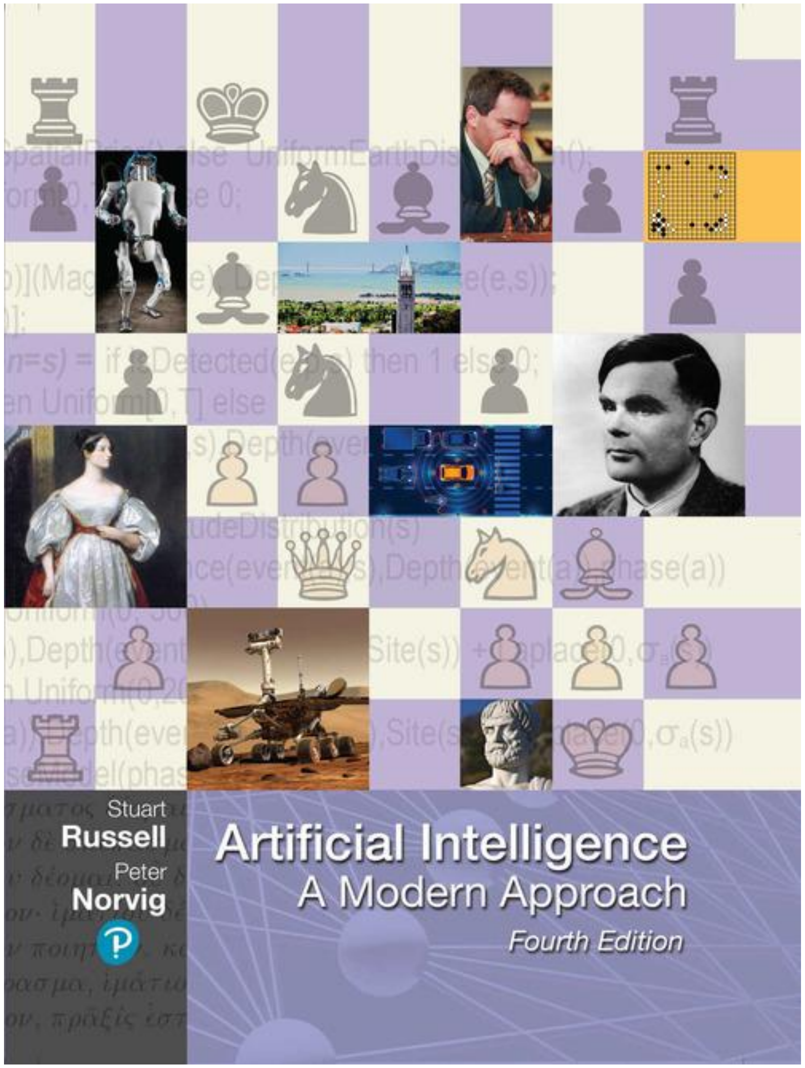
Artificial Intelligence Fundamentals

2024-2025



"Any sufficiently advanced technology is indistinguishable from magic."

- Arthur C. Clarke

AIMA Chapter 3

Solving Problems By Searching

Outline

- Problem-solving agents
- Example Problems
- Problem formulation
- Search Algorithms
- Uninformed Search Strategies
- Informed (Heuristic) Search Strategies
- Heuristic Functions

Problem-solving agents

When the correct action to take is not immediately obvious, an agent may need to plan ahead: to consider a **sequence of actions** that form **a path to a goal state**.

Such an agent is called a *problem-solving agent*, and the computational process it undertakes is called **search**.

We will cover several search algorithms. In this lecture, we consider only the simplest environments: *episodic, single agent, fully observable, deterministic, static, discrete, and known*.

We distinguish between **informed algorithms**, in which the agent can estimate how far it is from the goal, and **uninformed algorithms**, where no such estimate is available.

Problem-solving agents

Restricted form of general agent:

```
function Simple-Problem-Solving-Agent(percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← Update-State(state, percept)
    if seq is empty then
        goal ← Formulate-Goal(state)
        problem ← Formulate-Problem(state, goal)
        seq ← Search(problem)
    action ← Recommendation(seq, state)
    seq ← Remainder(seq, state)
    return action
```

Note: this is offline problem solving; solution executed "eyes closed."
Online problem solving involves acting without complete knowledge.

Example: Romania

On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate goal:

  be in Bucharest

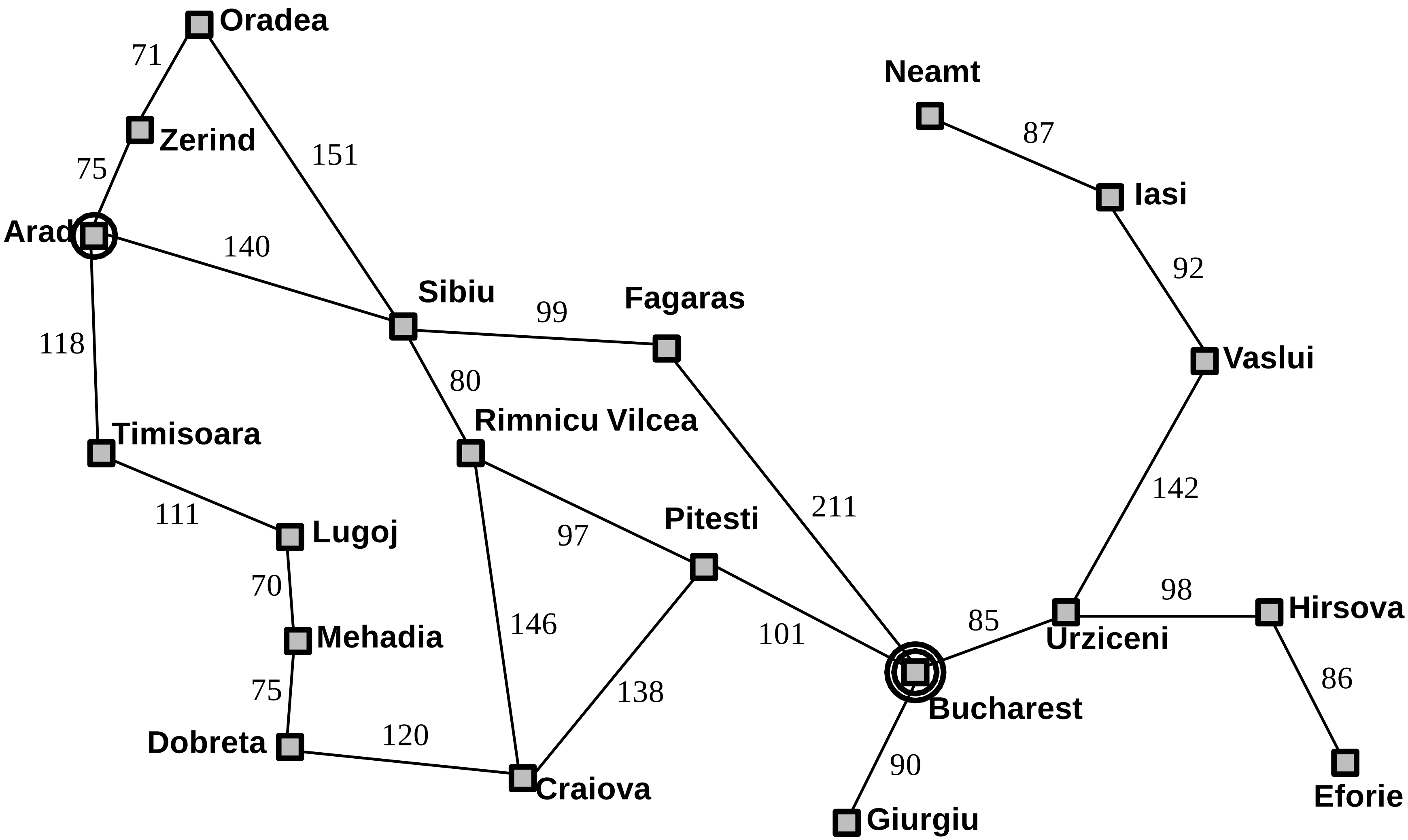
Formulate problem:

  states: various cities
  actions: drive between cities

Find solution:

  sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania



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Problem types

Deterministic, fully observable ⇒ single-state problem

Agent knows exactly which state it will be in; solution is a sequence

Non-observable ⇒ conformant problem

Agent may have no idea where it is; solution (if any) is a sequence

Nondeterministic and/or partially observable ⇒ contingency problem

percepts provide new information about current state

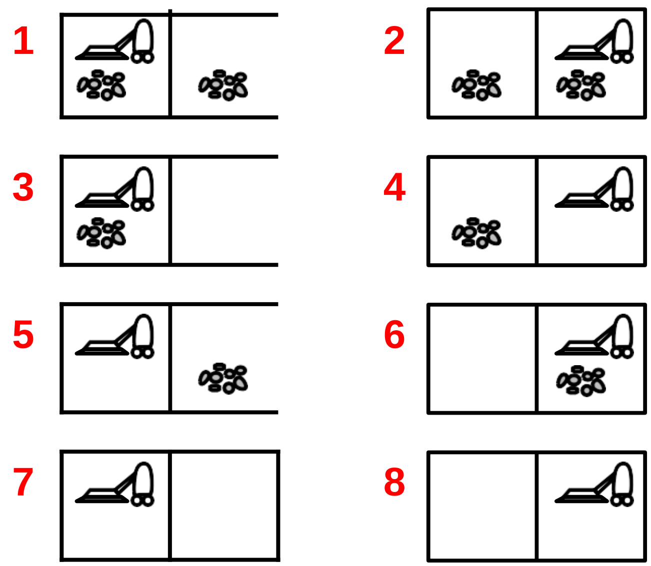
solution is a contingent plan or a policy

often interleave search, execution

Unknown state space ⇒ exploration problem ("online")

Example: vacuum world

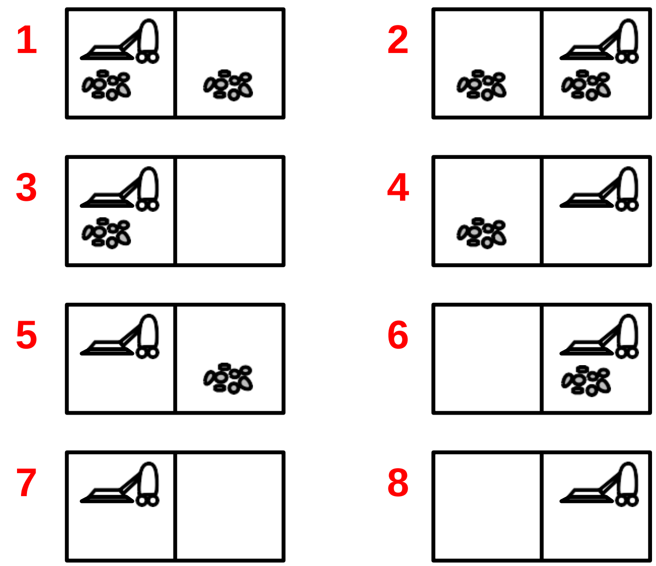
Single-state, start in #5. Solution?



Example: vacuum world

Single-state, start in #5. Solution?
[*Right*, *Suck*]

Conformant, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., *Right* goes to $\{2, 4, 6, 8\}$. Solution?



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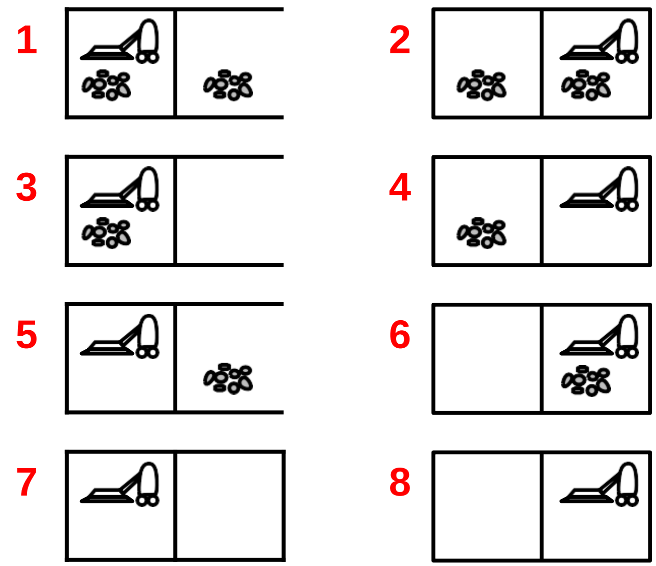
[*Right*, *Suck*, *Left*, *Suck*]

Contingency, start in #5
Murphy's Law: *Suck* can dirty a clean carpet
Local sensing: dirt, location only.
Solution?



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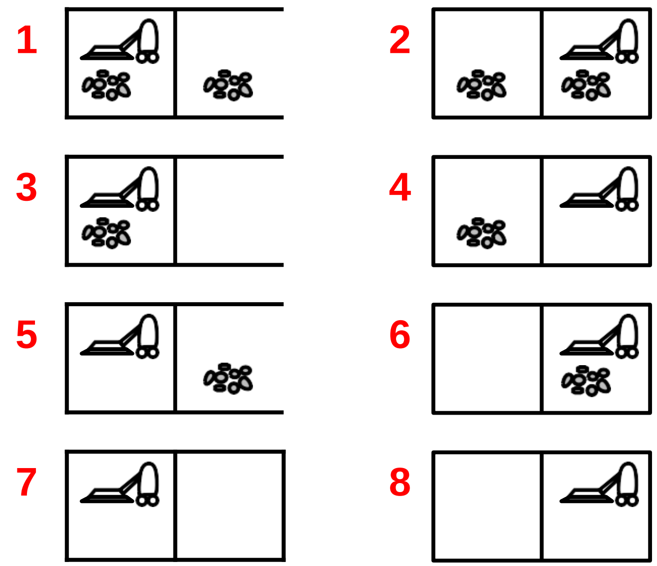
[*Right*, *Suck*, *Left*, *Suck*]

Contingency, start in #5
Murphy's Law: *Suck* can dirty a clean carpet
Local sensing: dirt, location only.
Solution?
[*Right*, if *dirt* then *Suck*]



Single-state problem formulation

A problem is defined by four items:

initial state e.g., "at Arad"

successor function $S(x)$ = set of action–state pairs

e.g., $S(Arad) = \{\langle Arad \rightarrow Zerind, Zerind\rangle, \ldots\}$

goal test, can be

explicit, e.g., x = "at Bucharest" implicit, e.g., $NoDirt(x)$

path cost (additive)

e.g., sum of distances, number of actions executed, etc.

