REPORT 3D MEDICAL ROBOTIC REGISTRATION

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1. Introduction

The present project concerns a puncture using a straight needle under the guidance of an endoscopic camera. The instruments used in this operative room are a needle, here called instrument, attached to a Cartesian robot (fitted with a passive wrist), an endoscopic camera, a localization system, an active mark attached to the instrument, another active mark attached to the camera and a target localized inside of the patient (Figure 1).

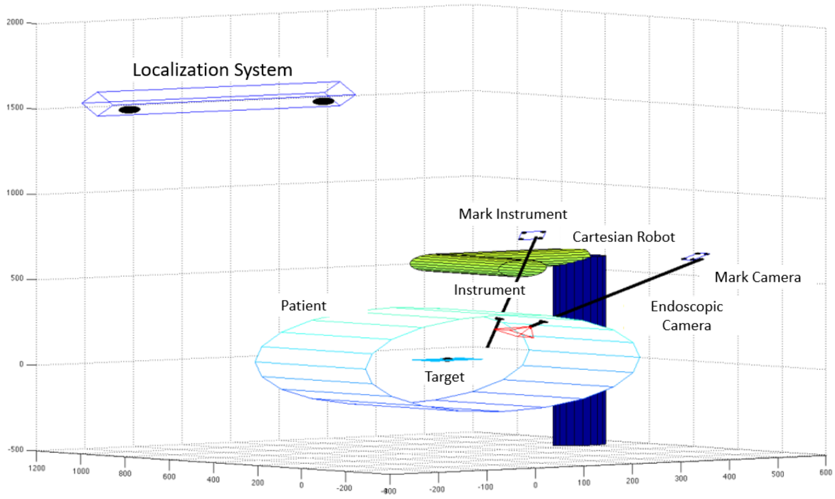


Figure 1 - Scheme of the setup.

With this setup, it is possible to automate the positioning of the instrument in the target. The target is defined manually for the surgeon through the endoscopic camera. With that aim, this report will describe and analyze four main tasks, the first task is to propose, to develop and to validate the registration and positioning procedure to bring the needle to the target using the localization system. The final goal of the second task is the same as the first one, except for not using the localization system. The third task concerns an important step of medical procedures, the assessment of errors propagation in the registration task. Finally, the last task aims to position the instrument in the target under a feedback control using the endoscopic camera.

In the registration procedure developed in this project, it used the Effector-Marker calibration (AX = XB) and the Triangulation. Both procedures are briefly detailed in the next subsections.

1.1. Effector-Marker Calibration

It is possible, in some cases, that the transformation between the end effector and the robotic arm marker is not previously known. Therefore, one of the possible ways to calibrate and determine this transformation is to use the localization system to solve the system of equations of the type AX= XB. Then, let ${}^{eff}T_{Mark\ Inst}$ be the transformation to be determined, i.e. the X of the equation, while the ($A = {}^{effi}T_{effj}$) will the the transformation matrix between the two positions of the end-effector and ($B = {}^{Mark\ Insti}T_{Mark\ Instj}$) the transformation matrix between the two mark positions. Therefore, one has the next system of equations:

$$ {}^{effi}T_{effj} \; {}^{eff}T_{Mark\,Inst} = {}^{eff}T_{Mark\,Inst} \; {}^{Mark\,Insti}T_{Mark\,Instj} \,. \tag{1} $$

To solve this system, it is necessary three different positions of the robot, and the matrix A and B are determined using the Forward Kinematic Model (FKM) of the robot and the localization system, as follows:

$$ {}^{effi}T_{effj} = {}^{effi}T_{base} \; {}^{base}T_{effj} \,, \tag{2} $$

$$ {}^{Mark\,Insti}T_{Mark\,Instj} = {}^{Mark\,Insti}T_{LocSys} \; {}^{LocSys}T_{Mark\,Instj} \,. \tag{3} $$

Then, after using the equation (2) and (3) and replacing them in the system (1), it is possible to split this system in two other systems, one to find the rotation matrix and other to find the translation vector, obtaining the following systems

$$ R^{ij}_{eff} R_x = R_x R^{ij}_{Mark\,Inst} \,, \tag{4} $$

$$ t^{ij}_{eff} + R^{ij}_{eff} t_x = t_x + R_x t^{ij}_{Mark\,Inst} \,. \tag{5} $$

