part 2: scenarios:

SCENARIO 1: Two Prophets, One Game

```
Average test error:        0.3067
Approximation error:       0.2000
Estimation error:          0.0940
Best prophet chosen:       53/100 times
```

SCENARIO 2: Two Prophets, Ten Games

```
Average test error:        0.2213
Approximation error:       0.2000
Estimation error:          0.0240
Best prophet chosen:       88/100 times
```

SCENARIO 3: Many Prophets, Ten Games

```
Average test error:        0.0930
Approximation error:       0.0064
Estimation error:          0.0852
Best prophet chosen:       3/100 times
Within 1% of best:         8/100 times
```

analysis: if the error rates were uniformly distributed between [0, 0.5] instead of [0, 1], the **approximation error would stay largely the same**, since we have haven't changes the lower bound on True risk. meanwhile, the **estimation error would be substantially smaller**, since even when we select a model whose true risk is greater than that of the best available model, the true risk of the chosen model will be closer to 0.

SCENARIO 4: Many Prophets, Many Games

```
Average test error:        0.0064
Approximation error:       0.0064
Estimation error:          0.0008
Best prophet chosen:       50/100 times
Within 1% of best:         98/100 times
```

analysis: if we evaluate the genralisation gap of a model based on the train set, we expect it to be **greater** than if we had masured generalisation gap based on the model's perfomrnace on the test. that is becuase the train set is a relatively small subset compared to the poplation and the model selection was biased in favor of a model performing well on the train set. no such bias affects the performance on the test set, and hence the test set provides a better approxiamtion of the gnerealisation gap.

SCENARIO 5: School of Prophets

```
Grid search completed
  k values (prophets):     [2, 5, 10, 50]
  m values (train games):  [1, 10, 50, 1000]
  Number of trials:        100
```

Scenario 5: School of Prophets Results

Average Test Error

	m=1	m=10	m=50	m=1000
k=2	0.0987	0.0861	0.0738	0.0703
k=5	0.1006	0.0658	0.0409	0.0294
k=10	0.1019	0.0665	0.0239	0.0187
k=50	0.0960	0.0653	0.0178	0.0034

Approximation Error

	m=1	m=10	m=50	m=1000
k=2	0.0643	0.0730	0.0683	0.0707
k=5	0.0346	0.0340	0.0351	0.0298
k=10	0.0184	0.0176	0.0155	0.0182
k=50	0.0036	0.0036	0.0034	0.0030

Estimation Error

	m=1	m=10	m=50	m=1000
k=2	0.0341	0.0126	0.0048	0.0003
k=5	0.0667	0.0332	0.0053	0.0002
k=10	0.0836	0.0486	0.0083	0.0003
k=50	0.0925	0.0623	0.0143	0.0004

the grid search shows clear patterns: as we increase **k** (class size) we increase two facotrs:

1. the chance of the available prophets including a prothet with a lower true risk, thus **decreasing approxiamtion error**` indeed k is almost entirely responsible for the approxiamtion error.
2. increasing the chance of including a prophet that happened to perform well on the trian set, while having a greater true risk, thus **increasing estiamton error and test error**. meanwhile, increasing **m** (the training set size) makes it harder for a model to perform well on the train set despite having a higher true risk thus, it **decreases the estimation error and the test error**.

indeed, we see that when we increased the number of games for ERM from scenario 1 to 2, we lowered the estimation and test error as expected.

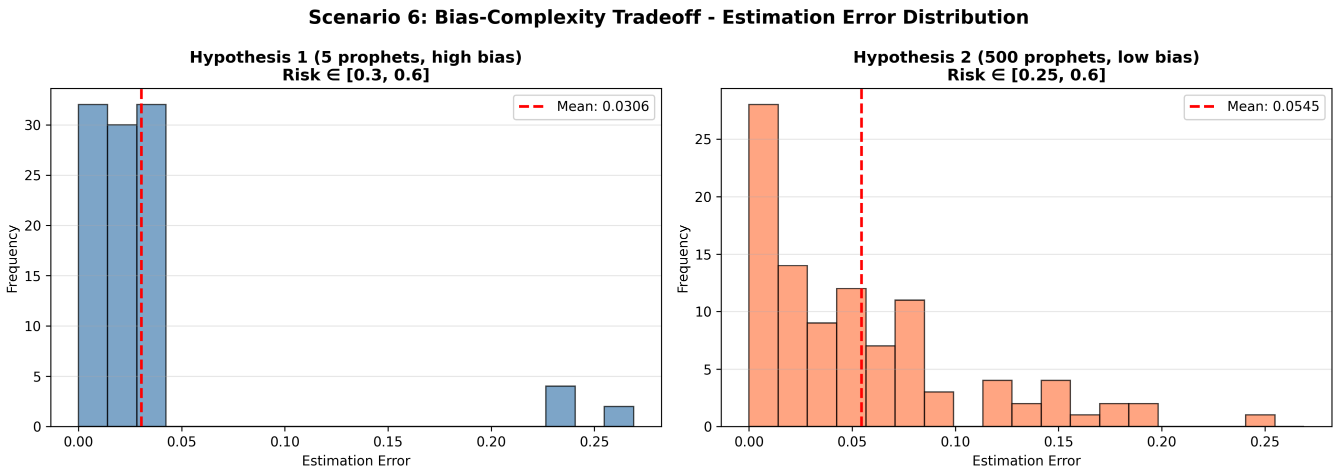
furthermore, scenarios 3 and 4 exactly show how the approxiamtion error is affected primarily by k, while incresing m led to lower test and estiamtion errors.