$c(x, a, y)$ is the step cost, assumed to be ≥ 0

A solution is a sequence of actions leading from the initial state to a goal state

Selecting a state space

Real world is absurdly complex
⇒ state space must be abstracted for problem solving

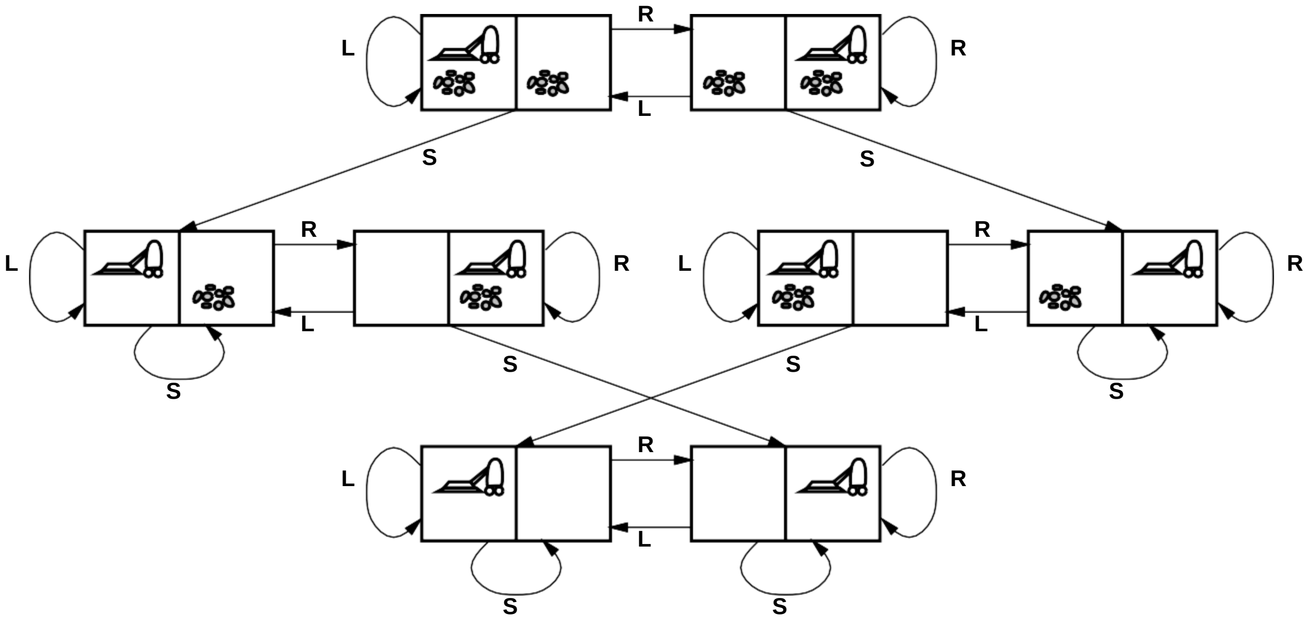
(Abstract) **state** = set of real states

(Abstract) **action** = complex combination of real actions e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc. For guaranteed realizability, any real state "in Arad" must get to some real state "in Zerind"

(Abstract) **solution** =
set of real paths that are solutions in the real world

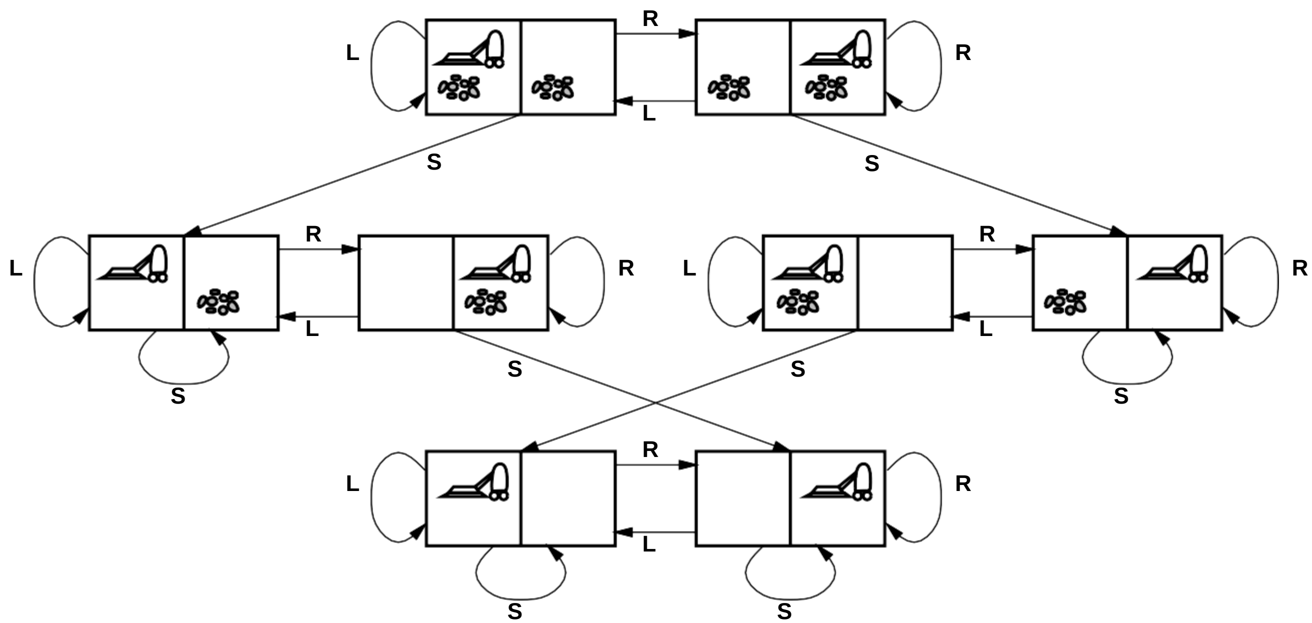
Each abstract action should be "easier" than the original problem!

Example: vacuum world state space graph



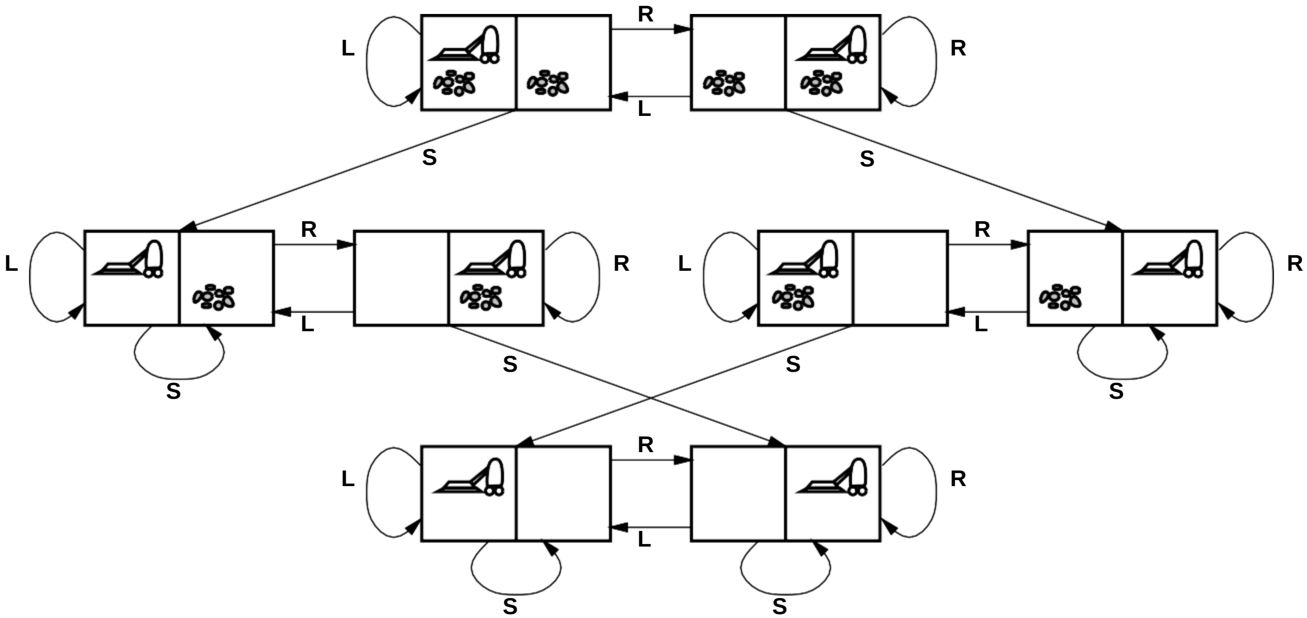
states?
actions?
goal test?
path cost?

Example: vacuum world state space graph



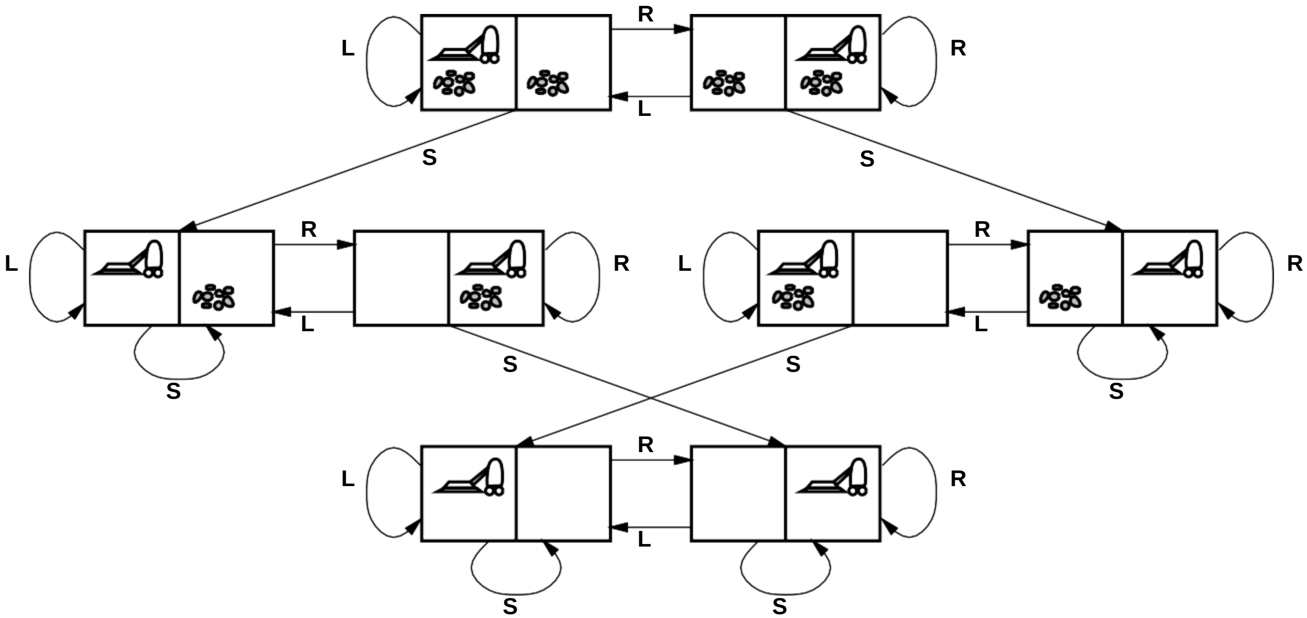
states?: integer dirt and robot locations (ignore dirt amounts etc.)
actions?
goal test?
path cost?

Example: vacuum world state space graph



states?: integer dirt and robot locations (ignore dirt amounts etc.)
actions?: *Left*, *Right*, *Suck*, *NoOp*
goal test?
path cost?

Example: vacuum world state space graph



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goal test?: no dirt
path cost?

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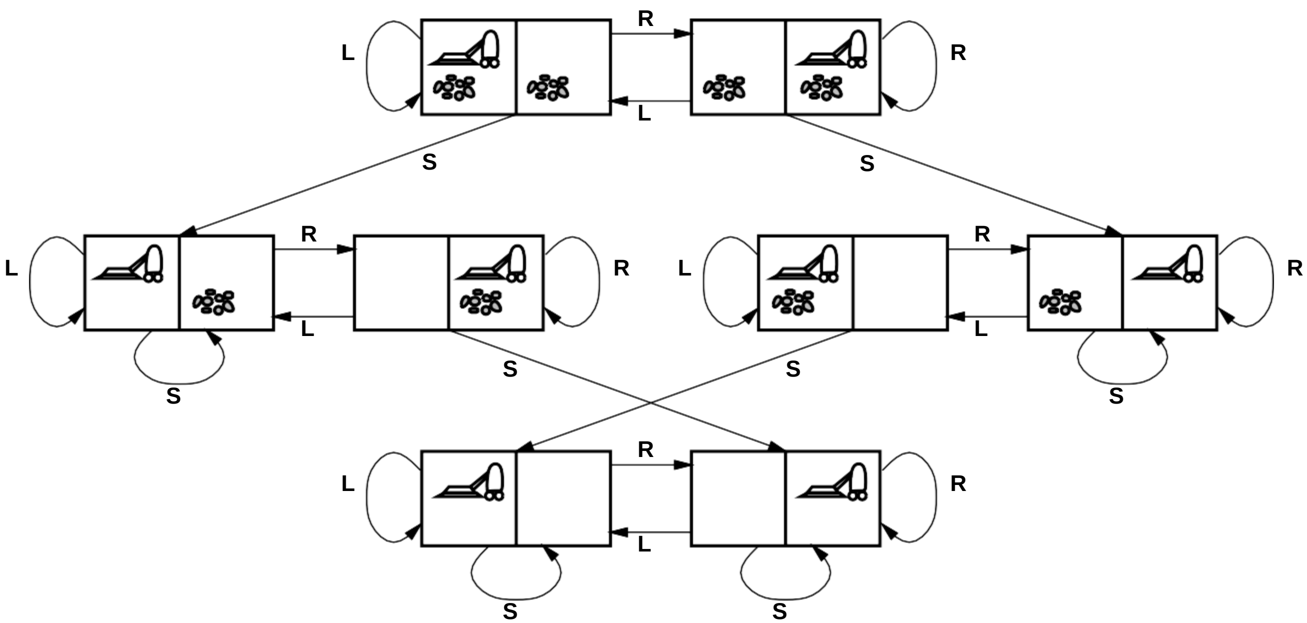

states?: integer dirt and robot locations (ignore dirt amounts etc.)
actions?: *Left*, *Right*, *Suck*, *NoOp*
goal test?: no dirt
path cost?: 1 per action (0 for *NoOp*)

Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

states?
actions?
goal test?
path cost?

Example: The 8-puzzle

7	2	4
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Start State

1	2	3
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Goal State

states?: integer locations of tiles (ignore intermediate positions)
actions?
goal test?
path cost?

Example: The 8-puzzle

7	2	4
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Start State

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Goal State

states?: integer locations of tiles (ignore intermediate positions)
actions?: move blank left, right, up, down (ignore unjamming etc.)
goal test?
path cost?

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7	2	4
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Start State

1	2	3
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Goal State

states?: integer locations of tiles (ignore intermediate positions)
actions?: move blank left, right, up, down (ignore unjamming etc.)
goal test?: = goal state (given)
path cost?

Example: The 8-puzzle

7	2	4
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Start State

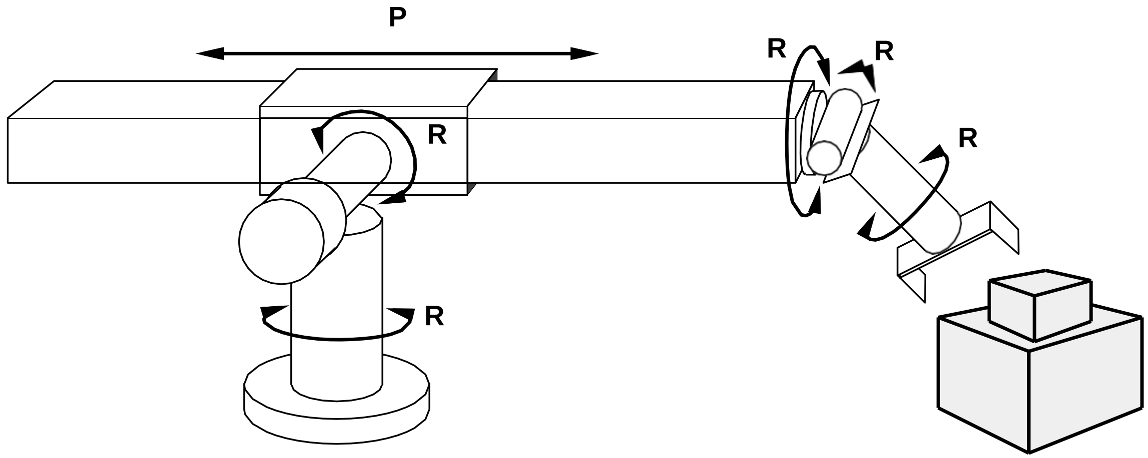
1	2	3
4	5	6
7	8	

Goal State

states?: integer locations of tiles (ignore intermediate positions)
actions?: move blank left, right, up, down (ignore unjamming etc.)
goal test?: = goal state (given)
path cost?: 1 per move

[Note: optimal solution of n-Puzzle family is NP-hard]

Example: robotic assembly



states?: real-valued coordinates of robot joint angles
parts of the object to be assembled

actions?: continuous motions of robot joints

goal test?: complete assembly with no robot included!

path cost?: time to execute

Tree search algorithms

Basic idea:

offline, simulated exploration of state space by generating successors of already-explored states (a.k.a. expanding states)

