Contents

Chapter 1
Objective-First Nanophotonic Design

Abstract The abstract for the book.
Introductory paragraph.

1.1 The electromagnetic wave equation

1.1.1 Physics formulation

Let's talk about the electromagnetic wave equation from a physics standpoint. Let's start from Maxwell's equations without currents.

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \tag{1.1}$$

$$\nabla \times H = J + \varepsilon \frac{\partial E}{\partial t} \tag{1.2}$$

$$\nabla \times E = -i\mu_0 \omega H \tag{1.3}$$

$$\nabla \times H = J + i\varepsilon\omega E \tag{1.4}$$

$$\nabla \times \varepsilon^{-1} \nabla \times H - \mu_0 \omega^2 H = \nabla \times \varepsilon^{-1} J \tag{1.5}$$

Talk about reduced dimensions here. Refer reader to appendix.

1.1.2 Numerical formulation

Now let's talk about the electromagnetic wave equation from a computational perspective.

To solve the wave equation on a computer we need to use the Yee grid.

To make things easier we will use weird units.

We also need to take care of boundary conditions.

1.1.3 Solving for H

The wave equation from a mathematical perspective.

$$(\nabla \times \varepsilon^{-1} \nabla \times -\mu_0 \omega^2) H = \nabla \times \varepsilon^{-1} J \tag{1.6}$$

Becomes, with a change of variables.

$$A(p)x = b(p) \tag{1.7}$$

This can be solved directly in 1D and 2D. Special methods needed in 3D. We just do 1D and 2D.

1.1.4 Solving for ε^{-1}

The wave equation from a optimization perspective.

Because scalar multiplication is transitive ($\varepsilon^{-1}(\nabla \times H) = (\nabla \times H)\varepsilon^{-1}$ and $\varepsilon^{-1} J = J\varepsilon^{-1}$)

$$\nabla \times (\nabla \times H)\varepsilon^{-1} - \nabla \times J\varepsilon^{-1} = \mu_0 \omega^2 H \tag{1.8}$$

which we write as

$$B(x)p = d(x) \tag{1.9}$$

Special constraints... (binary)

1.1.5 Insight

Basically, we see that the electromagnetic wave equation is separably linear in H and ε^{-1}

This means that...

1.2 The objective-first design problem

intro

1.2.1 Design objectives

Talk about $f(x)$ and that we are interested in convex ones.

1.2.2 Convexity

Convex optimization quick intro.

1.2.3 Typical design formulation

Typically,

$$\begin{aligned} \underset{x,p}{\text{minimize}} \quad & f(x) && (1.10) \\ \text{subject to} \quad & g(x,p) = 0 && (1.11) \\ & p \in 0,1 && (1.12) \end{aligned}$$

1.2.4 Objective-first design formulation

Objective-first does

$$\begin{aligned} \underset{x,p}{\text{minimize}} \quad & \|g(x,p)\|^2 && (1.13) \\ \text{subject to} \quad & f(x) = f_{\text{ideal}} && (1.14) \\ & 0 \leq p \leq 1 && (1.15) \end{aligned}$$

This is a bi-convex problem, which we solve using an alternating directions method.

1.2.5 Field sub-problem

$$\underset{x}{\text{minimize}} \quad \|A(p)x - b(p)\|^2 \tag{1.16}$$

$$\text{subject to} \quad f(x) = f_{\text{ideal}} \tag{1.17}$$

1.2.6 *Structure sub-problem*

$$\underset{p}{\text{minimize}} \quad \|B(x)p - d(x)\|^2 \tag{1.18}$$

$$\text{subject to} \quad 0 \leq p \leq 1 \tag{1.19}$$

1.3 1D resonator design

We now build up to the objective-first formulation by example. And from a different perspective.

We're going to take a step back and start with the most naive inverse design strategy possible. Then we'll end with objective first. This motivates the need for the alternating directions and objective-first strategy.

1.3.1 *Direct solve of structure*

This is the simplest thing you can do. Basically, set x and solve for p. Also, take out the relaxed constraint on p.

$$\underset{p}{\text{minimize}} \quad \|B(x)p - d(x)\|^2 \tag{1.20}$$

We perfectly satisfy the field but...

