Assignment #2

ELEC 5606

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Problem

A frequency modulated signal with $\Delta\omega = 2\pi(5x10^5)$ rad/s is applied to second order loop. As modulating frequnency increases, the phase error raches maximum of $\Omega = 2\pi(10^3)$ rad/s. At this same modulating frequency, the output u_2 provides a transfer function magnitude of $\frac{m_o}{m_i} = 2$dB.

Determine ω_n and ζ.

Solution

Solving for ω_n: From Blanchard: max power occurs at $x = 1 = \frac{\Omega}{\omega_n}$

Therefore $\omega_n = \Omega = 2\pi(10^3)$ rad/s.

$$\boxed{\textbf{Answer:} \quad \omega_n = 2\pi(10^3) rad/s}$$

Solving for ζ:

$\frac{m_o}{m_i} = |H(j\Omega)| = \sqrt{\frac{\omega_n^4 + 4\zeta^2\omega_n^2\Omega^2}{(\omega_n^2 - \Omega^2)^2 + 4\zeta^2\omega_n^2\Omega^2}} = 10^{\frac{2}{20}} = 1.259$

$$\begin{aligned} 1.259 &= \sqrt{\frac{\omega_n^4 + 4\zeta^2\omega_n^4}{4\zeta^2\omega_n^4}} \\ &= \sqrt{\frac{1+4\zeta^2}{4\zeta^2}} \end{aligned}$$

$$(1.259)^2 = \frac{1+4\zeta^2}{4\zeta^2}$$

Solving for ζ yields:

$$\boxed{\textbf{Answer:} \quad \zeta = 0.664}$$

Problem

A signal with an initial frequency offset of $5\pi(10^6)$ rad/s is applied to a loop having a synchronization bandwidth, $K = 4\pi(10^6)$ rad/s. The loop then tunes to a frequency offset of $3\pi(10^6)$rad/s.

Determine the type of loop.

Solution

This is a first order filter. Because initial offset, $\Delta\omega$, is greater than K, this loop will never reach lock. Instead, the loop exhibits frequency pulling, biasing the VCO toward the input and settling to a steady offset.

We can predict what this will be with the following:

$$\omega_{out} = \sqrt{\Delta\omega^2 - K^2)}$$
$$= \sqrt{(5\pi(10^6))^2 - (4\pi(10^6))^2}$$
$$= 3\pi(10^6)$$

Which exactly matches the problem description.

Problem

The phase detector output, u_1, for a first order loop was observed as the loop acquired lock to a 5MHz offset input.

Determine K_1 and K_3.

Solution

From the plot:

at t = 0, u_1 = 1V

at t = $0.035\mu s$, u_1 = -2.67V

at t= $0.07\mu s$, u_1 = 0V

and steady state u_1 = 2V

Solving for K_3:

$$u_{1sst} = 2V = \frac{\Omega_0}{K_3}$$
$$K_3 = \frac{2\pi(5e6)}{2v}$$
$$K_3 = 5\pi(10^6) \text{ rad/s/V}$$

$$\boxed{\textbf{Answer:} \quad K_3 = 5\pi(10^6) rad/s/V}$$

Solving for K_1:

$$K_1 = |u_{1sst}| = 2.67 \text{ V/rad}$$

$$\boxed{\textbf{Answer:} \quad K_1 = 2.67V/rad}$$

The above values are confirmed in MATLAB with the following plot:

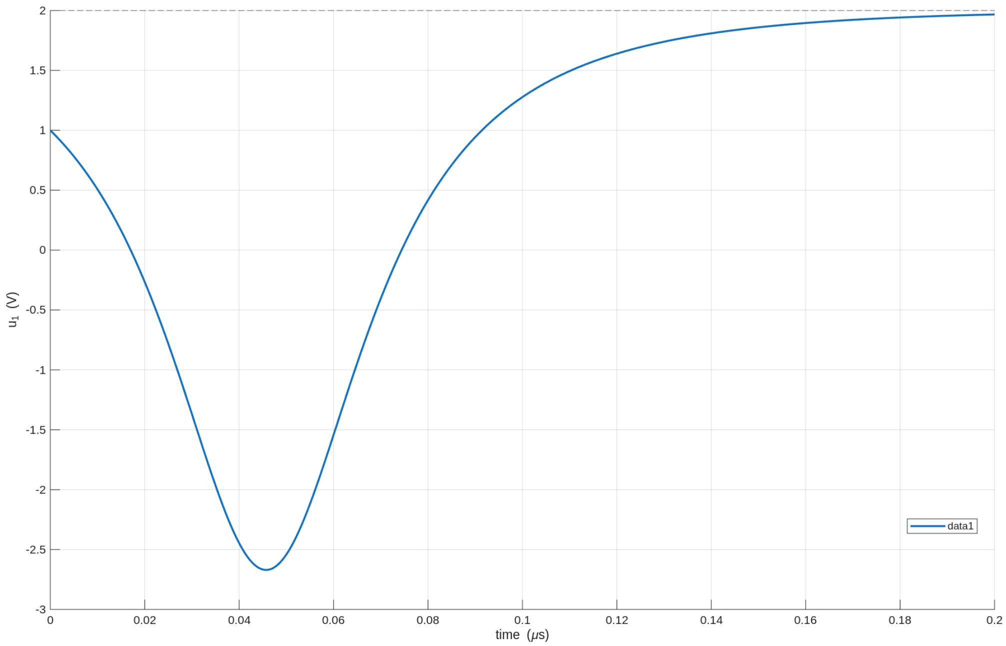

Figure 1: Plot of u_1 with K_1 and K_3 set to 2.67 and $5\pi(10^6)$ respectively

Problem

An incoming signal to a receiver is frequency modulated by a signal occupying the spectrum of 100Hz to 5KHz. The signal has power, S = -100dBm, the incoming Gaussian noise has one-sided PSD of $N_o = -168 \frac{dBm}{Hz}$.

Using a second order loop with filter $\frac{1+\tau_2 s}{1+\tau_1 s}$ such that $\zeta = \frac{1}{\sqrt{2}}$, determine K and and ω_n.

Determine Δf_i such that $\Delta\phi_{max} = 0.5$ radians.

Determine VCO phase variance, $\sigma^2_{\phi o}$ and SNR at discriminator output.

Solution

The solution process was largely based off of p.186 from Blanchard:

Solving K and ω_n:

I choose $\omega_n = 2\pi(5e3)$ rad/s, matching the upper bound of the occupying spectrum.

$$\boxed{\textbf{Answer:} \quad \omega_n = 2\pi(5e3) rad/s}$$

Subsequently, making K $\gg \omega_n$ simplifies the further analyses. Therefore I let K$= 100\omega_n = 2\pi(5e5)$ rad/s.

$$\boxed{\textbf{Answer:} \quad K = 2\pi(5e5) rad/s}$$

Solving for Δf_i:

$$\Delta\phi_{max} = \frac{\Delta\omega_i}{2\zeta\omega_n} = 0.5$$
$$\Delta f_i = \frac{0.5(2)\frac{1}{\sqrt{2}}2\pi(5e3)}{2}$$
$$\Delta f_i = 3.54 \text{ KHz}$$

$$\boxed{\textbf{Answer:} \quad \Delta f_i = 3.54 KHz}$$

Solving for $\sigma^2_{\phi o}$:

$$\sigma^2_{\phi o} = \frac{N_o}{2S}(2B_n)$$

where N$_o = -168 dBm = 1.585e - 17$ $\frac{mW}{Hz}$ and S $= -100 dBm = 1e - 10$ dBm.

