Assignment #1

ELEC 5606

Jesse Levine (101185127)

1

Problem

Require 50MHz first order PLL with a synchronous range of $\pm 5\,\text{kHz}$. A sinusoidal phase detector with $K_1 = 2\,\text{V/rad}$ and phase error $\Delta\phi = 0.2$ rad.

Find VCO sensitivity, K_3, and VCO frequency stability, $\frac{\Delta f_0}{f_0}$.

Solution

Solving for K:

synch. range = K

$$2\pi \times 5e3 = K_1 K_3$$

$$K_3 = 5\pi \times 10^3 \text{ rad/s/V} \Rightarrow 2.5 \text{ kHz/V}.$$

$$\boxed{\textbf{Answer:} \quad K_3 = 2.5kHz/V}$$

Solving for VCO frequency stability, $\frac{\Delta f_0}{f_0}$.

$$\Delta\phi = 0.2 \text{ rad} = \tfrac{2\pi\Delta f_0}{K}$$

Solving for f_0:

$$f_0 = 1 \text{ kHz}$$

$$\tfrac{\Delta f_0}{f0} = \tfrac{1kHz}{50MHz} = 2 \times 10^{-5}$$

$$\boxed{\textbf{Answer:} \quad \frac{\Delta f_0}{f0} = 2 \times 10^{-5}}$$

2

Problem

From Bode Plot of a PLL determine type of PLL, τ_1, τ_2 and K. Determine loop's ω_n and ζ and phase safety margin.

Solution

Type of loop: Second Order loop $\Rightarrow$ LPF with Phase-Lead Correction

Solving for τ_1:

At $\omega = 1$ rad/sec $\Rightarrow \omega = \frac{1}{\tau_1}$

$$\tau_1 = 1\text{s}$$

$$\boxed{\textbf{Answer:} \quad \tau_1 = 1s}$$

Solving for τ_2: At $\omega = 50$rad/sec $\Rightarrow \omega = \frac{1}{\tau_2}$

$$\tau_2 = 0.02\text{s}$$

$$\boxed{\textbf{Answer:} \quad \tau_2 = 0.02s}$$

Solving for K:

At $\omega = 1000$rad/sec $\Rightarrow \omega = \frac{K\tau_2}{\tau_1}$

$$\text{K} = 50 \times 10^3$$

$$\boxed{\textbf{Answer:} \quad K = 50 \times 10^3}$$

Solving for ω_n:

$$\omega_n^2 = \frac{K}{\tau_1}$$

$$\omega_n = 223.6 \text{ rad/s}$$

$$\boxed{\textbf{Answer:} \quad \omega_n = 223.6 rad/s}$$

Solving for ζ:

$$2\zeta\omega_n = \tfrac{1+K\tau_2}{\tau_1}$$

$$\zeta = 2.24$$

$$\boxed{\textbf{Answer:}\quad \zeta = 2.24}$$

Solving for phase safety margin:

$$\angle L(j\omega) = -90^\circ - \arctan(\omega\tau_1) + \arctan(\omega\tau_2)$$

Let ω be at the 0dB cross point: $\omega = 1000$ rad/s

$$\angle L(j\omega) = -90^\circ - 89.94 + 87.19 = -92.8^\circ$$

$$\text{Phase Margin} = 180 - 92.8 = 87.2^\circ \Rightarrow 1.52 \text{ rad.}$$

$$\boxed{\textbf{Answer:}\quad PhaseMargin = 87.2^\circ}$$

Confirmed using MATLAB below in Figure 1

3

Problem

the open loop TF of a PLL utilizing a LPF with phase lead correciton is given by:

$$G(j\omega) = \tfrac{K(1+j\omega\tau_2}{j\omega(1+j\omega\tau_1)}$$

Using Root Locus method plot LHS of real part of denominator of the Closed Loop TF when it is forced to be equal to 0 + 0j versus σ and explain why the loop is uncondotionally stable.

