Group 4 Problem Set

December 4, 2021

Problem 1

For a function h, its proximal mapping at $x \in \mathbb{R}$ with parameter η is defined as:

$$\text{prox}_{\eta h}(x) := \text{argmin}_{u \in X} h(u) + \frac{1}{2\eta}\|u - x\|^2$$

Suppose $h = 0$. Find a closed form for $\text{prox}_{\eta h}(x)$. Define the generalized gradient G to be the following function

$$G_\eta(x) := \arg\min_{u \in \mathcal{X}}(x - prox_{\eta h}(x - \eta \nabla f(x)))$$

Using the first part of the problem, show that the generalized gradient G reduces to $\nabla f(x)$ when $h = 0$.

Problem 2

Suppose

$$K \geq \frac{40(L(f(x_0) - f(x^*)))}{\epsilon^2}$$

for our optimization problem

$$\underset{x}{\text{argmin}} \quad f(x)$$

where K is the number of iterations, L is the Lipschitz constant of function f, x_0 is the initial guess for x, and x^* is the minimizer of $f(x)$. Let $L = 0.5, f(x_0) = 10, f(x^*) = 3.5$. What should be the smallest value of K so that we can get an error of 10^{-4}?