Solving *Hanoi* with ACT-R

Cognitive Modeling 2024–2025

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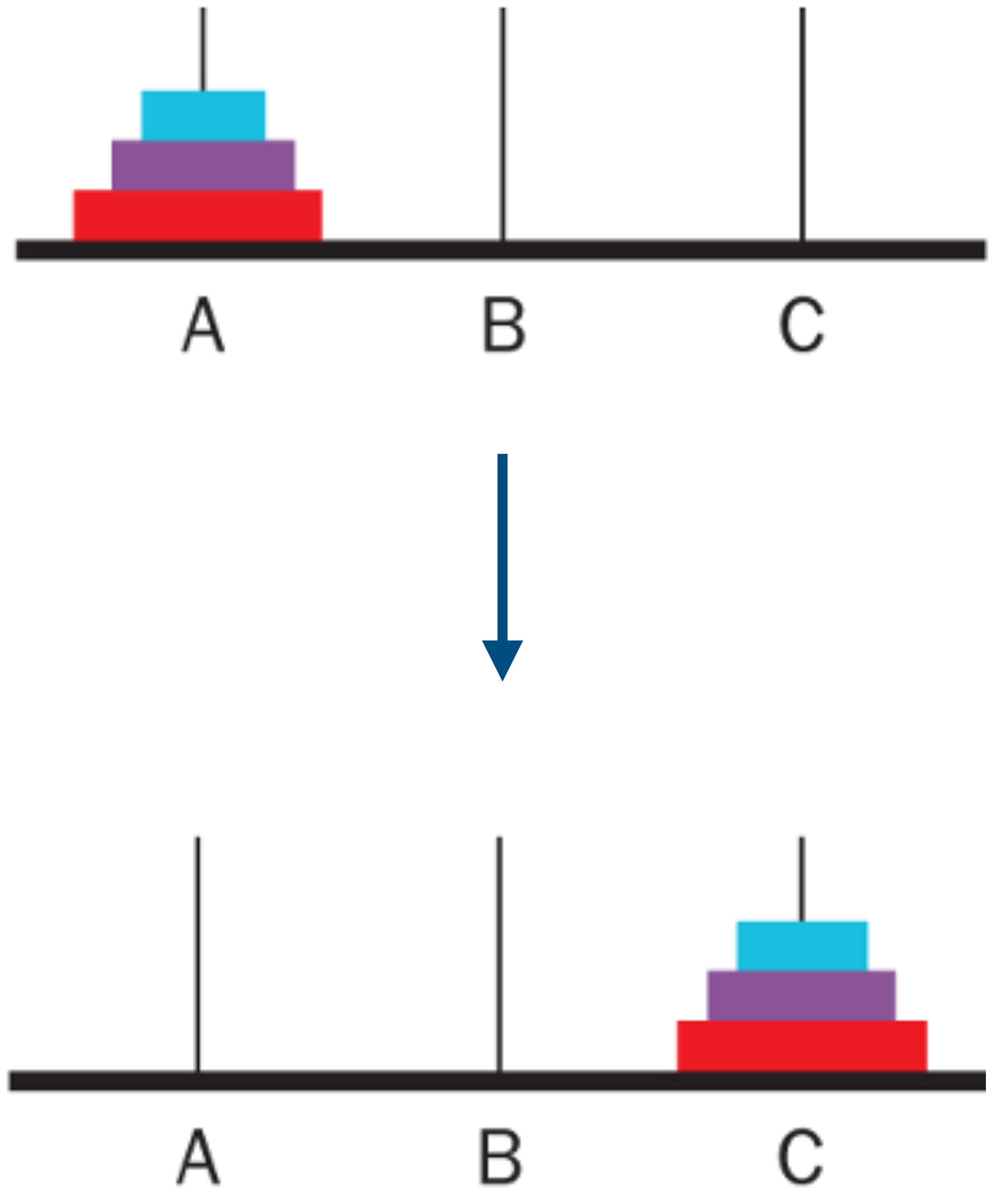
Introducing *Hanoi*

Tower of Hanoi

Information-processing approach to problem solving

- Move the discs from peg A to peg C.
- Only one disc can be moved at a time.
- Discs can only be moved if they are not below another disc.
- Larger discs cannot be placed on top of smaller discs.



States

Information-processing approach to problem solving

- How is a **state** defined?
- Dependent on the problem at hand.
- In Tower of Hanoi a state is a **configuration of the discs.**

Operators

Information-processing approach to problem solving

- You can transition from one state to another using **operators**.
  - e.g. move the small disc from peg A to peg B.
- You can get from the initial state to the goal state by sequentially applying operators.
- The question then is, **which** operators should we apply, and in which order?