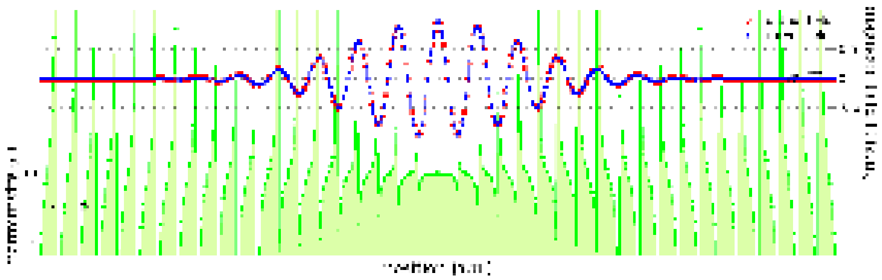

Fig. 1.1 test

1.3.2 Regularized solve of structure

We add a term to try to control p

$$\underset{p}{\text{minimize}} \quad \|B(x)p - d(x)\|^2 + \eta\,\|p - p_0\|^2 \tag{1.21}$$

Better, but a trade-off between field accuracy and structure variation.

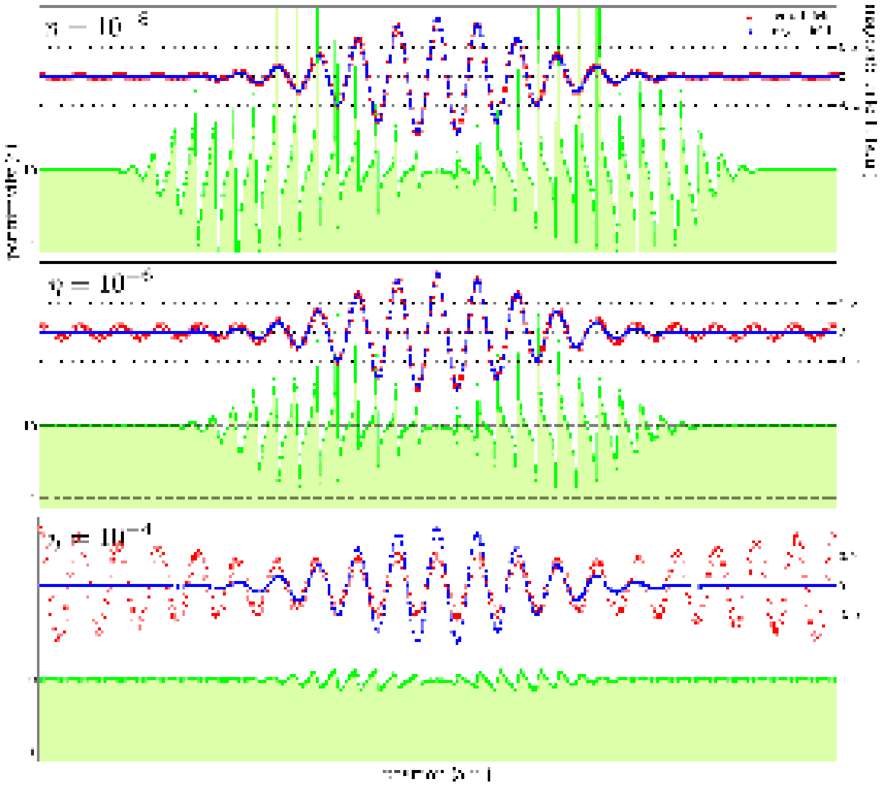

Fig. 1.2 test

1.3.3 Alternating directions solve

Now we actually do both fields.

$$\underset{p}{\text{minimize}} \quad \|B(x)p - d(x)\|^2 + \eta_1\|p - p_{\text{prev}}\|^2 \tag{1.22}$$

$$\underset{x}{\text{minimize}} \quad \|A(p)x - b(p)\|^2 + \eta_2 \|x - x_{\text{prev}}\|^2 \tag{1.23}$$

