Objective-first optimization for nanophotonics

- adjoint method
- objective-first approach
- field setup: boundary-value problem
- structure setup: level-set formulation
- example
- ongoing and future work

Jesse Lu, Jelena Vuckovic group, Stanford University

Adjoint method

Typical formulation of a structural optimization problem:

$$\begin{aligned} \text{decrease} \quad & f(x) & (1) \\ \text{subject to} \quad & g(x,p) = 0 & (2) \end{aligned}$$

- $f(x) : \mathbf{C}^n \to \mathbf{R}$ is the *design objective*
- $g(x,p) : \mathbf{C}^n \times \mathbf{R}^n \to \mathbf{C}^n$ is the *governing physics*
- $x \in \mathbf{C}^n$ is the field
- $p \in \mathbf{R}^n$ is the structure
- x is the dependant variable, p is the independant variable

- problem is non-convex, so optimize using first-order approximations

$$f(x_0 + dx) \approx f(x_0) + \frac{\partial f}{\partial x} dx \tag{3}$$

$$g(x_0 + dx, p_0 + dp) \approx g(x_0, p_0) + \frac{\partial g}{\partial x} dx + \frac{\partial g}{\partial p} dp \tag{4}$$

- assuming that $g(x_0, p_0) = 0$, the equality constraint is satisfied via

$$dx = -\left(\frac{\partial g}{\partial x}\right)^{-1} \frac{\partial g}{\partial p} dp \tag{5}$$

- now we can decrease $f(x_0 + dx)$,

$$
\begin{aligned}
f(x_0 + dx) &\approx f(x_0) + \frac{\partial f}{\partial x} dx && (6)\\
&\approx f(x_0) - \frac{\partial f}{\partial x} \left(\frac{\partial g}{\partial x} \right)^{-1} \frac{\partial g}{\partial p} dp && (7)\\
&\approx f(x_0) + \frac{\partial f}{\partial p} dp && (8)
\end{aligned}
$$

  by choosing dp in the direction of $dp \propto -\frac{\partial f}{\partial p}$

- computing $\frac{\partial f}{\partial p}$ can be reduced to a single field solve ($i.e.$ solving $g(x, p)$ for x given p)