States

Information-processing approach to problem solving

- **Initial** state
  - What is the state when we **start**?
- **Intermediate** state
  - Each state you encounter on the way from the initial state to the goal state.
- **Goal** state
  - What is the **required** state?
  - When do we say we have **solved** the problem?

Definitions

Information-processing approach to problem solving

- **State space** or **problem space**
  - All possible states the system can be in.
- Action space
  - All possible actions (operators) that can be taken to move from one state to the next.

State space complexity

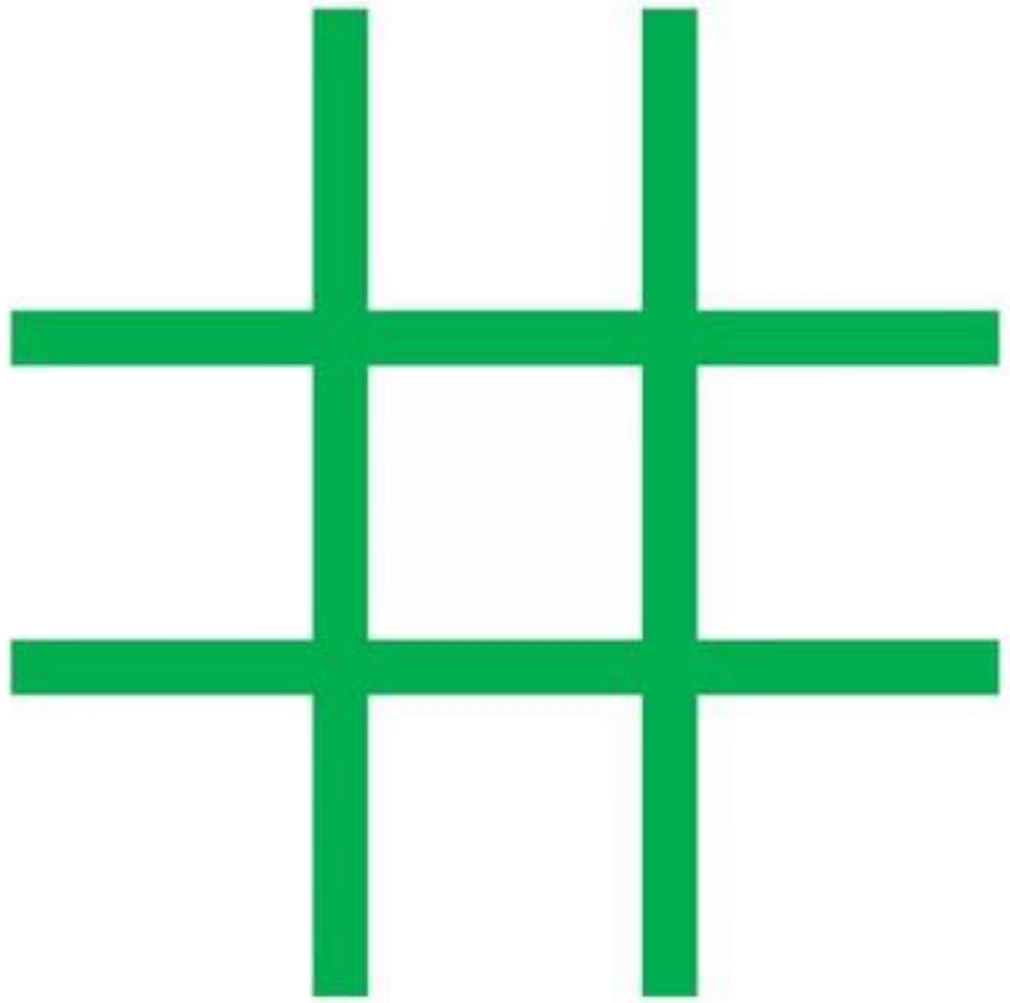
Information-processing approach to problem solving

- **How many possible states** are there in a certain problem?
- If the state space is larger, it will take more effort to find a solution.
- The main contribution of Newell and Simon was this **formal** and **objective** way of determining the difficulty of problems.
- The **state space complexity** is the $\log_{10}$ (order of magnitude) of the number of possible states.