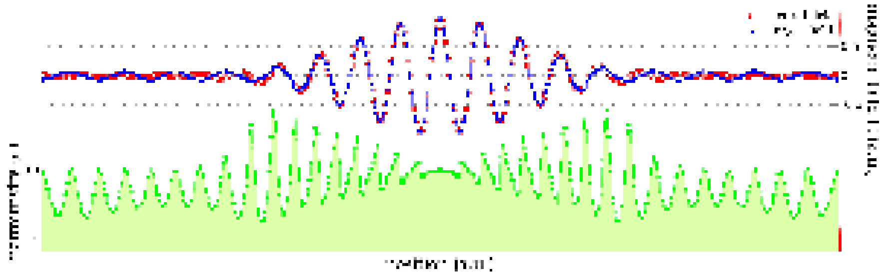

Fig. 1.3 test

1.3.4 Alternating directions solve with bounded p

$$\underset{p}{\text{minimize}} \quad \|B(x)p - d(x)\|^2 \tag{1.24}$$

$$\text{subject to} \quad 0 \leq p \leq 1 \tag{1.25}$$

$$\underset{x}{\text{minimize}} \quad \|A(p)x - b(p)\|^2 + \eta_2 \|x - x_{\text{prev}}\|^2 \tag{1.26}$$

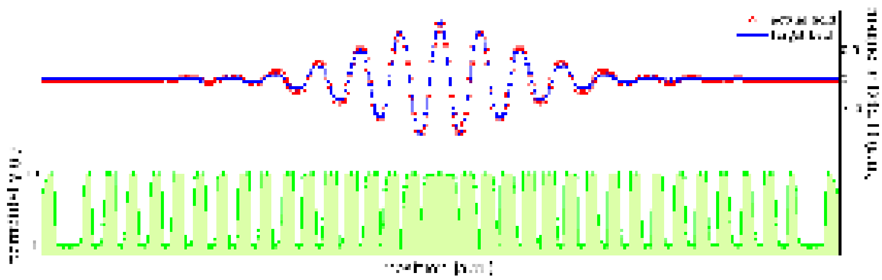

Fig. 1.4 test

1.4 Resonator design

Now that we see the usefulness of objective-first design strategy for a 1D resonator, we extend to 2D and approximate 3D as well.

We also transition design objectives: field everywhere, to certain field characteristics.

We continue to fill-out our understanding of objective-first, and we get a fuller flavor of what it can do.

1.4.1 "S" resonator

We construct an "S"-shaped field and create a resonator to make that field. This uses the previous equation...

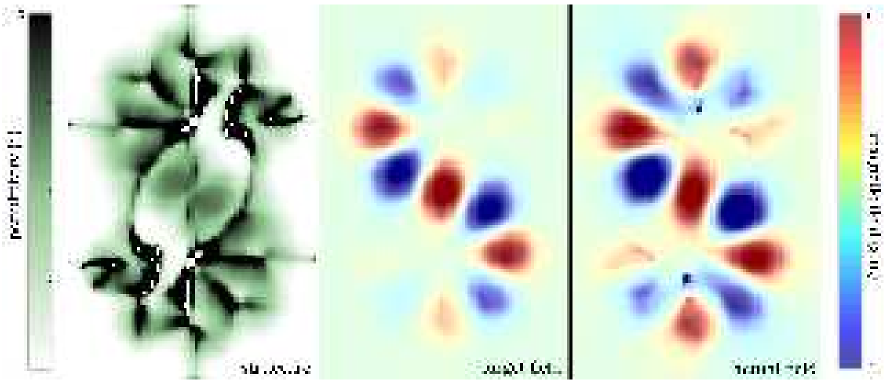

Fig. 1.5 caption

1.4.2 2D

Now we design for minimal mode-volume and maximal Q. Although Q^{-1} minimization should be a constraint we only do this in the next section.

