CHAPTER 3

Unit III

Fuzzy Systems

Syllabus Topic : Fuzzy Controller

3.1 Fuzzy Controller

- Most commercial fuzzy products use fuzzy knowledge-based controllers (FKBC).
- The principal structure of a FKBC is shown in Fig. 3.1.1.

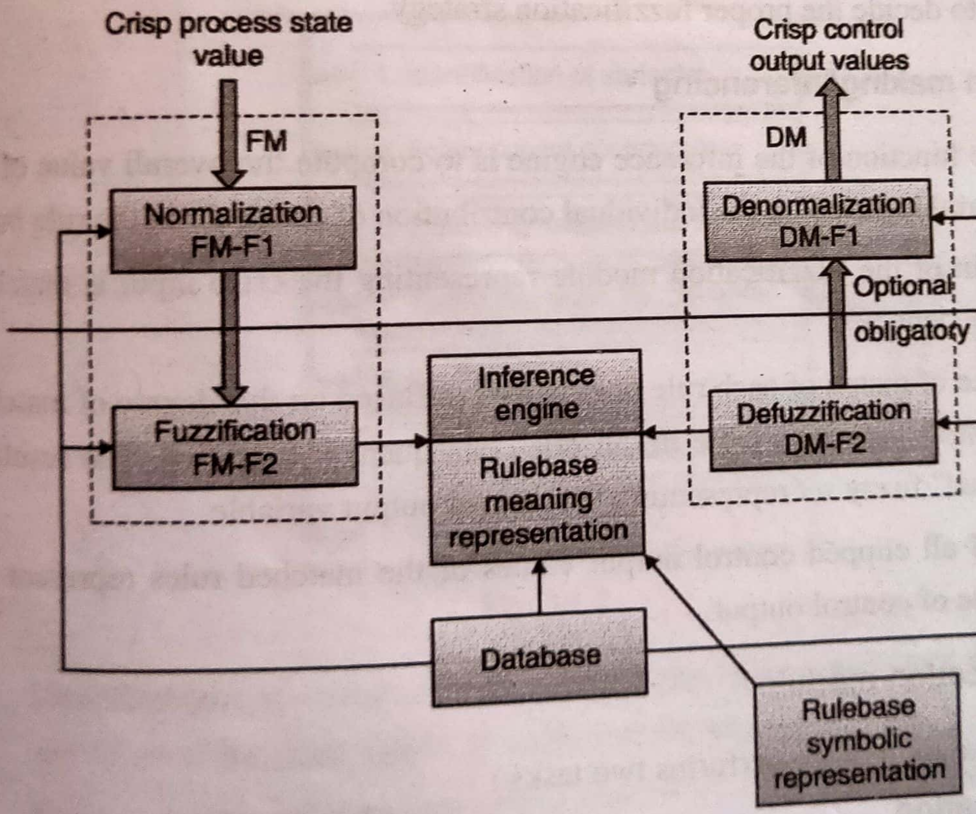

Fig. 3.1.1 : The structure of FKBC

- As shown in Fig.3.1.1, FKBC involves three important modules.

1. Fuzzification module

2. Decision making or Inferencing module
3. Defuzzification module

- In addition to this, it uses two more components
  – Data base and
  – Rule base
  } Knowledge base

Fuzzification module

Fuzzification module performs the following two functions.

1. **Normalization**

   This block performs a scale transformation which maps the physical values of input variables in to a normalized universe of discourse. This block is optional. If a non-normalized domain is used then this block is not required.

2. **Fuzzification**

   This block performs a fuzzification which converts a crisp input in to a fuzzy set. Here we need to decide the proper fuzzification strategy.

Decision making/Inferencing

- The basic function of the inference engine is to compute the overall value of the control output variable based on the individual contribution of each rule in the rule base.
- The output of the fuzzification module representing the crisp input is matched to each rule-antecedent.
- The degree of match of each rule is established. Based on this degree of match, the value of the control output variable in the rule-consequent is modified. The result is, we get the "clipped" fuzzy set representing the control output variable.
- The set of all clipped control output values of the matched rules represent the overall fuzzy value of control output.