State space complexity

Information-processing approach to problem solving

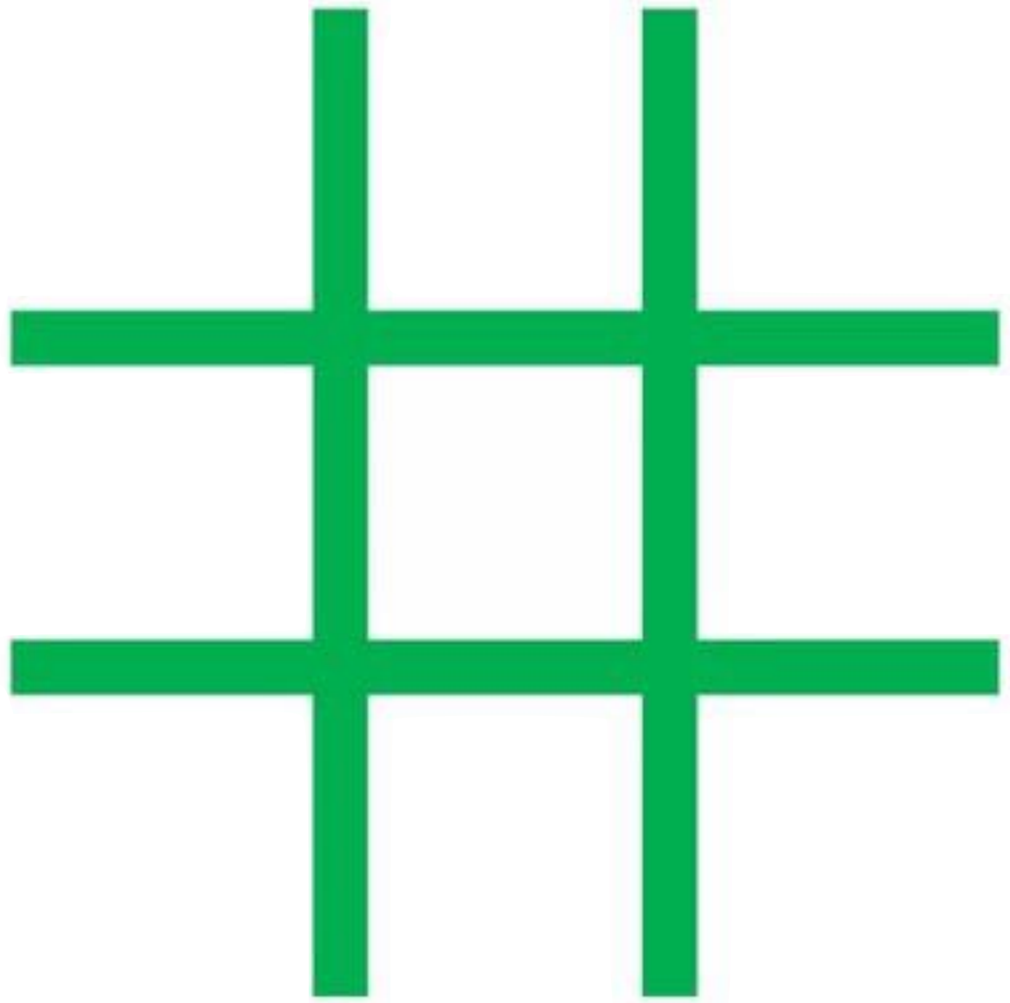
- Tic-tac-toe is played on a 3×3 grid.
- Players are assigned X or O and take turns, putting an X or an O in one of the cells.
- The game ends when one player gets three in a row or if all cells are filled.