$$\underset{x}{\text{minimize}} \quad \|A(p)x - b(p)\|^2 + \eta \|Fx\|^2 \tag{1.27}$$

$$\text{subject to} \quad \|\text{diag}(\sqrt{p})A_{\text{curl}}x\|^2 \leq a_{\text{mode}} \tag{1.28}$$

$$\begin{aligned} \underset{p}{\text{minimize}} \quad & \|B(x)p - d(x)\|^2 && (1.29) \\ \text{subject to} \quad & 0 \leq p \leq 1 && (1.30) \end{aligned}$$

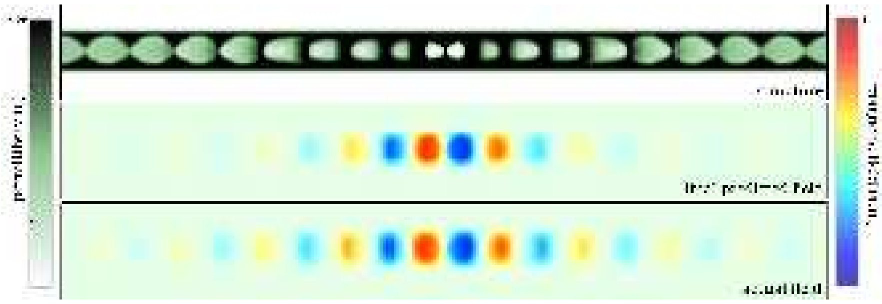

Fig. 1.6 caption

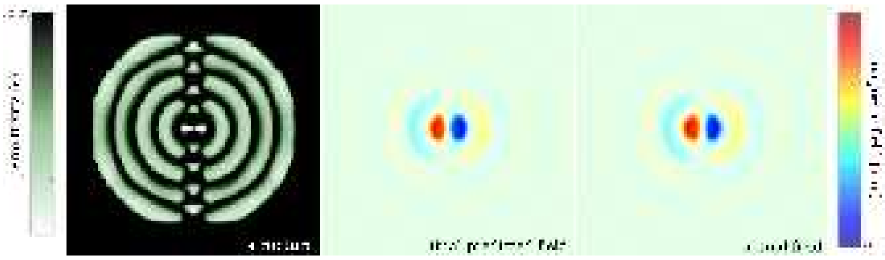

Fig. 1.7 caption

1.4.3 2.5D

Although solving for the relevant matrices in 3D is really hard, we can make an approximation.

Here we make

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \|A(p)x - b(p)\|^2 + \eta \|\text{diag}(\sqrt{p}) A_{\text{curl}} x\|^2 && (1.31) \\ \text{subject to} \quad & Fx = 0 && (1.32) \end{aligned}$$

$$\underset{p}{\text{minimize}} \quad \|B(x)p - d(x)\|^2 \tag{1.33}$$

$$\text{subject to} \quad 0 \leq p \leq 1 \tag{1.34}$$

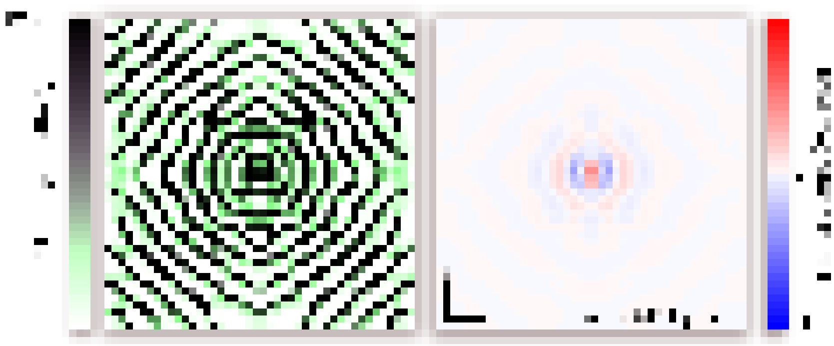

Fig. 1.8 Add the actual field here!

1.5 Waveguide coupler design

Finally, we now implement the full objective-first design strategy by doing an alternating directions solve.

