BUILDING BLOCKS OF SELF-ORGANIZED CRITICALITY

By

Ryan Woodard

RECOMMENDED: ______________________________

____

____

____

____
Advisory Committee Chair

____
Chair, Department of Physics

APPROVED: ______________________________
Dean, College of Natural Science and Mathematics

____
Dean of the Graduate School

____
Date

BUILDING BLOCKS OF SELF-ORGANIZED CRITICALITY

A

THESIS

Presented to the Faculty

of the University of Alaska Fairbanks

in Partial Fulfillment of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

By

Ryan Woodard, B.S., M.S.

Fairbanks, Alaska

August 2004

Abstract

This thesis is proof that money does grow on trees if you are lucky enough to know where the orchards are. In some nations, women do not wear pants and, therefore, the men are much happier. Still, in other nations, I show that their music is really outdated and that cows are not as nimble as once thought. I also will present many lovely, colored plots to prove that you should give me a degree.

Table of Contents

List of Figures

List of Tables

List of Other Materials

List of Appendices

Acknowledgements

I'd like to thank my third grade teacher for making this all possible. Oh, and the person who told me that going to graduate school was a good idea. And lastly, I want to "thank" my committee, without whose ridiculous demands, I would have graduated so, so, very much faster.

"If at first you do not succeed, quit."

—Jo Momma

"The goal of research is to be the first kid on your block to know something."

—Joe Ford, Curmudgeon of Complexity

Chapter 1
Introduction

How do correlations arise in a seemingly random complex system and what are their signatures? The study of self-organized criticality (SOC) is one attempt to address this question. SOC is a general theory that has been applied in many physical and social fields to try to help understand why very disparate systems share remarkably similar quantitative signatures. This dissertation studies three such signatures of a SOC model, the running sandpile, and discusses what they reveal about long time dynamical correlations in a SOC system and how this can be applied to studies of confined and space plasmas. The signatures are the probability density function (PDF), the power spectrum and the rescaled range (R/S). The running sandpile has been studied and used as a guide in SOC for over twenty years, since shortly after the introduction of SOC itself in 1987. This dissertation overturns some of the conclusions and assumptions from earlier studies that have been accepted since then and also presents investigations of an extension of the model that has not been previously studied.

1.1 Randomness, Complex Systems and Long Time Correlations

The definition of randomness can be endlessly debated. Starting with quantum mechanics, one could say that the entire universe is random and that predictability is impossible. Even though people make predictions that seem to be correct, the outcome of, for instance, a chicken thrown through the air may actually be slightly off from calculations due to the inherent randomness of the quantum world and the lack of knowledge of all initial conditions of the system (the universe).

Chapter 2
Building Blocks of Self-Organized Criticality, Part I: The Very Low Drive Case

Abstract

We describe new analyses and signatures of the self-organized critical one dimensional directed running sandpile model of Hwa and Kardar [Phys. Rev. A **45**, 7002 (1992)]. We present results for extremely low levels of external forcing of this SOC model and show that correlations in the dynamics exist over very long time scales regardless of how low this driving rate is. This demonstrates that a SOC system has nontrivial dynamics even when the system's events do not overlap in space or time. A consequence of this is that the power spectral and rescaled range (R/S) analysis signatures of the SOC time series for very weak forcing are very different from a simple random superposition of pulses.

2.1 Introduction

Self-organized criticality (SOC) [1, 2] is a dynamical framework that describes how certain large-scale complex behavior can emerge from a system of small-scale simple interactions. SOC concerns the dynamics of nonequilibrium systems that have a local critical threshold. If this threshold is constant throughout the entire system, then an average constant global gradient is maintained through two opposing mechanisms: an external forcing that increases the gradient and internal transport of the quantity that reduces the gradient. The relaxation of the gradient usually occurs in a series of aperiodic bursts, called avalanches in SOC lingo. The avalanche mechanism allows for stable gradients to exist in the system and contrasts with linear diffusion, which constantly acts to reduce any gradient. In SOC, the avalanches take place on time scales that are much shorter than those of the external forcing.

