Analysis and Control Design for Coupled Tank System

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Abstract: The coupled tank system is an excellent type of plant to apply and analyze control techniques, including for didactic purposes. Therefore, the present work will present two control techniques for the system: one using differential equations that describe the model and the other using a plant identification method through experimentally obtained data.

Keywords: Control; Coupled; Tanks; Identification; Decoupling;

1 Introduction

It is common to find chemical processes in industries that require control of the level and flow of fluids stored in coupled tanks. These systems can be used to regulate temperature and/or concentration, for example. Therefore, in this work two level control projects will be developed for two tanks that interact with each other through a pipe, in the configuration shown in Figure 1.

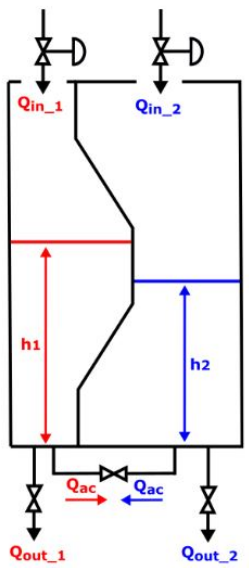

Figura 1. Coupled tank system.

And being described mathematically through the following equations:

$$\begin{cases} \dfrac{dh_1}{dt}A1 = Q_{in1} - Q_{out1} \pm Q_{ac}, & (1) \\ \dfrac{dh_2}{dt}A2 = Q_{in2} - Q_{out2} \mp Q_{ac}, & (2) \\ Q_{out2} = K_2\sqrt{h_2}, & (3) \\ Q_{out1} = K_1\sqrt{h_1}, & (4) \\ Q_{ac} = K_{ac}\sqrt{h_1 - h_2}, & (5) \end{cases}$$

where Q_{in1} and Q_{in2} are the input flows, Q_{out1} and Q_{out2} are the output flows, Q_{ac} is the coupling, which allows two directions depending on the level difference between the tanks, A_1 and A_2 are the tank areas, which depend on the level height of each tank, and h_1 and h_2 are the levels of tanks 1 and 2, respectively. Furthermore, K_1, K_2 and K_{ac} are proportionality constants that relate the output and coupling flow rates to the tank levels.

Therefore, the first stage of this work was the implementation of these equations in Matlab's SIMULINK environment. And, as the system is not linear, it was necessary to perform linearization around two points, one for each tank. In this case, the equilibrium points considered were $H_1 = 20cm$ and $H_2 = 17cm$. This process was used only in the first design strategy, since in the second strategy the system was identified experimentally.

2 First Project Strategy

In this first design strategy, the following steps were carried out: plant linearization, system decoupling, controller design for the linearized and decoupled system, testing and analysis of the controller in the real nonlinear system.

2.1 Linearization

In the linearization process, considering the equilibrium points $H_1 = 20\ cm$ and $H_2 = 17\ cm$, the system was represented in state space using the equation shown in Figure **??**, arriving at the result of Equation **??**.

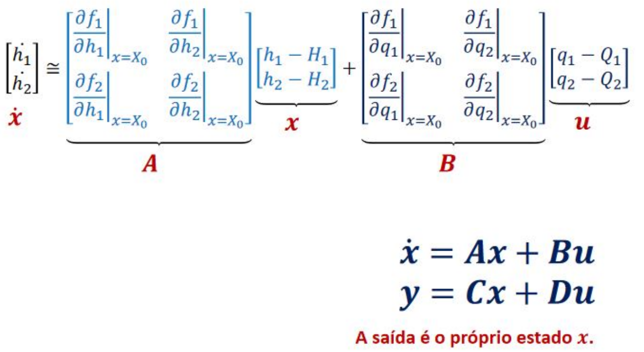

Figura 2. Representation of the system in linearized state space.

$$\begin{bmatrix}\dot{h}_1(t)\\ \dot{h}_2(t)\end{bmatrix} = \begin{bmatrix}-0.0608 & 0.0458\\ 0.0421 & -0.0642\end{bmatrix}\begin{bmatrix}h_1 - H_1\\ h_2 - H_2\end{bmatrix} + \begin{bmatrix}0.0079 & 0\\ 0 & 0.0073\end{bmatrix}\begin{bmatrix}q_1 - Q_1\\ q_2 - Q_2\end{bmatrix},$$

$$\begin{bmatrix}h_1(t)\\ h_2(t)\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}h_1 - H_1\\ h_2 - H_2\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}q_1 - Q_1\\ q_2 - Q_2\end{bmatrix}. \quad (6)$$

2.2 Decoupling

Once the plant was linearized, the system decoupling process was carried out following the steps:

(1) Determination of the transfer functions $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$ and $G_{22}(s)$ of Equation 7 through the Matlab command *ss(A,B,C,D)*, with A, B, C and D being the state space matrices from Equation 6, obtaining the system represented through transfer functions shown in Equation 8:

$$G(s) = \begin{bmatrix}G_{11}(s) & G_{12}(s)\\ G_{21}(s) & G_{22}(s)\end{bmatrix}, \quad (7)$$

$$G(s) = \begin{bmatrix}\frac{0.00794(s+0.06415)}{(s+0.11)(s+0.02213)} & \frac{0.00033}{(s+0.11)(s+0,02213)}\\ \frac{0.00033}{(s+0.11)(s+0.02213)} & \frac{0.00729(s+0.068)}{(s+0.11)(s+0,02213)}\end{bmatrix}. \quad (8)$$

(2) Determination of a decoupled system $T(s)$, whose system dynamics are present and which is a diagonal matrix, therefore we choose:

$$T(s) = \begin{bmatrix}G_{11}(s) & 0\\ 0 & G_{22}(s)\end{bmatrix}.$$

To do this, a matrix $D(s)$ is calculated that performs this ideal decoupling of the system, in which:

$$D(s) = T(s)G^{-1}(s), \quad (9)$$

getting:

$$D(s) = \begin{bmatrix}\frac{(s+0.11)(s+0.06415)(s+0.02213)}{(s+0.02213)(s+0.06415)(s+0.11)} & \frac{0.045821(s+0.11)(s+0.02213)}{(s+0.02213)(s+0.068)(s+0.11)}\\ \frac{0.04207(s+0.11)(s+0.02213)}{(s+0.02213)(s+0.06415)(s+0.11)} & \frac{(s+0.11)(s+0.068)(s+0.02213)}{(s+0.02213)(s+0.068)(s+0.11)}\end{bmatrix}, \quad (10)$$

simplifying:

$$D(s) = \begin{bmatrix}1 & \frac{0.045821}{(s+0.068)}\\ \frac{0.04207}{(s+0.06415)} & 1\end{bmatrix}, \quad (11)$$

(3) Therefore, once the controllers C_1 and C_2 have been determined for the systems $G_{11}(s)$ and $G_{22}(s)$, respectively, the total controller transfer function will be determined by Equation 12, so that the decoupled system operates in the configuration of Figure 3.

$$C_{TOTAL}(s) = C(s)D(s), \quad (12)$$

being

$$C(s) = \begin{bmatrix}C_1(s) & 0\\ 0 & C_2(s)\end{bmatrix}. \quad (13)$$

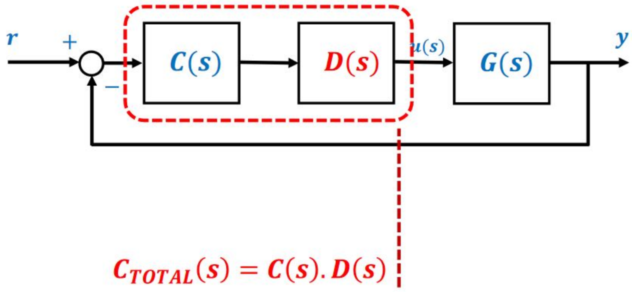

Figura 3. System decoupled.

2.3 Controller Project

Once these linearization and decoupling steps were carried out, the design of the PI controller by pole allocation could be carried out for the $G_{11}(s)$ and $G_{22}(s)$ systems. To then determine $C_{TOTAL}(s)$. In the case of the first project, as the $G_{11}(s)$ and $G_{22}(s)$ systems are second order, and the chosen controller is a PI, the three closed-loop poles were allocated for both systems (Ogata (2014)), obtaining:

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \quad (14)$$
$$s_3 = -\beta\zeta\omega_n \quad (15)$$

For the project, an overshoot less than 5%, a settling time of $t_s = 10s$ and a $\beta = 0.01$ were chosen for both $G_{11}(s)$ as for $G_{22}(s)$, obtaining the closed-loop poles:

$$s_{1,2} = -0.4 \pm j0.4195$$
$$s_3 = -0.004$$

Therefore, equating the polynomials of the closed-loop system with the desired poles, the following controllers were obtained:

$$C_1(s) = \frac{84.642(s+0.05379)}{s}$$
$$C_2(s) = \frac{92.204(s+0.05075)}{s}$$

2.4 Simulation results

Applying $C_{TOTAL}(s)$ in the SIMULINK nonlinear system, with references at $H_1 = 20cm$ and $H_2 = 17cm$, the following results were obtained:

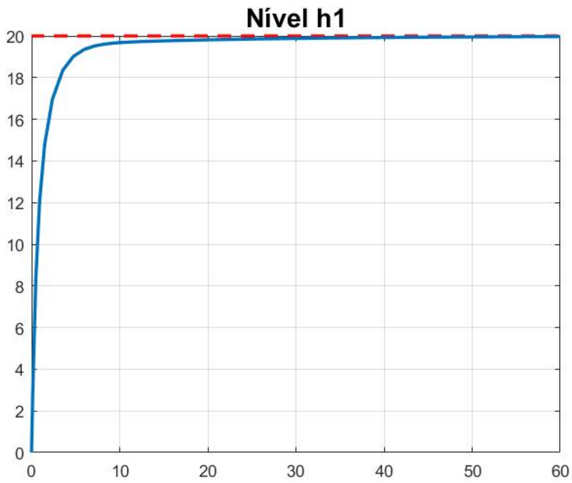

Figura 4. Output signal h_1 from tank 1.

