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Model Predictive Control of Vibration Systems

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Abstract

This article discusses the application of Model Predictive Control (MPC) on active vibration control. The advantages and drawbacks of the MPC approach if compared to its linear counterparts, in particular the ability of explicitly including actuator constraints and the increased computational cost, are considered. Finally, examples from the literature of Model Predictive Control of vibration systems are presented.

KEYWORDS:
Model Predictive Control, Vibration Systems, Active Vibration Control, Actuator Saturation

1 | INTRODUCTION

Active vibration control has seen numerous applications in various fields such as civil, aerospace and sound engineering [1, 2, 3]. Active vibration control consists in the use of sensors and actuators to mitigate vibrations on the system via closed-loop control.

Active control methods surge as an alternative to passive approaches, which involve modifying the system dynamic *a priori* through the installation of passive components [2]. While active control tends to be more complex and costly than its passive counterparts, it provides better performance, since it can react in real-time to system or disturbance alterations. Therefore, specially when the system knowledge is not exact and the frequencies are relatively low (which could result in impractically large dampeners), closed-loop control strategies are often applied.

Some of the more common active control strategies are position/velocity or acceleration/velocity linear feedback and Linear Quadratic Regulators (LQR). Such strategies are able to modify the position of unwanted poles, thus shaping the closed-loop response and avoiding resonance [4]. They, however, have no way of directly handling actuator saturation and performance degradation or even instability can result from operating close to the input limits.

Therefore, for the application of such linear control strategies the transducers need to be over-specified, such that the expected input dynamic is well within the actuator limits [5]. This, however comes with higher costs and even reliability problems, since there is still no guarantee that the actuators will not saturate.

Model Predictive Control (MPC) surges in this context as an interesting alternative, since hard input constraints can be easily introduced in this strategy through the MPC optimization problem [6]. The price to pay for this natural constraint handling and optimality is computational complexity, since a quadratic problem must be solved at each sampling instant.

This paper presents an overview of the application of MPC strategies on active vibration control and is organized as follows: Sections 2 and 3 present the active vibration and MPC control problems in more detail, while MPC applications on vibration control are presented in Section 4 and conclusions are drawn in Section 5.

2 | ACTIVE VIBRATION CONTROL

Active Vibration Control (AVC) is a control technique of vibration systems via force application in order to attenuate vibrations generated by external disturbances. These control systems involve several components, such as: position and velocity sensors, accelerometers, electronic amplifier systems and actuators, which can be electromagnetic, hydraulic or pneumatic [7, 3]. The basic functionality of the controller is to measure the system vibration and actively and instantly compensate, with the intent to keep the structure static. For example, if the external vibration is periodic, then the control system can cancel its effect by generating an opposite signal, such as a 180 degrees lagged sine function [3].

Passive Vibration Control (PVC) is not appropriate for all vibration systems, since the associated components, such as vibration dampers, shock absorbers and base isolation, may cause a high weight penalty, unviable for some applications, for example in space structures. Furthermore, AVC dynamically reacts to incoming vibrations, being more robust than PVC, which reduces disturbances simply due to the mechanical properties of its materials (open-loop) [1].

Active Vibration Control can be applied in a variety of vibration systems, for example in Large Space Structures (LSS) such as the International Space Station (ISS). In this station, the fundamental modal frequency is around 8 Hz and its intrinsic damping is only about 0.358% [3]. There, the use of passive dampers is infeasible, due to the increased associated weight. Another spatial example is the Hubble Telescope, a system that requires high pointing accuracy, where a tiny 10 micro-radian of filtering motion generates a 50 meters shift in the field of view for an imager at a distance of 500 km [3]. This precise control can only be achieved by AVC. However, Active Vibration Control is not limited to space structures, with other, more traditional, applications such as the control of vibrations generated by earthquakes, winds, unbalanced motors and waves. In these cases, the actuators have to be more robust, due to the necessity of larger actuation forces.

The basic control structures for AVC are Single Input-Single Output (SISO) vibration controller, state-space based multiple input and multiple output (MIMO) controller and Distributed vibration control. And one of the most common ways is to apply Active Vibration Control linearly [2], where the input has a linear dependance on the states and ODE with state feedback is given by

$$M\ddot{q}(t) + (C - BF)\dot{q}(t) + (K - BG)q(t) = 0, \tag{1}$$

where $M, C, K \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, with $M \succ 0$, $C \succcurlyeq 0$, $K \succcurlyeq 0$, $q \in \mathbb{R}^n$ is the position vector and the matrices $F, G \in \mathbb{R}^{m\times n}$ are the state feedback gains. In this way, it is possible to determine values of F and G in order to partially allocate the closed-loop poles with no *spill-over* [4]. Hence, the AVC done in this linear fashion allows the system to achieve stability and desired performance metrics, besides being relatively easy to implement.