State space complexity

Information-processing approach to problem solving

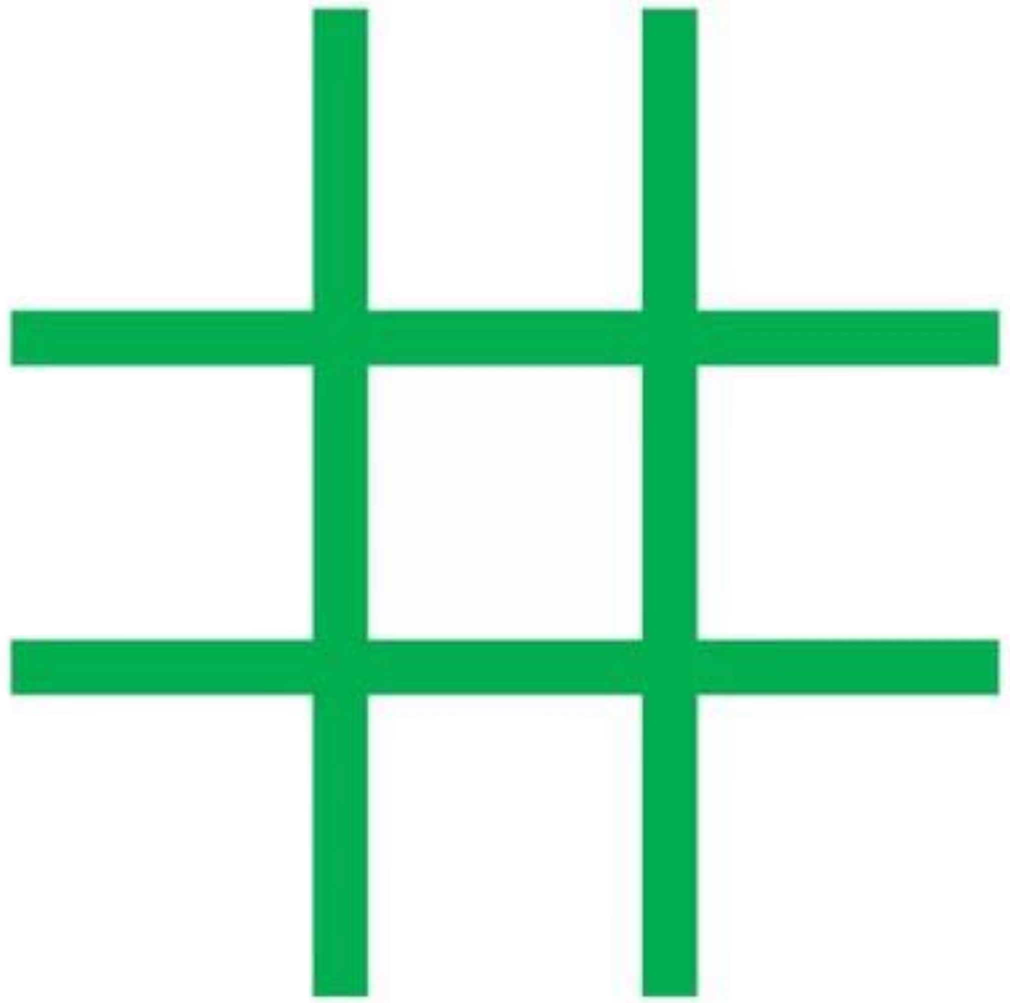
- What is the state space complexity of tic-tac-toe?
- Each cell can contain X, O, or empty.
- The number of possible states can then be found by $3^9 = 19{,}683$ giving a state space complexity of $\log_{10}(19{,}683) \approx 4$.



State space complexity

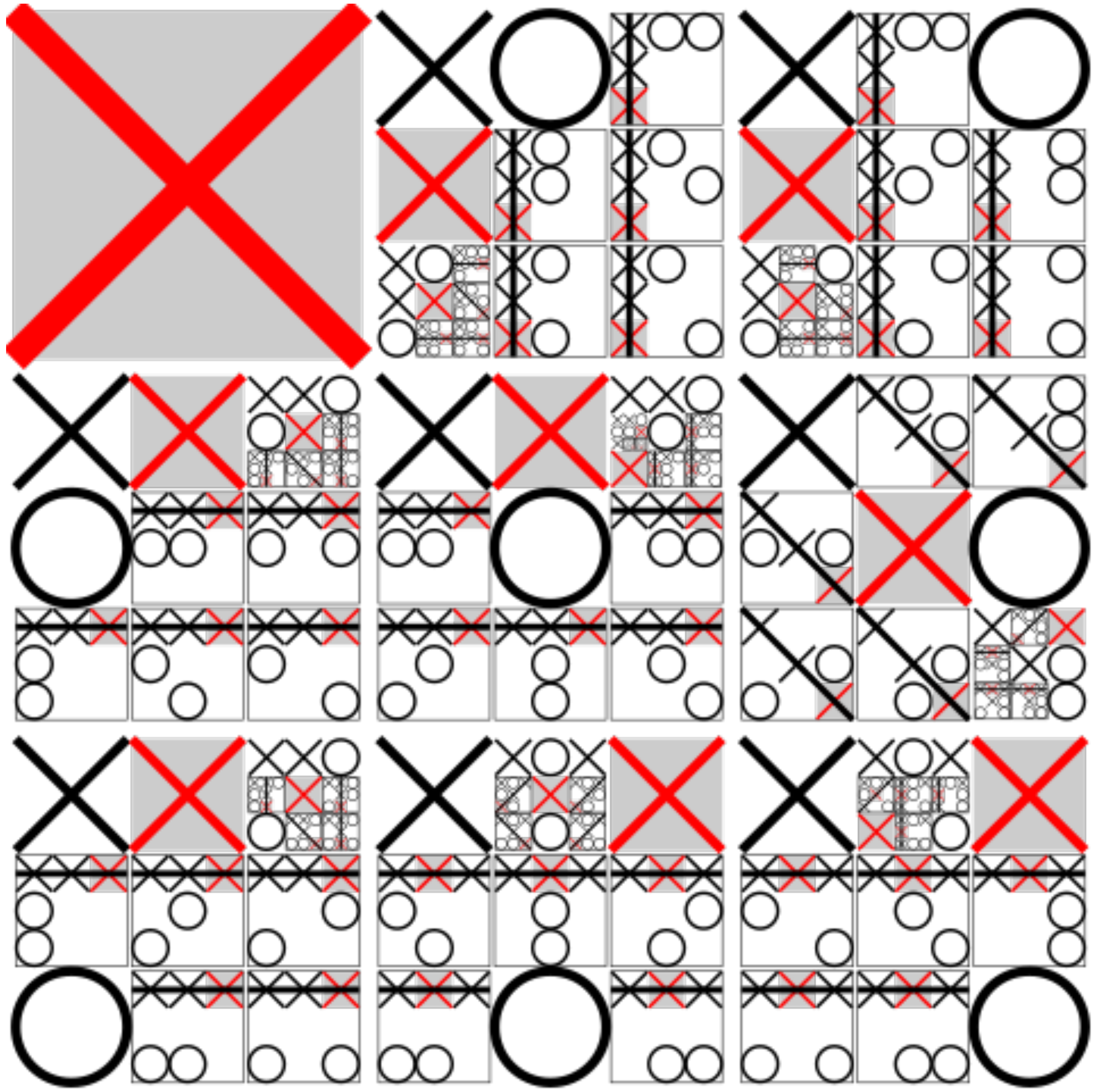
Information-processing approach to problem solving

