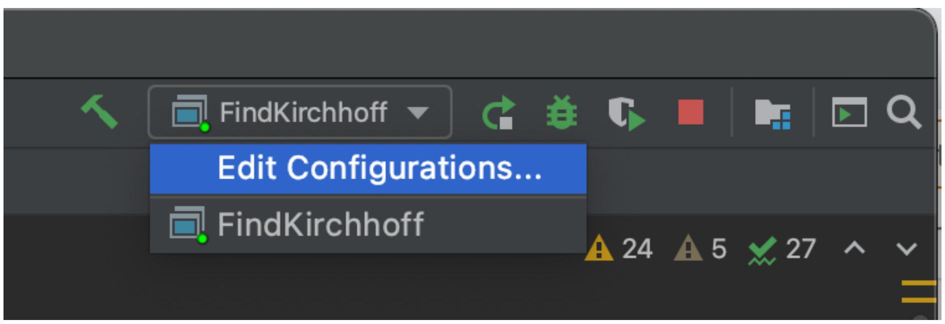
Downloading Application

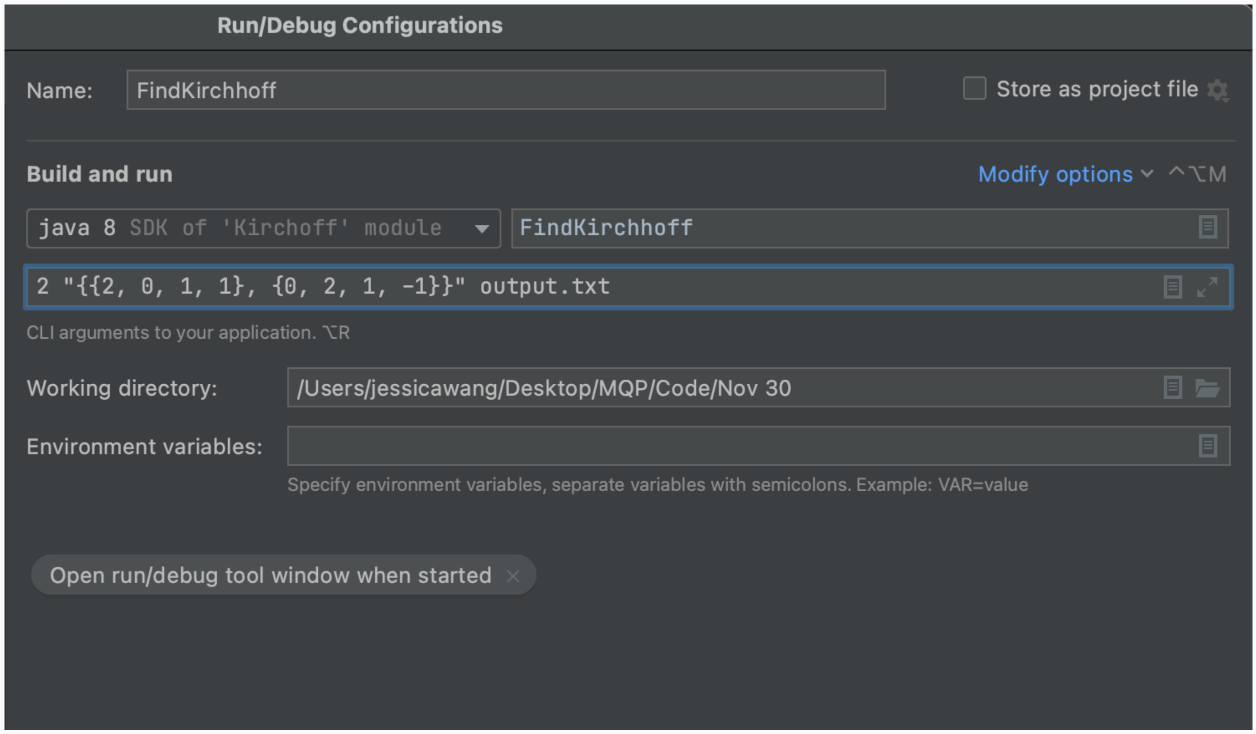
1. Download IntelliJ at https://www.jetbrains.com/idea/download/#section=mac, community version
2. Open the downloaded dmg, drag it into "Applications"
3. Open IntelliJ
4. From IntelliJ, open the entire "Kirchhoff Code" folder that I sent
5. Click "src", go to "FindKirchhoff.java"

