Some perspective on large numbers

Let's be honest: it can be hard to have an intuitive sense of large numbers. In an evolutionary sense, we were never exposed to numbers more than dozens or hundreds, and so we never got good at thinking with such large numbers[1] After a certain number of zeros, everything gets compressed in our thinking to "big"! A million? A billion? A trillion? They're just much larger than I can count. Or are they?

[1] Not until the invention of mathematics beyond simply counting anyway!

Have you ever tried counting to a million? I'm guessing you have better things to do with your time, but using the power of Math$_{\text{TM}}$ we could figure out how long it would take if you *did* count to a million, and a billion, and so on. So let's figure this out.

Once per second

If we make the simplifying assumption that you can count one number per second then counting to a million is pretty straight-forward: it takes a million seconds!

Is that not intuitive? Then let's try converting that to, say, hours[2].

[2] Notice that all of the units cancel out.

$$1,000,000 \text{ numbers} \times \frac{1 \text{ second}}{\text{number}} \times \frac{\text{minute}}{60 \text{ seconds}} \times \frac{\text{hour}}{60 \text{ minutes}} = 277.7\bar{7} \text{ hours}$$

Well that's helpful, right? No? Still not intuitive? Well let's convert it to days, then.

$$1,000,000 \text{ numbers} \times \frac{1 \text{ second}}{\text{number}} \times \frac{\text{minute}}{60 \text{ seconds}} \times \frac{\text{hour}}{60 \text{ minutes}} \times \frac{\text{day}}{24 \text{ hours}} =$$

I'll let you do the calculation. I'm so nice, right?

Then please repeat the calculation for a billion and a trillion, converting to units (e.g., days, weeks, …) that are more intuitive (e.g., less than a thousand).

But I count faster!

I agree, you can probably count faster than one number second (up to a point anyway). So let's see how much you can reduce this time to count to a million.

Please time yourself counting from one to one-hundred. Then convert that into a value for seconds/number. Now redo the calculation for how long it would take you to count to a million if you were able to keep up this same pace.

Now a billion

Let's repeat these calculations for a *billion*. And then a *trillion*.

Now do you have some sense for how large these numbers are? Remember, moving from a million to a billion is an increase of a thousand. Moving from a million to a trillion is an increase of a thousand-thousand, or a million. In other words, a trillion is a million-million. It is a much, much, much bigger number.

But why?

Why do we care? Because in science we often end up dealing with large numbers. And we need to be able to work with them. So, for instance, if I were to ask you if there are more humans on earth than (human) cells in the human body, what would you say?

The answer is that, as of this writing, there are an estimated 8,004,778,263 people on earth and an estimated 37.2×10^{12} cells in an average human body[3].

[3] This is a lovely little description of how they estimated this number... obviously no one wanted to *count* them all!

How many virus particles might be in an infected person? For SARS-CoV-2 one study estimated a person might carry between 1 billion and 100 billion virus particles (or virions). This is a lot, but not nearly as many virus particles as cells (as is implied in the very wrong headline of this news piece on the study). Amazingly the authors then estimated the *mass* of all of the SARS-CoV-2 virions in all of humans. Any guesses?[4]

[4] No, I'm not telling you! Follow the links for the answer.

Equations are tools...and you can be a clever tool user

The same way we can get a grip on large numbers, we can also play with equations of population growth to develop some intuition for how they work. Indeed, I've found that the best way to demystify equations is to plug in some numbers and do some calculations. The more the better! So...

A challenge

Let's think about a growing SARS-CoV-2 population. If we assume an infection starts with $N_0 = 1$ infectious virion and grows to a billion virions in the span of five days, what must its replication rate be?

Let's assume continuous, constant replication (and ignore the immune system fighting back and all of the other complexities), so we can use the exponential model of growth:

$$N(t) = N_0 e^{rt}$$

Start by assigning the known values I've given you to the variables in this equation. (Hint: we know $t = 5$ days, right?) Then remember that these are *equations* that can be manipulated any which way you like[5]. You should be able to estimate r. Give it a shot! (And check your work by plugging your estimate of r back into the equation to see if it gets you the right result.)

[5] But preferably according to the rules of mathematics.

Repeat the calculations assuming the population grows to ten-billion virions. Or starts from a founding population of 10 virions.

A final challenge

Now let's consider the number of cells in a human newborn baby. I found an estimate[6] of 1.25×10^{12} cells. We also know that the newborn began from a single cell roughly nine months earlier. So... if we assume that the cells all divide in two, in discrete steps[7], how many *times* does that one cell need to divide to produce the 1.25 trillion it is born with?

[6] Apparently based on a pretty old paper, so_ caveat emptor_

[7] This is only true for a little while, but let's run with this assumption.

It may be helpful to recall this equation:

$$N(t) = N_0 \times \lambda^t,$$

since we are assuming discrete time steps. Can you figure it out? Is it more or less than you expected?