

SCOA ASSIGNMENT - 1

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BE-2.

Q.1. Explain Classical set Vs Fuzzy set with example.

Ans →

— Classical set :-

1. Classical set is a collection of distinct objects. For example, a set of ~~subjects~~ students passing grades.
2. Each individual entity in a set is called a member or an element of the set.
3. The Classical set is defined in such a way that the universe of discourse is splitted into two groups, members and non-members. Hence, in case of classical sets, no partial membership exists.
4. Let A is a given set. The membership function can be used to define a set A is given by,
$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

5. For example,

The set of days of week unquestionably includes Tuesday, Wednesday & Saturday. And it just unquestionably excludes butter, liberty, shoe polish, and so on.

6. Operations on Classical set :-

For two sets A & B and universe X :-

i) Union :

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

This operation is called logical OR.

ii) Intersection:-

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

This operation is called logical AND.

iii) Complement:-

$$A' = \{x \mid x \notin A, x \in X\}$$

iv) Difference:-

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$

7. Properties of Classical sets:-

For two sets A & B and universe X,

i) Commutativity:-

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

ii) Associativity:-

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

ii) Distributivity:-

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

iii) Idempotency:-

$$A \cup A = A$$

$$A \cap A = A$$

iv) Identity:-

$$A \cup \emptyset = A$$

$$A \cap X = A$$

$$A \cap \emptyset = \emptyset$$

$$A \cup X = X$$

v) Transitivity:-

If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

- Fuzzy Set:-

1. Fuzzy set is a set having degrees of membership between 1 and 0. Fuzzy sets are represented with tilde character (\sim).
2. For example,
Numbers of cars following traffic signals at a particular time out of all cars present will have membership value between $[0, 1]$.
3. Partial membership exists when member of one fuzzy set can also be a part of other fuzzy sets in the same universe.
4. The degree of membership or truth is not same as probability, fuzzy truth represents membership in vaguely defined sets.
5. A fuzzy set \tilde{A} in the universe of discourse, U , can be defined by as a ^{set of} default set of ordered pairs & it is given by,

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

6. When the universe is discrete and finite, fuzzy set \tilde{A} is given by,

$$\tilde{A} = \sum_{i=1}^n \frac{\mu_{\tilde{A}}(x_i)}{x_i} = \frac{\mu_{\tilde{A}}(x_1)}{x_1} + \frac{\mu_{\tilde{A}}(x_2)}{x_2} + \dots + \frac{\mu_{\tilde{A}}(x_n)}{x_n}$$

$$\tilde{A} = \int \frac{\mu_{\tilde{A}}(x)}{x}$$

where 'n' is a finite value.

7. Fuzzy set also satisfy every property of classical sets.

8. Common operations of fuzzy sets are:-
Given two fuzzy sets \tilde{A} and \tilde{B} .

i) Union: Fuzzy set \tilde{C} is union of \tilde{A} & \tilde{B} :-
 $\tilde{C} = \tilde{A} \cup \tilde{B}$

$$\mu_{\tilde{C}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

ii) Intersection: Fuzzy set \tilde{D} is intersection of \tilde{A} & \tilde{B} :-
 $\tilde{D} = \tilde{A} \cap \tilde{B}$

$$\mu_{\tilde{D}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

iii) Compliment: Fuzzy set \tilde{E} is compliment of \tilde{A} :-
 $\tilde{E} = \tilde{A}^c$

$$\mu_{\tilde{E}}(x) = 1 - \mu_{\tilde{A}}(x)$$

9. Some Useful operations of Fuzzy Set:-

i) Algebraic sum:-

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

ii) Algebraic product:-

$$\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$$

iii) Bounded sum:-

$$\mu_{A \oplus B}(x) = \min\{1, \mu_A(x) + \mu_B(x)\}$$

iv) Bounded difference:-

$$\mu_{A \odot B}(x) = \max\{0, \mu_A(x) - \mu_B(x)\}$$

Q-2. Explain Union, Intersection, Complement operation of fuzzy set with example.

Ans →

- Union: Fuzzy set \tilde{C} is the union of fuzzy sets \tilde{A} and \tilde{B} .

$$\tilde{C} = \tilde{A} \cup \tilde{B}; \tilde{C} = \{(x, \mu_{\tilde{C}}(x)) | x \in X\}$$
 where

$$\mu_{\tilde{C}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

- Intersection: Fuzzy set \tilde{D} is the intersection of fuzzy sets \tilde{A} and \tilde{B} .

$$\tilde{D} = \tilde{A} \cap \tilde{B}$$

given by $\tilde{D} = \{(x, \mu_{\tilde{D}}(x)) | x \in X\}$ where,

$$\mu_{\tilde{D}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

- Complement: Fuzzy set \tilde{E} is the complement of fuzzy set \tilde{A} .

$$\tilde{E} = \complement_{\tilde{A}} X$$
 given by,

$$\tilde{E} = \{(x, \mu_{\tilde{E}}(x)) | x \in X\}$$
, where

$$\mu_{\tilde{E}}(x) = 1 - \mu_{\tilde{A}}(x).$$

- Example:-

Q Determine the union, intersection of the fuzzy sets \tilde{A} = "comfortable house of 4 persons = family" and \tilde{B} = "small house", where,

$$\tilde{A} = \{(1, 0.1), (2, 0.5), (3, 0.8), (4, 1.0), (5, 0.7), (6, 0.2)\}$$
 and

$$\tilde{B} = \{(1, 1), (2, 0.8), (3, 0.4), (4, 0.1)\}$$
.