Tree search algorithms

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offline, simulated exploration of state space by generating successors of already-explored states (a.k.a. expanding states)

```
function Tree-Search(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```


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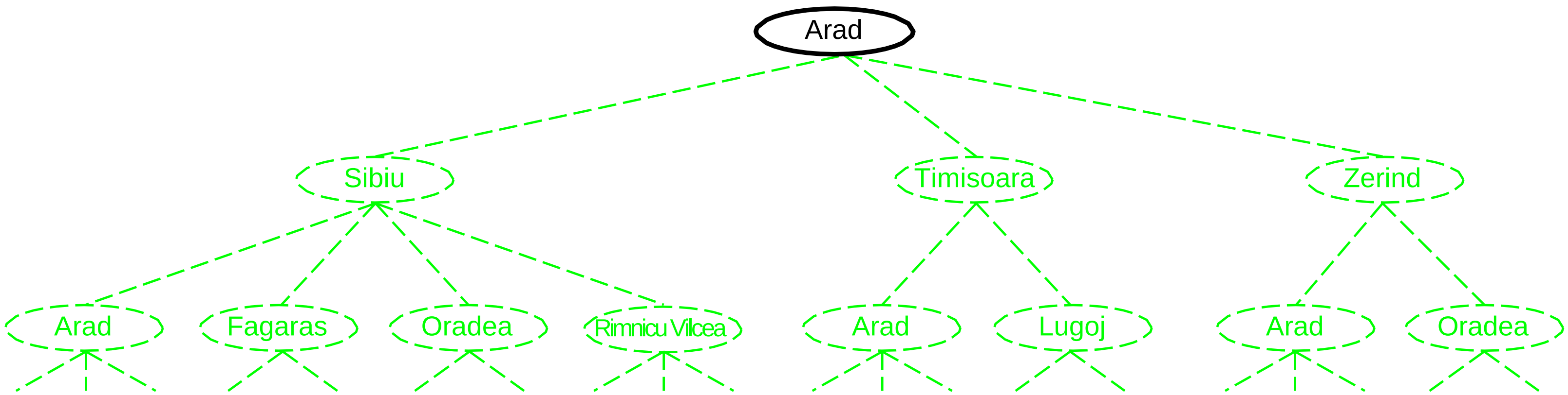
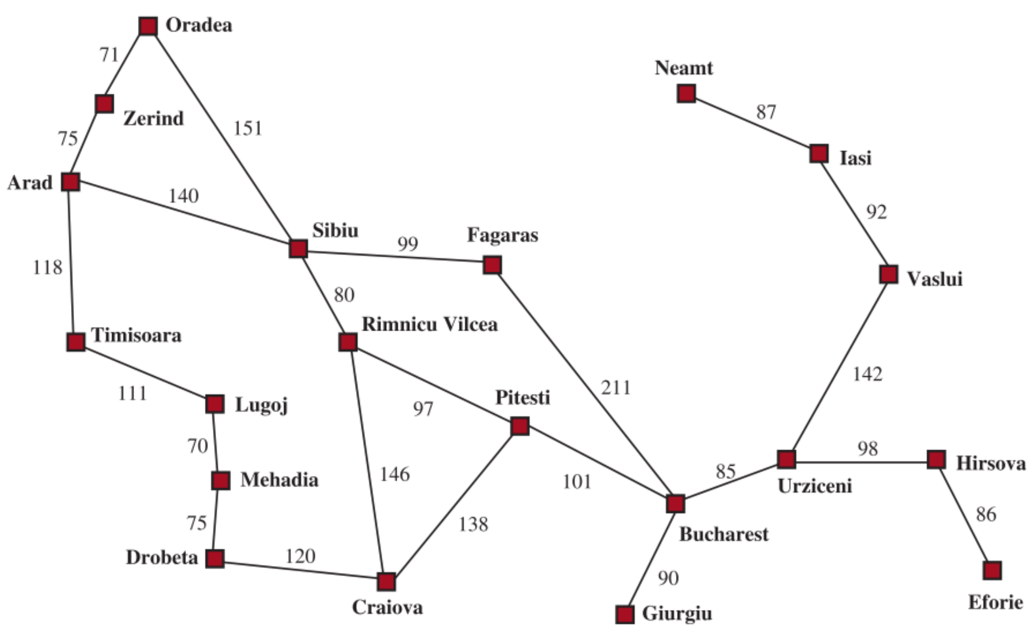
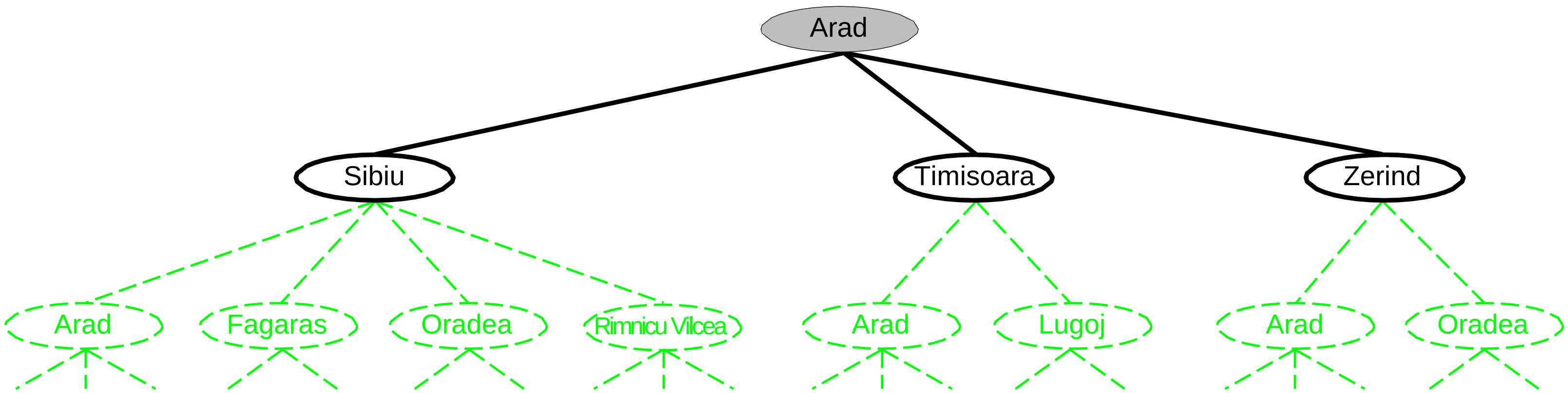
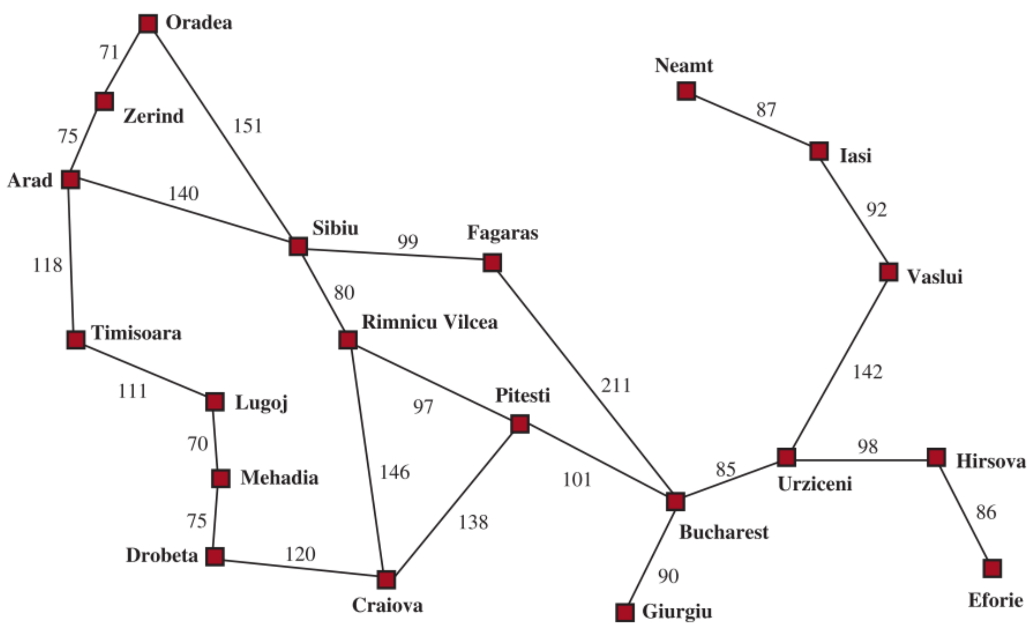
Tree search example





Tree search example





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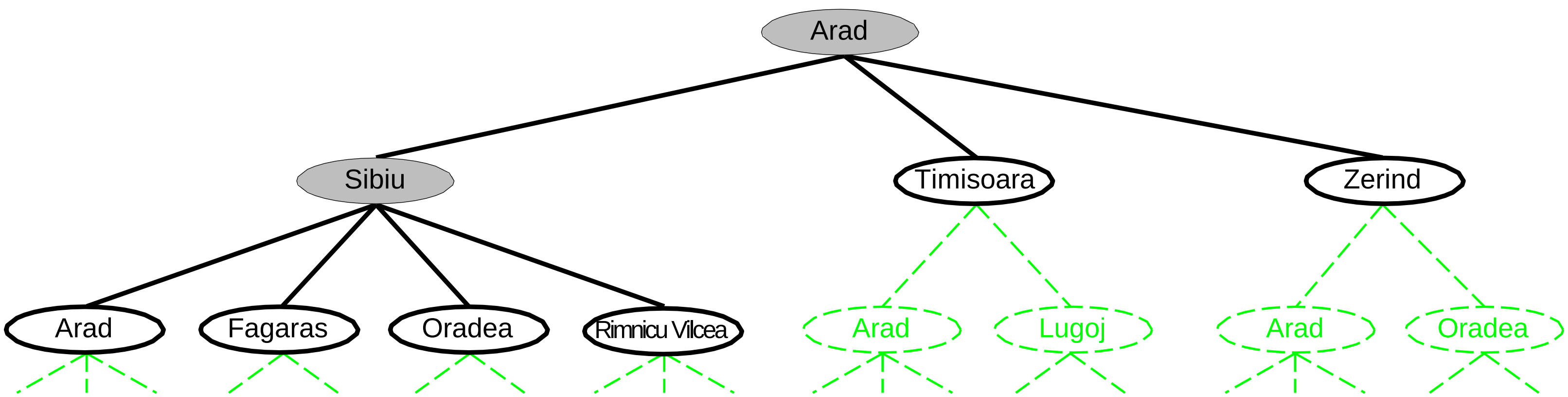
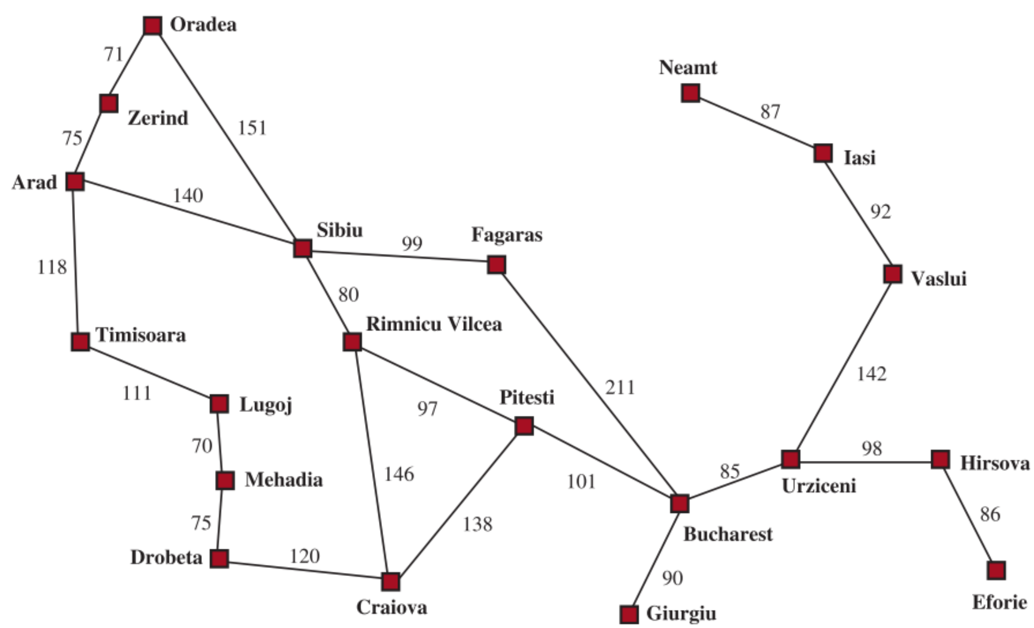
Tree search example





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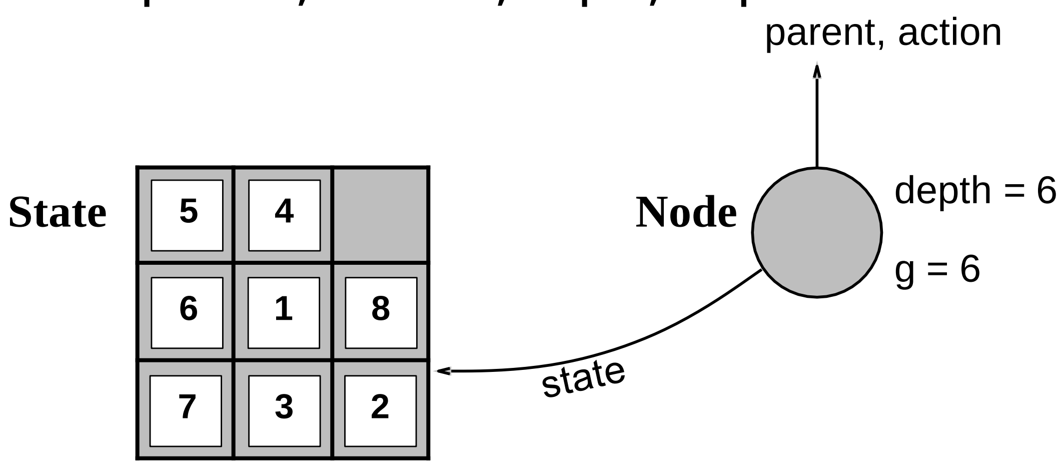
Implementation: states vs. nodes

A state is a (representation of) a physical configuration
A node is a data structure constituting part of a search tree
includes parent, children, depth, path cost $g(x)$
States do not have parents, children, depth, or path cost!



The Expand function creates new nodes, filling in the various fields and using the SuccessorFn of the problem to create the corresponding states.

Implementation: general tree search

```
function Tree-Search(problem, fringe) returns a solution, or failure
   fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← Remove-Front(fringe)
      if Goal-Test(problem, State(node)) then return node
      fringe ← InsertAll(Expand(node, problem), fringe)
```

