Assignment #3

ELEC 5606

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Problem

QPSK Costas Loop with input signal:

$$s(t) = x(t)\cos(\omega_i t + \theta_i) - y(t)\sin(\omega_i t + \theta_i)$$

Can the QPSK Costas Loop be used to syncrhonize onot a BPSK signal? if so, what signals emerge at the I and Q arms? How are they related?

Solution

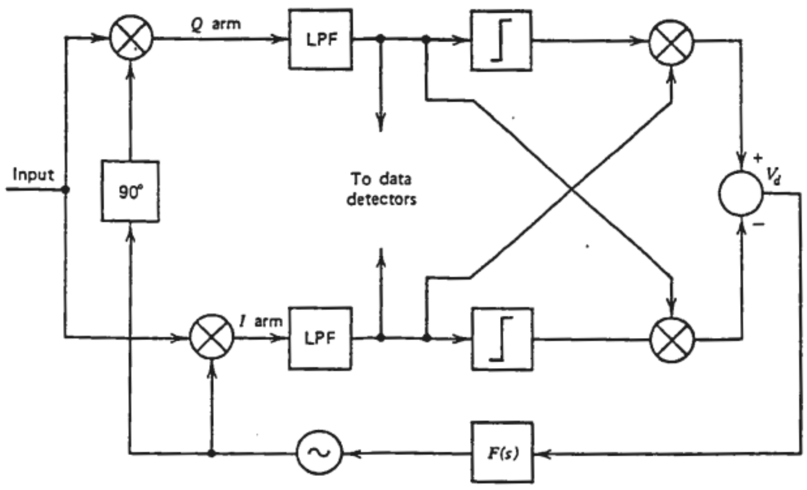

Figure 1: QPSK Costas Loop from course notes. This synchronizer recovers the carrier by splitting the input into in-phase (I) and quadrature (Q) arms, using cross-over multiplication between the linear and hard-limited outputs to generate the error voltage (V_d)1. The limiters provide the essential fourth-order nonlinearity required to drive the VCO to lock onto one of the four stable phases ($0°, 90°, 180°, 270°$).

The QPSK Costas Loop can be used to track/syncrhonize a BPSK input signal since the BPSK phases ($0°$ and $80°$) align with the QPSK stable lock points at $0°, 90°, 180°$ and $270°$. However there will be a fourfold ambiguity since the loop can also lock at $90°$ or $270°$ as well.

On top of that the QPSK loop introduces inefficiencies to handle the BPSK input signal. Aside from useless hardware, there is a penalty to the loop's SNR:

The loop's error signal,

$$v_d \approx sgn(I) \cdot Q - sgn(Q) \cdot I$$

relies on a Q-arm that, for BPSK, only contains noise. The hard limiter, sgn(Q), amplifies the Gaussian noise to a constant level which is then multiplied by a strong signal, I, injecting a high power noise term directly into the control signal which increases the noise jitter and therefore makes it harder for the loop to lock at lower SNR.

We can see this mathematically with the following derivation of each arm:

Given the input signal:

$$s(t) = x(t)\cos(\omega_i t + \theta_i) - y(t)\sin(\omega_i t + \theta_i)$$

Given BPSK signal: x(t) = m(t) and the y term: y(t) = 0. Therefore, including noise, s(t) becomes:

$$s(t) = m(t)\cos(\omega_i t + \theta_i) + n(t)$$

I-arm: I-arm multiplies input by recovered cosine carrier $2\cos(\omega_i t + \theta_o)$ and then Low Pass Filters it.

$$I(t) = LPF[(m(t)\cos(\omega_i t + \theta_i) + n(t)) \cdot 2\cos(\omega_i t + \theta_o)]$$
$$= LPF[2m(t)\cos(\omega_i t + \theta_i)\cos(\omega_i t + \theta_o) + 2n(t)cos(\omega_i t + \theta_o)]$$

Utilizing $2\cos A\cos B = \cos(A - B) + \cos(A + B)$

$$I(t) = LPF[m(t)(cos(\theta_i - \theta_o) + cos(2\omega_i t + \theta_i + \theta_o)) + 2n(t)cos(\omega_i t + \theta_o)]$$

Filtering out the high frequency terms and noise going to baseband:

$$I(t) = m(t)\cos(\theta_\epsilon) + n_I(t)$$

where $\theta_\epsilon = (\theta_i - \theta_o)$

Similarily, for the Q-arm:

$$Q(t) = -m(t)sin(\theta_\epsilon) + n_Q(t)$$

Now, at lock when $\theta_\epsilon \approx 0$:

$$I(t) = m(t) + n_I(t)$$

and

$$Q(t) = n_Q(t)$$

From these results above we can see that the Q-arm will only contain Gaussian noise which will be amplified by the hard limiter and then multiplied by signal in I-arm, thereby by using a QPSK Costas Loop to synchronize to a BPSK signal we force Q-arm to be a high power noise generator.