To use the code:

1. To find Kirchhoff graphs **with <u>row matrix</u>**:
   Go to top right, select "FindKirchhoff" and click "Edit Configurations"



   And type in to the box:



The format is:
`multiplicity "Row Matrix" output.txt`

So the line
`2 "{{2, 0, 1, 1}, {0, 2, 1, -1}}" output.txt`
means we want to find all Kirchhoff graphs of multiplicity 2 with matrix

$$R = \begin{bmatrix} 2 & 0 & 1 & 1 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$

and we want to put all the information of the Kirchhoff graph into a text file called "output.txt", which can be found in the same folder of the code.

2. Another way to use the code is to give it a list of **vertex cuts** instead of the row matrix. Again, make sure the appropriate line in "main" is lit up:

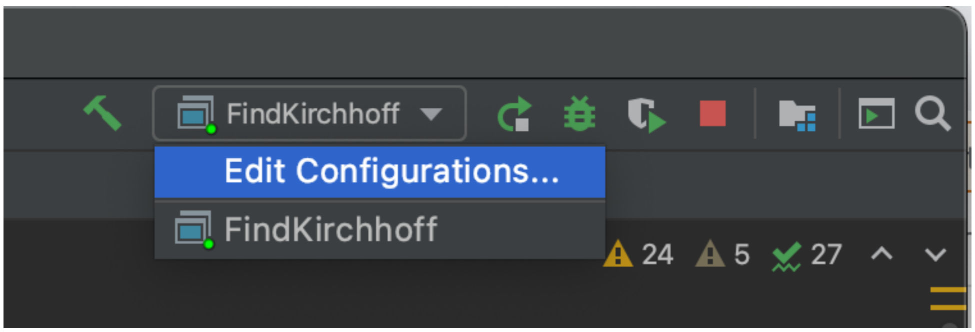
```
try(FileWriter writer = new FileWriter(outputFile, true)) {
    //find_kirchhoff_with_matrix(r, multiplicity, true, writer, outputFile);
    find_kirchhoff_with_cuts(r, store, multiplicity, true, writer,outputFile);
}
```

And change the first few lines like this:

```java
public static void main(String[] args) throws IOException {


    int multiplicity = Integer.parseInt(args[0]);
    int[][] r = stringToArray(args[1]);
    int[][] store = stringToArray(args[2]);
    File outputFile = new File(args[3]);
```

Now we can once again go to “edit configurations”



In the box, the format is now:

```
multiplicity "Row Matrix" "Vertex Cuts" output.txt
```

So this block of code

```
6
"{{2, 0, 1, 1}, {0, 2, 1, -1}}"
"{{-6, -6, -6, 0}, {-6, -4, -5 ,-1 }, {-6, -2, -4, -2}, {-6, 0, -3 ,-3},
{-6,2,-2,-4}, {-6,4,-1,-5}, {-6,6,0,-6}, {-4,-6,-5,1}, {-4,-4,-4,0},
{-4,-2,-3,-1}, {-4,0,-2,-2}, {-4,2,-1,-3}, {-4,4,0,-4}, {-4,6,1,-5},
{-2,-6,-4,2}, {-2,-4,-3,1}, {-2,-2,-2,0}, {-2,0,-1,-1}, {-2,2,0,-2},
{-2,4,1,-3}, {-2,6,2,-4}, {0,-6,-3,3}, {0,-4,-2,2}, {0,-2,-1,1}, {0,0,0,0},
{0,2,1,-1}, {0,4,2,-2}}"
output.txt
```

Means that we want to find all Kirchhoff graphs of multiplicity 6, associated with the row matrix $R = \begin{bmatrix} 2 & 0 & 1 & 1 \\ 0 & 2 & 1 & -1 \end{bmatrix}$, using the list of vertex cuts that we have inputted, and save all the information into a file called “output.txt”.

