

## ML Practical 2

## Title: Decision Tree Classification

1) Explain following terminologies related to decision tree building

a) Impurity:

It defines how well each classes are separated. In general, the impurity measure should satisfy the most when data are split evenly for attribute values.

$$P_i = \frac{1}{\text{No. of classes}}$$

No. of classes

Impurity should be 0 when all data belong to the same class.

b) Entropy:

The entropy of a random variable  $x$  is defined by

$$\text{Entropy}(H(x)) = - \sum p(x) \log p(x)$$

The entropy measures the expected uncertainty in  $x$ .

It has the following properties:

$H(x) \geq 0$ , entropy is always non-negative

$H(x) = 0$ , if and only if  $x$  is deterministic

c) Information Gain:

The expected information needed to classify a tuple in  $P$  is

given by,

$$\text{Info}_P(P) = - \sum_{i=1}^n P_i \log_2(P_i)$$

$$\text{Info}_A(P) = \sum_{i=1}^n \frac{|P_i|}{|P|} \times \text{Info}(P_i)$$

$$\text{Gain}(A) = \text{Info}(P) - \text{Info}_A(P)$$

2) What is Gini Index? Explain with formula.

Ans. 1) The Gini index is used in CART.

2) The Gini index measures the impurity of  $P$ , a data partition or set of training tuples as

$$\text{Gini}(P) = 1 - \sum_{i=1}^n P_i^2$$

3) The attribute that maximizes the reduction in impurity (or equivalent) has the maximum Gini Index and selected as splitting attribute.

4) Gini Index is a metric to measure how often a randomly chosen element would be incorrectly identified.

3) Solve the problem for given dataset in problem statement to explain how to find root node using entropy and information gain.

Ans.

Step 1: Finding the entropy

$$\text{Entropy}(E(S)) = - \sum p(x) \log_2 p(x)$$

∴ For the dataset, entropy is

$$\begin{aligned} \text{Entropy} &= -P(\text{yes}) \log_2(P(\text{yes})) - P(\text{No}) \log_2(P(\text{No})) \\ &= -\left(\frac{9}{14}\right) \log_2\left(\frac{9}{14}\right) - \left(\frac{5}{14}\right) \log_2\left(\frac{5}{14}\right) \end{aligned}$$

$$\approx 0.94$$

Step 2: Finding Information Gain of each attribute

Age						
< 21		21-35		> 35		
Yes	No	Yes	No	Yes	No	
2	2	3	1	4	2	

$$H(\text{AGE} < 21) = -\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right) = 0.971$$

$$H(\text{AGE } 21-35) = -\frac{4}{4} \log_2\left(\frac{4}{4}\right) - \frac{0}{4} \log_2\left(\frac{0}{4}\right) = 0$$

$$H(\text{AGE} > 35) = -\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) = 0.971$$

$$\therefore \text{Information gain (age)} = \frac{-5 \times 0.971}{14} + \frac{4 \times 0}{14} + \frac{5 \times 0.971}{14}$$

$$\text{Info (age)} = 0.693$$

$$\text{Gain (Age)} = E(s) - \text{Info (age)}$$

$$= 0.94 - 0.693$$

$$= 0.247$$

Income						
High		Low		Medium		
Yes	No	Yes	No	Yes	No	
2	2	3	1	4	2	

$$H(\text{INCOME HIGH}) = -\frac{2}{4} \log_2\left(\frac{2}{4}\right) - \frac{2}{4} \log_2\left(\frac{2}{4}\right) = 1$$

$$H(\text{INCOME LOW}) = -\frac{3}{4} \log_2\left(\frac{3}{4}\right) - \frac{1}{4} \log_2\left(\frac{1}{4}\right) = 0.811$$

$$H(\text{INCOME MED}) = -\frac{4}{6} \log_2\left(\frac{4}{6}\right) - \frac{2}{6} \log_2\left(\frac{2}{6}\right) = 0.918$$

$$\therefore \text{Information Gain (Income)} = \frac{4 \times 1}{14} + \frac{4 \times 0.811}{14} + \frac{6 \times 0.918}{14} = 0.911$$

$$\text{Gain (Income)} = E(s) - \text{Info (Income)} = 0.0291$$

Gender				
Male		Female		
Yes	No	Yes	No	
3	4	6	1	



$$H(\text{Gender MALE}) = -\frac{3}{4} \log_2\left(\frac{3}{4}\right) - \frac{1}{4} \log_2\left(\frac{1}{4}\right) = 0.985$$

$$H(\text{Gender FEMALE}) = -\frac{6}{7} \log_2\left(\frac{6}{7}\right) - \frac{1}{7} \log_2\left(\frac{1}{7}\right) = 0.91885$$

$$\text{Information gain (GENDER)} = \frac{7}{14} \times 0.985 + \frac{7}{14} \times 0.91885 = 0.949$$

$$\text{Gain (Gender)} = 0.94 - 0.949 = 0.1515$$

Marital Status			
Single		Married	
Yes	No	Yes	No
5	2	4	3

$$H(\text{MS SINGLE}) = -\frac{5}{7} \log_2\left(\frac{5}{7}\right) - \frac{2}{7} \log_2\left(\frac{2}{7}\right) = 0.863$$

$$H(\text{MS MARRIED}) = -\frac{4}{7} \log_2\left(\frac{4}{7}\right) - \frac{3}{7} \log_2\left(\frac{3}{7}\right) = 0.985$$

$$\text{Information gain (M.S)} = \frac{7}{14} \times 0.863 + \frac{7}{14} \times 0.985 = 0.924$$

$$\text{Gain (marital status)} = 0.94 - 0.924 = 0.016$$

Attributes	Gain
Age	0.247
Income	0.0291
Gender	0.1515
Marital status	0.016

Since the attribute "Age" has highest gain, it is selected as root node.