- Many **illegal** positions.
  - All Xs or Os, etc.
- Also, **rotations** are considered unique.
- When we remove rotations, reflections, and illegal states, we get to a state space of size 5,478 or a complexity of $\log_{10}(5{,}478) \approx 3$.



State space complexity

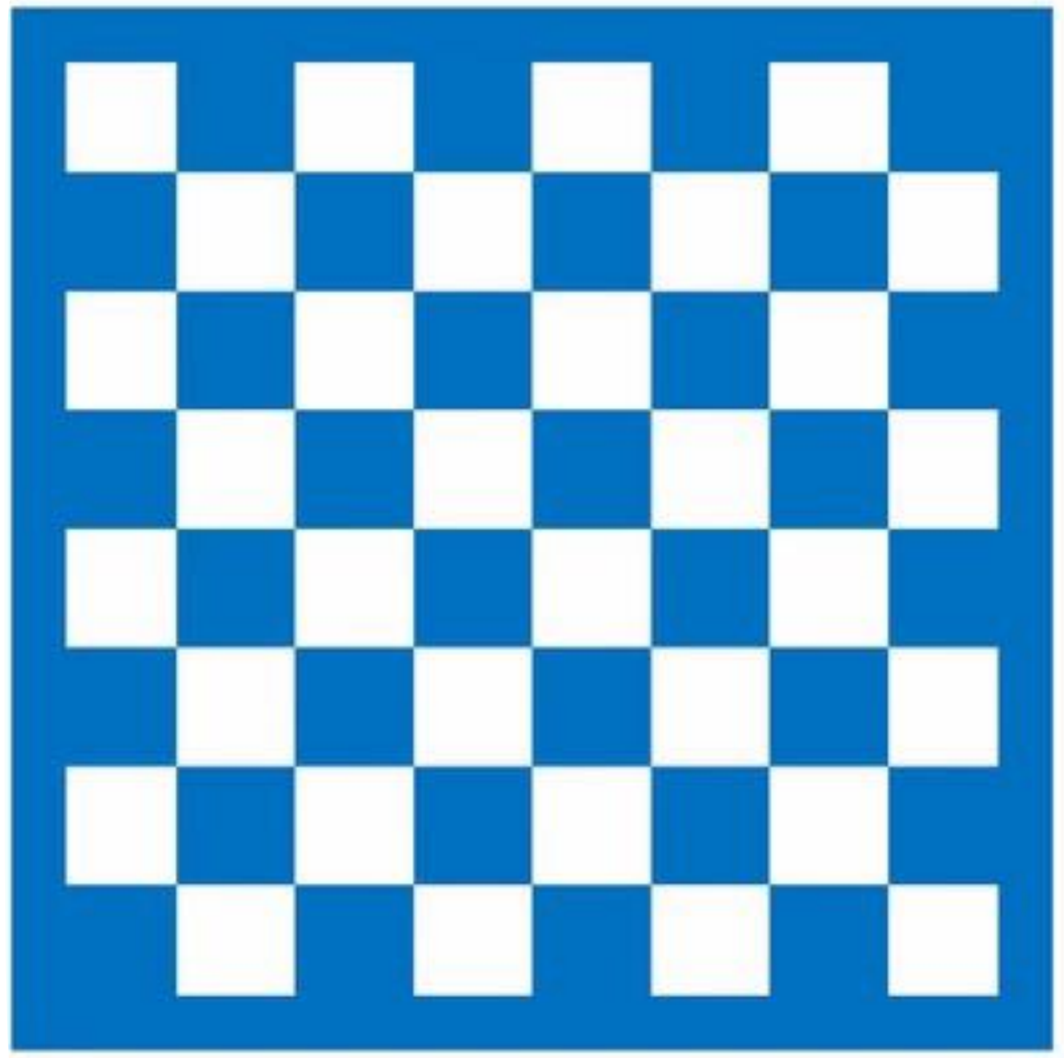
Information-processing approach to problem solving



State space complexity

Information-processing approach to problem solving

- What about **chess?**
- State space complexity of around 47.
- Go has a complexity of 170.