Defining v^{ij}_{eff} as the eigenvector of R^{ij}_{eff} and $v^{ij}_{Mark\,Inst}$ as the eigenvector of $R^{ij}_{Mark\,Inst}$, it can be proved that the system (4) can be rewritten by

$$ r_x = V^{-1} v \,, \tag{6} $$

where r_x is a vector with the components of the rotation matrix R_x, V is a matrix and v is a vector, both are given by

$$ V = \begin{bmatrix} v^{21}_{\text{Mark Inst}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & v^{21}_{\text{Mark Inst}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v^{21}_{\text{Mark Inst}} \\ v^{31}_{\text{Mark Inst}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & v^{31}_{\text{Mark Inst}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v^{31}_{\text{Mark Inst}} \\ v^{32}_{\text{Mark Inst}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & v^{32}_{\text{Mark Inst}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v^{32}_{\text{Mark Inst}} \end{bmatrix} , \tag{7} $$

$$ v = [v^{21}_{eff} \quad v^{31}_{eff} \quad v^{32}_{eff}]^T . \tag{8} $$

Then, after to find the vector r_x, the rotation matrix R_x is found just reshaping r_x. To the system (5), some algebraic manipulations can shows that

$$ t_x = \begin{bmatrix} R^{12}_{eff} - I \\ R^{31}_{eff} - I \\ R^{32}_{eff} - I \end{bmatrix}^{+} \begin{bmatrix} R_x\, \mathrm{t}^{12}_{\text{Mark Inst}} + \mathrm{t}^{12}_{\text{eff}} \\ R_x\, \mathrm{t}^{31}_{\text{Mark Inst}} + \mathrm{t}^{31}_{\text{eff}} \\ R_x\, \mathrm{t}^{32}_{\text{Mark Inst}} + \mathrm{t}^{32}_{\text{eff}} \end{bmatrix} \tag{9} $$

1.2. Triangulation

Consider a point P seen for two different cameras as shown in Figure 2.

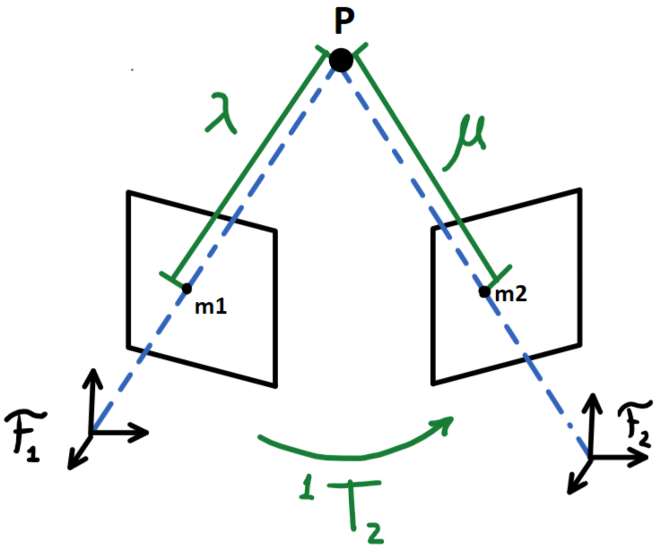


Figure 2 - Scheme of the triangulation.

The position of point P with respect to the Frame 1 (F_1) and Frame 2 (F_2) can be determined by equating the equations below

$$P = \lambda m_1, \tag{10}$$

$$P = {}^{Cam1}t_{Cam2} + \mu\, {}^{Cam1}R_{Cam2} m_2, \tag{11}$$

which results, after some algebraic manipulations, in

$$[\lambda\ \mu]^T = [m_1 \quad - {}^{Cam1}R_{Cam2} m_2]^+ \ {}^{Cam1}t_{Cam2}. \tag{12}$$

Finally, the position in the Camera 1, for instance, is given by

$$^{Cam1}t_P = \lambda m_1, \tag{13}$$

2. Registration using the Localization System

This section will primarily list all the necessary steps to carry out the first registration task, and then show details and the results of some steps. Thereby, the following steps were performed:

1. The robot effector was moved to three different positions;
2. The pose $^{eff}T_{base}$ and $^{LocSys}T_{Mark\ Inst}$ for each position were storage in variables;
3. Then, the procedure of Effector-Marker described in the subsection (1.1) was applied to find the $^{eff}T_{Mark\ Inst}$ transformation matrix;