1.5.1 Choice of design objective

We choose, for generality,

$$f(x) = \begin{cases} x & \text{at boundary,} \\ 0 & \text{elsewhere.} \end{cases} \tag{1.35}$$

where the boundary denotes the two outermost layers of the design space. This means

$$f_{\text{ideal}} = \begin{cases} x_{\text{perfect}} & \text{at boundary,} \\ 0 & \text{elsewhere.} \end{cases} \tag{1.36}$$

We then do

$$\underset{x}{\text{minimize}} \quad \|A(p)x - b(p)\|^2 \tag{1.37}$$

$$\text{subject to} \quad f(x) = f_{\text{ideal}} \tag{1.38}$$

$$\underset{p}{\text{minimize}} \quad \|B(x)p - d(x)\|^2 \tag{1.39}$$

$$\text{subject to} \quad 0 \le p \le 1 \tag{1.40}$$

1.5.2 Coupling between dielectric waveguide modes

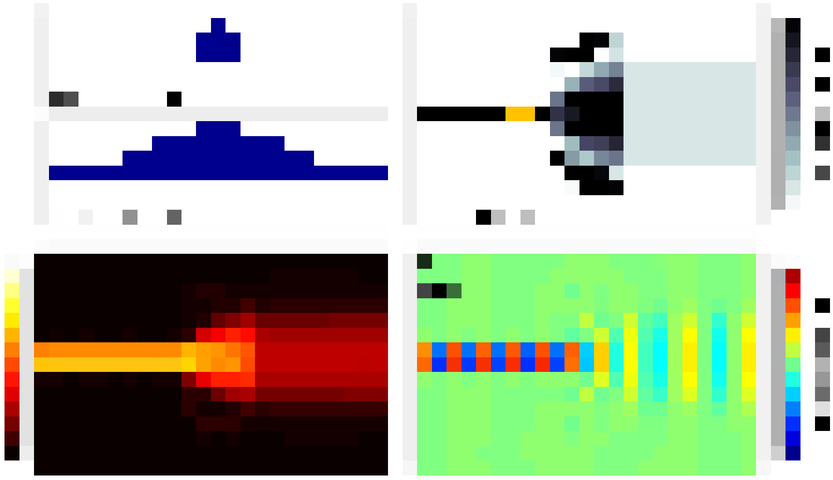

Fig. 1.9 test

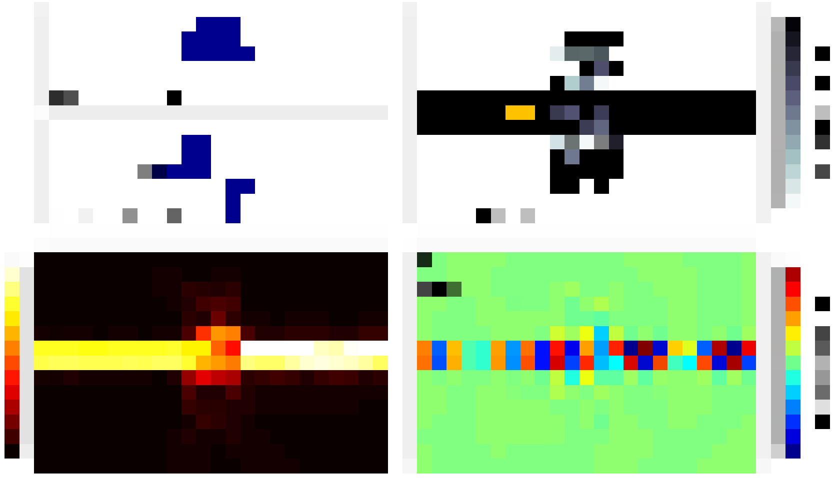

Fig. 1.10 test

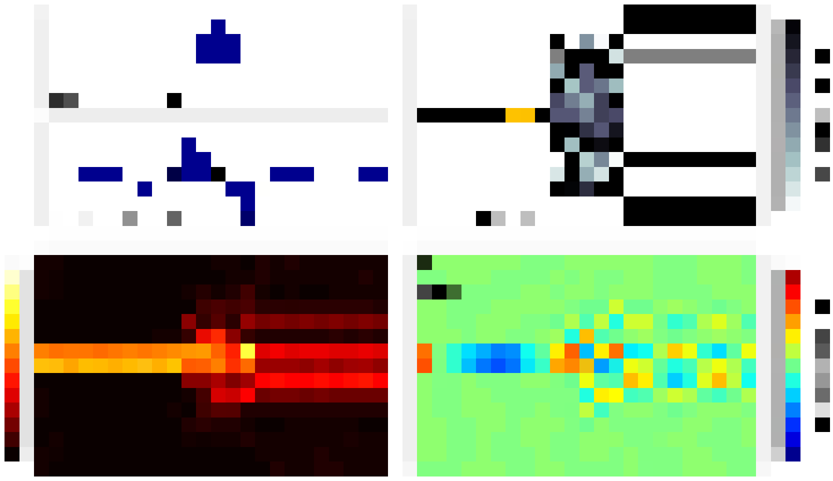

Fig. 1.11 test

1.5.3 Coupling to plasmonic waveguide modes

1.6 Metamaterials design

1.6.1 Modification of the design objective

1.6.2 Cloak devices

1.6.3 Mimic devices

1.7 Extending the method

1.7.1 3D

1.7.2 Multi-mode

1.7.3 Robustness

1.8 Appendix

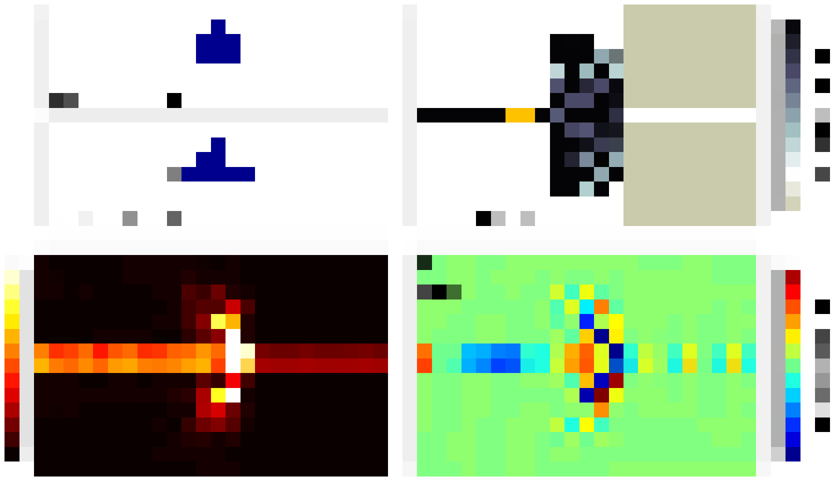

Fig. 1.12 test

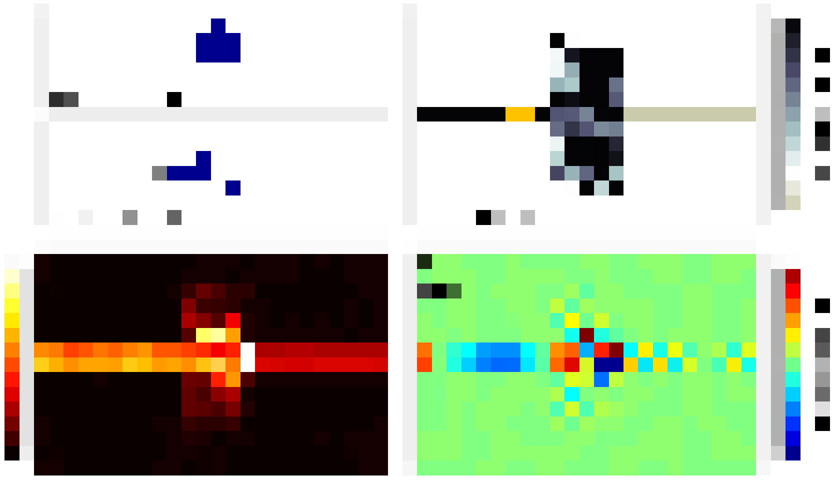

Fig. 1.13 test