Notes on the Bi-Conjugate Gradient Method

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1 Without a preconditioner

Found in section 3.6.1 of [1]. Initialize as follows:

$$r = b - Ax, \hat{r} = r, \rho_0 = 1, \hat{p} = p = 0, k = 0. \tag{1}$$

Repeat the following until a termination condition such as $\|r\|_2 < \epsilon \|b\|_2$ is satisfied,

$$k = k + 1 \tag{2a}$$

$$\rho_k = \hat{r}^T r, \beta = \rho_k / \rho_{k-1} \tag{2b}$$

$$p = r + \beta p, \hat{p} = \hat{r} + \beta \hat{p} \tag{2c}$$

$$v = Ap \tag{2d}$$

$$\alpha = \rho_k / (\hat{p}^T v) \tag{2e}$$

$$x = x + \alpha p \tag{2f}$$

$$r = r - \alpha v, \hat{r} = \hat{r} - \alpha A^T \hat{p}. \tag{2g}$$

References

[1] C. T. Kelley, "Iterative Methods for Linear and Nonlinear Equations", SIAM (1995).

[2] http://en.wikipedia.org/wiki/Biconjugate_gradient_method