$$\beta^2 \begin{bmatrix} E_x \\ E_y \end{bmatrix} = A \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$= (A - \beta^2) \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \omega^2 \mu_y \epsilon_x + \partial_x \frac{1}{\epsilon_z} \partial_x \epsilon_x + \mu_y \partial_y \frac{1}{\mu_z} \partial_y - \beta^2 & \partial_x \frac{1}{\epsilon_z} \partial_y \epsilon_y - \mu_y \partial_y \frac{1}{\mu_z} \partial_x \\ \partial_y \frac{1}{\epsilon_z} \partial_x \epsilon_x - \mu_x \partial_x \frac{1}{\mu_z} \partial_y & \omega^2 \mu_x \epsilon_y + \partial_y \frac{1}{\epsilon_z} \partial_y \epsilon_y + \mu_x \partial_x \frac{1}{\mu_z} \partial_x - \beta^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$= \left(\begin{bmatrix} \omega^2 \mu_y \epsilon_x + \partial_x \frac{1}{\epsilon_z} \partial_x \epsilon_x & \partial_x \frac{1}{\epsilon_z} \partial_y \epsilon_y \\ \partial_y \frac{1}{\epsilon_z} \partial_x \epsilon_x & \omega^2 \mu_x \epsilon_y + \partial_y \frac{1}{\epsilon_z} \partial_y \epsilon_y \end{bmatrix} + \begin{bmatrix} \mu_y \partial_y \frac{1}{\mu_z} \partial_y - \beta^2 & -\mu_y \partial_y \frac{1}{\mu_z} \partial_x \\ -\mu_x \partial_x \frac{1}{\mu_z} \partial_y & \mu_x \partial_x \frac{1}{\mu_z} \partial_x - \beta^2 \end{bmatrix} \right) \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$= \begin{bmatrix} \omega^2 \mu_y E_x + \partial_x \frac{1}{\epsilon_z} \partial_x E_x & \partial_x \frac{1}{\epsilon_z} \partial_y E_y \\ \partial_y \frac{1}{\epsilon_z} \partial_x E_x & \omega^2 \mu_x E_y + \partial_y \frac{1}{\epsilon_z} \partial_y E_y \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \end{bmatrix}$$

$$+ \begin{bmatrix} \mu_y \partial_y \frac{1}{\mu_z} \partial_y - \beta^2 & -\mu_y \partial_y \frac{1}{\mu_z} \partial_x \\ -\mu_x \partial_x \frac{1}{\mu_z} \partial_y & \mu_x \partial_x \frac{1}{\mu_z} \partial_x - \beta^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = A \begin{bmatrix} \epsilon_x \\ \epsilon_y \end{bmatrix} + B \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

to include the PML, we simply let

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} \epsilon_x^{real} \\ \epsilon_y^{real} \end{bmatrix}$$

Maybe we can separate out the different components of ϵ

The eigenvalue equation is

$$\left(\left(\begin{bmatrix} \omega^2\mu_y & 0 \\ 0 & \omega^2\mu_x \end{bmatrix} + \begin{bmatrix} \partial_x \frac{1}{\epsilon_z}\partial_x & \partial_x \frac{1}{\epsilon_z}\partial_y \\ \partial_y \frac{1}{\epsilon_z}\partial_x & \partial_y \frac{1}{\epsilon_z}\partial_y \end{bmatrix}\right)\begin{bmatrix} \epsilon_x & 0 \\ 0 & \epsilon_y \end{bmatrix} + \begin{bmatrix} \mu_y \partial_y \frac{1}{\mu_z}\partial_y & -\mu_y \partial_y \frac{1}{\mu_z}\partial_x \\ -\mu_x \partial_x \frac{1}{\mu_z}\partial_y & \mu_x \partial_x \frac{1}{\mu_z}\partial_x \end{bmatrix} - \beta^2\right)\begin{bmatrix} E_x \\ E_y \end{bmatrix} = 0$$

Changing variables to $D = \epsilon E$ gives us $\begin{bmatrix} \epsilon_x E_x \\ \epsilon_y E_y \end{bmatrix} = \begin{bmatrix} D_x \\ D_y \end{bmatrix}$

and $\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \frac{1}{\epsilon_x} & 0 \\ 0 & \frac{1}{\epsilon_y} \end{bmatrix}\begin{bmatrix} D_x \\ D_y \end{bmatrix}$

∂

the equation is now

$$\left(\begin{bmatrix} \omega^2\mu_y & \\ & \omega^2\mu_x \end{bmatrix} + \begin{bmatrix} \partial_x \frac{1}{\epsilon_z}\partial_x & \partial_x \frac{1}{\epsilon_z}\partial_y \\ \partial_y \frac{1}{\epsilon_z}\partial_x & \partial_y \frac{1}{\epsilon_z}\partial_y \end{bmatrix} + \left(\begin{bmatrix} \mu_y \partial_y \frac{1}{\mu_z}\partial_y & -\mu_y \partial_y \frac{1}{\mu_z}\partial_x \\ -\mu_x \partial_x \frac{1}{\mu_z}\partial_y & \mu_x \partial_x \frac{1}{\mu_z}\partial_x \end{bmatrix} - \beta^2\right)\begin{bmatrix} \frac{1}{\epsilon_x} & \\ & \frac{1}{\epsilon_y} \end{bmatrix}\right)\begin{bmatrix} D_x \\ D_y \end{bmatrix} = 0$$

∂

∂