simulink. The prediction step updated the priori error covariance matrix $\hat{\mathbf{P}}^-$, and the priori estimate $\mathbf{x}^-$ based on the model. The correction step computes the posteriori estimate $\hat{\mathbf{x}}$ and error covariance $\hat{\mathbf{P}}$. This is done using the measurement and the matrix $\mathbf{Q_d}$, where $\mathbf{Q_d}$ is to be chosen. In order for the timing to be precise, a time delay block was added between the prediction step output, and the correction step input.

5.2 Test-plan and hypothesis

In the lab-preparation we planned to test at least three choices of values along the diagonal of $\mathbf{Q_d}$. The Kalman filter trade-off between prioritizing the model and the measurement meant that we could not plan exact values for $\mathbf{Q_d}$. The behaviour depends on the size of $\mathbf{R_d}$, which needed to be measured in the lab. The different behaviours we wanted to achieve was:

- An estimate dominated by the model, $\mathbf{Q_d} \gg \mathbf{R_d}$
- An estimate dominated by the measurement, $\mathbf{Q_d} \ll \mathbf{R_d}$
- A compromise, by having $\mathbf{Q_d} \approx \mathbf{R_d}$

When the estimate was model dominated, we believed that the helicopter would become very vulnerable to disturbances. The estimates are not able to correct itself based on changes caused by phenomenons outside the model.

On the other hand, when having an estimate dominated by the measurement, we believed that the system would be able to control itself. Even with a weak model the system should be able to control itself as long as the measurements it receives are informative enough. As experienced in section 4, the measurements from the IMU are very noisy, but since they are non-biased our hypothesis is that the helicopter would still be controllable.

In most cases, we believe that the compromise solution will give best system. This also applies to the helicopter. Therefore we believed that a compromise would work best when estimating the states using Kalman filter. Contrary to the measurement dominated approach, we believed that the behaviour with $\mathbf{Q_d} \approx \mathbf{R_d}$ would be easy to control.

5.3 Results and observations

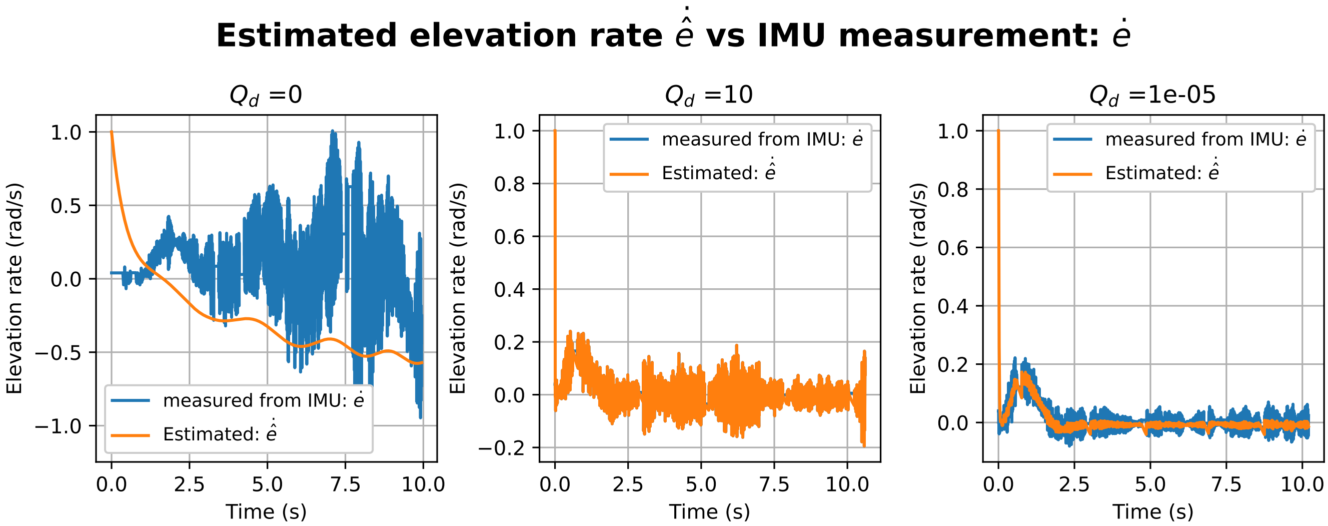

Figure 11: Plot of the estimated elevation rate against the IMU measured elevation rate.