Remark

“find_kirchhoff_with_matrix” will take longer to run than “find_kirchhoff_with_cuts”, because the matrix function must first find all the appropriate vertex cuts before trying to find any graph.

Structure of the code

Below is a short description of the backtracking exhaustive search algorithm for a given matrix R and multiplicity $m_{\max}$.

1. Find all possible vertex cuts with entries between $-m_{\max}$ and $m_{\max}$ by finding all linear combinations of the row vectors of R. Let Λ be the set of all possible vertex cuts with an arbitrary order. Initialize $\mathbb{T}$ to be an empty list which we will add vertices into. This will serve as our "to-do" list.

2. Construct an anchor vertex, assign the first vertex cut in Λ to the anchor vertex. Add the set of edges according to the vertex cut. If doing so results in vertices to have coordinates (x_1, x_2) where $x_1 < x_2$, then we abandon this vertex cut and remove all the vertices that were constructed. Add all vertices neighboring to the anchor vertex to $\mathbb{T}$.

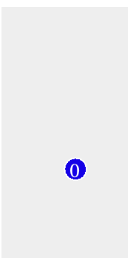
3. Go to the next vertex in the graph (according to the order in $\mathbb{T}$), assign an appropriate vertex cut to it. Delete this vertex in $\mathbb{T}$ and add its neighboring vertices to $\mathbb{T}$. If the current vertex cut is not in Λ or doing so results in having $m(\boldsymbol{G})$ greater than $m_{\max}$, then we abandon this vertex and goes back to the previous vertex, and assign the next vertex cut in Λ to it.

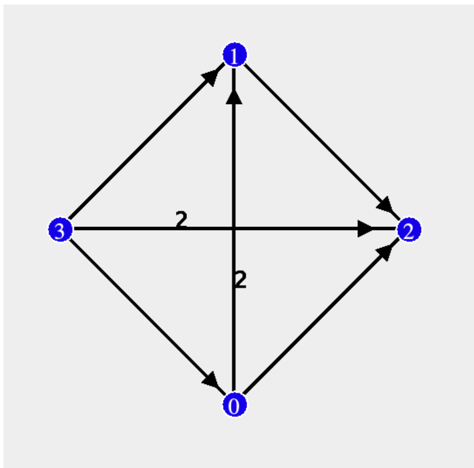
4. We repeat step 3 until either
   - we find a graph with all vertices assigned to a vertex cut in S, which means we have likely found a Kirchhoff graph, or
   - we have exhausted all cuts in S, which means there is no Kirchhoff graph with multiplicity n, $n < m_{\max}$ associated to R.

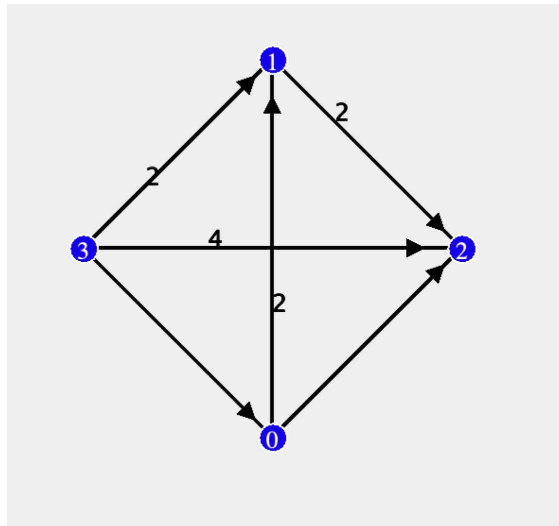
5. If a Kirchhoff graph is found, we add it to a list of graphs, and continue the process with step 2 to find the next possible graph until $\mathbb{T}$ is empty.