However, as explained in the introduction, this linear form does not work well with input restrictions and saturation of the control signal may lead to performance degradation or even instability. Model Predictive Control, further discussed in Section 3, thus surges as a viable alternative to linear AVC in problems where actuator saturation cannot be disregarded.

3 | MODEL PREDICTIVE CONTROL

Model Predictive Control (MPC) is an a advanced control strategy which uses a model of the dynamic system to predict future states and solves, for each time instant, an optimization problem in order to find the best sequence of future inputs according to a given cost function and satisfying the necessary constraints [**?**].

Some of the advantages of the MPC strategies include the natural inclusion of constraints [6] and inherent optimality, while its main drawback is the computational stress and time associated with solving an optimization problem at each sampling instant.

One of the main MPC techniques uses a linear state-space model of the system to make predictions and considers a quadratic cost and linear constraints [8]. Consider a prediction horizon $N \in \mathbb{N}$ and the following linear discrete-time system model:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k, \end{aligned} \tag{2}$$

at each time instant the controller measures (or estimates) the current state and solves the following optimization problem:

$$\begin{aligned} \min_{\hat{\mathbf{u}}} \sum_{j=1}^{N} y_{k+j|k}^T Q y_{k+j|k} &+ \sum_{j=0}^{N-1} u_{k+j|k}^T R u_{k+j|k} \\ s.t. \quad x_{k+j|k} \in \mathcal{X},\ u_{k+j|k} \in \mathcal{U}&, \end{aligned} \tag{3}$$

where $\hat{\mathbf{u}} = (u_{k|k}, \dots, u_{k+N|k})$ are the future inputs, variables of the optimization problem, $x_{k+j|k}, y_{k+j|k}$ are the predicted states and outputs, Q, R are positive definite matrices that define state and input cost and $\mathcal{X}, \mathcal{U}$ are polyhedral sets that represent hard linear constraints.

Notice that (3) is a convex quadratic problem, since the future states and outputs have an affine dependence on $\hat{\mathbf{u}}$. Therefore, interior-point and gradient based algorithms are guaranteed to arrive at the global minimum, and computational cost is significantly reduced if compared to general non-linear and non-convex problems.

Following the receding horizon policy [8, 6], problem (3) is solved at each sampling instant, and the MPC control law is given by

$$u_k = \kappa_{MPC}(x_k) = u_{k|k}^*, \tag{4}$$

where $\hat{\mathbf{u}}^*(x_k) = (u_{k|k}^*, \dots, u_{k+N|k})^*$ is the solution of (3).

Notice that for this MPC control strategy a state-space representation of the controlled system is needed. For vibration systems, with the mild assumption of a full-rank mass matrix, the matrices A and B can be obtained from the second order representation $M\ddot{q} + D\dot{q} + Kq = B_s u$ as

$$A = \begin{pmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ M^{-1}B_s \end{pmatrix}, \quad (5)$$

where the state vector $x = \begin{pmatrix} q & \dot{q} \end{pmatrix}^T$ represents both the system position and velocity.

It is worth noting that other system models can be used in the prediction of the output trajectory. In particular, a linear relationship from $\hat{\mathbf{u}}$ to $y_{k+j|k}$ can be obtained from the transfer matrix $G(s)$ (with $Y(s) = G(s)U(s)$) or the step responses of each input-output pair.

Model Predictive Control strategies can then be developed based on such predictions, called General Predictive Control (GPC) and Dynamic Matrix Control (DMC), respectively [9]. Such strategies have the benefit of not needing an explicit state-space system representation, which is specially interesting for distributed-parameter models, where state-space approximations may yield complex models, with a considerably large number of states.

4 | APPLICATIONS OF MPC ON VIBRATION CONTROL

In this section we present some applications and associated performance evaluations of MPC on active vibration control found in the control literature.