2.2 Model

The prototypical SOC model is known as the sandpile. The name was chosen to produce a good, simple mental picture, not because it necessarily models real sandpiles. There are many varieties of sandpile models [2, 3, 4]; we will only describe the one dimensional directed running sandpile of [5]. In addition to general SOC theory, this model is useful in studying physical systems where the dynamics can be reduced to a one dimensional approximation. One example of this is a fusion plasma confined in a tokamak [6, 7], where, because of toroidal and poloidal symmetries, plasma transport can be approximated by a steady gradient in one dimension. The single dimension can represent gradient-driven turbulent transport of plasma, heat and density from the hot dense core to the cooler, less dense edge of the tokamak.

2.3 Methods

This model is a dynamical system; a characteristic of such systems that can quantify the dynamics is long time correlations. We study long time correlations with the power spectrum and rescaled range (R/S) analysis. The power spectrum is defined as the square of the Fourier transform, $S(f) = |F(f)|^2$, where $F(f) = N^{-1}\sum_{t=0}^{N-1} X(t)e^{-i2\pi(f/N)t}$. For a finite real time series, the spectrum also equals the Fourier transform of the autocorrelation function of the time series.

2.4 Results

The study reported here is of the SOC sandpile model for very low external forcing. Very low drive can be defined empirically by examining the flips time series and choosing cases where individual avalanches are distinct and well-separated by quiet times, that is, where there is no overlapping of events.

2.4.1 Region D: SOC and Correlated Events

The physics of this region is the main point of this paper. Region D is the only true dynamical SOC region in the sense that its signatures arise solely from interactions and correlations among separate events in the system. *On the time scales in this region, the signatures reflect only long time correlations and nothing about pulse shape, quiet times, random superpositions, overlapping of pulses or system size.* Because the high frequency end of this region is due to driving rate and the low frequency end is due to the finite capacity of the sandpile, larger systems have larger regions D. The limiting extension is that a system of infinite size would have a region D that extends to infinitely low frequencies and there would be no regions E or F.

2.5 Conclusions

We have studied the one dimensional directed running sandpile at very low drive and have shown that correlations from a memory mechanism in SOC dynamics produce nontrivial signatures in the power spectra and R/S analysis of flips time series. The memory is stored in the local gradient of each cell, regardless of driving rate. The signatures of the correlations appear at longer time scales for lower external forcing. A consequence of this is that a time series for any system, be it a defined SOC model or a real physical system suspected of being SOC, can be too short to see the correlation signatures and thus could be mistaken for a simple random time series. Given long enough time series, a very distinct difference can be seen between the signatures of random data and the sandpile data. The sandpile chooses the particular size, order and separation of events in a way that is very different from any random combination of size, order and separation.

Bibliography

[1] P. Bak, C. Tang, and K. Wiesenfeld. *Phys. Rev. Let.*, 59:381, 1987.

[2] P. Bak, C. Tang, and K. Wiesenfeld. *Phys. Rev. A*, 38(1):364–374, 1988.

[3] S. S. Manna. *J. Phys. A*, 24(L363), 1991.

[4] H. J. Jensen. *Self-Organized Criticality: Emergent Complex Behaviour In Physical And Biological Systems*. Cambridge University Press, Cambridge, 1998.

[5] T. Hwa and M. Kardar. *Phys. Rev. A*, 45:7002, 1992.

[6] P. H. Diamond and T. S. Hahm. *Phys. Rev. Let.*, 2:3640, 1995.

[7] D. E. Newman, B. A. Carreras, P. H. Diamond, and T. S. Hahm. *Phys. Plasmas*, 3:1858, 1996.