Defuzzification module

Defuzzification module performs two tasks :

1. **Defuzzification**

   It performs defuzzification which converts the overall control output into a single crisp value.

2. **Denormalization module**

This block maps the crisp value of the control output into the physical domain. This block is optional. It is used only if normalization is performed during the fuzzification phase.

The knowledge base basically consists of a database and a rule base.

- The **database** provides the necessary information for proper functioning of the fuzzification module, the rule base and the defuzzification module.
- The information in the database includes :
  - Fuzzy MFs for the input and output control variables
  - The physical domains of the actual problems and their normalized values along with the scaling factors.

3.1.1 Steps in Designing FLC

Following are the steps involved in designing FLC

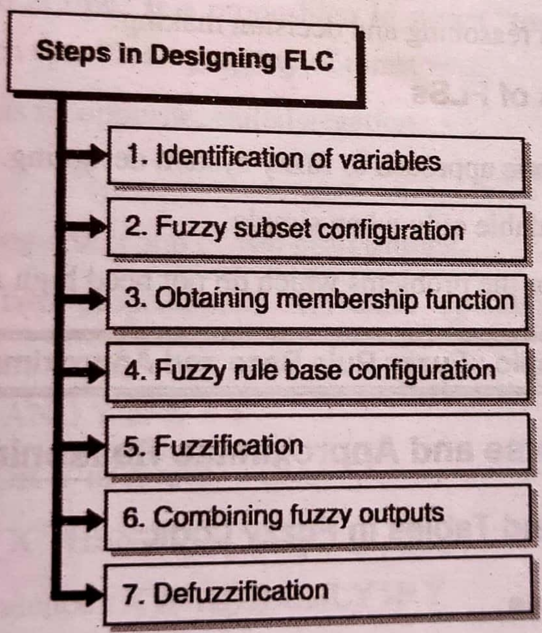

Fig. 3.1.2

➔ 1. **Identification of variables :** Here, the input, output and state variables must be identified of the plant which is under consideration.

➔ 2. **Fuzzy subset configuration :** The universe of information is divided into number of fuzzy subsets and each subset is assigned a linguistic label. Always make sure that these fuzzy subsets include all the elements of universe.

➔ 3. **Obtaining membership function :** Now obtain the membership function for each fuzzy subset that we get in the above step.

→ 4. **Fuzzy rule base configuration :** Now formulate the fuzzy rule base by assigning relationship between fuzzy input and output.

→ 5. **Fuzzification :** The fuzzification process is initiated in this step.

→ 6. **Combining fuzzy outputs :** By applying fuzzy approximate reasoning, locate the fuzzy output and merge them.

→ 7. **Defuzzification :** Finally, initiate defuzzification process to form a crisp output

3.1.2 Advantages of FLSs

- It uses very simple Mathematical concepts for reasoning.
- An FLS can be modified by just adding or deleting rules due to flexibility of fuzzy logic.
- Fuzzy logic Systems can take imprecise, distorted, noisy input information.
- FLSs are easy to construct and understand.
- Fuzzy logic is a solution to complex problems in all fields of life, including medicine, as it resembles human reasoning and decision making.

3.1.3 Disadvantages of FLSs

- There is no systematic approach to fuzzy system designing.
- They are understandable only when simple.
- They are suitable for the problems which do not need high accuracy.

Syllabus Topic : Fuzzy Rule Base and Approximate Reasoning

3.2 Fuzzy Rule Base and Approximate Reasoning

3.2.1 Truth Values and Tables in Fuzzy Logic

Linguistic Variables

- Fuzzy logic uses linguistic variables. By a linguistic variable, we mean a variable whose values are words or sentences in a natural or artificial language.
- For example 'age' is a linguistic variable if it takes values such as young, middle, old, very old etc.
- The linguistic variable provides approximate characterization of a complex problem.
- A linguistic variable is characterization of a complex problem.
- A linguistic variable is characterized by,
  - o Name of a variable-x

- Term set of the variable-t(x)
- Syntacting rules used to generate values of x
- Semantic rules for associating each value of x with its meaning.

Linguistic hedges

Linguistic hedges can be considered as an operator which acts on the fuzzy set representing the meaning of its operand. For example, consider the composite term "very tall man". The operator **very** acts on the fuzzy meaning of the term **tall man**.