4. After that, it is possible to find the transformation between the base and the localization system ${}^{base}T_{LocSys}$;
5. Using the principle of triangulation (subsection 1.2), the position of the instrument trocar with respect to the base is determined;
6. And then, also using the triangulation, now applied to the endoscopic camera, the pose of the target with respect to the base is obtained.
7. Finally, using the direction of the instrument, it is possible to determine the position of the end-effector needed to bring the tip of the instrument to the target.
8. Therefore, the last step is to apply this position in the function *MoveEffPosition(position)*

For the first step, the random positions were chosen

$$
{}^{base}t_{eff1} = [-200\ \ -50\ \ 1050]^T,
$$
$$
{}^{base}t_{eff2} = [-190\ \ -30\ \ 1065\ ]^T,
$$
$$
{}^{base}t_{eff3} = [-170\ \ -100\ \ 1055]^T.
$$

Then, the respective transformations ${}^{eff}T_{base}$ and ${}^{Mark\ Inst}T_{LocSys}$ for the three positions were obtained through the functions *GetRobotCurrentPosition()* and *GetLocalizerInformation()*, respectively. After that, the system of equation (1) was solved through the process described in the subsection (1.1). Which leads to

$$
{}^{eff}T_{Mark\ Inst} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -20 \\ 0 & 1 & 0 & -50 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{14}
$$

So, it is easy to find the ${}^{base}T_{LocSys}$, which is given by

$$
{}^{base}T_{LocSys} = {}^{base}T_{eff}\ {}^{eff}T_{Mark\ Inst}\left({}^{LocSys}T_{Mark\ Inst}\right)^{-1}, \tag{15}
$$

$$
{}^{base}T_{LocSys} = \begin{bmatrix} -1 & 0 & 0 & -500 \\ 0 & 0.7071 & -0.7071 & 800 \\ 0 & -0.7071 & -0.7071 & 2000 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{16}
$$

To find the trocar position, a triangulation process was performed, using the instrument length and direction in the ${}^{base}t_{eff1}$ and ${}^{base}t_{eff2}$. Where the position of the trocar can be calculate considering that

$$
{}^{base}t_{trocar} = {}^{base}t_{eff1} + \lambda\, z_{eff1}, \tag{17}
$$

$$
{}^{base}t_{trocar} = {}^{base}t_{eff2} + \mu\, z_{eff2}, \tag{18}
$$

where z_{effi} is the direction of the z axis in the frame of effector in the ${}^{base}t_{effi}$, i.e., the third column of the matrix ${}^{effi}T_{base}$. And the parameters λ and μ is given by:

$$[\lambda\ \mu]^T = [z_{eff1} \quad -z_{eff2}]({}^{base}t_{eff2} - {}^{base}t_{eff1}), \tag{19}$$

Then, replacing the result of the equation (19) in (17) and (18), the trocar position is given by:

$$ {}^{base}t_{trocar} = [-350 \quad -100 \quad 800]^T . \tag{20}$$

To obtain the target position with respect to the base, the triangulation process (subsection 1.2) is used as well. First, the camera was moved in two different positions, in order to, using the localization system to find the transformation between the camera positions ${}^{Cam1}T_{Cam2}$.

$$ {}^{Cam1}T_{Cam2} = ({}^{LocSys}T_{Cam1})^{-1}\ {}^{LocSys}T_{Cam2} \tag{21}$$

Then, the points m_1 and m_2 were calculated through the images obtained from the endoscopic camera, as shown below

$$m_1 = K^{-1}\ {}^{Image\,1}t_{target}, \tag{22}$$

$$m_2 = K^{-1}\ {}^{Image\,2}t_{target}. \tag{23}$$

where, ${}^{Image\,i}t_{target}$ is the point in pixels seen by the Camera i, and K is the intrinsic parameter matrix. So, following the equations (12) and (13), the target position with respect to the first position of the camera is given by

$$ {}^{Cam1}t_{target} = [-25 \quad 13.3975 \quad 203.3013]^T, \tag{24}$$

then, changing the base frame, one obtain the target position with respect to the base through to next equation

$$ {}^{base}t_{target} = {}^{base}T_{LocSys}\ {}^{LocSys}T_{Cam1}[\ {}^{Cam1}t_{target} \quad 1]^T . \tag{25}$$

Therefore, the last step was to find the end effector position that positions the tip of the instrument on the target (${}^{base}t_{effx}$). To accomplish this task, a unit vector was found whose direction is aligned with the target and the trocar. This vector is given by

$$direction = ({}^{base}t_{target} - {}^{base}t_{trocar}) / \left| {}^{base}t_{target} - {}^{base}t_{trocar} \right| \tag{26}$$