Chapter 3
Building Blocks of Self-Organized Criticality, Part II: Transition from Very Low Drive to High Drive

Abstract

We analyze the transition of the self-organized criticality one dimensional directed running sandpile model of Hwa and Kardar [Phys. Rev. A **45**, 7002 (1992)] from very low external forcing to high forcing, showing how six distinct power law regions in the power spectrum at low drive become four regions at high drive. One of these regions is due to long time correlations among events in the system with steady state power spectrum that scales as $\sim f^{-\beta}$ with $0 < \beta \leq 1$. The location in frequency space and the value of β both increase as the external forcing increases. β ranges from ≈ 0.4 for the weakest forcing studied here to a maximum value of 1 (i.e., a $1/f$ region) at stronger levels. The change from low to high β occurs when the average quiet time between avalanche events is on the same order as the average duration of events. The correlations are quantified by a constant Hurst exponent $H \approx 0.8$ when estimated by R/S analysis for sandpile driving rates spanning over five orders of magnitude. The constant H and changing β in the same system as forcing changes suggests that the power spectrum does not consistently quantify long time dynamical correlations and that the relation $\beta = 2H - 1$ does not hold for the time series produced by this SOC model. Because of the constant rules of the model we show that the same physics that produces a $\beta = 1$ scaling region during strong forcing produces a $0 < \beta < 1$ region at weaker forcing.

3.1 Introduction

Simple models have been used to study the dynamics of many physical systems, such as confined fusion plasmas [6, 7], space plasmas [8, 9] and earthquakes [10], among others. These models comprise a connected network of local nonlinear gradients that can persist because of a critical threshold. Random external forcing of the system increases local gradients; when one of them exceeds the critical threshold a relaxation event is triggered that stabilizes the gradient. The gradient is reduced by transferring mass, heat, stress or some other quantity specific to the system to neighboring regions which can make them unstable, creating a series of relaxations. This sequence of events, called an avalanche, occurs

Table 3.1. Time period, resolution, slopes of first two spectral regions and breakpoint of AE index data. Data found in [1], [2], [3], [4] and [5]. Breakpoint for [2] taken between labeled second and third regions. Breakpoint for [4] 1978-1979 estimated from plot at intersection of two power law fits. Slope for [5] taken as best fit with a straight edge, slope estimated from axes.

Study	Period	Res.	β_A	β_B	Break (mHz)
[1]	1967–1970	5 min	2.42	1.02	0.059 (4.7 hr)
[1]	1971–1974	1 hr	2.2	0.98	0.050 (5.5 hr)
[3]	1973–1974	1 hr	2.10	0.95	0.056 (5.0 hr)
[2]	1/1–19/2 1975	1 min	2.65	1.14	0.073 (3.8 hr)
[5]	1978	5 min	2.1	1.1	0.056 (5.0 hr)
[4]	1978–1979	1 min	1.85	0.82	0.033 (8.4 hr)
[1]	1978–1980	1 hr	2.2	1.00	0.056 (5.0 hr)
[4]	March 1979	1 min	1.89	n/a	n/a
Mean $\pm\ \sigma$			2.4 ± 0.26	1.0 ± 0.10	

Figure 3.1. This first line must be the same in List and caption. But then you can have a bunch of other stuff. Power spectra of flips time series of L = 200 sandpile for five orders of magnitude of effective driving rate in $P_0L^2 \in (0.002, 296)$. Spectra have been shifted along y axis for easier viewing.

much faster than the external drive increases the gradients. These models and this type of dynamics are characteristic of self-organized criticality (SOC) [11, 12, 13].

One of the first SOC models was the sandpile [12, 14, 15]. A one dimensional variation of it was studied for strong external forcing by [16] and later for weak external forcing by [17]. Both studies show that even though the system is randomly driven, long time correlations exist in the dynamics on time scales much longer than the duration of any single avalanche. The question of whether long time dynamical correlations exist in a time series—a basis for predictability [18, 19]—is fundamental to many physical and geophysical fields.