Solution

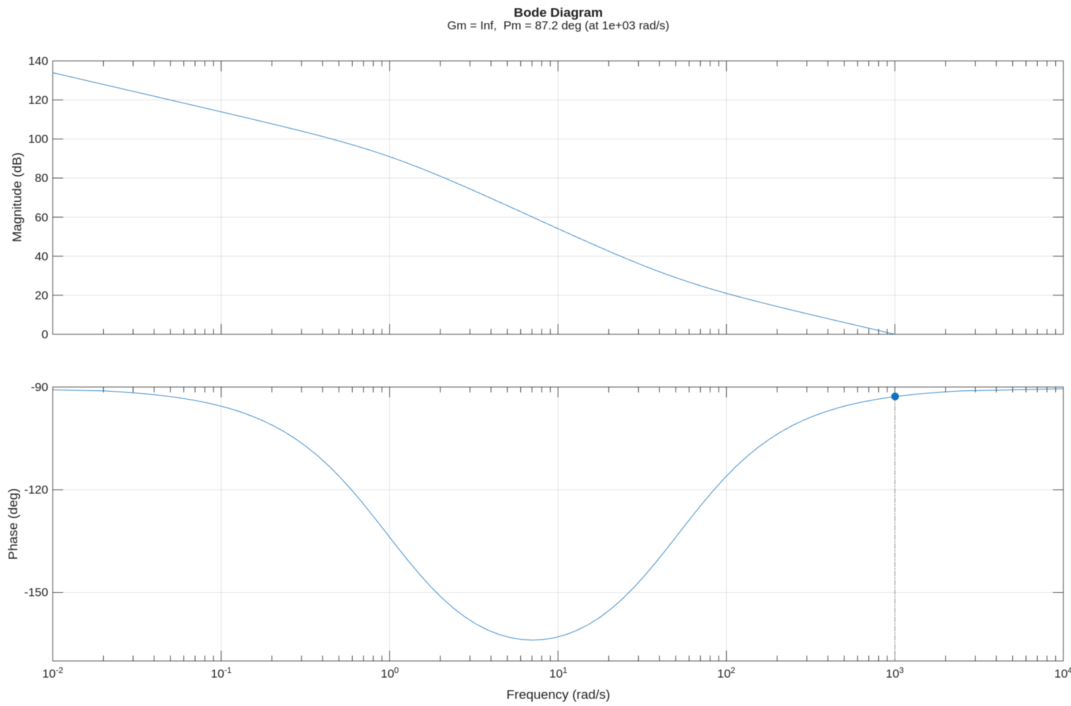

Figure 1: Bode Plot of PLL from computed values

$$G(s) = \frac{K(1+s\tau_2}{s(1+s\tau_1)} \Rightarrow H(s) = \frac{G(s)}{1+G(s)}$$

The denominator of H(s) is therefore:

$$\mathrm{D(s)} = \tau_1 s^2 + (1 + K\tau_2)s + K$$

let $s = \sigma + j\omega$

Real:

$$\tau_1(\sigma^2 - \omega^2) + \sigma(1 + K\tau_2) + K = 0$$

Imaginary:

$$2\tau_1\sigma\omega + \omega(1 + K\tau_2) = 0j$$

We must isolate K and for nontrivial roots we therefore obtain the following for the Imaginary part:

$$2\tau_1\sigma + 1 + K\tau_2 = 0$$

and therefore:

$$K = \frac{-2\tau_1\sigma - 1}{\tau_2}$$

Subbing this back into the Real part:

$$\tau_1(\sigma^2 - \omega^2) + \sigma(1 + \frac{-2\tau_1\sigma - 1}{\tau_2}\tau_2) + \frac{-2\tau_1\sigma - 1}{\tau_2}\tau_2 = 0$$

$$\tau_1\sigma^2 - \tau_1\omega^2 + \sigma(-2\tau_1\sigma) + \frac{-2\tau_1\sigma - 1}{\tau 2} = 0$$

$$\omega^2 = \sigma^2 + \frac{\sigma(-2\tau_1\sigma}{\tau_1} + \frac{-2\tau_1\sigma - 1}{\tau_1\tau_2}$$

$$\omega^2 = -\sigma^2 + \frac{-2\tau_1\sigma - 1}{\tau_1\tau_2} \ (1)$$

Setting $\tau_1 = 1$ and $\tau_2 = 0.02$, the above is plotted in MATLAB as seen in Figure 2.