Once this is done, the end effector must be in this direction, displaced by 350 mm (value obtained from the matrix ${}^{eff}T_{Inst}$). The result is

$$ {}^{base}t_{effx} = {}^{base}t_{target} - 350 * direction \tag{27}$$

$$ {}^{base}t_{effx} = [-\ 305.0199 \quad -\ 70.0133 \quad 859.9735]^{T} \tag{28} $$

Then, it is possible to see the tip of the instrument touching the target in the figures below

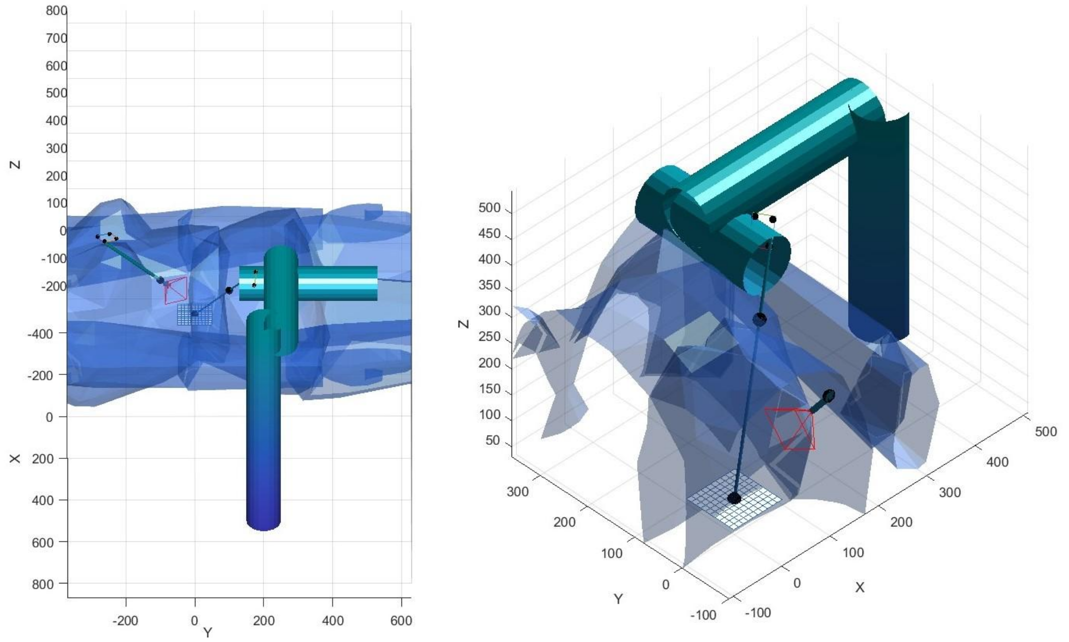


Figure 3 - The instrument in the target.

The error obtained through the function *ComputeTRE()* was

$$ Error \ = \ 6.54\ x\ 10^{-11} \tag{29} $$

3. Registration Without Localization System

The second task is to obtain the same result shown in Figure 3, but without using the localization system. Thereby, the camera will localizate the instrument through some markers placed, that should be at least four non-coplanar points. The steps to reach this task were

1. Obtain the transformation ${}^{Cam1}T_{Cam2}$ using the new transformation ${}^{Cam1}T_{Inst}$ obtained with the function *GetInstrumentPosition()*.

$$ {}^{Cam1}T_{Cam2} \ = \ {}^{Cam1}T_{Inst}({}^{Cam2}T_{Inst})^{-1} \tag{30} $$

2. Do the same triangulation with the camera, and calculus to obtain the position of the end effector that positions the tip of the instrument on the target. However, the position of target with respect to the base is given by

$$ {}^{base}t_{target} \ = \ {}^{base}T_{eff}\ {}^{eff}T_{inst}({}^{Cam1}T_{inst})^{-1}\ [\ {}^{Cam1}t_{target} \quad 1]^{T}. \tag{31} $$

Finally, it obtained almost the same result of the first task, but with the following error

$$ Error \ = \ 8.1026\ x\ 10^{-12} \tag{32} $$

One observes that the error in the registration process that does not involve the localization system is smaller than the error using the localization system. This makes sense, such that the number of iterations is smaller in the second case, i.e. the error propagation is smaller. Therefore, this second approach has the advantage of using less computation and more precision, however, the fact that the instrument has to always be seen by the camera demonstrates the main drawback of this type of process.