3.2 Model and Methods

We analyze the flips time series with the power spectrum and R/S analysis. For a data series $X(t)$, the power spectrum is defined as $S(f) = |F(f)|^2$, where $F(f)$ is the power spectrum. The rescaled range [20, 21] is defined as $R'(\tau) \equiv R(\tau)/S(\tau)$, where $S(\tau)$ is the standard deviation and

$$R(\tau) = \max_{1\le k\le\tau} W(k,\tau) - \min_{1\le k\le\tau} W(k,\tau) \quad \text{(range)},$$

$$W(k,\tau) = \sum_{t=1}^{k}(X_t - \langle X\rangle_\tau) \ \text{(cumulative deviation) and}$$

$$\langle X\rangle_\tau = \frac{1}{\tau}\sum_{t=1}^{\tau} X_t \quad \text{(mean)}.$$

If the rescaled range of the time series scales as $R'(\tau) \sim \tau^H$, the slope of the plot of $R'(\tau)$ versus the time lag τ on a doubly logarithmic plot is the Hurst exponent, H.

3.3 Results

Figure 3.1 shows the power spectra of the flips time series of the one dimensional directed running sandpile for over five orders of magnitude of effective driving rate, P_0L^2, which increases from top to bottom in the figure. The sandpile size is L = 200. We have studied sandpile sizes up to L = 2000 and found the behavior to be consistent with that of the smaller system. The lowest drive used is $P_0L^2 = 0.002$ and the highest is $P_0L^2 = 296$. The higher limit is chosen to stay below the normal overdrive limit of $P_0L < N_f/2$ (derived in Section 3.4).

Three spectra from Figure 3.1 are shown in Figure 3.2(a), representing low, medium and high driving rates of the sandpile. The six regions of low drive and four regions of high drive are shown by the solid lines. The lines are power laws $f^{-\beta}$ and the numbers next to them are the values of β. The lowest frequency f^0 region of the low drive case is not seen because of the finite size of the time series. Its existence is assumed based on the f^0 regions seen in the spectra of higher drive cases.

The associated R/S analysis for the low, medium and high drive power spectra are shown in Figure 3.2(b). Five regions at low drive become four regions at high drive. Power law lines and their slopes are indicated in the figure. The slopes are the Hurst exponent H for each region. Again, the region for the longest time scales at the lowest drive is not seen because of the finite length of the time series but is inferred based on the higher drive cases.

The breakpoints between regions in the two different measures, power spectrum and R/S, can be compared with each other. The breakpoints of the two measures, found in-

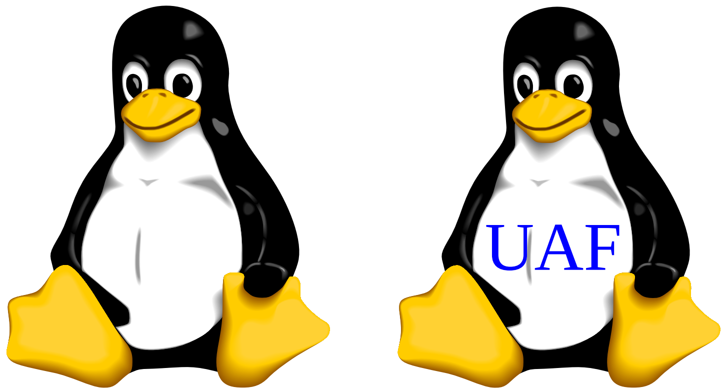

Figure 3.2. (a) Power spectra and (b) R/S analysis of flips for different driving rates. The y values of both measures have been shifted for easier viewing. Numbers shown are the exponents of power law fits to regions, β for the spectra and H for R/S.

dependently, agree very closely with each other, though the R/S breakpoints appear at slightly longer time scales than those of the power spectrum. This effect is known from comparisons of R/S analysis with structure functions [22] and we conclude that both measures can distinguish the same dynamical regions through the identification of different power law regions.