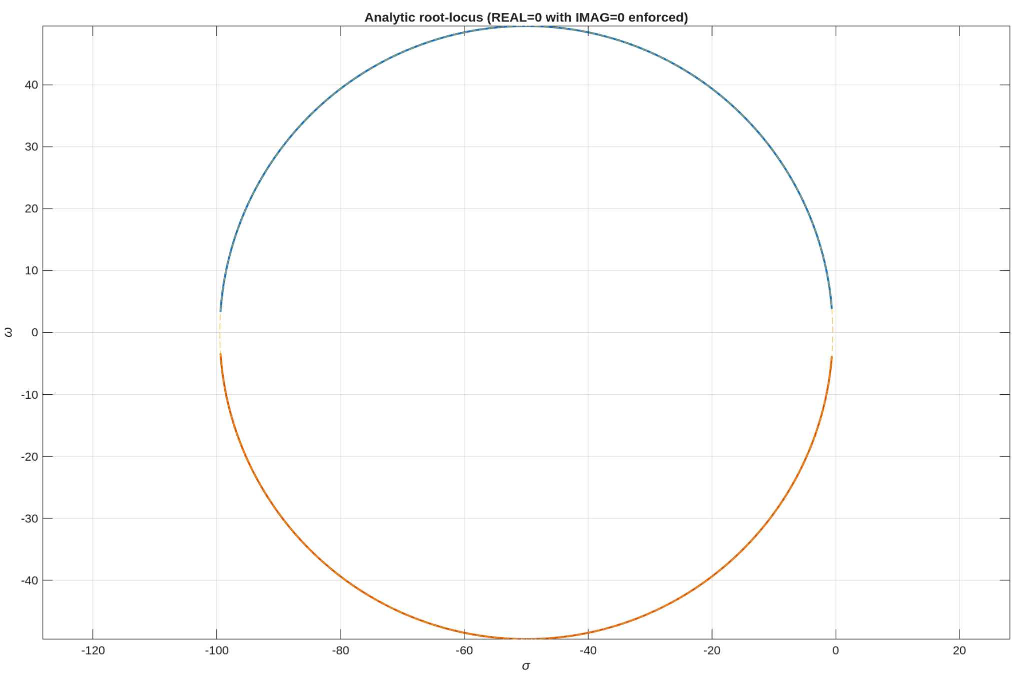

Figure 2: LHS of real part of D(s) plotted vs σ

From Figure 2 and from equation 1 for any K>0 (with $\tau_1 > 0$, $\tau_2 > 0$ and $\tau_1 \geq \tau_2$), we see that the loop is unconditionally stable.

The entire circle lies strictly in the left half-plane, never crossing the imaginary axis.

4

Problem

When excited by a frequency step, $\Delta\omega$, of 200 rad/sec, the phase response for a second order loop incorporating a LPF with phase lead correction with damping constant of 1 is shown in the diagram.

Determine loop gain, K, and natural frequency, ω_n.

Solution

From the phase response: Steady state error, SSE = 0.1 rad

Solving for K:

$$K = \frac{\Delta\omega}{SSE} = 2000$$

$$\boxed{\textbf{Answer:} \quad K = 2000}$$

Solving for ω_n: From Blanchard P.93, eqn. 5.10:

$$\phi(t) = \frac{\Delta\omega}{K} + \frac{\Delta\omega}{\omega_n} e^{-\omega_n t}(\omega_n t - \frac{\omega_n^2}{K} t - \frac{\omega_n}{K})$$

This becomes:

$$\phi(t) = \frac{\Delta\omega}{K} + \Delta\omega e^{-\omega_n t}(t - \frac{\omega_n}{K} t - \frac{1}{K})$$

Differentiating $\phi(t)$ for $\dot{\phi}(t)$:

$$\dot{\phi}(t) = \Delta\omega e^{-\omega_n t}(1 - \omega_n t(1 - \frac{\omega_n}{K}))$$