...probably could've answered this more concisely.. :)

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Problem

BPSK Costas Loop wioth branch filters having a noise equivalent BW of $\mathrm{B}_i = 10^3$Hz and a loop noise equivalent BW $2\mathrm{B}_L = 10^2$Hz is in lock at t = 0 with zero static phase error.

The incoming signal power is -80dBm and the one sided noise PSD is -120dBm/Hz.

Find the VCO phase variance, the bit rate the loop can receive without deviating significantly from ideal performance and the bit error probability due to the noise performance for the Costas Loop at this bit rate.

Solution

Solving for VCO phase variance:

Based on P.14-13 in the Notes we can use the σ_ϵ^2 derived from the squaring-loop analysis as the phase jitter for Squaring loop and Costas loop are identical (in certain scenarios).

$$\sigma_\epsilon^2 = 4(\tfrac{B_L N_o}{A^2})(1 + \tfrac{1}{2\rho_i})$$

where $\rho_i = \frac{A^2/2}{N_o B_i}$ and $\frac{A^2}{2} = S$

First solving for ρ_i:

$$\rho_i = \tfrac{S}{N_o B_i}$$
$$S = -80\text{dBm} = 1e-8,\ N_o = -120\text{dBm/Hz} = 1e-12\text{mW/Hz}$$
$$\rho_i = \tfrac{1e-8}{1e-12 \cdot 10^3}$$
$$= 10$$

Now solving for σ_ϵ^2:

$$\sigma_\epsilon^2 = 4(\tfrac{B_L N_o}{2S})(1 + \tfrac{1}{2\rho_i})$$
$$= 4(\tfrac{(10/2)^2(1e-12)}{2(1e-8)})(1 + \tfrac{1}{2(10)})$$
$$= 0.0105\ \text{rad}^2$$

$$\boxed{\textbf{Answer:}\quad \sigma_\epsilon^2 = 0.0105 rad^2}$$

Solving for Bit Rate:

From P.14-26, using eqn. 42, we know:

$$\delta = \tfrac{1}{B_L T} \tfrac{1}{1 + \frac{B_i T}{2E_b/N_o}} > 5$$

The above needs to be greater than 5 for practically no degradation.

We can rearrange: $\frac{E_b}{N_o} = \frac{SN_o}{f_b}$

Therefore:

$\frac{B_i T}{2E_b/N_o} = \frac{B_i f_b}{f_b \cdot 2S/N_o} = \frac{B_i}{2S/N_o}$

And subsequently:

$$\delta = \frac{1}{B_L T}\frac{1}{1+\frac{B_i}{2S/N_o}}$$
$$= \frac{1}{50T}\frac{20}{21} = \frac{2}{105T}$$
$$\frac{2}{105T} > 5$$
$$T = \frac{2}{525} \rightarrow f_b = 262.5\text{bps}$$

$$\boxed{\textbf{Answer:} \quad f_b = 262.5bps}$$

Solving for Bit Error Probability:

At $f_b = 262.5$bps, we can solve this two ways: either graphically or using Q function.

$P_b = Q(\sqrt{\frac{2E_b}{N_o}})$

$\frac{E_b}{N_o} = \frac{S/N_o}{f_b} = 38.1 = 15.8\text{dB}$

Using Q function:

$$P_b = Q(\sqrt{2(38.1)}) = 1.3e-18$$

Negligible.

$$\boxed{\textbf{Answer:} \quad P_b = 1.3e-18}$$

We can also confirm using Fig.12-25 on P.14-26:

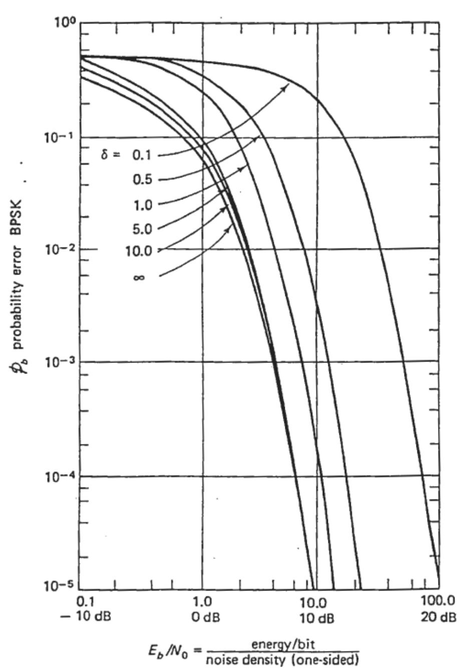

Fig. 12-25 BPSK error probability $\mathcal{P}_b$ versus E_b/N_0 for various values of δ, the normalized bandwidth factor [Lindsey, 1966]

Tracing the $\delta = 5$ curve, it is off the graph at 15.8dB.

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Problem

The bit error probability of a 10^6bps detector incroporating an Early-Late gate synchronizer must be better than 10^{-4} for an SNR of 9dB (7.943).

The synchronizer PD characteristic is linear, providing an output which ranges from $+\pi/2$ to $-\pi/2$ over time-offsets ranging from -T/4 sec to T/4 sec.

The synchronizer incorporates an Integrator with correction to realize a damping constant of 0.5 and the VCO has sensitivity of $2\pi \times 10^5$rad/sec/V.

Determine max. noise equivalent BW, ω_n and the time constants, τ_1, τ_2.

Solution

Solving for max noise equivalent BW, $2B_L$

From eqn. 21 on P.15-10 we can solve for $2B_L$. We will also need to consult Fig.14-10 on P.15-10 to determine σ_ϵ.

$$\sigma_{\epsilon t}^2 = \frac{N_o B_L}{2} = \frac{B_L T}{2E_b/N_o}$$

Rearranging for B$_L$:

$$B_L = \frac{2\sigma_{\epsilon t}^2 E_b/N_o}{T}$$

Now to determine σ_ϵ:

We use Fig.14-10 and also the fact that with $\frac{\sigma_\epsilon}{T} \leq 0.05$ only about 0.5dB of degradation is realized for any bit error probability. Therefore, I use that, 0.05, as my design goal.

We know $T = \frac{1}{10^6}$, SNR $= 9\text{dB} = 7.943 = \frac{E_b}{N_o}$

Therefore:

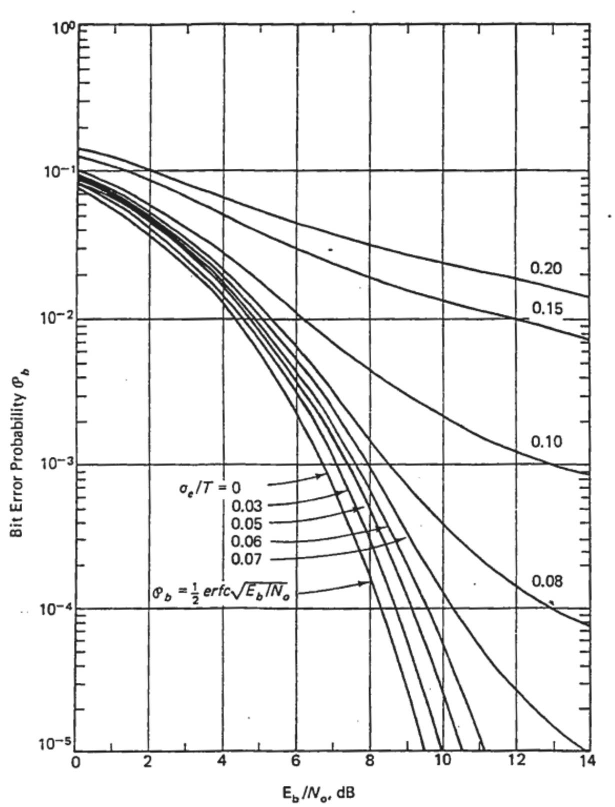

Fig. 14-10 Average probability of bit error versus E_b/N_0 with standard deviation of the symbol sync error σ_ϵ as a parameter (NRZ) [From Lindsey and Simon, 1973, 469, Fig. 9-37]

$$B_L = \frac{2(0.05)^2(7.943)}{10^{-6}}$$

$$= 397154 \text{ Hz}$$

$$2B_L = 79.4\text{kHZ}$$

$$\boxed{\textbf{Answer:} \quad 2B_L = 79.4kHz}$$

Solving for ω_n:

Given that the synchronizer uses an Integrator with correction and $\zeta = 0.5$, we can use equation on P.7-11:

$$2B_L = \frac{\omega_n}{4\zeta}(1 + 4\zeta^2)$$

$$\omega_n = \frac{79.4e3(4(0.5))}{1+4(0.5)^2} = 79.4\text{kHz}$$

$$\boxed{\textbf{Answer:} \quad \omega_n = 79.4kHz}$$

Solving for τ_1 and τ_2:

From P.3-5:

$$\omega_n^2 = \frac{K}{\tau_1}$$

We need to solve for K_1 to get K:

$$K_1 = \frac{dv_d}{d\phi_i}$$

We know dv_d is $\pi/2$ and $d\phi_i = 2\pi\frac{T/4}{T} = \pi/2$

Therefore $K_1 = \frac{\pi/2}{\pi/2} = 1$ V/rad

Solving for τ_1:

$$\tau_1 = \frac{K}{\omega_n^2} = 99.7\ \mu\text{s}$$

$$\boxed{\textbf{Answer:}\quad \tau_1 = 99.7\mu s}$$

And solving for τ_2:

$$2\zeta\omega_n = \frac{K\tau_2}{\tau_1}$$

$$\tau_2 = \frac{2\zeta\omega_n\tau_1}{K} = 12.6\ \mu\text{s}$$

$$\boxed{\textbf{Answer:}\quad \tau_2 = 12.6\mu s}$$

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Problem

An Early-Late Gate employs a LPF with correction as its loop filter with the following TF: $F(j\omega) = \frac{1+j\omega(10^{-2})}{1+j\omega(2)}$. It employs a VCO having a center frequency of 5MHz and a sensitivity of $K_3 = 10^4$ Hz/V.

The branch integrators each have a time constant $T_1 = 50$ nsec and the bit stream applied to the synchronizer is bipolar having amplitude $V_s = \pm 5$ V.

Determine the one-sided syncrhonization BW K(Hz) for the Early-Late Gate, ω_n and ζ, the acquisition BW and acquisition time.

Solution

Solving for K.

$K = K_1 K_3$, we have $K_3 = 10^4$ Hz/V.

We can determine K_1 from P.15-7, using Fig.11-15 where $K_1 =$ slope $\times$ T.

slope $= \frac{A}{T_1}$ and T $= \frac{1}{5e6}$

Therefore:

$$K = \frac{5}{50e{-}9}\frac{1}{536} \times 10^4$$
$$= 200\text{kHz} = 400\pi \text{ krad/sec}$$

$$\boxed{\textbf{Answer:} \quad K = 200kHz}$$

Solving for ω_n and ζ:

To determine ω_n and ζ we will need τ_1 and τ_2 which can be found within the TF of the filter:

$F(j\omega) = \frac{1+j\omega(10^{-2})}{1+j\omega(2)} \rightarrow \tau_1 = 2, \tau_2 = 10^{-2}$

$$\omega_n^2 = \frac{K}{\tau_1}$$
$$omega_n = \sqrt{\frac{400\pi e3}{2}} = 792.7 \text{ rad/sec.}$$

$$\boxed{\textbf{Answer:} \quad \omega_n = 792.7 rad/s}$$

$$2\zeta\omega_n = \frac{1+K\tau_2}{\tau_1}$$
$$\zeta = \frac{1+400\pi e3(10^{-2})}{2(2)(792.7)} = 3.96$$

$$\boxed{\textbf{Answer:} \quad \zeta = 3.96}$$

Which is quite overdamped but stable...will simply lock very slowly.

Solving for Acq. BW:

From P.10-13:

$$\Omega_{acq} = 2\sqrt{K\zeta\omega_n}$$

$$= 125 \text{ krad/sec} \approx 20\text{kHz}$$

$$\boxed{\textbf{Answer:} \quad \Omega_{acq} \approx 20kHz}$$

And solving for T_{acq}:

$$T_{acq} = \frac{\Omega_{acq^2}}{2\zeta\omega_n^3}$$

$$= \frac{(125e3)^2}{2(3.96)(792.7)^3} \approx 4 \text{ sec}$$

$$\boxed{\textbf{Answer:} \quad {}_{acq} \approx 4sec}$$