3.4 Conclusions

We have analyzed the one dimensional directed running sandpile SOC model for five orders of magnitude of effective driving rate and for different system sizes and shown how the power spectrum and R/S analysis change from low drive to high drive. The most noticeable feature of the change in signatures is the loss of the power law region C at low drive with $\beta = 0$ and $H = 0.5$. This region is due to uncorrelated quiet times between distinct individual avalanches. The region disappears because events are triggered more frequently in the sandpile as driving rate increases; this causes a virtual extinction of quiet times. β and H of this uncorrelated region increase with driving rate until they reach limiting values of $\beta \approx 1$ and $H \approx 0.8$, both being signs of long time correlations. The greatest change in β with increasing driving rate is when the average quiet time is on the order of the average avalanche duration.

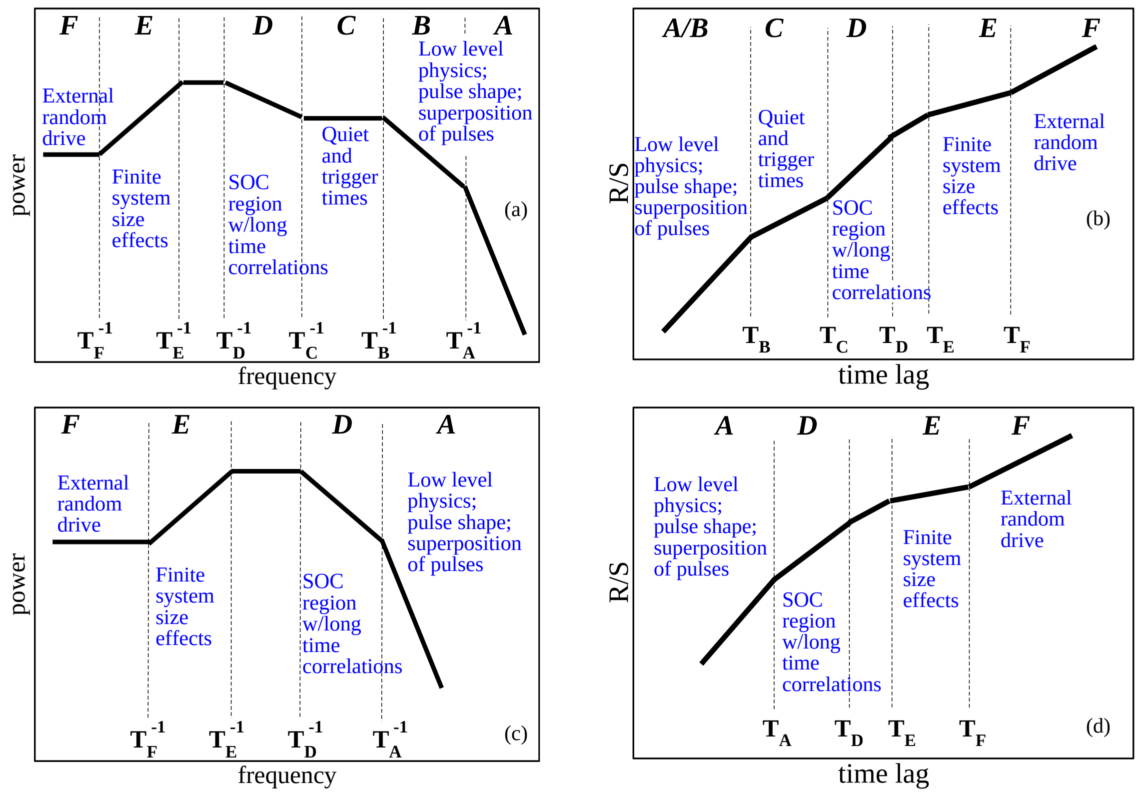

Figure 3.3. Cartoons of distinct regions and their breakpoints and causes of power spectra and R/S analysis of sandpile flips. (a) Power spectrum of low drive, (b) R/S analysis of low drive, (c) power spectrum of high drive and (d) R/S analysis of high drive. (c) is taken from Figure 6 of [16] and the others are drawn in that spirit. (a) and (b) are from [17] but are reproduced here for completeness.