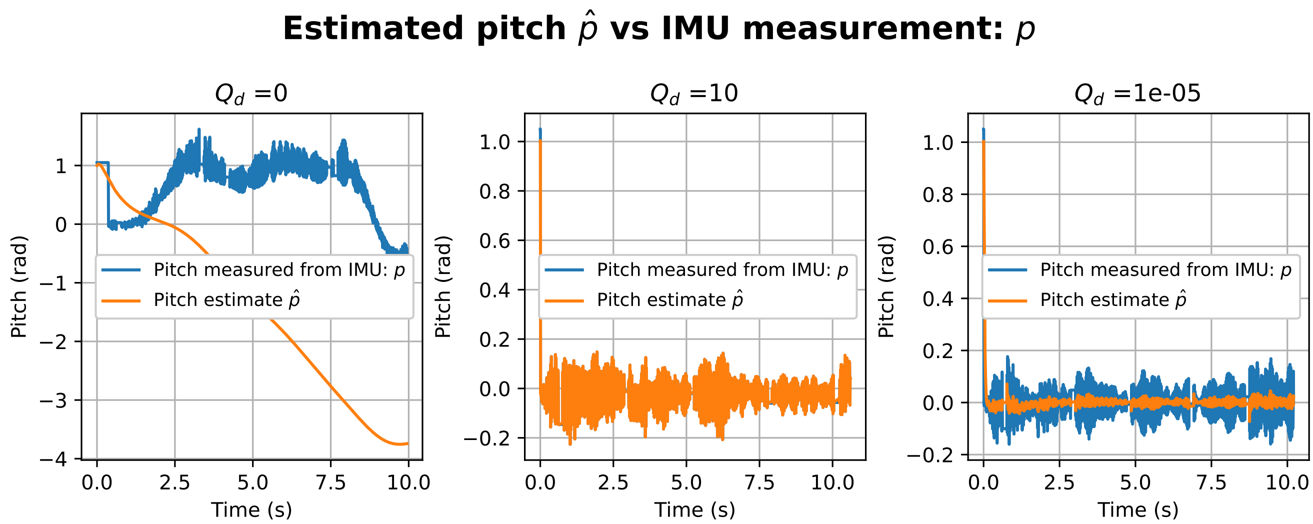

Figure 12: Plot of the estimated pitch against the IMU measured pitch.

From 11 and 12 we can observe that the model dominated estimate, when $\mathbf{Q_d}$ is $\mathbf{0}$, deviates greatly from the measurements. The estimate is in this case noise free. The deviation from the measurement, could indicate that our hypothesis is correct. Disturbances not accounted for in the model, can result in the helicopter moving away from the local linearization points, where our model no longer explains the system dynamics. Being unable to measure this results in the helicopter control failing.

In the case where $\mathbf{Q_d} \ll \mathbf{R_d}$ we see that the estimate is similar to the measurement. This results in a noisy estimate, that follows the measurement well. The helicopter could be controlled to some degree, but the high amount of estimate noise resulted in a behaviour where the helicopter was vibrating aggressively around the reference. This is in accordance with our hypothesis.

When we chose $\mathbf{Q_d}$ to be similar to $\mathbf{R_d}$ we obtained a compromise between estimator noise and following the measurement. This resulted in an estimator that worked well in our controller. Having a low amount of noise compared to large $\mathbf{Q_d}$, but still following the measurement substantially better than $\mathbf{Q_d} = \mathbf{0}$.

5.4 Conclusion

To conclude, using a Kalman filter is a powerful way to estimate states for the helicopter model. The primary advantage of applying the Kalman filter lies in its ability to optimally combine noisy measurements with the system's dynamic model. This flexibility makes it possible to produce accurate state estimates even in the presence of uncertainty. From observing the estimate change based on different values of $\mathbf{Q_d}$, we were able to observe how the theoretical concepts of kalman filtering, allowed us to produce state estimates that could be used in controlling the helicopter by combining two uncertain aspects of our system. Both the noisy measurement signals from the IMU, and the inaccuracies caused by the simplification of our model.

References

[1] Department of Engineering Cybernetics NTNU. *Helicopter lab assignment*. 2024.

[2] Department of Engineering Cybernetics NTNU. *Helicopter lab preparation*. 2024.