State space complexity

Information-processing approach to problem solving

game	state space complexity
tic-tac-toe	3
Connect Four	13
checkers	20
chess	47
backgammon	20
go	170

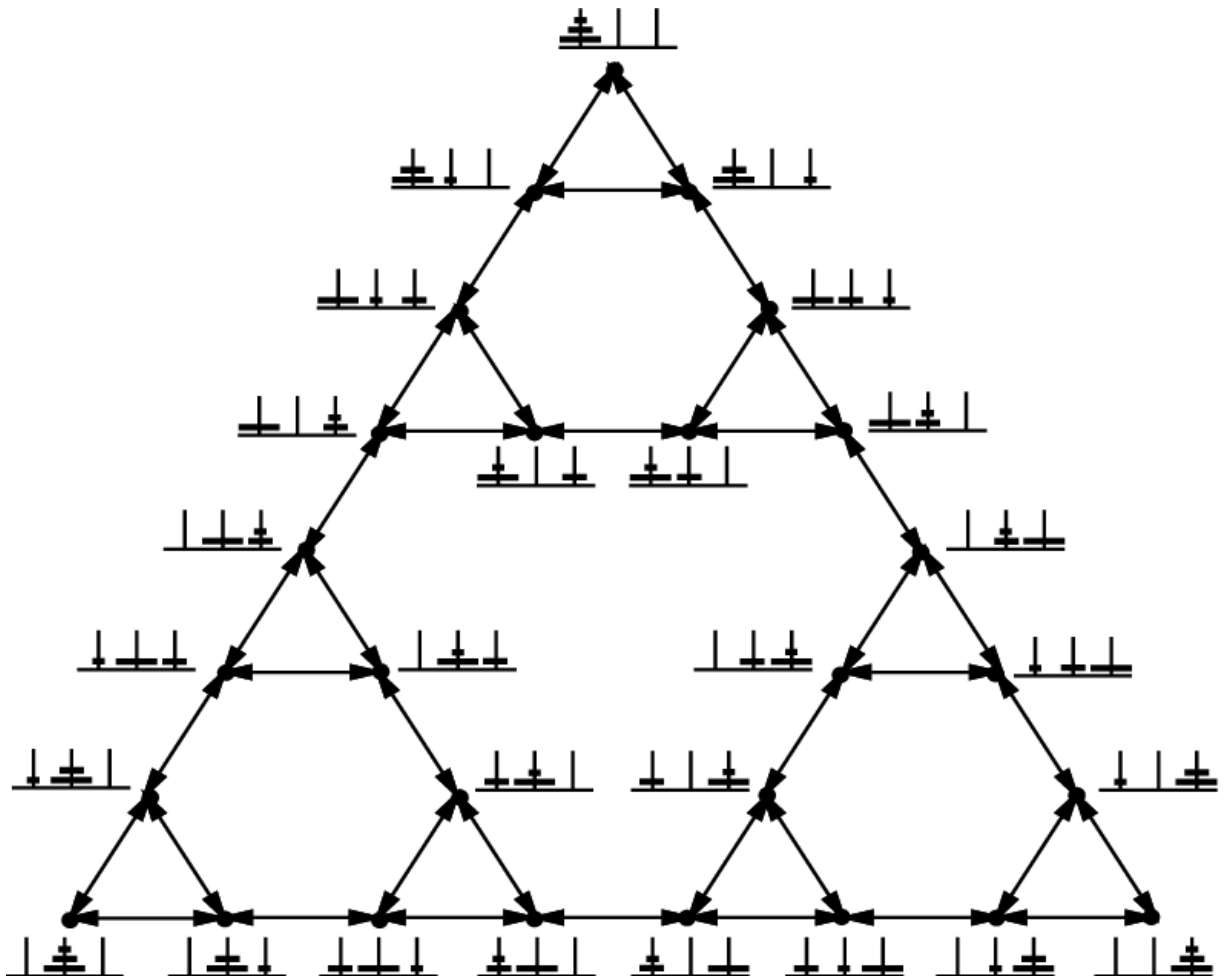
State space complexity

Information-processing approach to problem solving

- What about **Tower of Hanoi?**
- Three discs can be on one of three pegs, so state space is of size $3^3 = 27$ with a complexity of $\log_{10}(27) \approx 1$.
- We can even draw the complete state space!