4. Assessment of Error Propagation

The third task aims to evaluate the impact of the error in the trocar position with respect to the registration process through two different ways: using variance propagation and simulation with noise. To do this, one considers that the error in ${}^{world}t_{trocar}$ is isotropic and with an amplitude $1\,mm^2$, therefore, $E_x = 3I$. The first step of the variance propagation is to determinate the function that varies with the position of the trocar, it is given by

$$f({}^{world}t_{trocar}) = {}^{world}t_{eff} - 350\frac{{}^{world}t_{trocar} - {}^{world}t_{eff}}{\left|{}^{world}t_{trocar} - {}^{world}t_{eff}\right|} \quad (33)$$

Then, it used the matlab function *jacobian()* to determinate the Jacobian matrix, given by

$$J = \begin{bmatrix} 0.8790 & -0.1014 & -0.5071 \\ -0.1014 & 0.7071 & -0.1690 \\ -0.5071 & -0.1690 & 0.3381 \end{bmatrix} \quad (34)$$

Finally, calculating $E_y = JE_xJ^T$, it obtained

$$E_y = \begin{bmatrix} 3.12 & -0.36 & -1.80 \\ -0.36 & 4.08 & -0.60 \\ -1.80 & -0.60 & 1.2 \end{bmatrix} \quad (35)$$

Using the numeric process with 999,999 iterations, it is shown in the matlab program. The ellipsoid is shown in the figure below

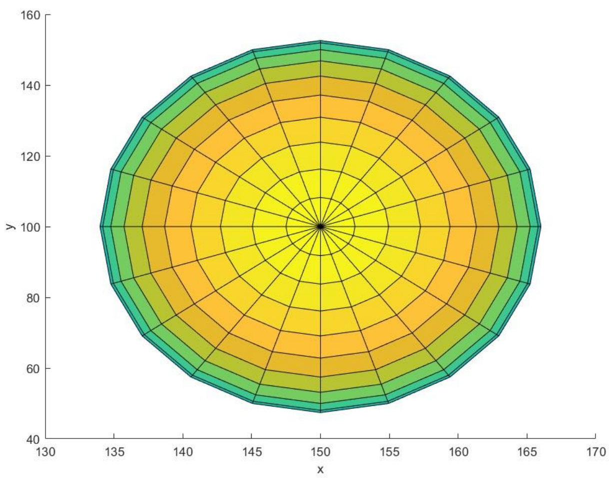


Figure 4 - Ellipsoid

5. Positioning under the Feedback of the Camera

The final task consisted of implementing a closed-loop control system to guide the instrument towards the target based only on the feedback of the endoscopic camera. That is, instead of assessing the position error directly from the information provided by the FKM, this error needed to be calculated from the images only. The control loop implemented is schematized in Figure 3, and each step will be explained in the following.

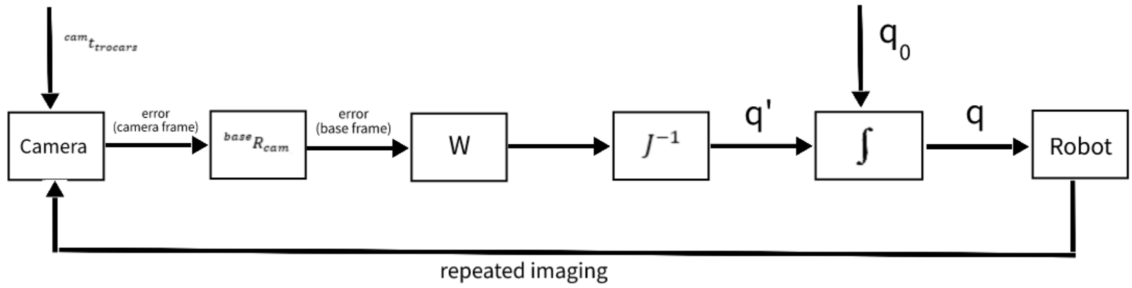


Figure 3 - Control loop based on the camera feedback.

There are some steps that are done before entering the loop process.

The position of the target point in the camera frame is obtained by triangulation (as before), moving the camera once (in the direction of the vector $[5 \quad 0 \quad -25]^T$) to simulate a stereoscopic view of the placed target. After this step the camera stays fixed. The position of the instrument can be obtained by identifying in the images the marker printed on its body, which is a task that is only simulated in this project.