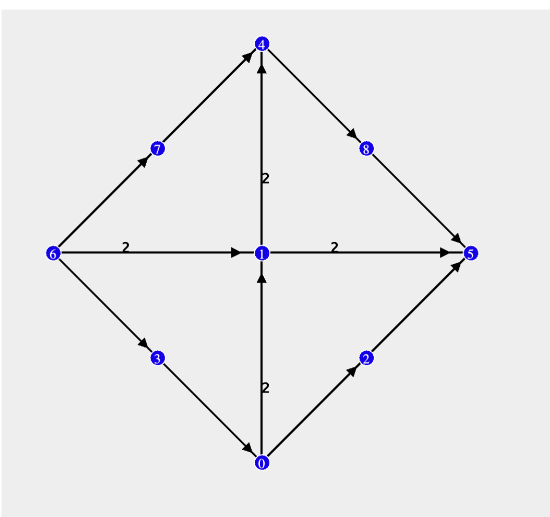
Examples

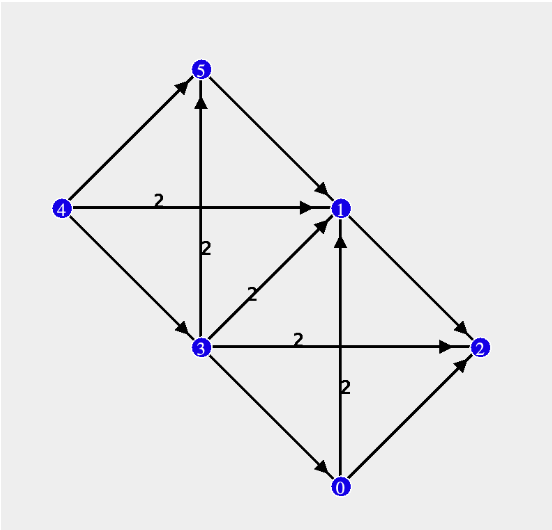
Both examples from the section above mean we want to find all prime and composite graphs of multiplicity 4 with the given row matrix and list of cuts.
It produces the following 19 graphs:







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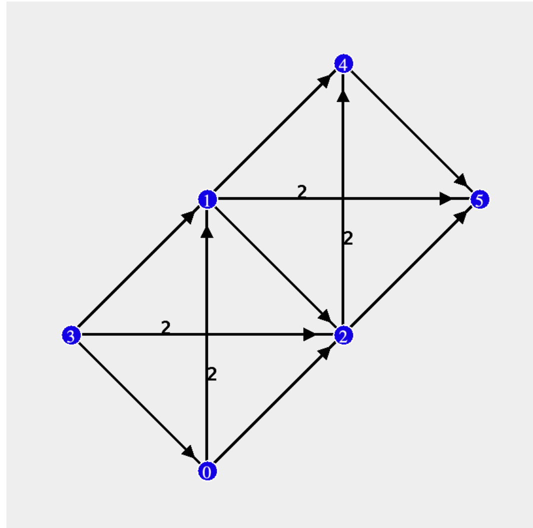
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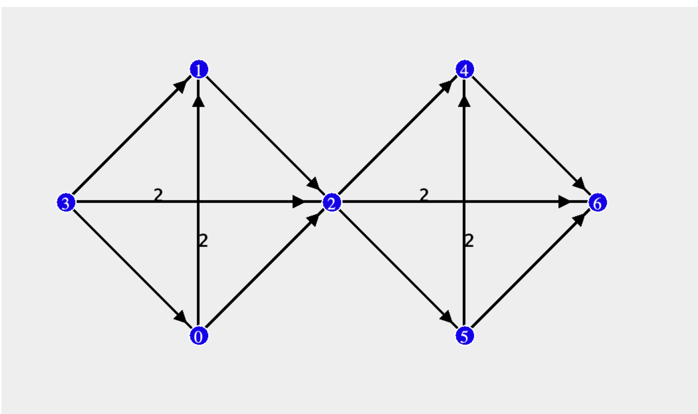
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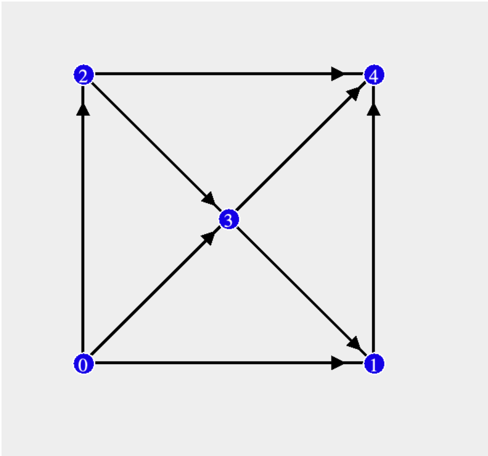
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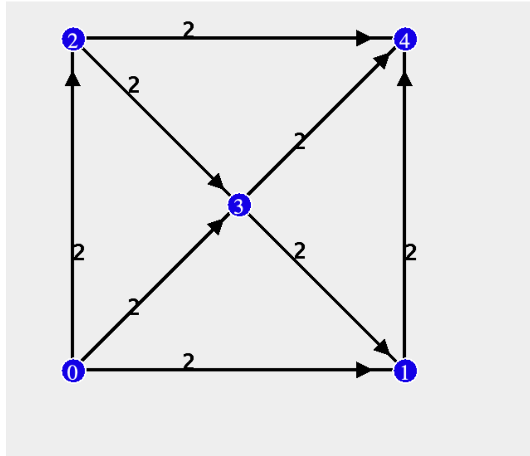
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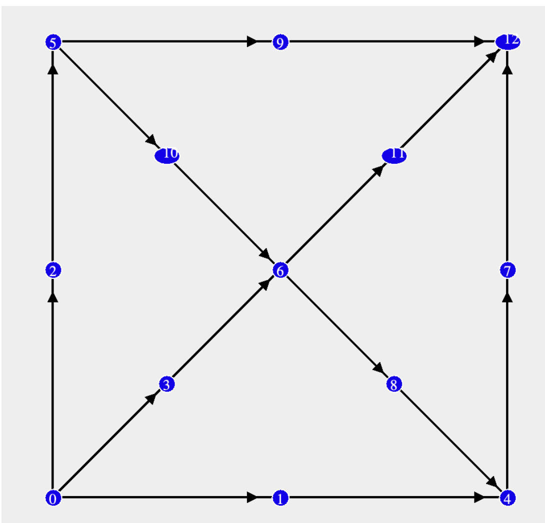
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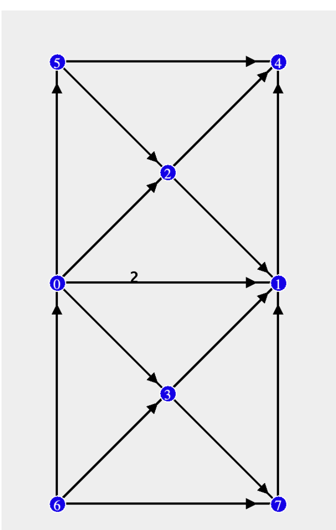
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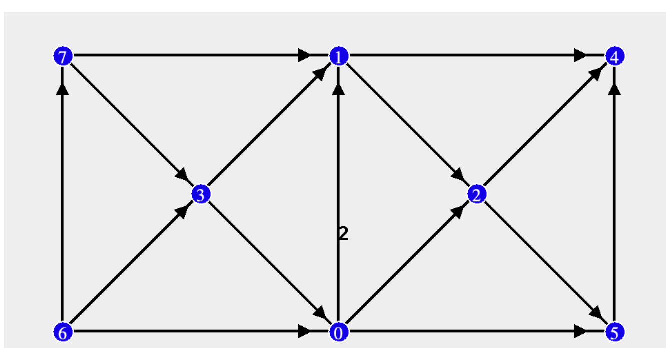