State space complexity

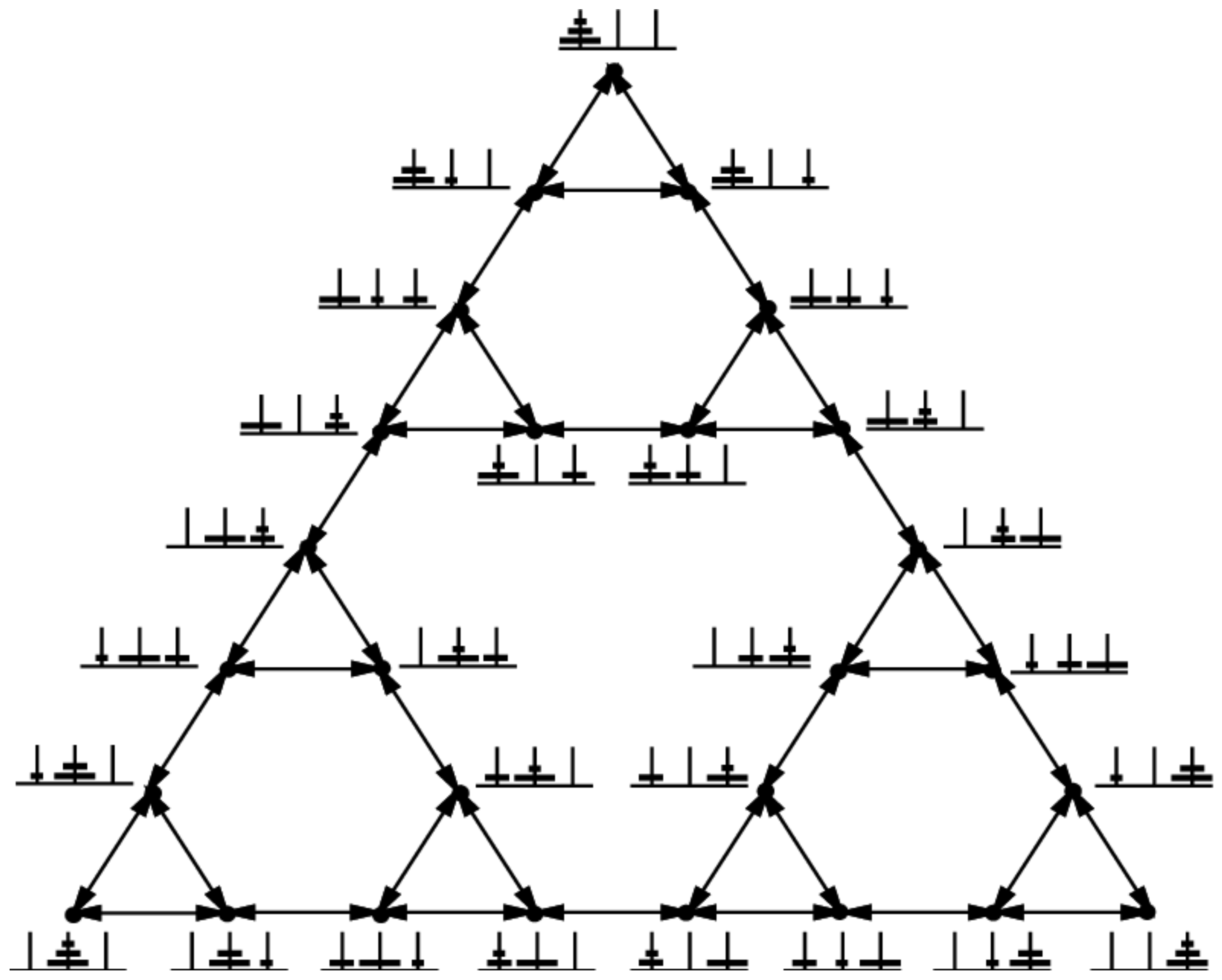
Information-processing approach to problem solving



