

$$(AD)^T = DA^T \quad (1)$$

$$(AD)_{ij} = A_{ij}d_j \quad (2)$$

$$(DA^T)_{ij} = d_i(A^T)_{ij} = A_{ji}d_i \quad (3)$$

Find  $d$  such that  $AD$  is symmetric where

$$A = \nabla \times \nabla \times \quad (4)$$

for a non-regular cartesian grid. In other words, find  $d$  such that

$$A_{ij}d_j = A_{ji}d_i, \quad \text{for all } i, j. \quad (5)$$

FIGURE HERE

For the figure above,

$$A_{ba} = -\frac{1}{\Delta x_b \Delta y_b} \quad (6)$$

$$A_{ab} = -\frac{1}{\Delta x_a \Delta y_a} \quad (7)$$

$$d_i \propto \frac{1}{\Delta x_i \Delta y_i} \quad (8)$$

Also, we will have

$$A_{ca} = -\frac{1}{\Delta x_b \Delta z_b} \quad (9)$$

$$A_{ac} = -\frac{1}{\Delta x_a \Delta z_a} \quad (10)$$

$$d_i \propto \frac{1}{\Delta x_i \Delta z_i} \quad (11)$$

Lastly, we already have  $A_{aa'} = A_{a'a}$  for  $a$  and  $a'$  in the same direction.

The conclusion, then, is

$$d_i = \frac{1}{\Delta x_i \Delta y_i \Delta z_i} \quad (12)$$