SCENARIO 6: Bias-Complexity Tradeoff

```
Hypothesis 1 (5 prophets, high bias):
  Average test error:      0.3570
```

```
  Approximation error:     0.3230
  Estimation error:        0.0306

Hypothesis 2 (500 prophets, low bias):
  Average test error:      0.3044
  Approximation error:     0.2500
  Estimation error:        0.0545
```



we see that while class 2 has models sampled from a 'better' range of true risk (lower bound 0.25 as opposed to 0.3), the estiamtion error is higher. this is as expected, since given a larger calss size there is a greater likelihood of choosing a sample with low error on the train set despite higher true risk. meanwhile, the fact that the models in calss 2 on average have lower true risk, leads to a lower test error.

part 3: pac learning analyis:

in this part we use the formula relating the number of samples to accuracy and cofidence: `m = 2 * np.log(2 * h / delta) / epsilon`, which can also be expressed as: `m = (2 / epsilon) * (np.log(2)+np.log(h)-np.log(delta))` we note that m is inversely proportional to epsilon and to the log of delta. it is proportional to the log of h.

QUESTION 1: Compute minimal number of samples

```
  Input Parameters:
    |H| (hypothesis class size) = 100
    epsilon (desired accuracy)  = 0.05
    delta (confidence level)    = 0.01


Output: m (rounded up) = 397
```

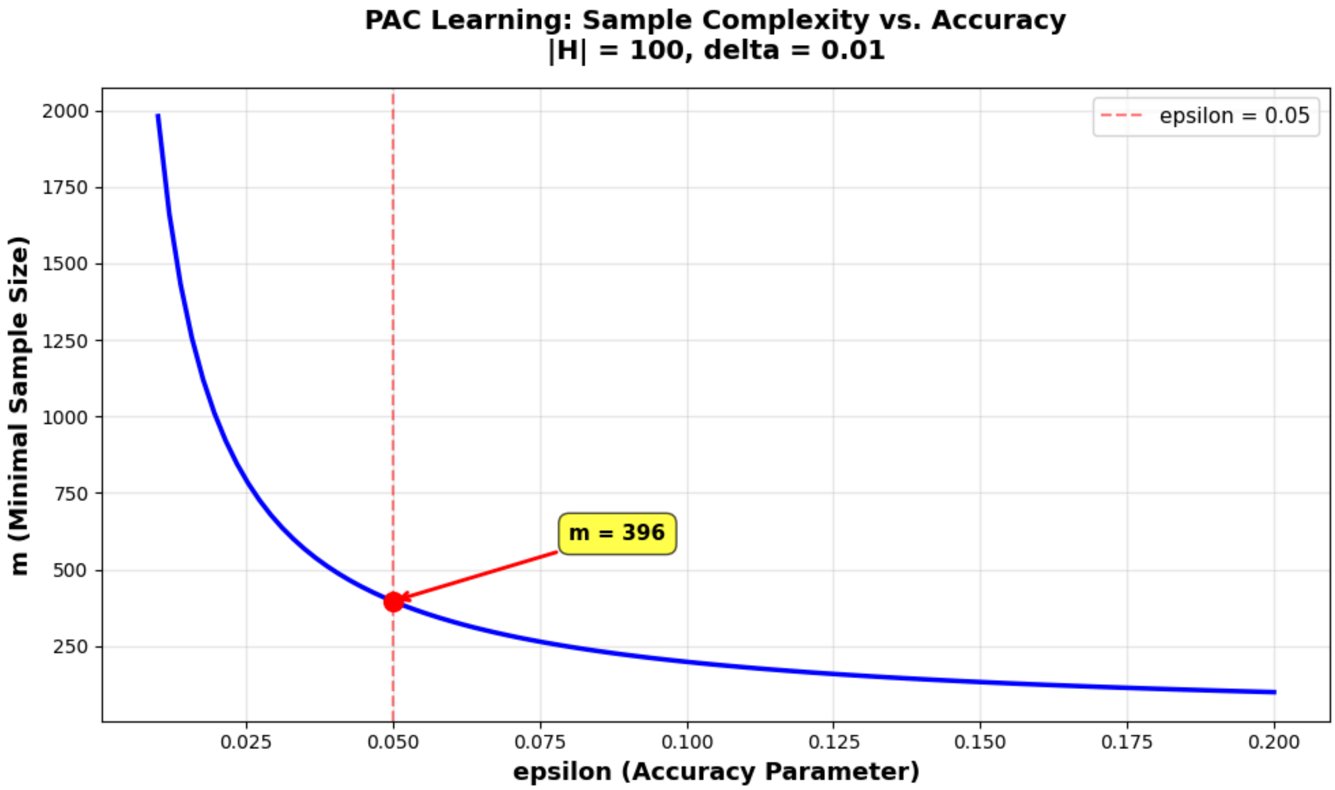
QUESTION 2: Analyze change when |H| is doubled

```
Input Parameters:
  |H| (hypothesis class size) = 200 (doubled)
```

```
Output: m (rounded up) = 424
```