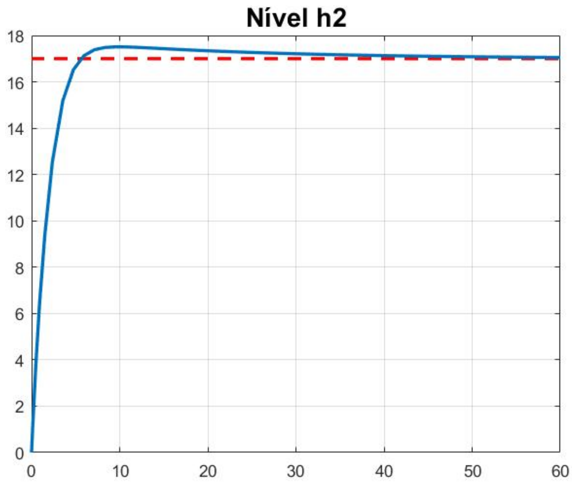

Figura 5. Output signal h_2 from tank 2.

3 Second Project Strategy

In the second strategy, the models considered were experimental, in which the transfer functions $G_{11}(s)$ and $G_{22}(s)$ were identified using the Smith method, to then design two PI controllers also by pole allocation.

3.1 System Identification

In the identification process, applying the flow signals in Figures 6 and 7, the variation in tank levels shown in Figure **??**. As the obtained signal contains noise mainly coming from the level sensors, a moving average filter was applied to the signal, obtaining the result in Figure 9. Thus, applying the Smith identification method analyzing the Matlab vectors, the approximate first-order experimental models of Equations 16 and 17 were determined.

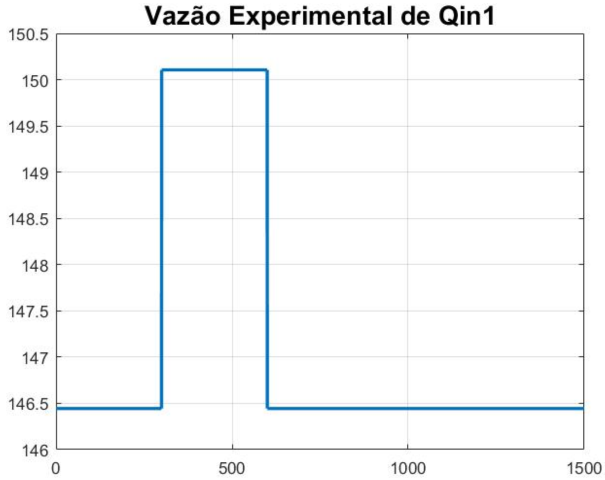

Figura 6. Flow Signal applied to tank 1.

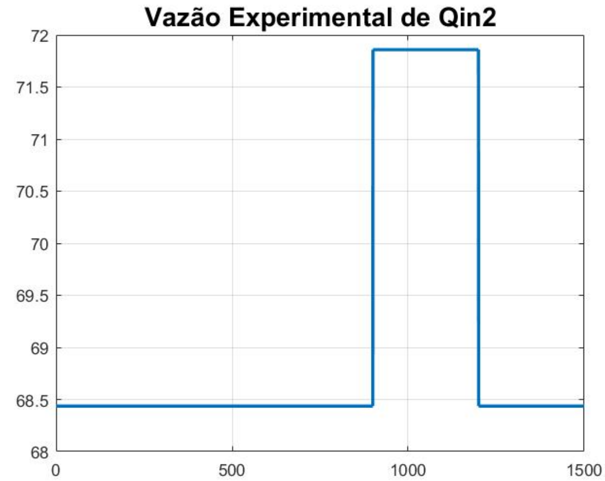

Figura 7. Flow Signal applied to tank 2.

