we want to find gradiant $h(\sigma)$

evaluated at:

$$f(\boldsymbol{\sigma}) = \begin{bmatrix} \sigma_1 \cdot \sigma_2 \\ \sigma_1^2 + \sigma_2^2 \\ \sigma_1 \end{bmatrix}$$

$$\boldsymbol{\sigma} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

by the chainrule:

$$h'(\sigma) = g'(r(\sigma)) \cdot r'(\sigma)$$

$$\downarrow$$

$$h'(\sigma) = g'(r(\sigma)) \cdot f'(\sigma)$$

$$= \frac{3}{2} \|r(\sigma)\| \cdot r(\sigma) \cdot f'(\sigma)$$

$$= \frac{3}{2} \|r(\sigma)\| \cdot J_r(\sigma)^T \cdot r(\sigma)$$

let us evaluate:

$$\sigma = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad y = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$f(\sigma) = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \rightarrow f'(\sigma) = \begin{pmatrix} 2 & 1 \\ 2 & 4 \\ 1 & 0 \end{pmatrix}$$

$$\downarrow$$

$$r(\sigma) = \begin{matrix} 1 \\ 4 \\ 0 \end{matrix}$$

$$h'(\sigma) = \frac{3}{2} \cdot \left\| \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix} \right\| \cdot \begin{pmatrix} 2 & 2 & 1 \\ 1 & 4 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$$

$$\boxed{= \frac{3}{2} \cdot \sqrt{17} \cdot \begin{pmatrix} 10 \\ 17 \end{pmatrix}}$$