In [5], MPC is shown to be a feasible and appealing alternative to linear AVC in the presence of input saturation. This is demonstrated via its application on an experimental cantilever beam apparatus [10] (which can be viewed as a simplified representation of many vibration systems). The proposed MPC presents a reduced settling time and input/output energy signals in the presence of input constraints. The higher computational cost of the MPC is handled by manually implementing all online algorithms in assembly, in order to minimize overhead. This way, MPC was shown to be viable for faster systems, with computation time less than $150\mu s$

Similarly, in [11], the application of the MPC strategy to the control of vibration is done to a flexible structure, now with multiple inputs and multiple outputs (MIMO) using piezoelectric actuators and strain gauge sensing. There, it was concluded that the use of a controller capable of predicting the beam tip displacement in the future, mainly because the vibration has an oscillatory character, makes this strategy superior to other control techniques.The MPC strategy resulted in suppression of beam tip displacement to less than $1mm$ in $15s$.

It is further shown in [12] that the vibration suppression of an one-link flexible manipulator via MPC proposed in [11] can be applied through finite-element based modeling, rather than empirical models obtained via input-output responses. The finite-element modeling resulted in more accurate predictions and thus better control performance than the empirical approach. Finite-element modeling and MPC control can therefore be suitably combined for active vibration suppression.

Expanding to other types of processes, [13] brings up the application of MPC with constraints to plants whose dynamical behavior is both nonlinear and fast changing, which is different from traditional slow process applications. The system chosen to evaluate the effectiveness of this control is a four-link closed loop planar mechanism. As a result of this article, the MPC control, when compared with standard PID control, has proved to be very effective for the reference position tracking and vibration suppression, besides showing good robustness both in the presence of plant uncertainties and incompatibilities between the actual and measured control variables.

5 | CONCLUSION

It can be seen that, while the literature on Model Predictive Control applied to active vibration suppression is still limited, the results present it as a valid alternative to more classic methods, particularly in the presence of constraints, nonlinearities or model uncertainties. Furthermore, its main drawback, computational cost, is becoming less of a concern due to hardware and software developments.

Notice that the MPC applications on active vibration control found in the literature use a state-space prediction model. In this light, future research could consider the application of other prediction models which directly apply the second-order structures that naturally arise from vibration systems in the Model Predictive Control problem.

References

1. A. Preumont, *Vibration control of active structures an introduction*, 4 ed., Springer, Brussels, 2011.
2. Daniel J. Inman, *Vibration with control*, Wiley, Virginia Tech, USA, 2006.
3. Indian Institute of Technology Kanpur, *Introduction to active vibration control*.
4. Yun-Feng Cai, Jiang Qian, and Shu-Fang Xu, *The formulation and numerical method for partial quadratic eigenvalue assignment problems*, Numerical Linear Algebra with Applications **18** (2011), no. 4, 637–652.
5. Adrian G. Wills, Dale Bates, Andrew J. Fleming, Brett Ninness, and S. O. Reza Moheimani, *Model predictive control applied to constraint handling in active noise and vibration control*, IEEE

Transactions on Control Systems Technology **16** (2008), no. 1, 3–12.

6. J. Maciejowski, P. Goulart, and E. Kerrigan, *Constrained control using model predictive control*, Springer, 2007.
7. Anshul Sharma, Rajeev Kumar, Rahul Vaish, and Vishal S. Chauhan, *Active vibration control of space antenna reflector over wide temperature range*, Composite Structures **128** (2015), 291–304.
8. Basil Kouvaritakis and Mark Cannon, *Model predictive control*, Springer, 2016.
9. Ammar Ramdani and Said Grouni, *Dynamic matrix control and generalized predictive control, comparison study with imc-pid*, International Journal of Hydrogen Energy **42** (2017), no. 28, 17561–17570, Special Issue on The 4th European Conference on Renewable Energy Systems (ECRES 2016), 28-31 August 2016, Istanbul, Turkey.
10. Tarsicio Beléndez, Cristian Neipp, and Augusto Beléndez, *Large and small deflections of a cantilever beam*, European Journal of Physics **23** (2002), no. 3, 371–379.
11. M. Hassan, R. Dubay, C. Li, and R. Wang, *Active vibration control of a flexible one-link manipulator using a multivariable predictive controller*, Mechatronics **17** (2007), no. 6, 311–323.
12. Rickey Dubay, Marwan Hassan, Chunying Li, and Meaghan Charest, *Finite element based model predictive control for active vibration suppression of a one-link flexible manipulator*, ISA Transactions **53** (2014), no. 5, 1609–1619, ICCA 2013.
13. P. Boscariol, A. Gasparetto, and V. Zanotto, *Model predictive control of a flexible links mechanism*, Intell Robot Syst **58**, 2010.