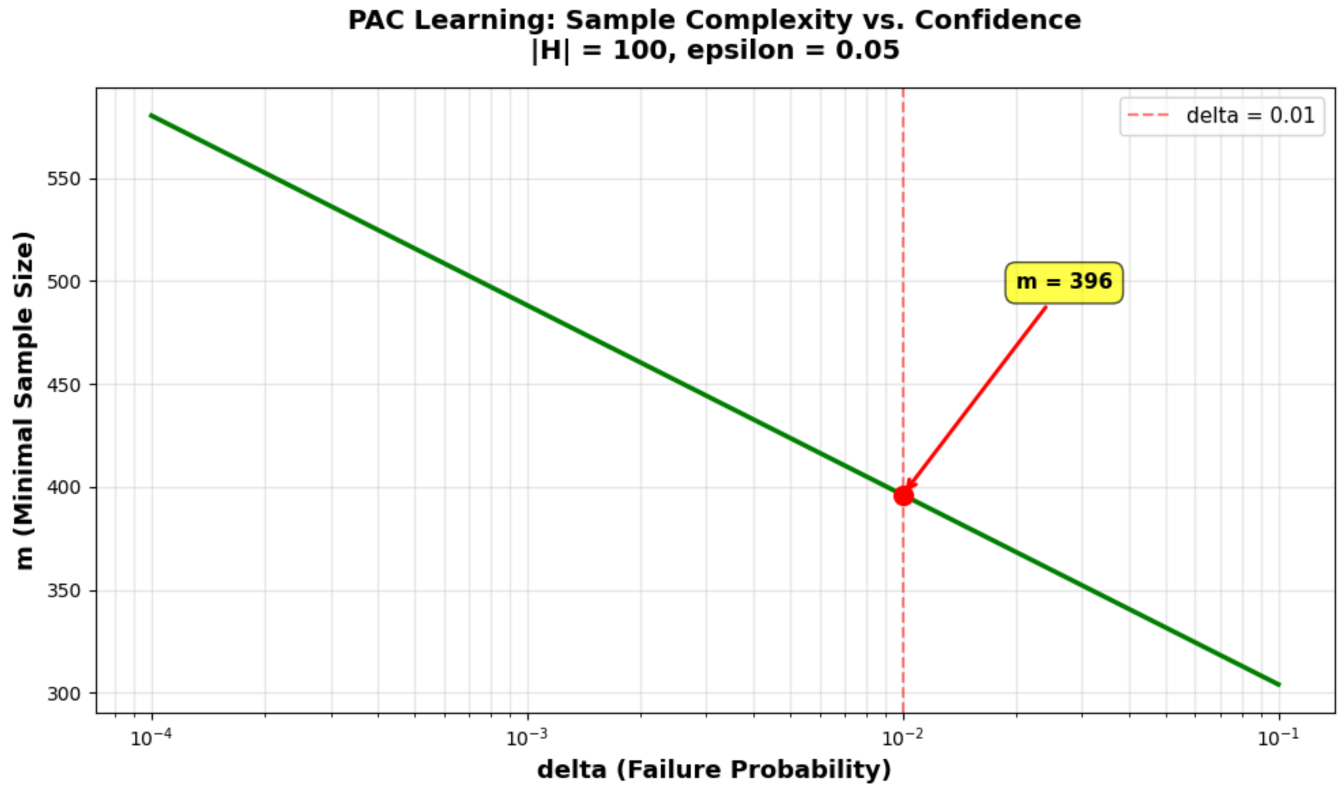
we see that if we double |H| we only need an additional 27 samples. if we refactor the equation we get `m = orignial_val + (2 / epsilon) * np.log(2)' which comes to 27

QUESTION 3: Plot m as a function of epsilon in [0.01, 0.2]



we clearly see m increase in inverse proportion to epsilon. hence, to halve the error, we need to double the number of samples.

QUESTION 4: Plot m as a function of delta in [10^-4, 0.1] (log scale)



we clearly see that curve follows an exact logarimic scale. hence, to increase the confidence 10 fold, we need only increase the numerator by a factor of log(10), making this scale well.