Need to first find $2B_n$

Because K $\gg \omega_n$, $2B_n$ simplifies to the following:

$$2B_n \approx \frac{\omega_n}{4\zeta}(1 + 4\zeta^2)$$
$$2B_n = 33321.6$$

And therefore:

$$\sigma^2_{\phi o} = \frac{1.585e-17}{2(1e-10)}(33321.6)$$
$$2.64 \times 10^{-3} \text{ rad}^2.$$

$$\boxed{\textbf{Answer:}\quad \sigma^2_{\phi o} = 2.64 \times 10^{-3} rad^2}$$

Solving for SNR at Discriminator Output:

$$(\tfrac{S}{N})_D = \tfrac{\Delta f_i^2}{2}\tfrac{\sqrt{2}(2S)}{N_o \pi F_m^3}$$
$$= \tfrac{(3.54e3)^2}{2}\tfrac{\sqrt{2}(2)(1e-10)}{(1.585e-17)\pi(5000)^2}$$
$$= 284.73 \rightarrow 24.5\text{dB}$$

$$\boxed{\textbf{Answer:}\quad (\frac{S}{N})_D = 24.5 dB}$$

Problem

A signal applied to a PLL demodulator has phase modulation with a PSD of $S(\frac{W}{Hz}) = \frac{5.75\times10^{-9}}{f^2+1}$. This signal is accompanied by white additive gaussian noise, n$'$(t), with PSD of $S(\frac{W}{Hz}) = 4 \times 10^{-17}$.

Determine optimum TF (defined by Weiner), the loop geometry and K.

Solution

From Blanchard, $\text{H}_{opt} = 1 - \frac{\sqrt{\eta}}{\Psi(j\omega)}$

I begin by first switching to angular frequency:

$\omega = 2\pi f$

$$S_{\phi_i}(f) = \tfrac{5.75\times10^{-9}}{f^2+1}$$
$$S_{\phi_i}(\omega) = \tfrac{1}{2\pi}\tfrac{5.75\times10^{-9}}{(\frac{\omega}{2\pi})^2+1}$$
$$= \tfrac{2\pi 5.75e-9}{\omega^2+(2\pi)^2}$$

Now form $\text{S}_y(\omega)$:

$$S_y(\omega) = S_{\phi_i}(\omega) + S_{n'}(\omega$$
$$= \tfrac{2\pi 5.75e-9}{\omega^2+(2\pi)^2} + \tfrac{4e-17}{2\pi}$$

Here I let $\omega_0 = 2\pi$, $\text{A} = 2\pi 5.75e - 9$ and $\text{S}_0 = \frac{4e-17}{2\pi}$

$$\begin{aligned} S_y(\omega) &= \tfrac{A}{\omega^2+\omega_0^2} + S_0 \\ &= \tfrac{A+S_0(\omega^2+\omega_0^2)}{\omega^2+\omega_0^2} \\ &= \tfrac{S_0(\frac{A}{S_0}+\omega^2+\omega_0)}{\omega^2+\omega_0^2} \end{aligned}$$

Here I now let $\omega_a^2 = \frac{A}{S_0} + \omega_0^2$

$$S_y(\omega) = \tfrac{S_0(\omega_a^2+\omega^2)}{\omega^2+\omega_0^2}$$

Now, forming $\Psi(j\omega)$:

$$\Psi(j\omega) = \sqrt{S_0}\tfrac{j\omega+\omega_a}{j\omega+\omega_0}$$

$$\Psi(s) = \sqrt{S_0}\tfrac{s+\omega_a}{s+\omega_0}$$

And forming H_{opt}:

$$\begin{aligned} H_{opt} &= 1 - \tfrac{\sqrt{S_0}}{\sqrt{S_0}\frac{s+\omega_a}{s+\omega_0}} \\ &= 1 - \tfrac{s+\omega_0}{s+\omega_a} \\ &= \tfrac{\omega_a-\omega_0}{s+\omega_a} \end{aligned}$$

With $\omega_a = \sqrt{\frac{A}{S_0} + \omega_0^2} = 75.3\text{e}3$ rad = 12KHz.

Therefore:

$$\boxed{\textbf{Answer:} \quad H_{opt} = \frac{75.3e3 - 2\pi}{s + 75.3e3}}$$

This is a First Order Loop with DC gain nearly exactly at 1.

And for Synchronization BW:

$$\boxed{\textbf{Answer:} \quad K = 12KHz}$$