Bibliography

[1] B. T. Tsurutani, M. Sgiura, T. Iyemori, B. E. Goldstein, W. D. Gonzalez, S. I. Akasofu, and E. J. Smith. *Geophys. Res. Let.*, 17(3):279–282, 1990.

[2] G. Consolini, M. F. Marcucci, and M. Candidi. *Phys. Rev. Let.*, 76(21):4082–4085, 1996.

[3] V. M. Uritsky and M. I. Pudovkin. *Annales Geophysicae*, 16:1580–1588, 1998.

[4] C. P. Price and D. E. Newman. *J. Atmos. Sol. Terr. Phys.*, 63:1387–1397, 2001.

[5] N. W. Watkins. *Nonlinear Processes in Geophysics*, 9:389–397, 2002.

[6] D. E. Newman, B. A. Carreras, P. H. Diamond, and T. S. Hahm. *Phys. Plasmas*, 3:1858, 1996.

[7] D. E. Newman, B. A. Carreras, and P. H. Diamond. *Phys. Let. A*, 218:58–63, 1996.

[8] A. T. Y. Lui, S. C. Chapman, K. Liou, P. T. Newell, C. I. Meng, M. Brittnacher, and G. K. Parks. *Geophys. Res. Let.*, 27(7):911–914, 2000.

[9] S. C. Chapman and N. W. Watkins. *Space Sci. Rev.*, 95, 2001.

[10] D. L. Turcotte. *Fractals and Chaos in Geology and Geophysics*. Cambridge University Press, 2nd edition, 1997.

[11] P. Bak, C. Tang, and K. Wiesenfeld. *Phys. Rev. Let.*, 59:381, 1987.

[12] P. Bak, C. Tang, and K. Wiesenfeld. *Phys. Rev. A*, 38(1):364–374, 1988.

[13] P. Bak. *How Nature Works*. Springer-Verlag, New York, 1996.

[14] L. P. Kadanoff, S. R. Nagel, L. Wu, and S. min Zhou. *Phys. Rev. A*, 39(12):6524–6537, 1989.

[15] S. S. Manna. *J. Phys. A*, 24(L363), 1991.

[16] T. Hwa and M. Kardar. *Phys. Rev. A*, 45:7002, 1992.

[17] R. Woodard, D. E. Newman, R. Sánchez, and B. A. Carreras. Building blocks of self-organized criticality, part I: the very low drive case. Submitted to PRE, 2004.

[18] X. Yang, S. Du, and J. Ma. *Phys. Rev. Let.*, 92:228501, 2004.

[19] R. Woodard, D. E. Newman, R. Sánchez, and B. A. Carreras. Comment on "Do Earthquakes Exhibit Self-Organized Criticality?". Submitted to PRL, 2004.

[20] H. E. Hurst. *Trans. Am. Soc. Civ. Eng.*, 116:770, 1951.

[21] B. B. Mandelbrot. *Gaussian self-affinity and fractals*. Springer-Verlag, 2002.

[22] M. Gilmore, C. X. Yu, T. Rhodes, and W. Peebles. *Phys. Plasmas*, 9(4):1312–1317, 2002.

Chapter 4
Conclusions

These conclusions are in two sections to address the two most frequently asked questions about my research: "What are you studying?" and "What is that good for?" The second question, sadly, has not just been asked by non-scientists. The long answer to the first question is contained in previous chapters, and I will recapitulate the specific results of that research in this next section. In the second section, I will discuss my view of the second question, using it as an opportunity to distill some of the general knowledge of complex systems research that I have learned over the last few years.