Setting the above to 0 and solving for ω_n when $\Delta\omega$ = 200, t = 8ms and K = 2000:

$$\omega_n = 133.97 \text{ rad/s}$$

$$\boxed{\textbf{Answer:} \quad \omega_n = 133.97 rad/s}$$

The above are confirmed with MATLAB in Figure 3

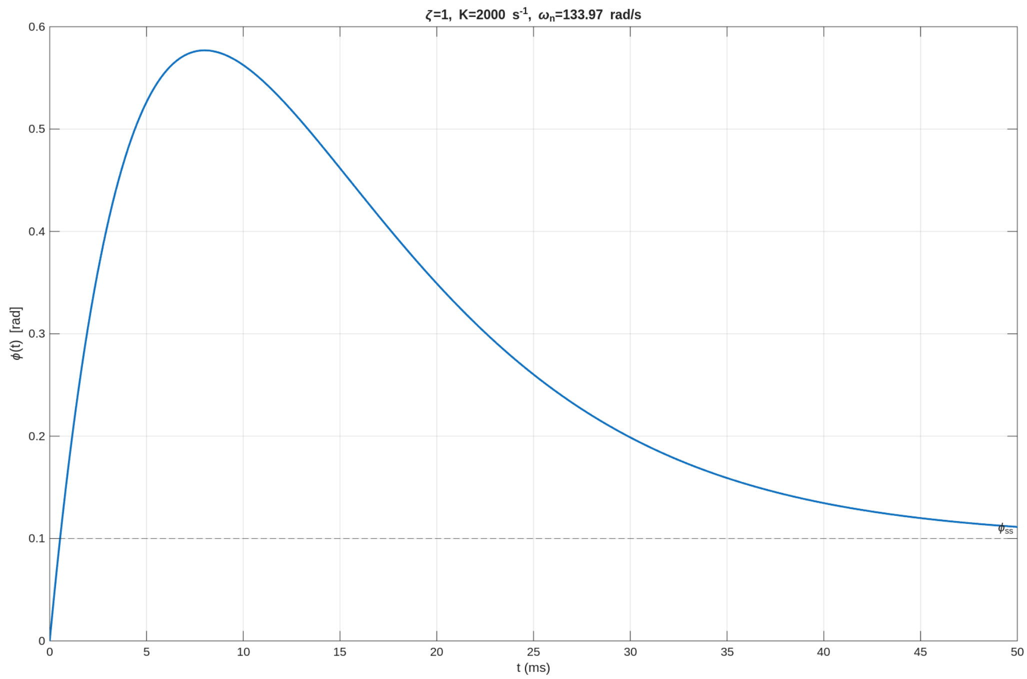

Figure 3: Phase Response of Loop with K = 2000 and ω_n = 133.97 rad/s

5

Problem

Second order loop with integrator and phase lead correction. Input signal is a 4MHz frequency modulated signal with a frequency deviation of 500 Hz. Maximum phases error, $\Delta\phi$, is 0.35 rad and occurs at modulating frequency Ω = 1kHz.

Determine ζ, ω_n and for what frequency deviation does the maximum phase serror remain below 0.2 rad?

Solution

We know the maximum occurs at ω_n, therefore:

$$\omega_n = 2\pi(1kHz) = 6283 \text{ rad/s}$$

Solving for ζ:

$$\Delta\phi_M = \frac{\Delta\omega_i}{s\zeta\omega_n}$$

$$0.35 = \frac{s\pi \times 500}{2\zeta(6283)} \Rightarrow \zeta = 0.714$$

$$\boxed{\textbf{Answer:} \quad \zeta = 0.714}$$

Now for what $\Delta\omega_i$ does $\Delta\phi$ remain below 0.2 rad:

$$\Delta\omega_i = 2\Delta\phi_M\zeta\omega_n = 1794 \text{ rad/ or } 286 \text{ Hz.}$$

$$\boxed{\textbf{Answer:} \quad \Delta\omega_i = 286Hz}$$