```
function Expand(node, problem) returns a set of nodes
   successors ← the empty set
   for each action, result in Successor-Fn(problem, State[node]) do
      s ← a new Node
      Parent-Node[s] ← node;  Action[s] ← action;  State[s] ← result
      Path-Cost[s] ← Path-Cost[node] + Step-Cost(node, action, s)
      Depth[s] ← Depth[node] + 1
      add s to successors
   return successors
```

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Search strategies

A strategy is defined by picking the order of node expansion
Strategies are evaluated along the following dimensions:

- completeness — does it always find a solution if one exists?
- time complexity — number of nodes generated/expanded
- space complexity — maximum number of nodes in memory
- Optimality — does it always find a least-cost solution?

Time and space complexity are measured in terms of:

b — maximum branching factor of the search tree

d — depth of the least-cost solution

m — maximum depth of the state space (may be ∞)

Uninformed search strategies

Uninformed strategies use only the information available in the problem definition:

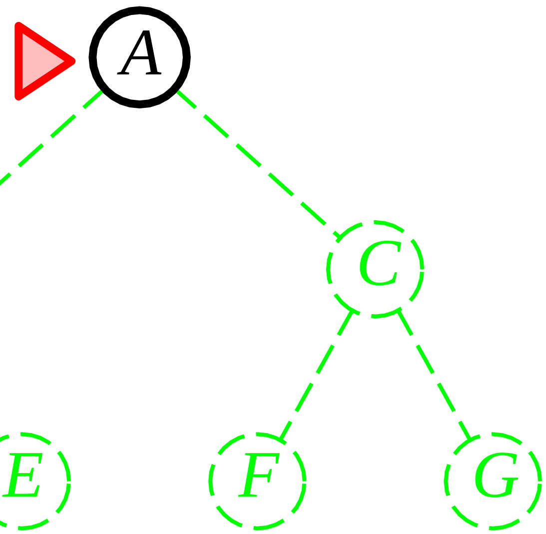
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end



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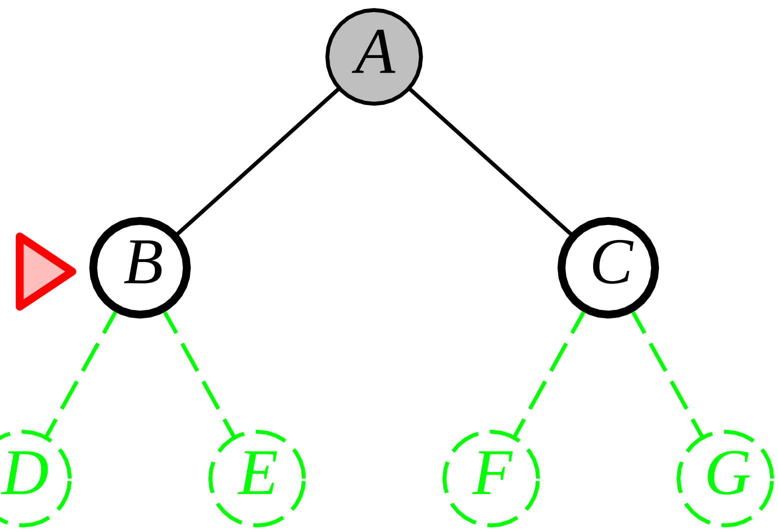

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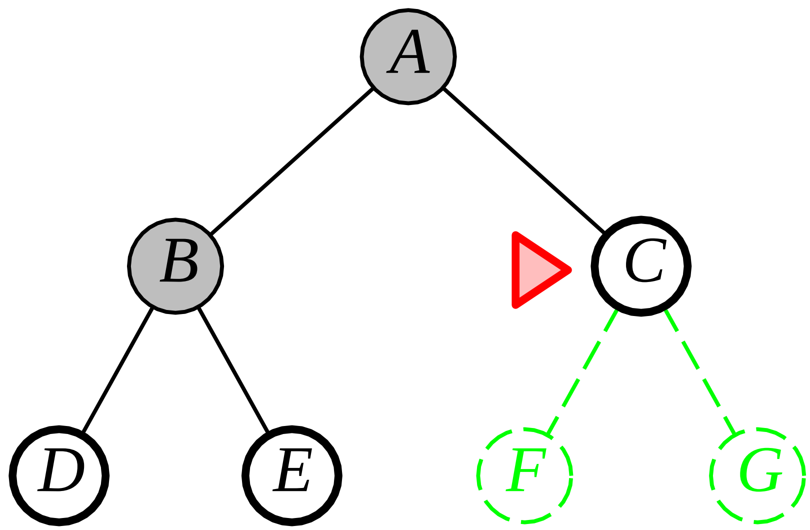

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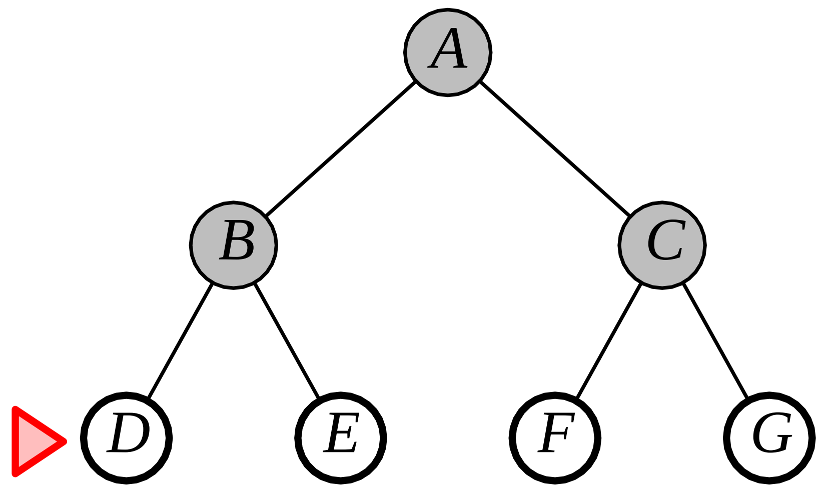


Properties of breadth-first search

Complete?

Properties of breadth-first search

Complete? Yes (if b is finite)

Time?

Properties of breadth-first search

Complete? Yes (if b is finite)

Time? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?

Properties of breadth-first search

Complete? Yes (if b is finite)

Time? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space? $O(b^{d+1})$ (keeps every node in memory)

Optimal?

Properties of breadth-first search

Complete? Yes (if b is finite)

Time? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space? $O(b^{d+1})$ (keeps every node in memory)

Optimal? Yes (if cost = 1 per step); not optimal in general

Space is the big problem; can easily generate nodes at 100MB/sec
so 24hrs = 8640GB.

Uniform-cost search

Expand least-cost unexpanded node. called **Dijkstra's algorithm** by the theoretical computer science community, and uniform-cost search by the AI community

Implementation:

fringe = queue ordered by path cost, lowest first

Equivalent to breadth-first if step costs all equal

Complete? Yes, if step cost $\geq \epsilon$ (min step cost, >0)

Time? # of nodes with $g \leq C^*$, $O(b^{1+C^*/\epsilon})$
where C^* is the cost of the optimal solution

Space? # of nodes with $g \leq C^*$, $O(b^{1+C^*/\epsilon})$

Optimal? Yes—nodes expanded in increasing order of $g(n)$

can be much greater than b^d

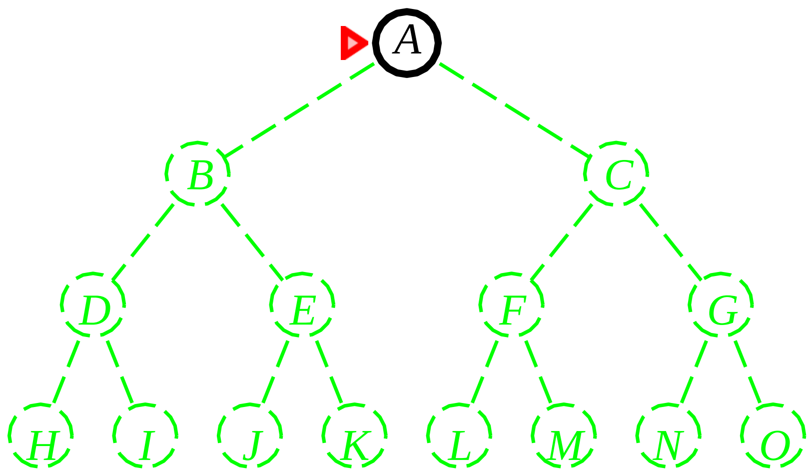
can explore large trees of actions with low costs before exploring paths involving a high-cost and perhaps useful action

Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front



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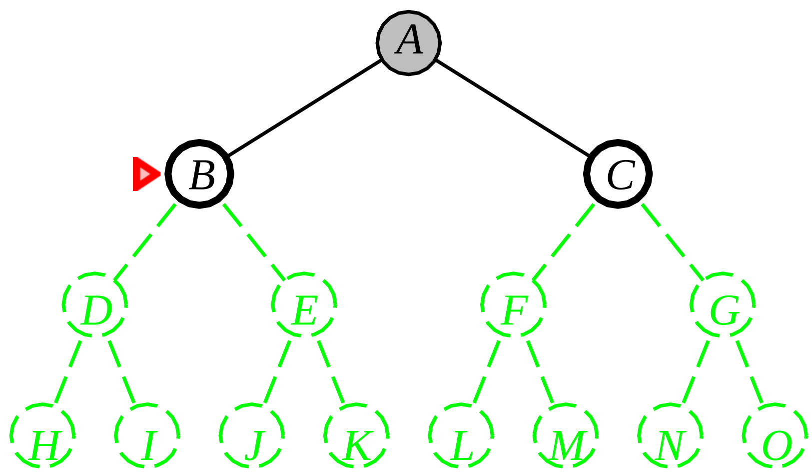

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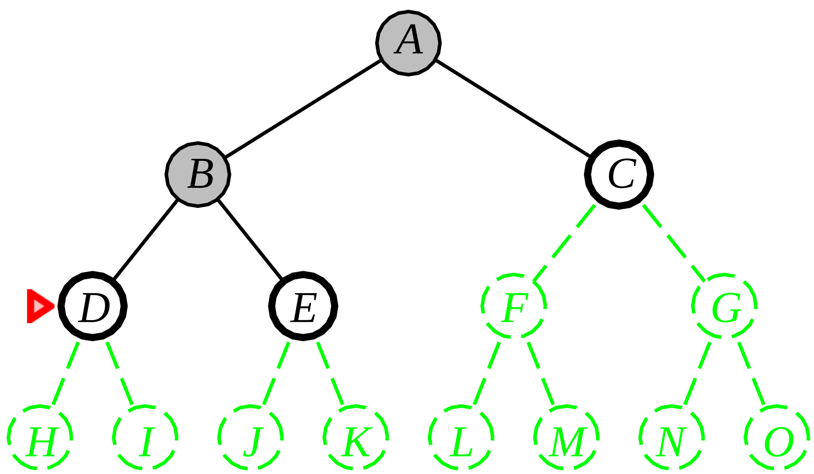

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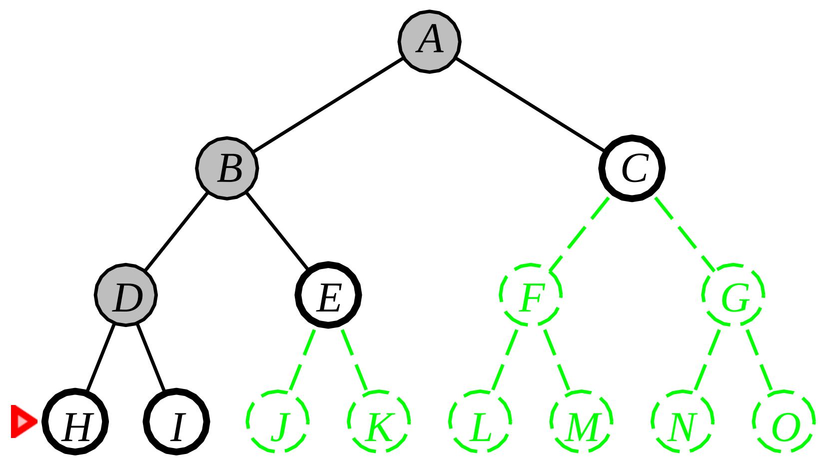

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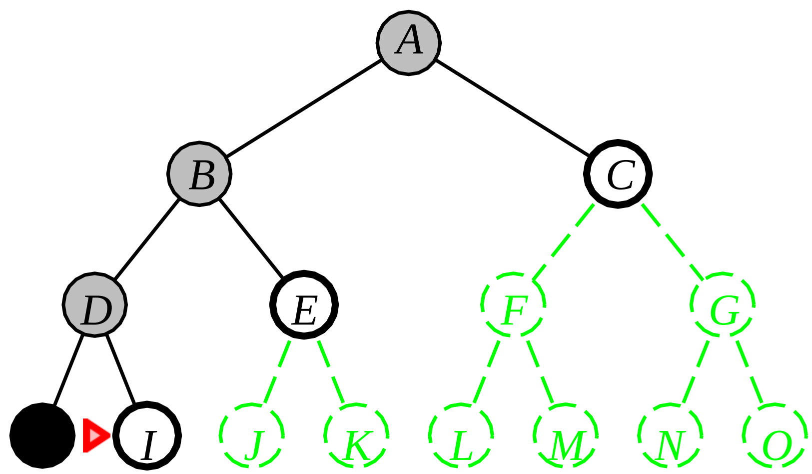

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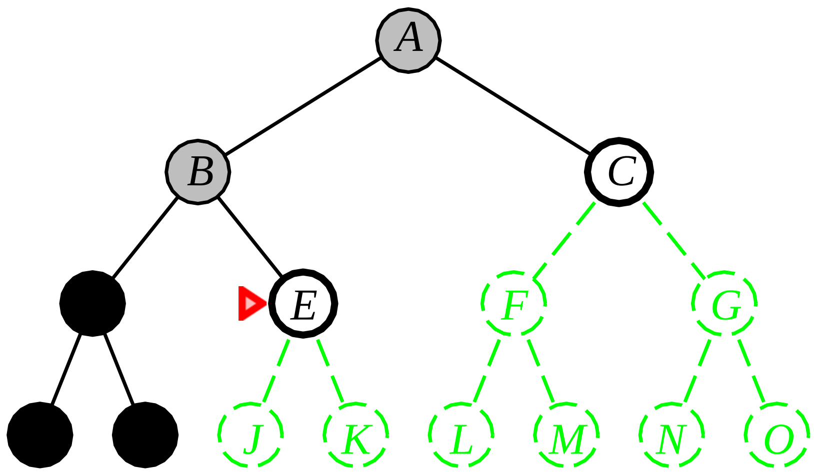

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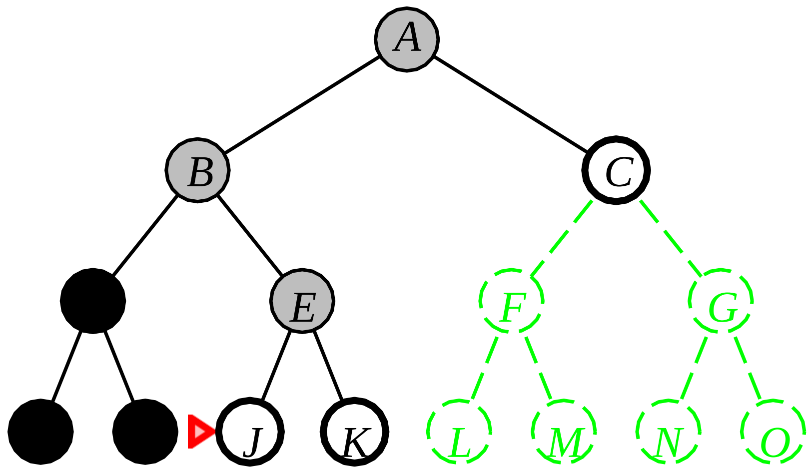

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Expand deepest unexpanded node

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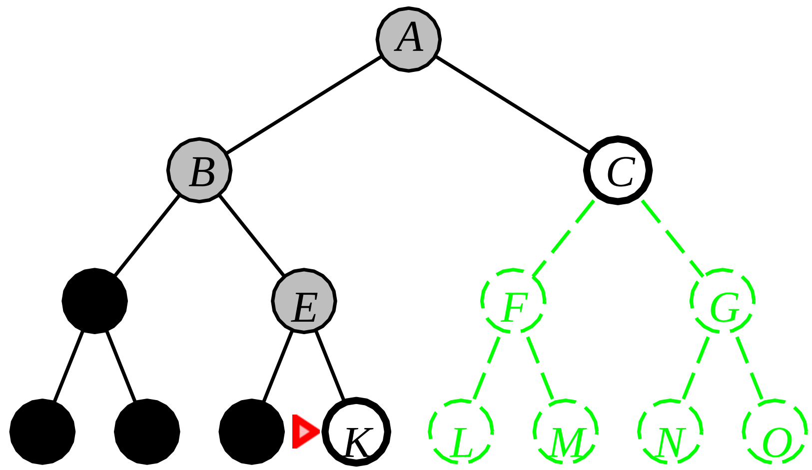

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Expand deepest unexpanded node

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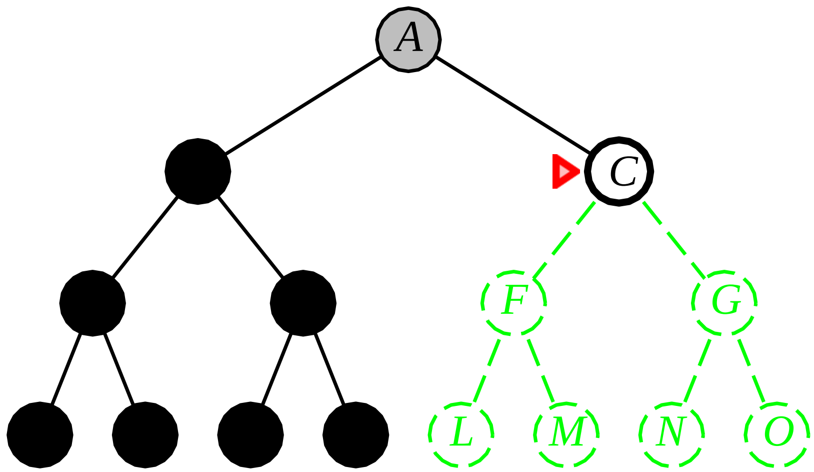

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Expand deepest unexpanded node

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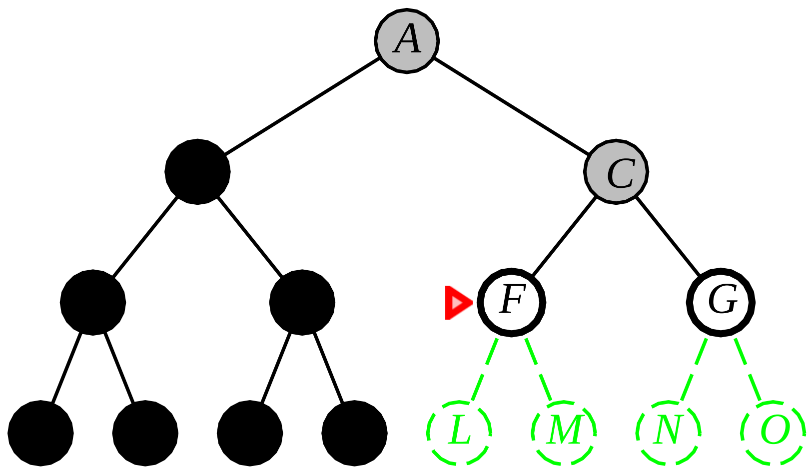

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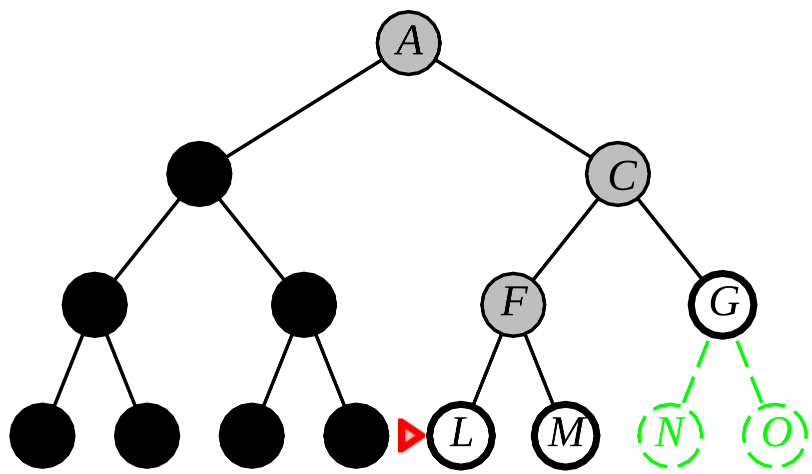

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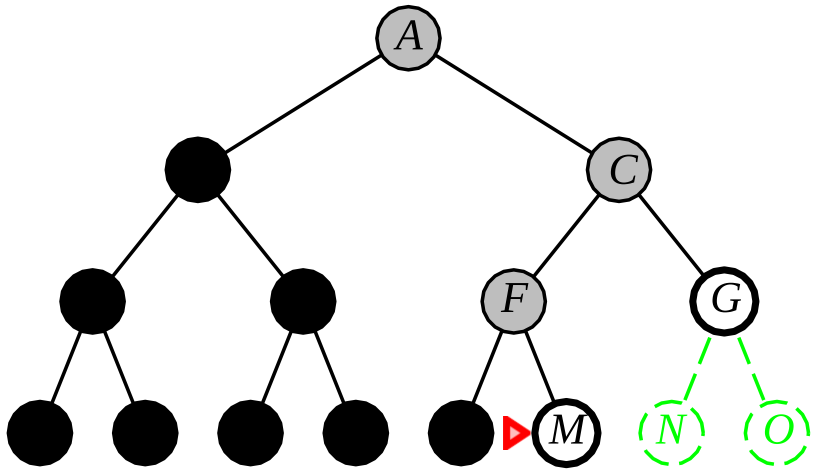


Properties of depth-first search

Complete?

Properties of depth-first search

Complete? No: fails in infinite-depth spaces, spaces with loops

Modify to avoid repeated states along path

$\Rightarrow$ complete in finite spaces

Time?

Properties of depth-first search

Complete? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first
(«m» is the maximum depth of the tree)

Space?

Properties of depth-first search

Complete? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
⇒ complete in finite spaces

Time? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space? $O(bm)$, i.e., linear space!

Optimal?

Properties of depth-first search

Complete? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space? $O(bm)$, i.e., linear space!

Optimal? No

Depth-limited search

= depth-first search with depth limit *l*,
i.e., nodes at depth *l* have no successors

Choosing the right l is key!

Recursive implementation:

```
function Depth-Limited-Search(problem,limit) returns soln/fail/cutoff
    Recursive-DLS(Make-Node(Initial-State[problem]),problem,limit)

function Recursive-DLS(node,problem,limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if Goal-Test(problem,State[node]) then return node
   else if Depth[node] = limit then return cutoff
   else for each successor in Expand(node,problem) do
        result ← Recursive-DLS(successor,problem,limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result /= failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

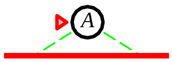
Iterative deepening search

```
function Iterative-Deepening-Search(problem) returns a solution
    inputs: problem, a problem

    for depth ← 0 to ∞  do
        result ← Depth-Limited-Search(problem, depth)
        if result /= cutoff then return result
    end
```

Iterative deepening search $l = 0$

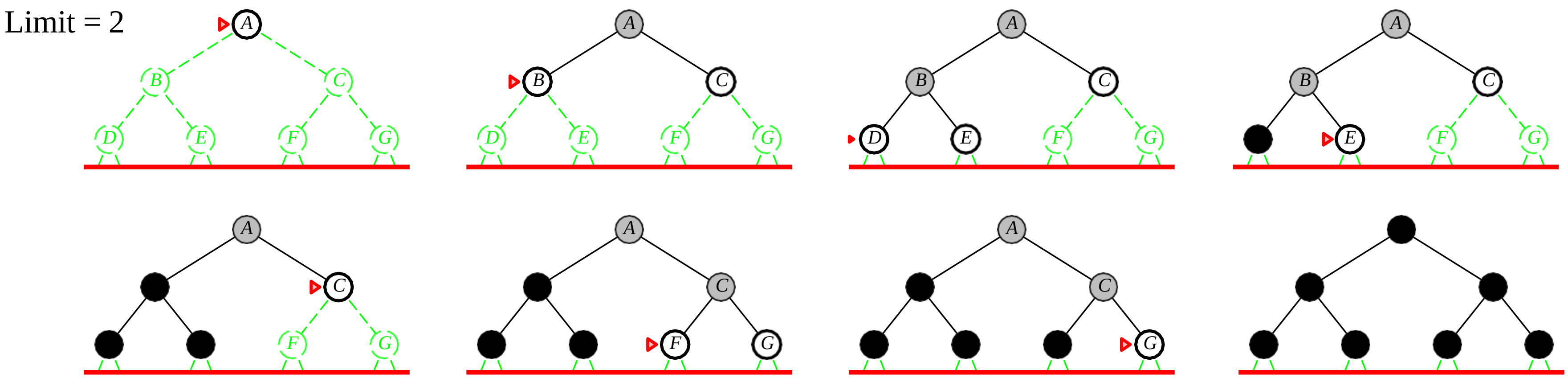
Limit = 0



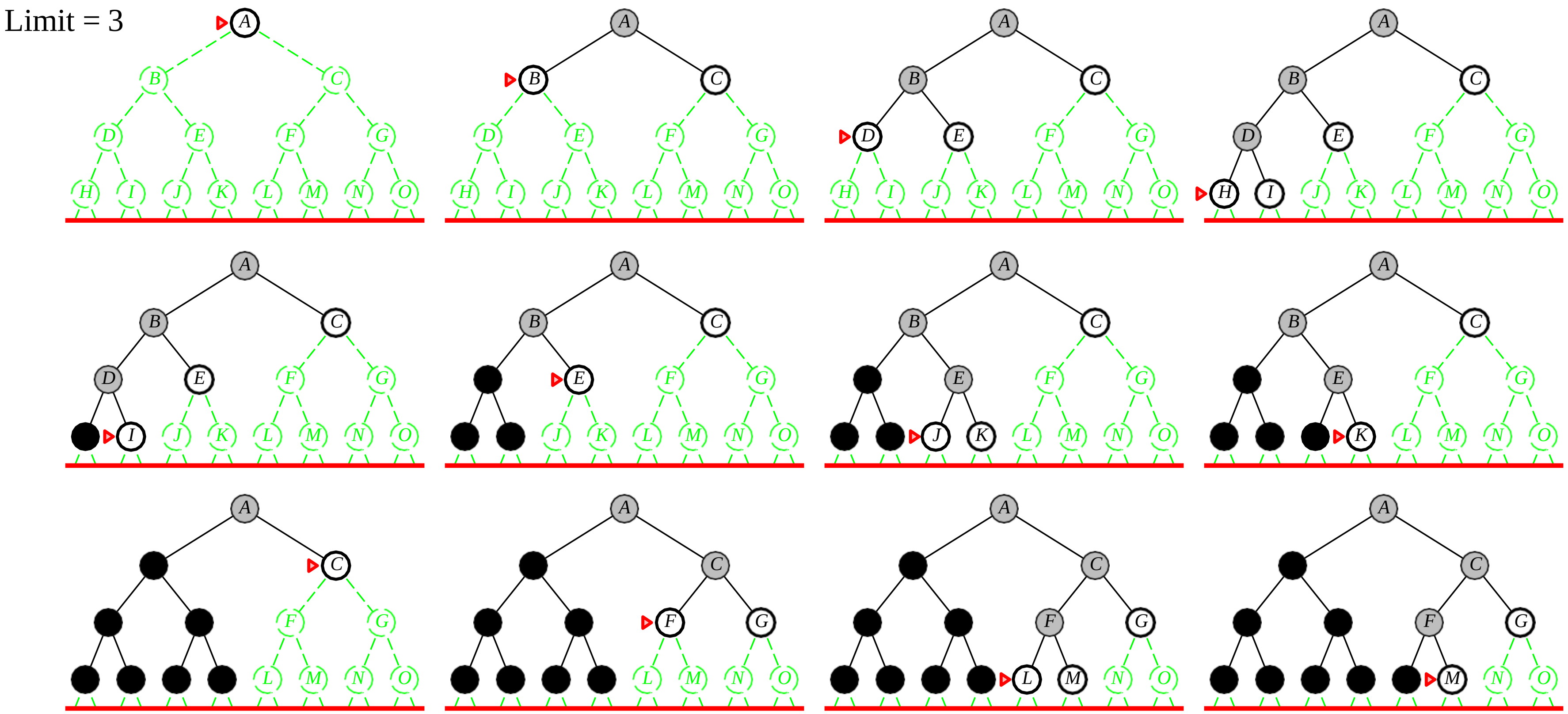
Iterative deepening search $l = 1$

Limit = 1

Iterative deepening search $l = 2$



Iterative deepening search $l = 3$



Properties of iterative deepening search

Complete?

Properties of iterative deepening search

Complete? Yes

Time?

Properties of iterative deepening search

Complete? Yes

Time? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?

Properties of iterative deepening search

Complete? Yes

Time? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space? $O(bd)$

Optimal?

Properties of iterative deepening search

Complete? Yes

Time? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space? $O(bd)$

Optimal? Yes, if step cost = 1
Can be modified to explore uniform-cost tree

Numerical comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$
$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

IDS does better because other nodes at depth d are not expanded

BFS can be modified to apply goal test when a node is generated

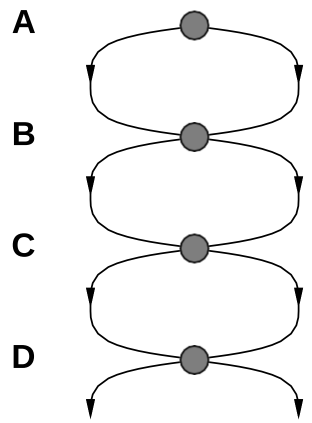
Summary of Uninformed Search algorithms

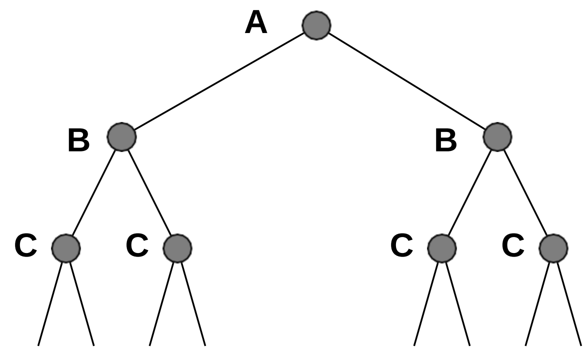
Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes[1]	Yes[1,2]	No	No	Yes[1]	Yes[1,4]
Optimal cost?	Yes[3]	Yes	No	No	Yes[3]	Yes[3,4]
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$

Figure 3.15 Evaluation of search algorithms. b is the branching factor; m is the maximum depth of the search tree; d is the depth of the shallowest solution, or is m when there is no solution; ℓ is the depth limit. Superscript caveats are as follows: [1] complete if b is finite, and the state space either has a solution or is finite. [2] complete if all action costs are $\geq \epsilon > 0$; [3] cost-optimal if action costs are all identical; [4] if both directions are breadth-first or uniform-cost.

Repeated states

Failure to detect repeated states can turn a linear problem into an exponential one!





Graph search

```
function Graph-Search(problem, fringe) returns a solution, or failure
    closed ← an empty set of explored states
    fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test(problem, State[node]) then return node
        if State[node] is not in closed then
            add State[node] to closed
            fringe ← InsertAll(Expand(node, problem), fringe)
    end
```

Review: Tree search

```
function Tree-Search(problem, fringe) returns a solution, or failure
    fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test[problem] applied to State(node) succeeds return node
        fringe ← InsertAll(Expand(node, problem), fringe)
```

A strategy is defined by picking the order of node expansion

Informed (Heuristic) Search Strategies

Idea: use an evaluation function for each node – estimate of "**desirability**"

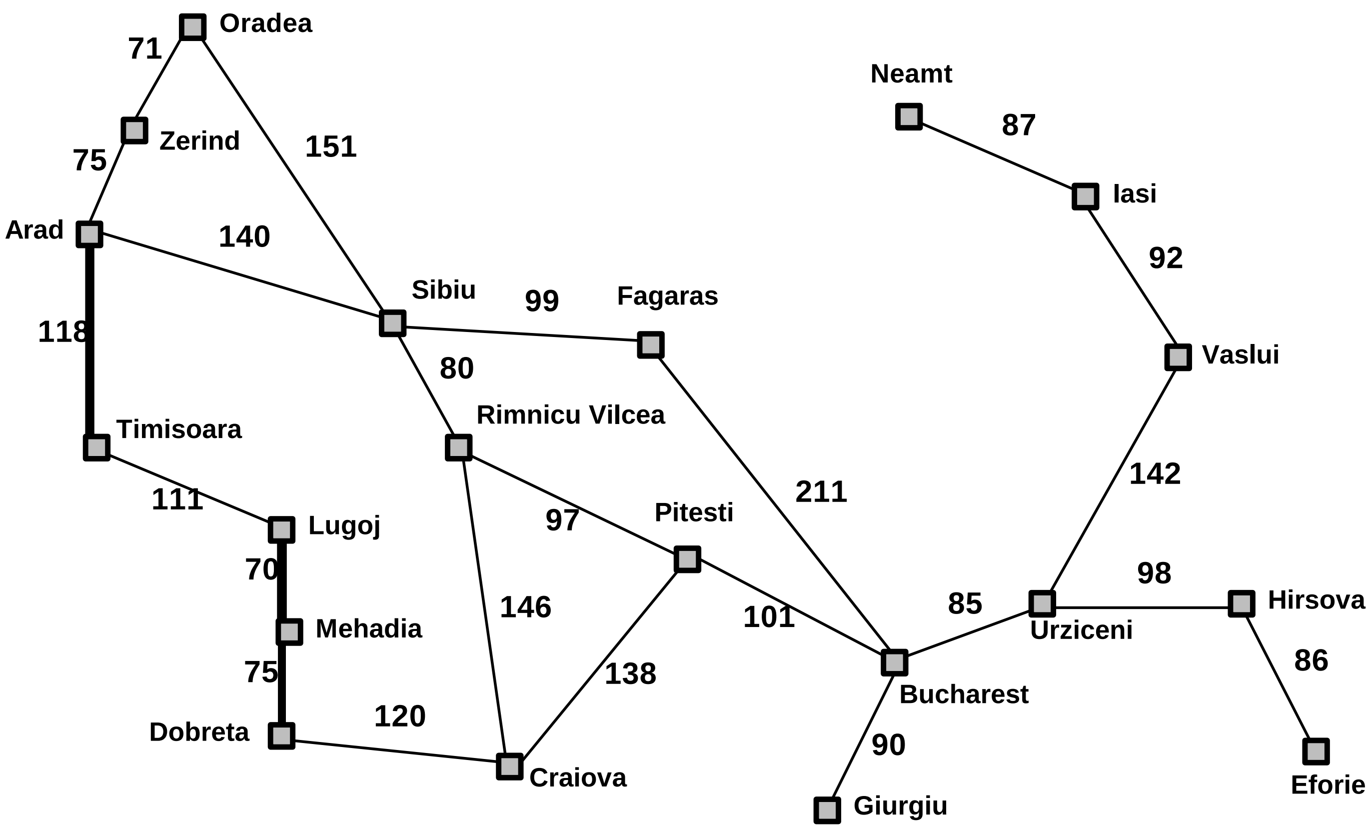
⇒ Expand most desirable unexpanded node

Implementation: *fringe* is a queue sorted in decreasing order of desirability

«Best-first» search:

- Greedy search
- A* search

Romania with step costs in km



Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

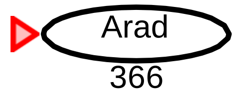
Greedy search

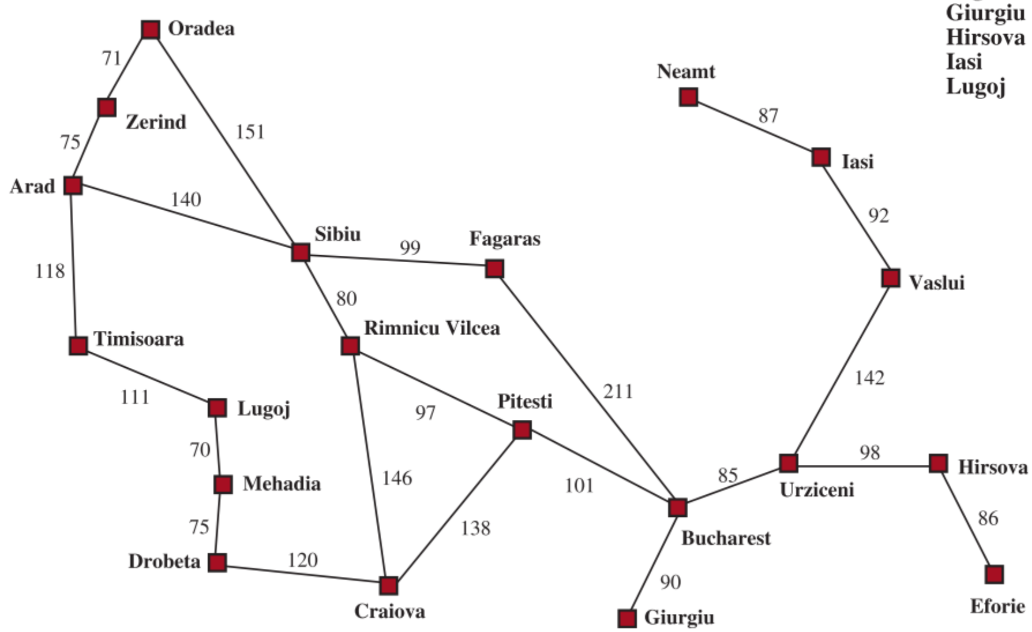
Evaluation function $h(n)$ (heuristic)
= estimate of cost from n to the closest goal

E.g., $h_{\mathrm{SLD}}(n)$ = straight-line distance from n to Bucharest

Greedy search expands the node that appears to be closest to goal

Greedy search example





Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
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Drobeta	242	**Pitesti**	100
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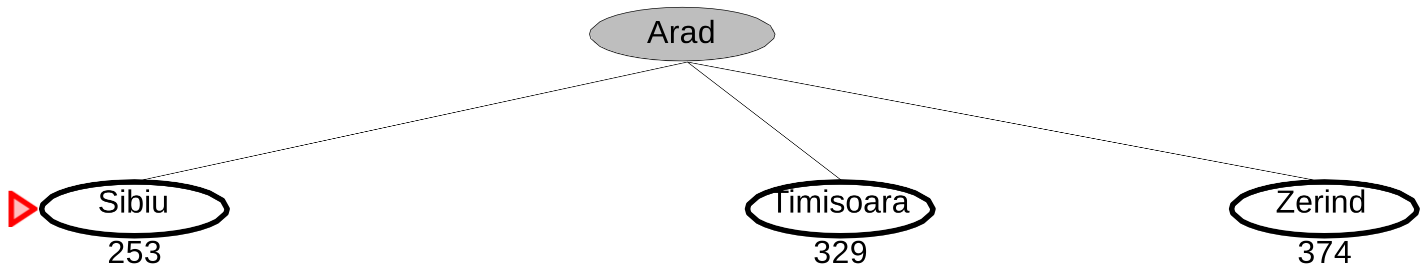
Greedy search example



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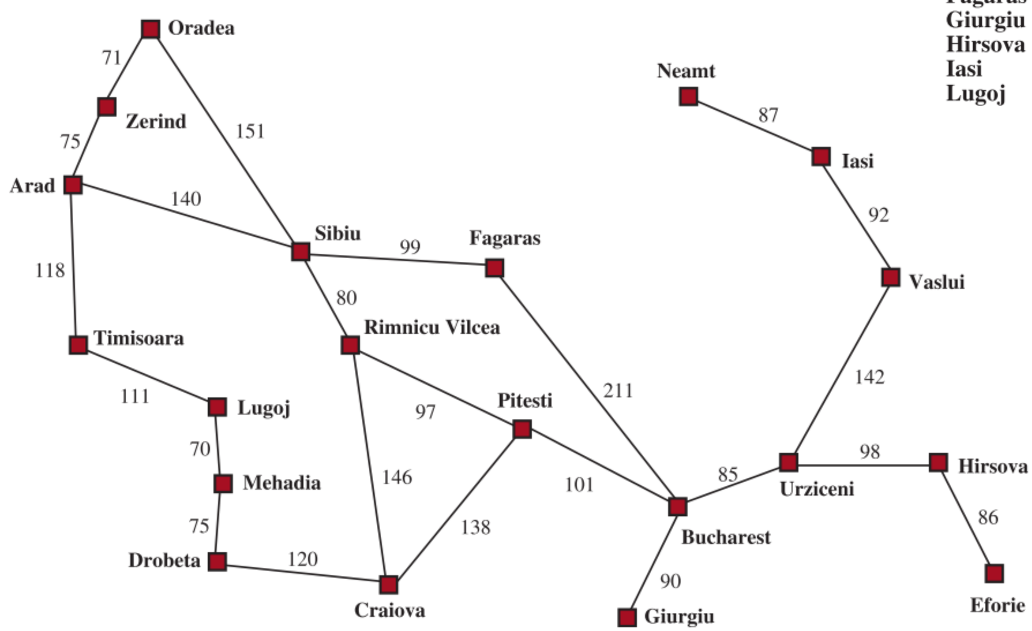
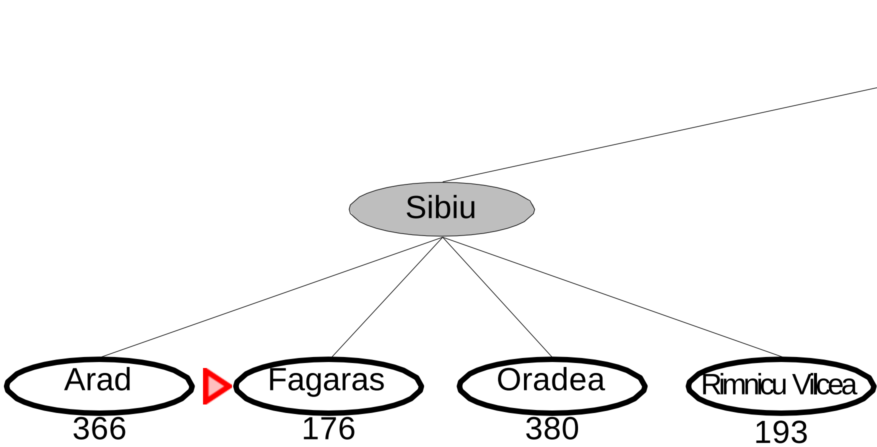


Pearson

Greedy search example



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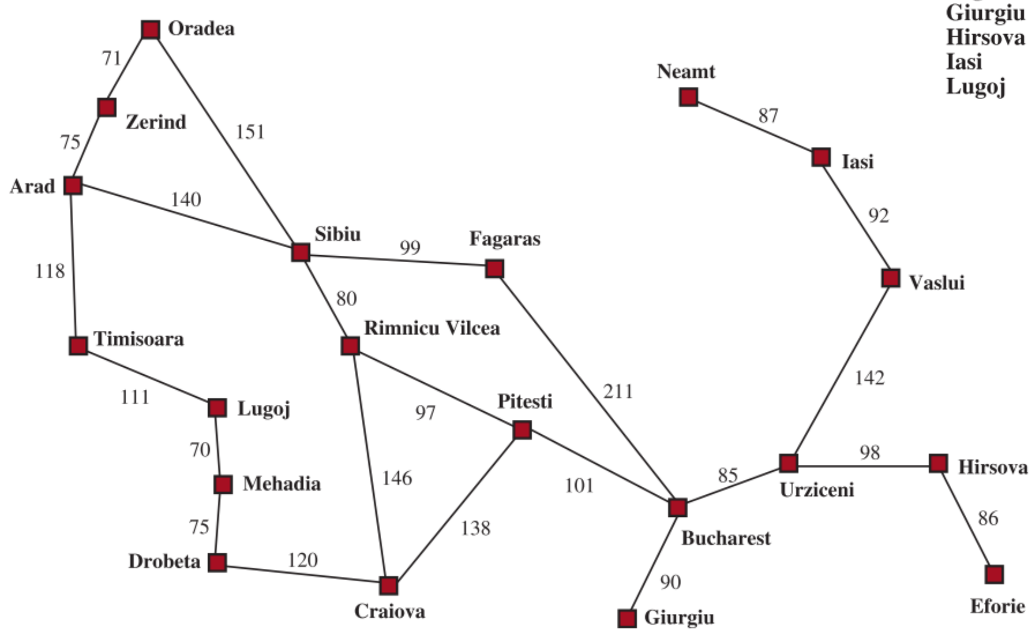
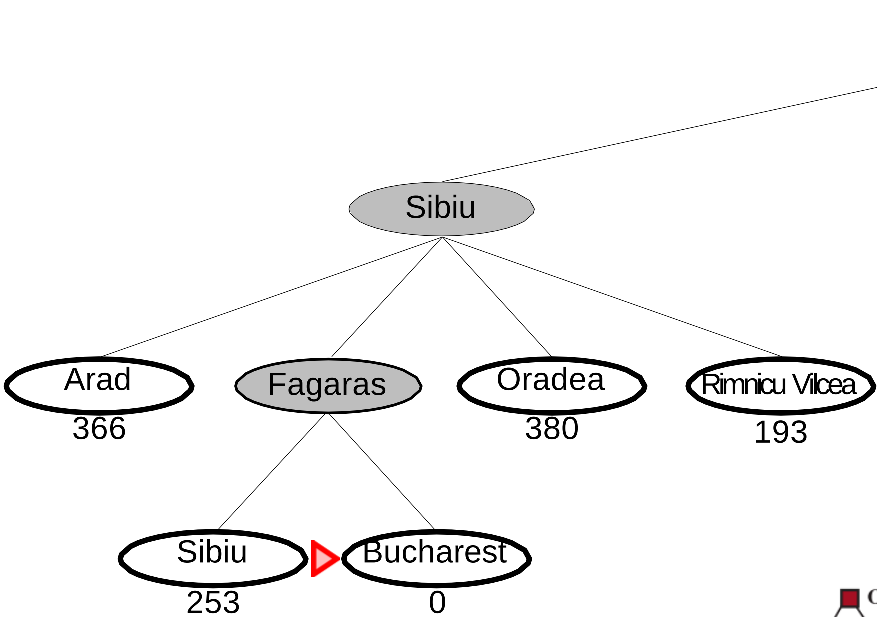


Pearson

Greedy search example



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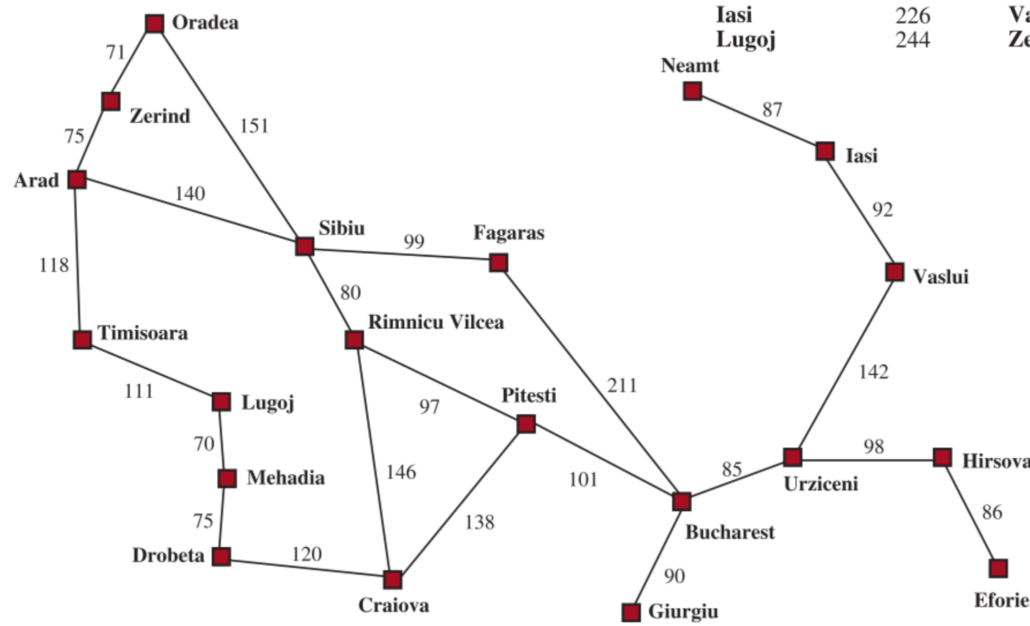


Properties of greedy search

Complete?

Properties of greedy search

Complete? No–can get stuck in loops, e.g., with Oradea as goal,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time?

Properties of greedy search

Complete? No–can get stuck in loops, e.g.,

Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?

Properties of greedy search

Complete? No–can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

Time? $O(b^m)$, but a good heuristic can give dramatic improvement

Space? $O(b^m)$—keeps all nodes in memory

Optimal?