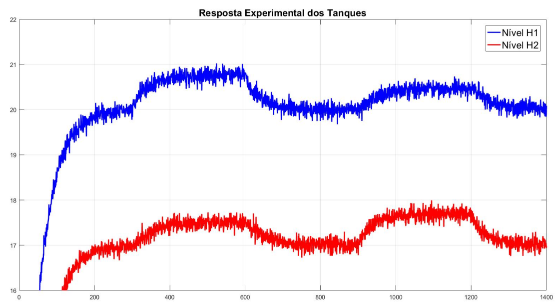

Figura 8. Level sensor output signal of h_1 and h_2.

$$G_{11}(s) = \frac{0.2036}{44.85s + 1}, \tag{16}$$

$$G_{22}(s) = \frac{0.1985}{37.58s + 1}. \tag{17}$$

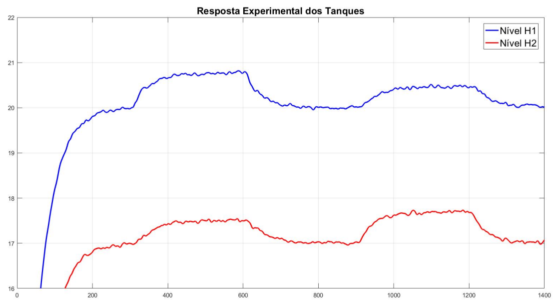

Figura 9. Filtered level output signal.

3.2 Controller Project

After identifying the experimental models $G_{11}(s)$ and $G_{22}(s)$, two controllers $C_1(s)$ and $C_2(s)$ were designed Proportional-Integral by pole allocation. However, in this case, the model obtained is approximate and first order, so the allocation is made of only two closed-loop poles. In this case, using the Equation 14, choosing an overshoot smaller than 1% and a settling time of $t_s = 10s$ for the 2% criterion (Ogata (2014)), obtained the following closed-loop poles for both models:

$$s_{1,2} = -0.4 \pm j0.2729$$

Therefore, comparing the polynomials, the controllers were obtained:

$$C_1(s) = \frac{171.68(s + 0.3015)}{s}$$
$$C_2(s) = \frac{146.48(s + 0.3032)}{s}$$

3.3 Simulation results

Applying the $C_1(s)$ and $C_2(s)$ controllers directly to the SIMULINK nonlinear system, the following results were obtained:

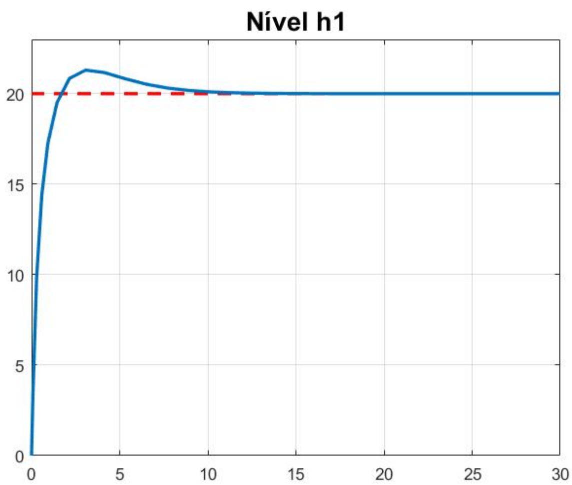

Figura 10. Output signal h_1 from tank 1.

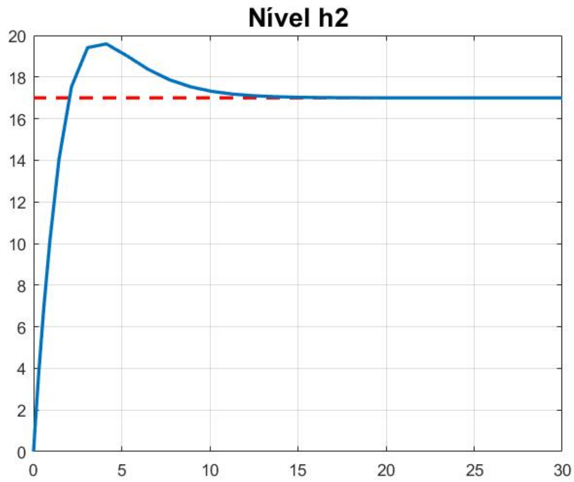

Figura 11. Output signal h_2 from tank 2.

4 Conclusion

The development of control projects in this work provided practical growth in control techniques seen during the course. When comparing the system response in the first project in relation to the second, it is observed that tank 1 managed to respond without overshoot and with a settling time of 20 seconds for the 2% criterion. Tank 2 presented a discrete overshoot of approximately 2% and a settling time of 35 seconds. This is a better performance, in relation to the outstanding performance, when looking at the results of the second project. In which, by tuning the controllers well, an overshoot of 6.5% and a settling time of 10 seconds were obtained in tank 1, and in tank 2, an overshoot of 15% and a settling time of accommodation of 15 seconds. Therefore, the second project presented a better settling time. However, there was greater difficulty in tuning the second project, since the system was experimentally approximated by a first-order system, and the second project was modeled by white box, using the differential equations that describe the physical system.

Referências

Ogata, K. (2014). *Engenharia de Controle Moderno*. Pearson, São Paulo.