Contents

Chapter 1
Objective-First Nanophotonic Design

Abstract The abstract for the book.
Introductory paragraph.

1.1 The electromagnetic wave equation

1.1.1 Physics formulation

Let's talk about the electromagnetic wave equation from a physics standpoint. Let's start from Maxwell's equations without currents.

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \tag{1.1}$$

$$\nabla \times H = J + \varepsilon \frac{\partial E}{\partial t} \tag{1.2}$$

$$\nabla \times E = -i\mu_0 \omega H \tag{1.3}$$

$$\nabla \times H = J + i\varepsilon\omega E \tag{1.4}$$

$$\nabla \times \varepsilon^{-1} \nabla \times H - \mu_0 \omega^2 H = \nabla \times \varepsilon^{-1} J \tag{1.5}$$

1.1.2 Numerical formulation

Now let's talk about the electromagnetic wave equation from a computational perspective.

To solve the wave equation on a computer we need to use the Yee grid.
To make things easier we will use weird units.
We also need to take care of boundary conditions.

1.1.3 Solving for H

The wave equation from a mathematical perspective.

$$(\nabla \times \varepsilon^{-1} \nabla \times - \mu_0 \omega^2) H = \nabla \times \varepsilon^{-1} J \tag{1.6}$$

Becomes, with a change of variables.

$$A(p)x = b(p) \tag{1.7}$$

This can be solved directly in 1D and 2D. Special methods needed in 3D. We just do 1D and 2D.

1.1.4 Solving for ε^{-1}

The wave equation from a optimization perspective.
Because scalar multiplication is transitive ($\varepsilon^{-1}(\nabla \times H) = (\nabla \times H)\varepsilon^{-1}$ and $\varepsilon^{-1} J = J\varepsilon^{-1}$)

$$\nabla \times (\nabla \times H)\varepsilon^{-1} - \nabla \times J\varepsilon^{-1} = \mu_0 \omega^2 H \tag{1.8}$$

which we write as

$$B(x)p = d(x) \tag{1.9}$$

Special constraints... (binary)

1.1.5 Insight

Basically, we see that the electromagnetic wave equation is separably linear in H and ε^{-1}
This means that...

1.2 The objective-first design problem

intro

1.2.1 Design objectives

Talk about $f(x)$ and that we are interested in convex ones.

1.2.2 Convexity

Convex optimization quick intro.

1.2.3 Typical design formulation

Typically,

$$\begin{aligned} \underset{x,p}{\text{minimize}} \quad & f(x) && (1.10)\\ \text{subject to} \quad & g(x,p) = 0 && (1.11)\\ & p \in 0,1 && (1.12) \end{aligned}$$

1.2.4 Objective-first design formulation

Objective-first does

$$\begin{aligned} \underset{x,p}{\text{minimize}} \quad & \|g(x,p)\|^2 && (1.13)\\ \text{subject to} \quad & f(x) = f_{\text{ideal}} && (1.14)\\ & 0 \le p \le 1 && (1.15) \end{aligned}$$

This is a bi-convex problem, which we solve using an alternating directions method.

1.2.5 Field sub-problem

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \|A(p)x - b(p)\|^2 && (1.16)\\ \text{subject to} \quad & f(x) = f_{\text{ideal}} && (1.17) \end{aligned}$$

1.2.6 Structure sub-problem

$$\begin{aligned} \underset{p}{\text{minimize}} \quad & \|B(x)p - d(x)\|^2 && (1.18) \\ \text{subject to} \quad & 0 \leq p \leq 1 && (1.19) \end{aligned}$$

1.3 1D resonator design

We now build up to the objective-first formulation by example. And from a different perspective.

1.3.1 Least-squares

1.3.2 Regularized least-squares

1.3.3 Alternating directions

1.4 Resonator design

1.4.1 Unbounded ε

1.4.2 2D

1.4.3 2.5D

1.5 Waveguide coupler design

this should be more straightforward.

1.6 Metamaterials design