Properties of greedy search

Complete? No–can get stuck in loops, e.g., Iasi →
Neamt → Iasi → Neamt →
Complete in finite space with repeated-state checking

Time? $O(b^m)$, but a good heuristic can give dramatic improvement

Space? $O(b^m)$—keeps all nodes in memory

Optimal? No

A^* search

Idea: avoid expanding paths that are **already expensive**

Evaluation function $f(n) = g(n) + h(n)$

$g(n)$ = cost so far to reach n

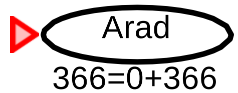
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A^* search uses an admissible heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n. (Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Theorem: A^* search is optimal

A* search example



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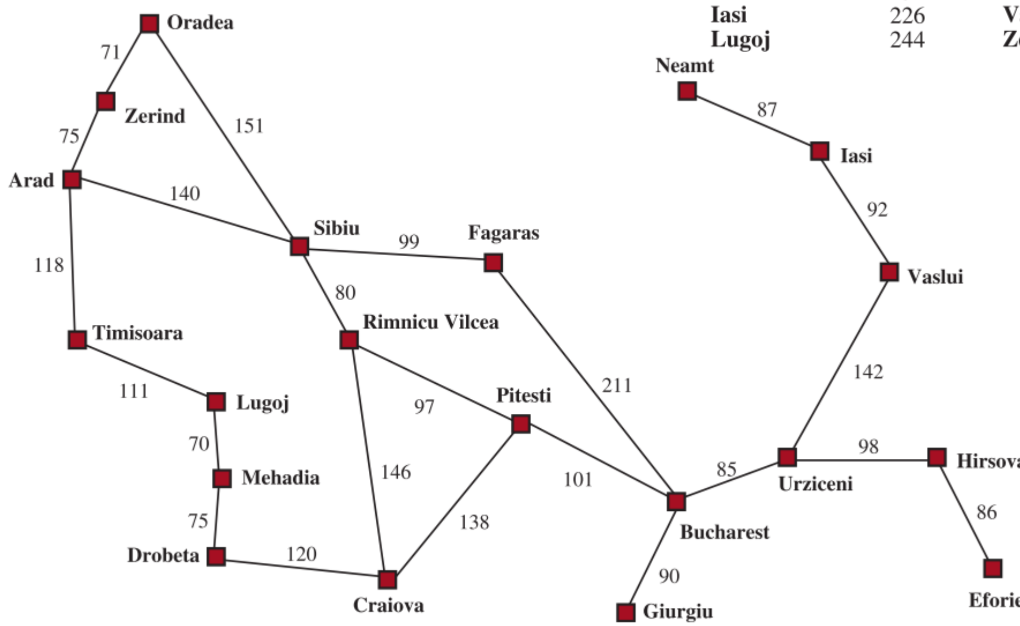
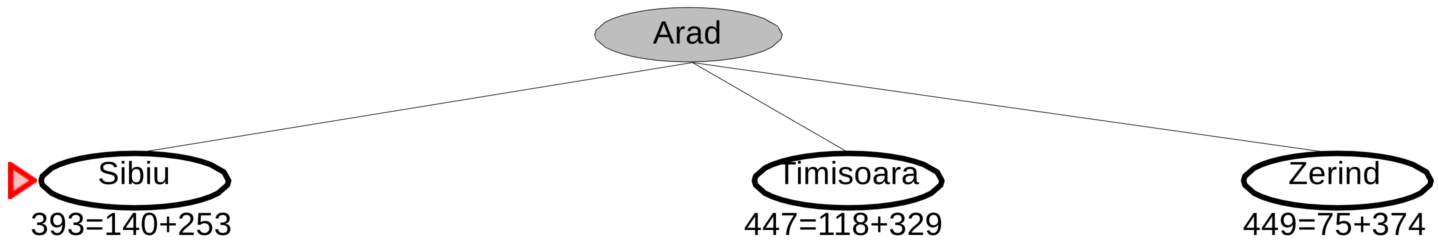


A* search example



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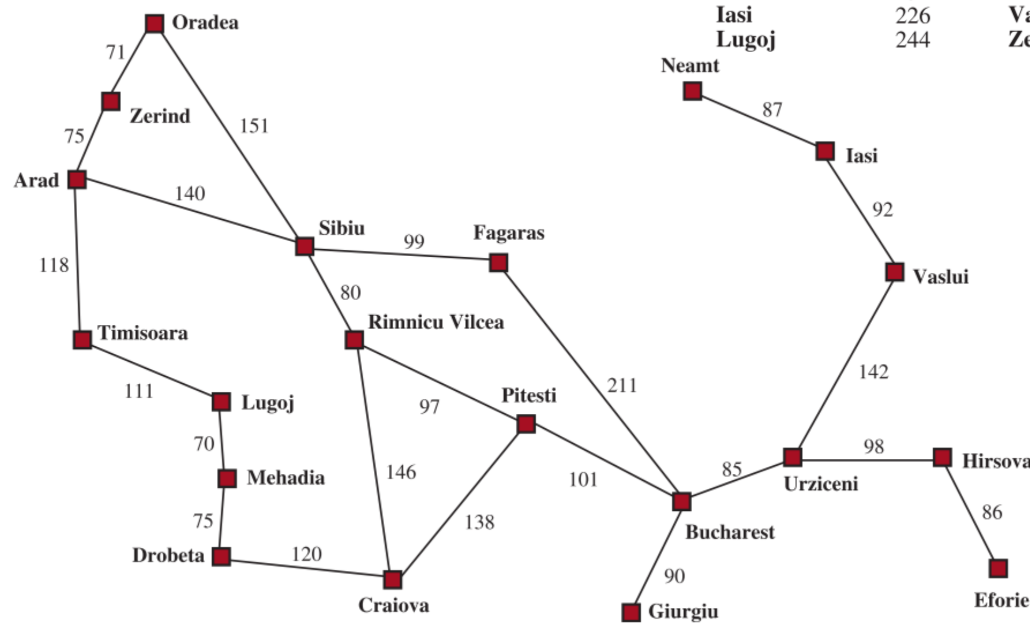
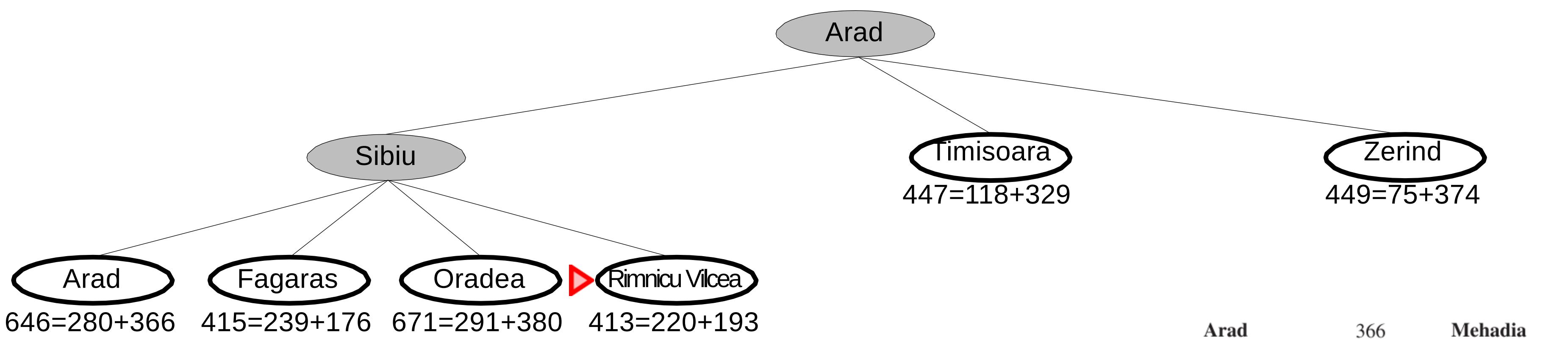


A* search example



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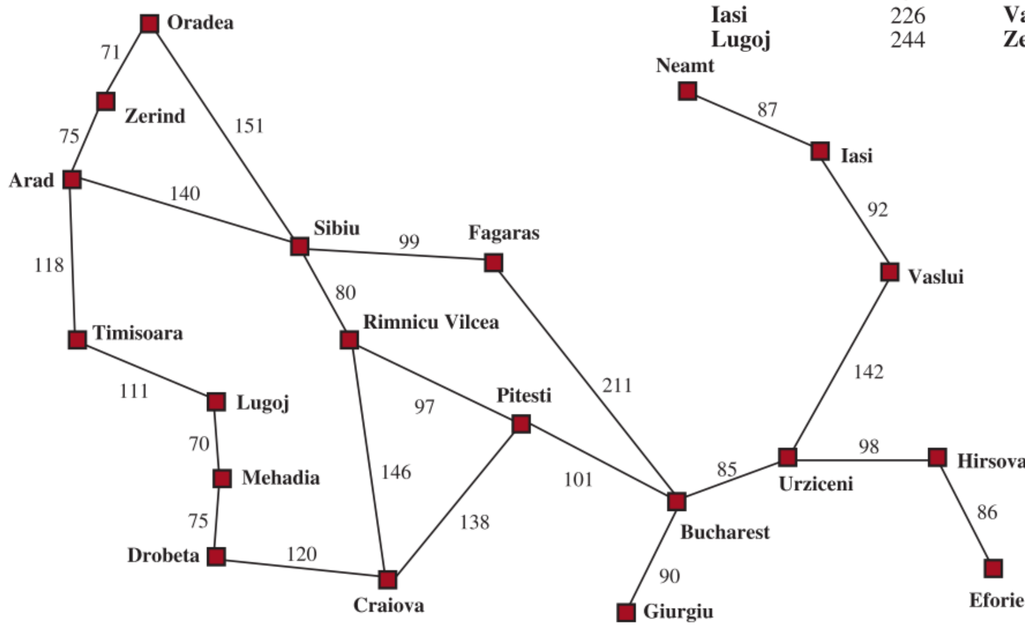
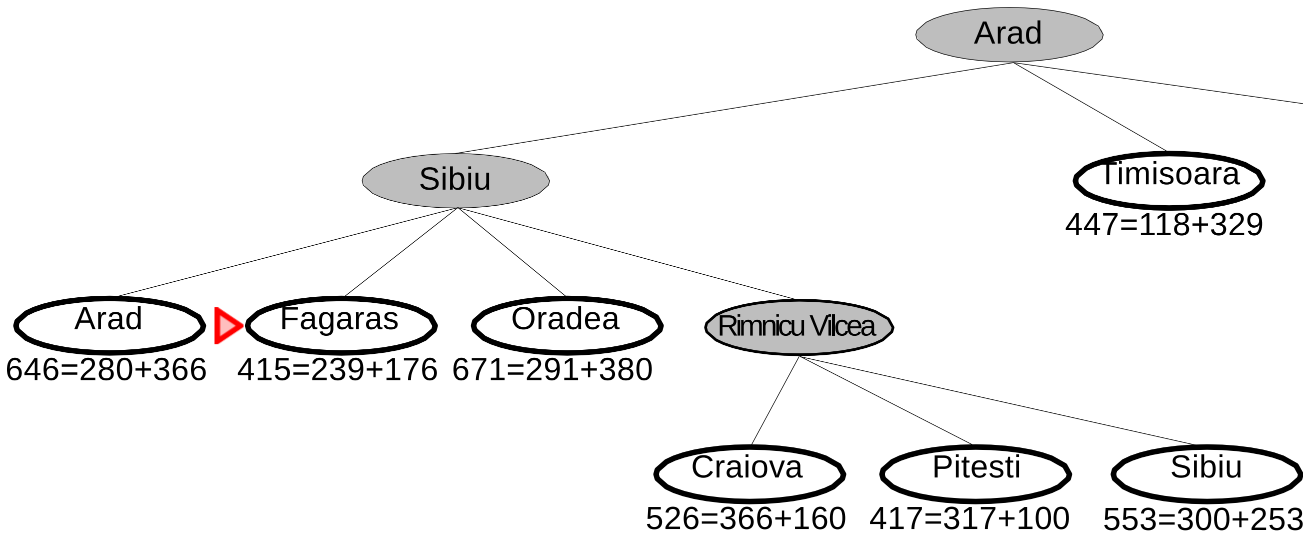


Pearson

A* search example



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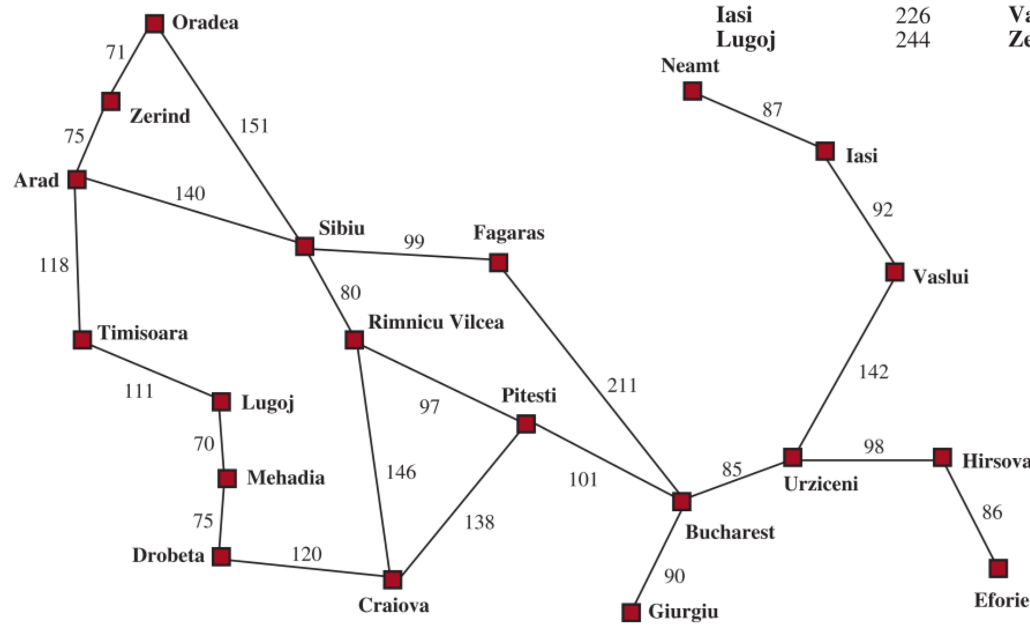
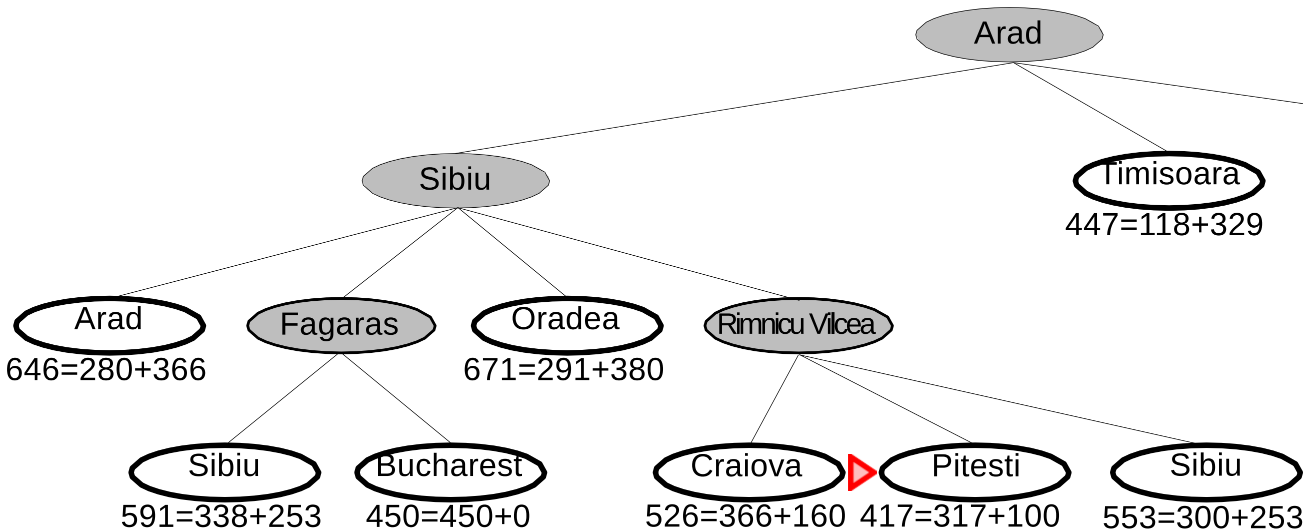