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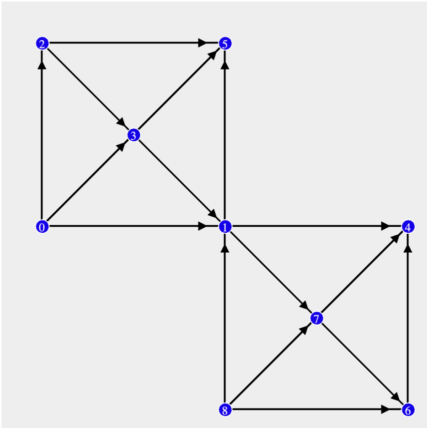
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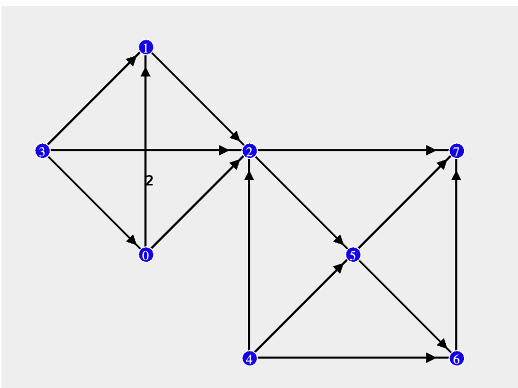
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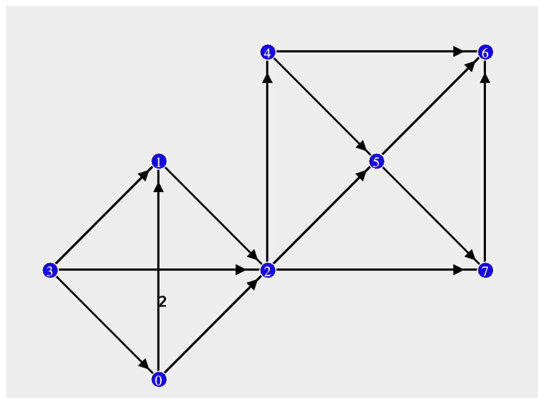
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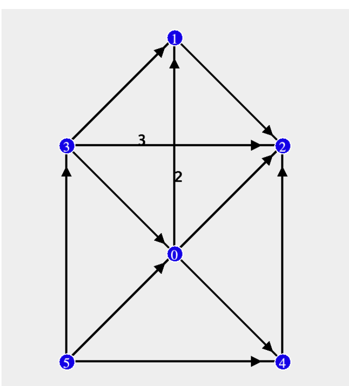
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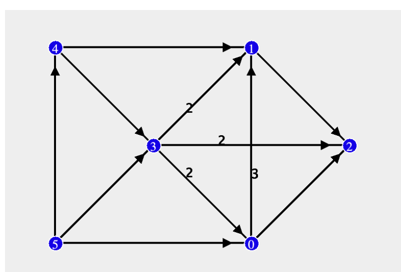
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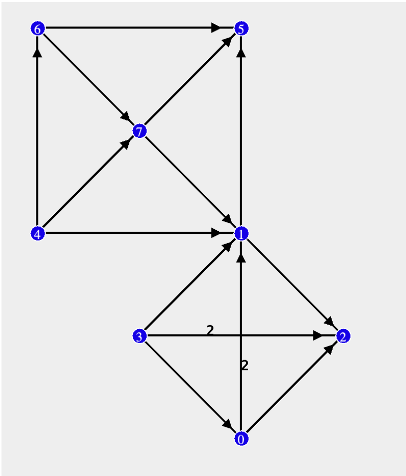
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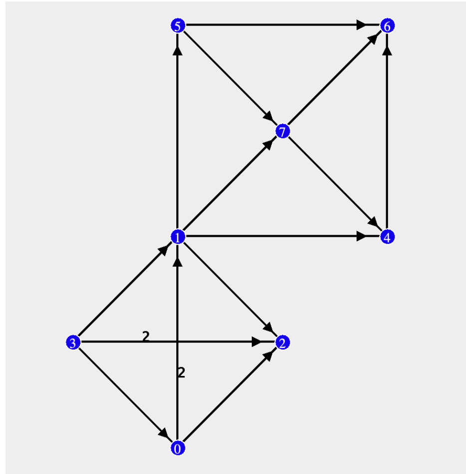
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