Solving *Hanoi*

Information-processing approach to problem solving

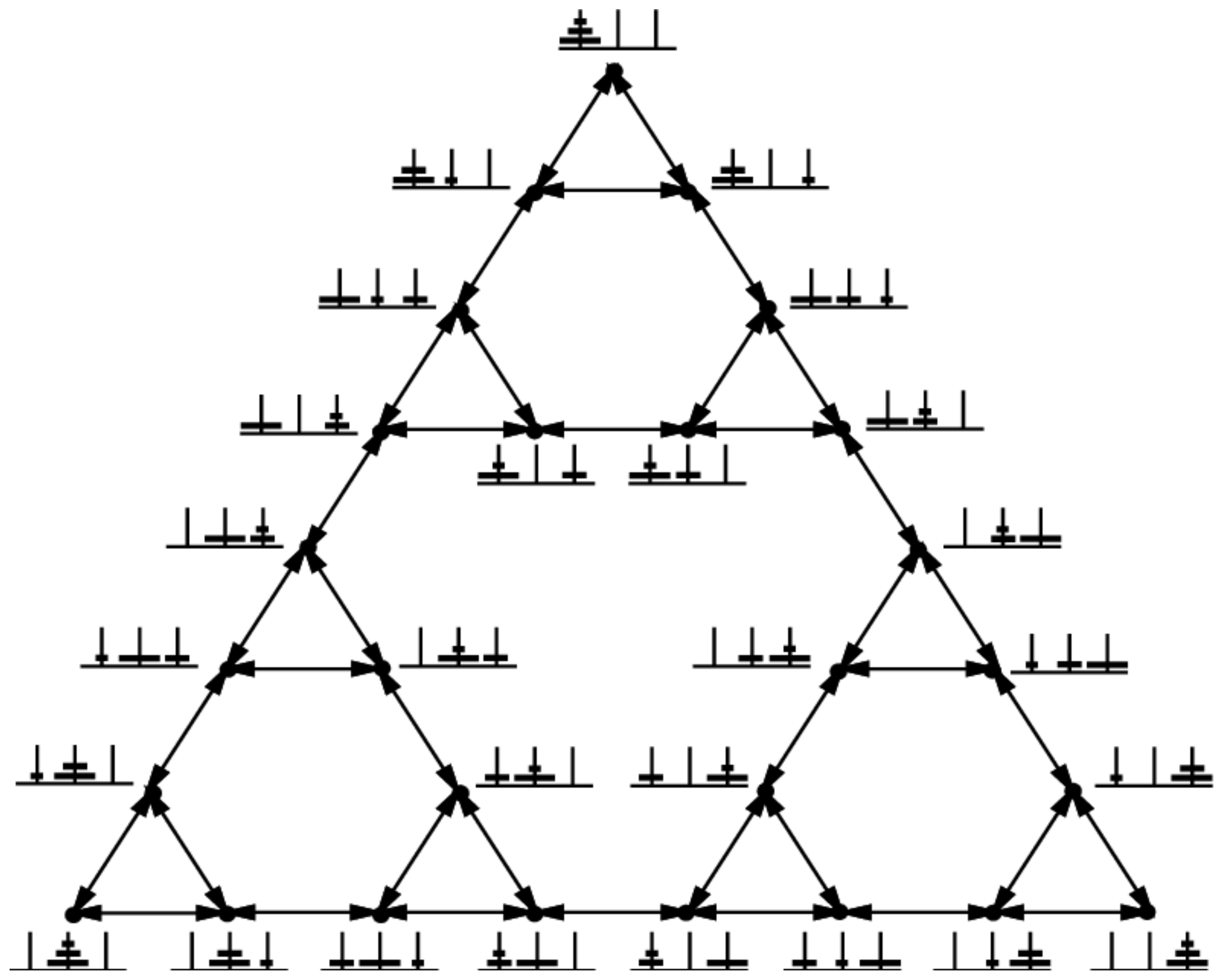
- In order to solve this problem, we find the shortest path from the initial state to the goal state.
- Notice that this only works for small problems!
- For larger problems, humans seem to use **hill-climbing-like** search techniques.



Solving *Hanoi*

Information-processing approach to problem solving

- Defining subgoals.
  - What are suitable subgoals?
- From the initial position, there are only two possible next states.
- Which one to choose?



The assignment (lab report): Solving Hanoi using ACT-R

Solve *Tower of Hanoi* using the ACT-R framework

Assignment

- How to map a real-world problem to a computational framework.
- Create groups of ~3 students.
- For a passing grade, your implementation does **not** need to resemble human decision making, it should merely be able to successfully solve *Hanoi* from the initial state.
- If you explain how humans solve Hanoi, and take that as inspiration for your solution, you will receive bonus marks.
- We will refer to the pegs as [A, B, C], and the disks from large to small [1, 2, 3].

Solve *Tower of Hanoi* using the ACT-R framework

Assignment

- For every step, print both action and state to console:
  - `“Disk x was moved to peg y.”`
  - `“Peg A has disks [], peg B has disks [], peg C has disks [].”`
- Use your own creativity and ingenuity to come up with a solution!