A* search example



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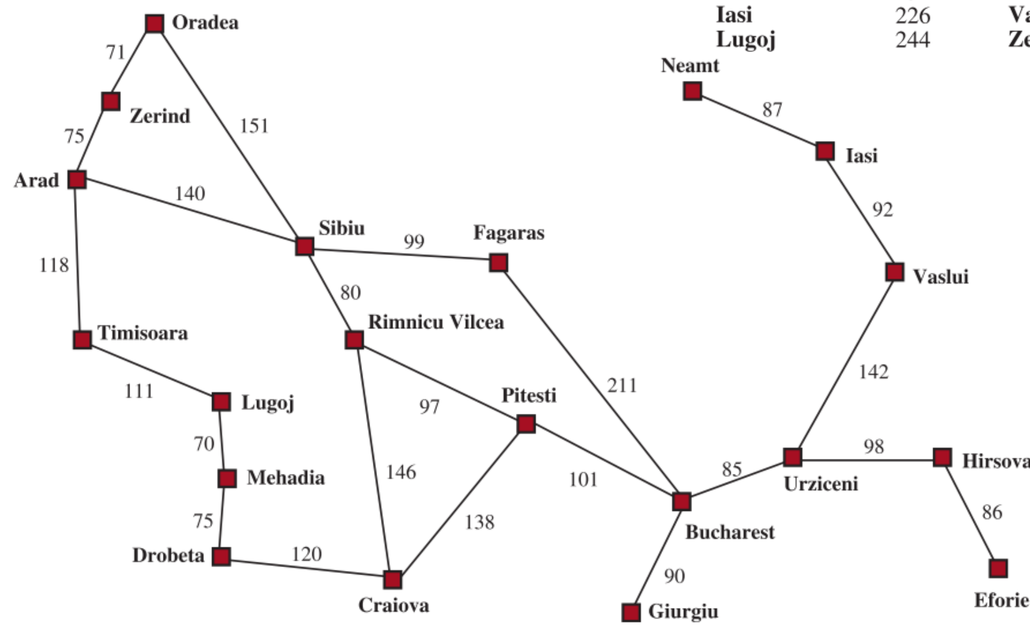
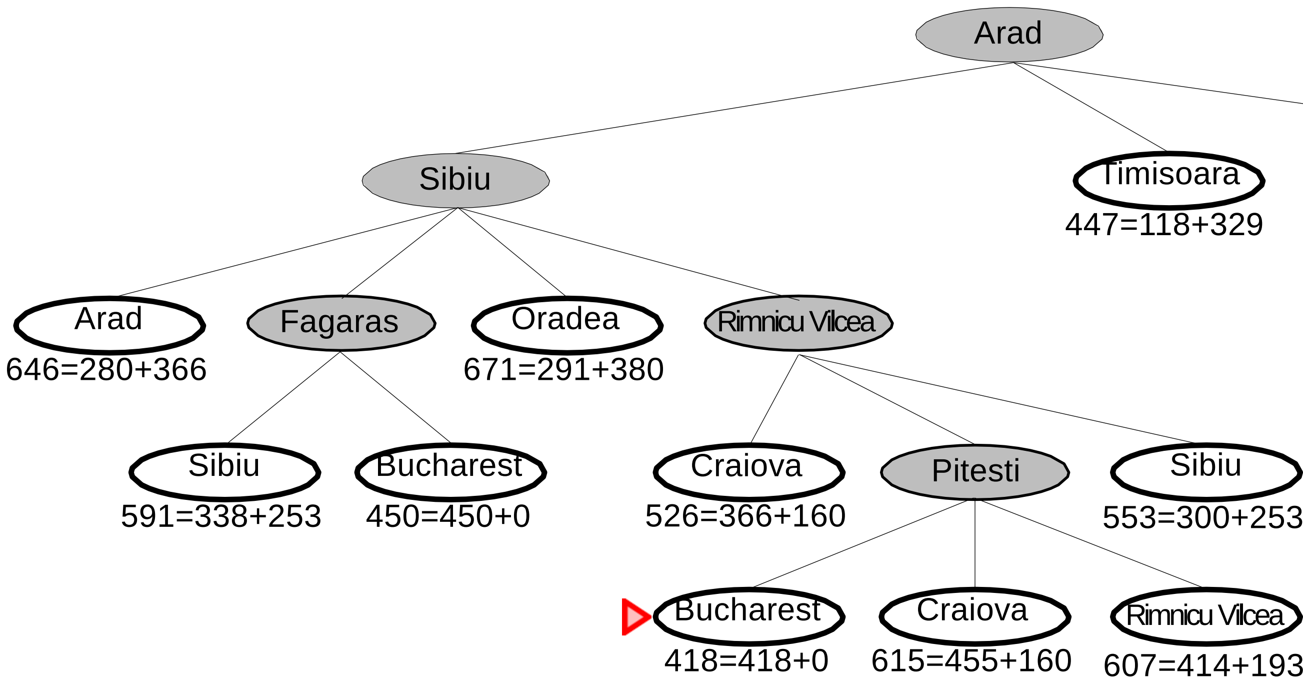
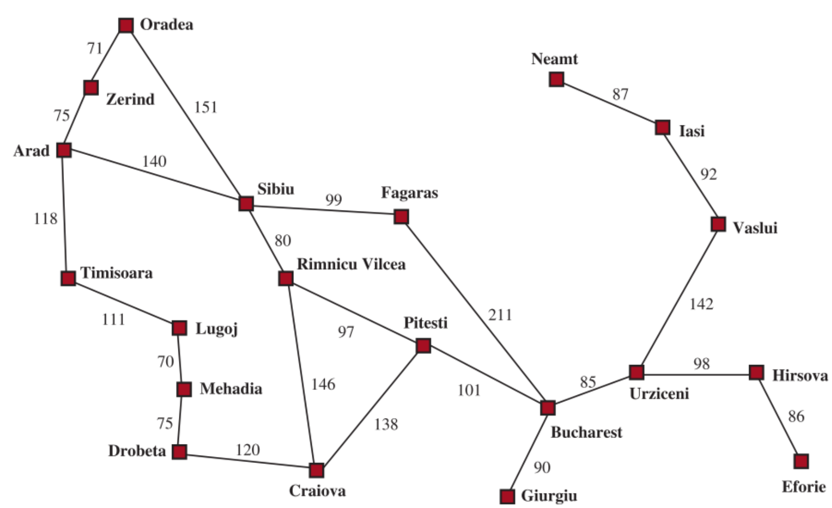


A* search example





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Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
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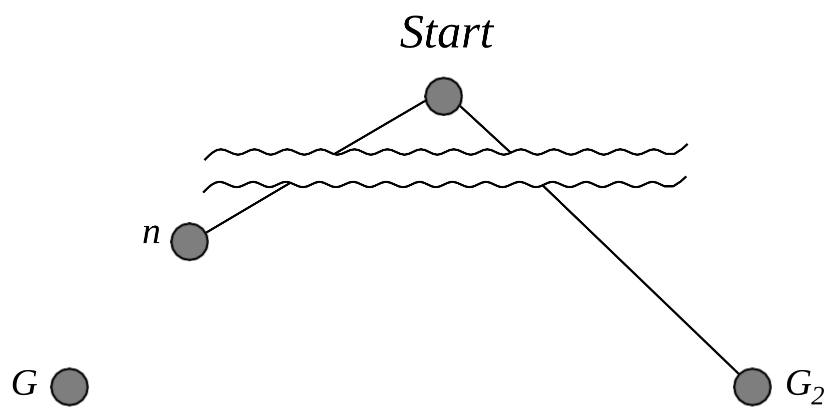
Pearson

Optimality of A∗ (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.



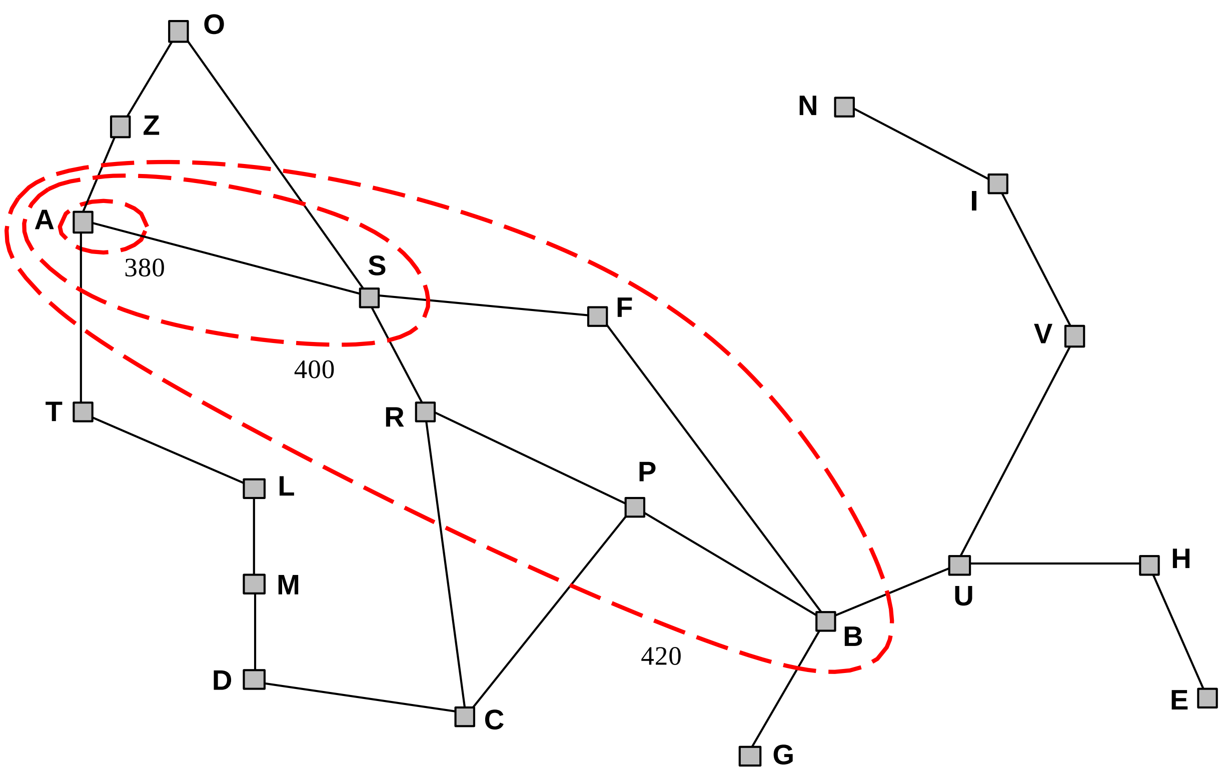
$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A*

Complete?

Properties of A*

Complete? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?

Properties of A*

Complete? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time? Exponential in [relative error in $h \times$ length of soln.]

Space?

Properties of A*

Complete? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time? Exponential in [relative error in $h \times$ length of soln.]

Space? Keeps all nodes in memory

Optimal?

Properties of A*

Complete? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time? Exponential in [relative error in $h \times$ length of soln.]

Space? Keeps all nodes in memory

Optimal? Yes—cannot expand f_{i+1} until f_i is finished

A$*$ expands all nodes with $f(n) < C*$

A$*$expands some nodes with $f(n) = C*$

A$*$expands no nodes with $f(n) > C*$

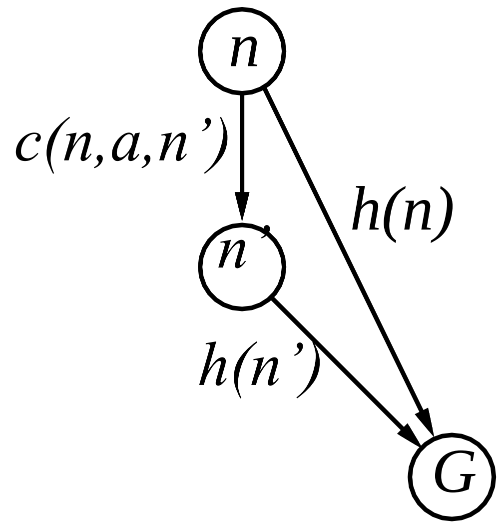
Proof of lemma: Consistency

A heuristic is consistent if

$$h(n) \leq c(n, a, n\ ) + h(n\ )$$

If h is consistent, we have

$$\begin{aligned} f(n\ ) &= g(n\ ) + h(n\ ) \\ &= g(n) + c(n, a, n\ ) + h(n\ ) \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

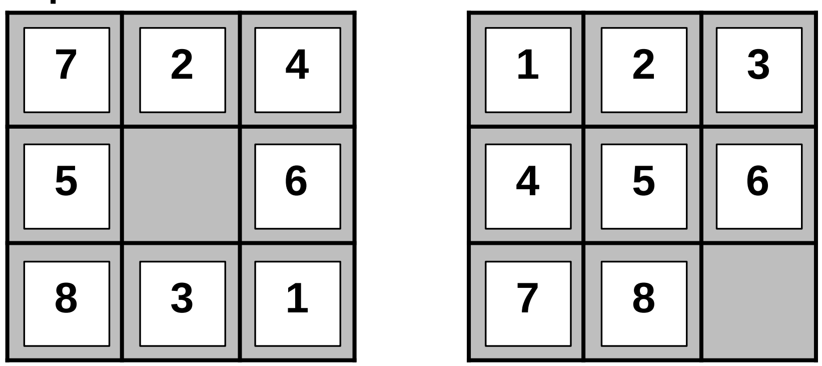


I.e., $f(n)$ is nondecreasing along any path.

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State **Goal State**

$h_1(S)$ =??
$h_2(S)$ =??

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$h_1(S)$ =?? 6
$h_2(S)$ =?? 4+0+3+3+1+0+2+1 = 14

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible), then h_2 dominates h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A$^*(h_1)$ = 539 nodes A$^*(h_2)$ = 113 nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
A$^*(h_1)$ = 39,135 nodes A$^*(h_2)$ = 1,641 nodes

Given any admissible heuristics h_a, h_b, $h(n) = \max(h_a(n), h_b(n))$

is also admissible and dominates h_a, h_b

Relaxed problems

Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem

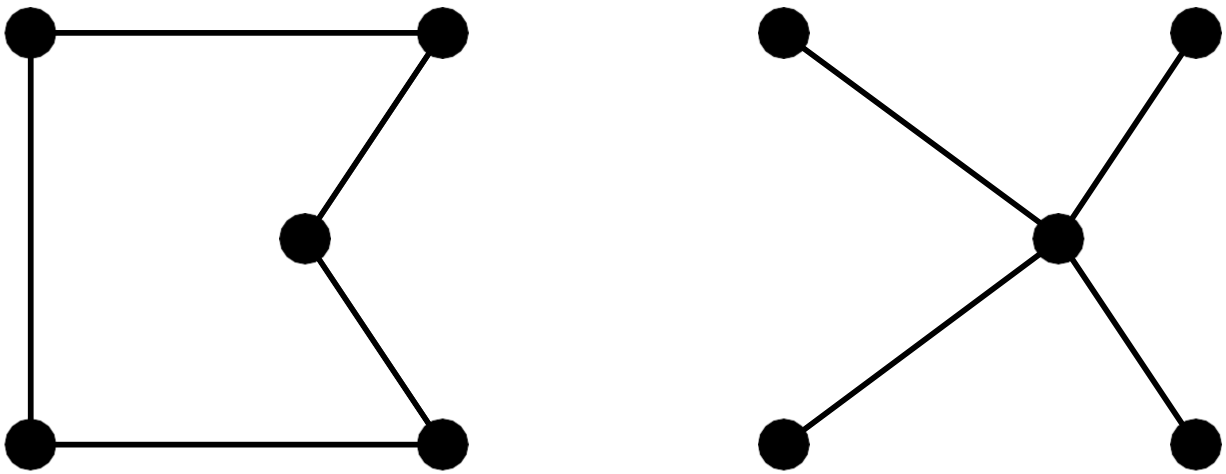
If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Key point: *the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem*

Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP) Find the shortest tour visiting all cities exactly once



Minimum spanning tree can be computed in $O(n^2)$ and is a lower bound on the shortest (open) tour

Summary

A problem consists of five parts: the **initial state**, a set of **actions**, a **transition model** describing the results of those actions, a set of **goal states**, and an **action cost function**.

Uninformed search methods have access only to the **problem definition**. Algorithms build a search tree in an attempt to find a solution.

Informed search methods have access to a **heuristic** function h(n) that estimates the cost of a solution from n.

In the next lecture...

- Local Search and Optimization Problems
  - Hill-climbing
  - Simulated annealing
  - Genetic algorithms
- Local search in continuous spaces
- Search with Nondeterministic Actions
- Search in Partially Observable Environments