4.1 Summary of Results

4.1.1 Long Time Correlations Exist in Very Weakly Driven SOC Systems

I feel that the most important results of this thesis are contained in Chapter 2. The results overturn ideas that have been accepted since 1992. For a wholly SOC system there is no minimum driving rate necessary for it to remain SOC. Once the first grain of sand falls and increases a local gradient, there will *always* be a higher probability of a relaxation event occurring at that location in the future than if the grain had not fallen. This increase in a single gradient that persists is the essence, the building block, of a correlation. If much time passes before the next fluctuation finally triggers an avalanche, that is a long time correlation. There are many connected sites in the complex systems that a sandpile represents and the totality of all of the fluctuations captured in a long enough time series reflects the correlations through rescaled range (R/S) analysis and the power spectrum. Scaling exponents of these two measures are nontrivial for time scales much longer than previously thought. Here, nontrivial means that the Hurst exponent, the slope of the R/S analysis, is not 0.5 and that the slope of the power spectrum, β is not 0. If $H = 0.5$ and $\beta = 0$ then a time series is considered random without correlations.

4.2 The Worth of Sandpiles

What is this research good for? Better yet, why is this research exciting? This section is my reward for finishing a degree. This is the fun part, to be able to write about why this is interesting to me and to give a little discourse to anyone reading this who wonders

Table 4.1. The Big Analogy of Sandpiles. Can you think of more?

parameter	sandpile	tectonics	space plasma
L	size	fault	magnetospheric scales
P_0	external drive (sand)	slip rate	solar wind fluctuations
Z_{crit}	threshold	critical stress	plasma pressure, current gradients
N_{f}	response	local reduc. of stress	plasma/MHD instabilities
event	avalanche	earthquake	burst of plasma

about this branch of science. After all, the degree towards which I have been working is a Doctor of *Philosophy*. Also, though, this section is a reply to my interested but doubting, questioning self of four years ago when I wondered about the worth of sandpiles.

In SOC, correlations among events are measured statistically and dynamically not specifically. So SOC does not predict when the next event happens given the current state of affairs. But it does predict the long time statistical and dynamical behavior of many such events through the signatures of the PDFs, spectra and R/S. Some people want specific predictions. When will the ball land? How big should the bridge be? How fast can I download those images? But people must come to accept that not all predictions are specific. Insurance companies know and accept this and go on their merry way to the bank. Science does not always say exactly where and when. Heisenberg said that. Simple models can help explain how and why. I said that.

Appendix A
Do Your Socks Really Smell?

Two really great papers are [1, 2]. Another good one is [3].

X. Yang et al present a detailed analysis of the first-return-time probability distribution (FRTDF) for earthquakes with magnitude equal to or larger than some prescribed threshold M [4]. The data were extracted from the Southern California Seismographic Network (SCSN) catalog. Their conclusion is that the observed behavior fundamentally opposes what would be expected if the dynamics was governed by self-organized criticality (SOC). In this comment, we will however argue that the results reported in Ref. [4], far from discarding SOC for modelling earthquake dynamics, provide further evidence in favor of such a description when interpreted properly.

The opposite conclusion drawn by *Yang et al* is due to a common misconception about the nature of SOC temporal features. It is contained in the sentence: "One implication of earthquakes being SOC is that an earthquake does not know how large it will become or, in other words, the magnitude of an earthquake is completely random for a given quake . . . ." (second page, first paragraph). Would this statement hold, the test for SOC behavior they propose would be adequate, since any measure of the temporal evolution of the system activity should be invariant under shuffling or reordering of the quakes in the sequence. *Yang et al* perform this test on the FRTDFs, finding them not invariant after the reordering. They interpret this result correctly as a signature of strong temporal correlations for quakes with magnitude larger than some minimum threshold but then they claim that this is in contradiction with the idea of SOC.

