Research internship

Notes on project in DiVincenzo group

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Abstract

Magnus expansion

Contents

1. The truncated Magnus expansion

The Magnus expansion[1–4] is defined as

$$\Omega(t,t_0) = \sum_{n=1}^{\infty} \Omega_n(t,t_0), \tag{1}$$

$$\Omega_1(t,t_0) = \int_{t_0}^{t} \mathrm{d}t_1 \tilde{H}(t_1), \tag{2}$$

$$\Omega_2(t,t_0) = -\frac{1}{2}\int_{t_0}^{t} \mathrm{d}t_2 \left[\Omega_1(t_2,t_0), \tilde{H}(t_2)\right] = \frac{1}{2}\int_{t_0}^{t} \mathrm{d}t_1 \int_{t_0}^{t_1} \mathrm{d}t_2 \left[\tilde{H}(t_1), \tilde{H}(t_2)\right]. \tag{3}$$

We use this series to approximate the trajectory of a two-level qubit driven by an oscillating signal proportional to $\boldsymbol{\sigma}_x$ in the lab frame. To this end, we use the following rotating frame Hamiltonian:

$$\boldsymbol{H}(t) = \frac{E(t)}{4}\left(\boldsymbol{\sigma}_x + \cos(2\omega t)\boldsymbol{\sigma}_x - \sin(2\omega t)\boldsymbol{\sigma}_y\right), \tag{4}$$

where we set the detuning Δ and phase offset ϕ of the drive both to zero.

For now, we restrict ourselves to the first order Magnus term, which is equivalent to the rotating wave approximation for constant drive. We begin by studying the case of linear drive, given by

$$E(t) = E_0 + E_1 t. \tag{5}$$

We then integrate equation (2) over a full period $t_c = \pi/\omega$ of the drive Hamiltonian 4, starting from t_0:

$$\begin{aligned}
\Omega_1(t,t_0) &= \int_{t_0}^{t_0+t_c} \frac{E(t)}{4}\left(\boldsymbol{\sigma}_x + \cos(2\omega t)\boldsymbol{\sigma}_x - \sin(2\omega t)\boldsymbol{\sigma}_y\right) \mathrm{d}t \\
&= \int_{t_0}^{t_0+t_c} \frac{E(t)}{4}\boldsymbol{\sigma}_x \,\mathrm{d}t + \int_{t_0}^{t_0+t_c} \frac{E(t)}{4}\left(\cos(2\omega t)\boldsymbol{\sigma}_x - \sin(2\omega t)\boldsymbol{\sigma}_y\right) \mathrm{d}t \\
&= \frac{E_0 t_c + E_1(t_0 t_c + t_c^2/2)}{4}\boldsymbol{\sigma}_x + \frac{E_1 t_c \sin(2\omega t_0)}{8\omega}\boldsymbol{\sigma}_x + \frac{E_1 t_c \cos(2\omega t_0)}{8\omega}\boldsymbol{\sigma}_y \\
&= t_c \frac{E_0 + E_1 t_0}{4}\boldsymbol{\sigma}_x + t_c^2 \frac{E_1}{8}\boldsymbol{\sigma}_x + \frac{t_c}{\omega}\left(\frac{E_1 \sin(2\omega t_0)}{8}\boldsymbol{\sigma}_x + \frac{E_1 \cos(2\omega t_0)}{8}\boldsymbol{\sigma}_y\right) \\
&= t_c\left[\frac{E_0 + E_1 t_0}{4}\boldsymbol{\sigma}_x + \frac{\pi E_1}{8\omega}\left(\boldsymbol{\sigma}_x + \sin(2\omega t_0)\boldsymbol{\sigma}_x + \cos(2\omega t_0)\boldsymbol{\sigma}_y\right)\right]
\end{aligned}$$

References

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