The input for each iteration will be the calculated error between the desired position of the effector and its current position. In order to calculate the error in the effector position, the position of the trocars in the camera frame needs to be known. It is calculated by first obtaining the transformation between the camera frame and the base frame, and then by multiplying its inverse by the position of the trocar in the base frame (already known). This is represented in equations 5.1 and 5.2

$$^{base}T_{cam} = {^{base}T_{eff}} \; {^{eff}T_{inst}} \left[{^{cam}T_{inst}}\right]^{-1} , \tag{5.1}$$

$$^{cam}t_{trocar} = \left[{^{base}T_{cam}}\right]^{-1} \; {^{base}t_{trocar}} . \tag{5.2}$$

This calculation needs not be done more than once, as both the camera and the trocar stay fixed. With the trocar position in the camera frame, the current effector position in the same frame can be calculated using also the known instrument position in the camera frame:

$$^{cam}t_{eff} = {^{can}t_{inst}} + 350 \, \frac{\left({^{cam}t_{inst}} - {^{cam}t_{trocars}}\right)}{\left\| {^{cam}t_{inst}} - {^{cam}t_{trocars}} \right\|} . \tag{5.3}$$

The same logic applied to the target position yields the desired effector position in the camera frame:

$$^{cam}t^{*}_{eff} = {^{cam}t_{target}} + 350 \frac{(^{cam}t_{target} - {^{cam}t_{trocars}})}{\left\| ^{cam}t_{target} - {^{cam}t_{trocars}} \right\|} \quad . \tag{5.4}$$

And the error in the camera frame is obtained by their difference:

$$^{cam}e(t) = {^{cam}t^{*}_{eff}} - {^{cam}t_{eff}} \quad . \tag{5.5}$$

The error in the base frame is:

$$^{base}e(t) = {^{base}R_{cam}} \, {^{cam}e(t)} \quad . \tag{5.6}$$

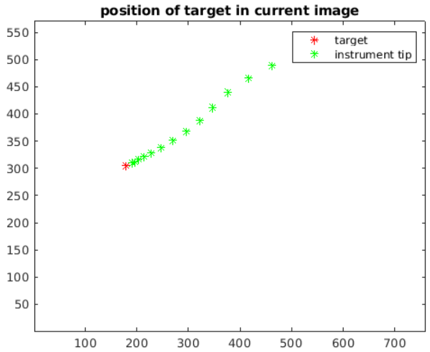
Even though it would be possible in theory to simply move once the robot in the direction of the target, this cannot be done. Firstly, a harsh movement of the robot could cause injuries to the patient. Moreover, this iterative method can compensate for errors present in the process.

The movement is thus made in small steps, whose size is determined by the matrix W shown in the control loop (Figure 5.1). This matrix consists in the identity matrix multiplied by a scalar factor — the bigger this factor, the quicker the movement of the instrument converges to the desired position. The value used for the scaling is 1/3.

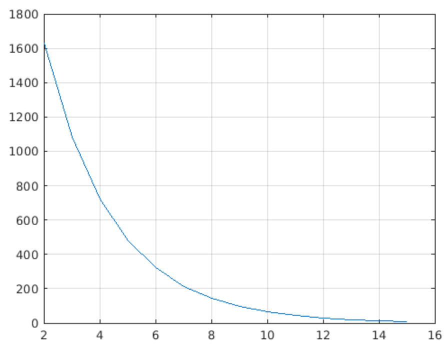
The estimated and scaled error vector is then multiplied by the inverse of the Jacobian to get the derivative of q, the configuration vector of the robot.

$$q' = J^{-1} * W * {^{base}e(t)} \quad . \tag{5.7}$$

To finish the loop step, q' is added to the current vector q and the desired position of the effector is obtained from the geometric model. The loop iterates until the measured error is below the arbitrary threshold of 10.



(a)



(b)

Figure 4 - Results of the simulation. (a) Movement of the instrument towards the target. (b) Evolution of the error along the iterations.

6. Conclusion

In this project it was possible to compare the advantages and disadvantages of the choices that need to be made when implementing a real-life solution to a medical task. In Sections 2 and 3 the same problem was solved using two different methods. Although using only the endoscopic camera can be cheaper it is a more robust solution to use the navigation system to perform the registration task.