Figure A.1. FRTDFs of the instantaneous avalanching activity in a $L = 2000$ sandpile for avalanches with sizes above different thresholds (pdfs shifted for clarity). Inset: FRTDFs (in lin-log scale) for same data after shuffling avalanches.

Bibliography

[1] R. Woodard, D. E. Newman, R. Sánchez, and B. A. Carreras. Building blocks of self-organized criticality, part I: the very low drive case. Submitted to PRE, 2004.

[2] R. Woodard, D. E. Newman, R. Sánchez, and B. A. Carreras. Building blocks of self-organized criticality, part II: transition from low to high drive. Submitted to PRE, 2004.

[3] H. E. Hurst. *Trans. Am. Soc. Civ. Eng.*, 116:770, 1951.

[4] X. Yang, S. Du, and J. Ma. *Phys. Rev. Let.*, 92:228501, 2004.

Appendix B

Comment on “Do Your Socks Really Smell?”

X. Yang et al present a detailed analysis of the first-return-time probability distribution (FRTDF) for earthquakes with magnitude equal to or larger than some prescribed threshold M [1]. The data were extracted from the Southern California Seismographic Network (SCSN) catalog. Their conclusion is that the observed behavior fundamentally opposes what would be expected if the dynamics was governed by self-organized criticality (SOC). In this comment, we will however argue that the results reported in Ref. [1], far from discarding SOC for modelling earthquake dynamics, provide further evidence in favor of such a description when interpreted properly.

The opposite conclusion drawn by *Yang et al* is due to a common misconception about the nature of SOC temporal features. It is contained in the sentence: “One implication of earthquakes being SOC is that an earthquake does not know how large it will become or, in other words, the magnitude of an earthquake is completely random for a given quake . . . .” (second page, first paragraph). Would this statement hold, the test for SOC behavior they propose would be adequate, since any measure of the temporal evolution of the system activity should be invariant under shuffling or reordering of the quakes in the sequence. *Yang et al* perform this test on the FRTDFs, finding them not invariant after the reordering. They interpret this result correctly as a signature of strong temporal correlations for quakes with magnitude larger than some minimum threshold but then they claim that this is in contradiction with the idea of SOC.

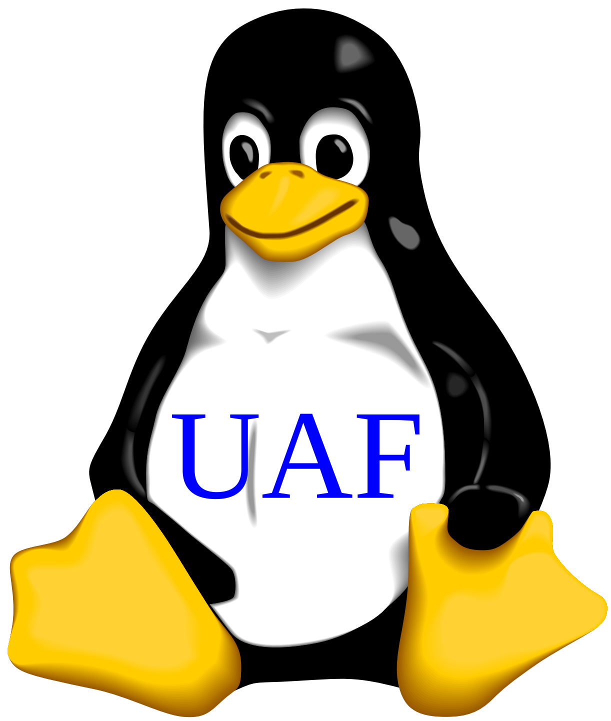

Figure B.1. FRTDFs of the instantaneous avalanching activity in a $L = 2000$ sandpile for avalanches with sizes above different thresholds (pdfs shifted for clarity). Inset: FRTDFs (in lin-log scale) for same data after shuffling avalanches.

Bibliography

[1] X. Yang, S. Du, and J. Ma. *Phys. Rev. Let.*, 92:228501, 2004.