Also determine $\complement_{\tilde{A}} X$ i.e. complement of \tilde{A} where $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$: ("non-comfortable house for a 4-person-family").

→ 1. Union :-

$$\begin{aligned}\tilde{A} \cup \tilde{B} &= \{(1, \max(0.1, 1)), (2, \max(0.5, 0.8)), \\ &\quad (3, \max(0.8, 0.4)), (4, \max(1, 0.1)), \\ &\quad (5, \max(0.7, 0)), (6, \max(0.2, 0))\} \\ &= \{(1, 1), (2, 0.8), (3, 0.8), (4, 1), (5, 0.7), (6, 0.2)\}\end{aligned}$$

2. Intersection :-

$$\begin{aligned}\tilde{A} \cap \tilde{B} &= \{(1, \min(0.1, 1)), (2, \min(0.5, 0.8)), \\ &\quad (3, \min(0.8, 0.4)), (4, \min(1, 0.1)), \\ &\quad (5, \min(0.7, 0)), (6, \min(0.2, 0))\} \\ &= \{(1, 0.1), (2, 0.5), (3, 0.4), (4, 0.1), \\ &\quad (5, 0), (6, 0)\}\end{aligned}$$

$\tilde{A} \cup \tilde{B}$ can be read as "comfortable house of 4-
persons - family or small" and,
 $\tilde{A} \cap \tilde{B}$ as "comfortable house of 4 person family
and small".

3. Compliment :-

$$\begin{aligned}C\tilde{A} \times &= \{(1, 1-0.1), (2, 1-0.5), (3, 1-0.8), \\ &\quad (4, 1-1), (5, 1-0.7), (6, 1-0.2), \\ &\quad (7, 1-0), (8, 1-0), (9, 1-0), (10, 1-0)\} \\ &= \{(1, 0.9), (2, 0.5), (3, 0.2), \\ &\quad (4, 0), (5, 0.3), (6, 0.8), \\ &\quad (7, 1), (8, 1), (9, 1), (10, 1)\}.\end{aligned}$$

Q.3. Explain types of Hybrid System & explain.

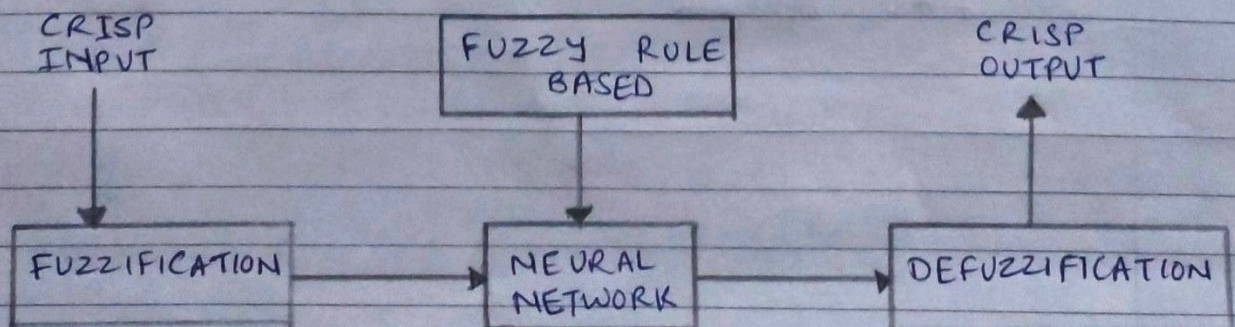
Ans →

A Hybrid system is an intelligent system which is framed by combining atleast two intelligent technologies like Fuzzy Logic, Neural networks, Genetic algorithm, etc. The combination of different techniques in one computational model make these systems possess an extended range of capabilities.

- Types of Hybrid Systems :-

1. Neuro Fuzzy Hybrid Systems :-

- Neuro fuzzy system is based on fuzzy system which is trained on the basis of working of neural network theory. The learning process operates only on the local information and causes only local changes in the underlying fuzzy system.
- A neuro-fuzzy system can be seen as a 3-layer feed forward neural network.
- The first layer represents input variables, the middle (hidden) layer represents fuzzy rules and the third layer represents output variables.
- Fuzzy sets are encoded as connection weights within the layers of the network, which provides functionality in processing and training model.



2. Neuro Genetic Hybrid System:-

- A Neuro Genetic hybrid system is a system that combines Neural networks: which are capable to learn various tasks from examples, classify objects and establish relation between them and Genetic algorithm: which serves important search and optimization techniques.
- Genetic algorithms can be used to improve the performance of Neural network and they can be used to decide the connection weights of the inputs.
- These algorithms can also be used for topology selection and training network.

3. Fuzzy Genetic Hybrid systems:-

A Fuzzy Genetic Hybrid system is developed to use fuzzy logic based techniques for improving and modelling Genetic algorithm and vice-versa.

Genetic algorithm has proved to be a robust and efficient tool to perform tasks like generation of fuzzy rule base, generation of membership function, etc.

Three approaches that can be used to develop such system are:-

1. Michigan Approach
2. Pittsburgh Approach
3. IRL Approach