Other examples of hedges are slightly, moderately, fairly, extremely etc.

Propositions

Propositions are text sentences expressed in any language. The general form of proposition is, **z is p**.

Where, z represents the subject and p represents the predicate.

Example : "Tomato is ripe" is a proposition in which "tomato" is the subject and "is ripe" is the predicate, which specifies a property of tomato.

- Every proposition has its opposite, called negation.

Truth table

Truth table defines logic function of two propositions.

Let X and Y be two propositions. The basic operations that can be applied to propositions are

(1) Conjunction ($\wedge$) : X AND Y or $X \wedge Y$

(2) Disjunction ($\vee$) : X OR Y or $X \vee Y$

(3) Implication ($\Rightarrow$) : IF X THEN Y

(4) Bidirectional / equivalence : X IF AND ONLY IF Y

Inference rules

Based on the operations on propositions, inference rules can be formed.

For example,

(1) $[X \wedge (X \Rightarrow Y)] \Rightarrow Y$

(2) $[\bar{X} \wedge (\bar{Y} \vee X)] \Rightarrow \bar{X}$

- The truth values of propositions in fuzzy logic can take any value over the unit interval [0, 1].

- The truth value of the proposition "z is A" is defined by a point in [0, 1] (called the numerical truth value) or a fuzzy set in [0, 1] (called the linguistic truth value)
- The truth value of a proposition can be obtained from the truth value of other propositions.

For example :

(1) tv (X AND Y) $= tv(X) \wedge tv(Y)$ (Intersection)

$= \min \{tv(X), tv(Y)\}$

(2) tv (X OR Y) $= tv(X) \vee tv(Y)$ (Union)

$= \max \{tv(X), tv(Y)\}$

(3) tv (NOT X) $= 1 - tv(X)$ (Complement)

3.2.2 Fuzzy Propositions

Propositions in fuzzy logic include the following :

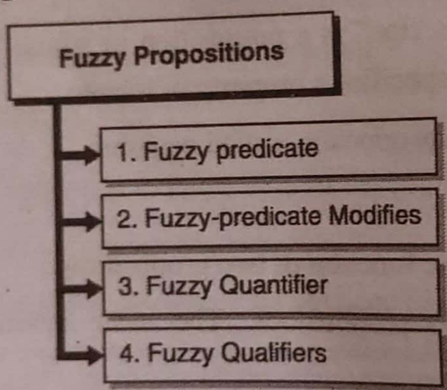

Fig. 3.2.1

→ 1. Fuzzy predicate

Almost every predicate in natural language is fuzzy in nature. For example, tall, short, quick, hot etc.

→ 2. Fuzzy-predicate Modifies

Fuzzy-predicate modifiers act as hedges for linguistic variables. For example, the words very, slightly, extremely are modifiers and the proposition can be "Water is **very** hot"

In the above example "hot" is a linguistic variable and "very" act as hedge.

→ 3. Fuzzy Quantifier

A fuzzy quantifier is defined as a fuzzy number which provides an imprecise characterization of the cardinality of one or more fuzzy or non-fuzzy sets. Examples of fuzzy quantifiers are words like "most", "many", "few" etc. In other words, fuzzy quantifiers gives us an approximate idea of the number of elements of a subset fulfilling a certain conditions.

4. Fuzzy Qualifiers

A fuzzy qualifiers is also a proposition of fuzzy logic. There are four modes of qualifiers.

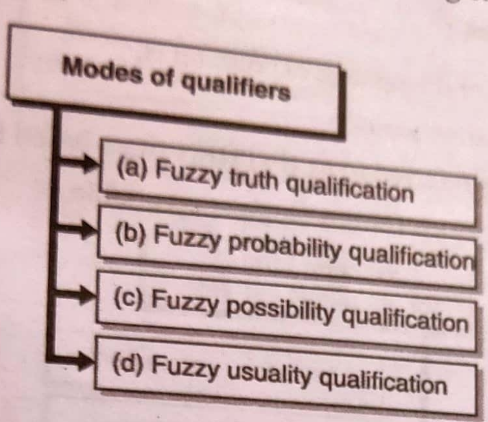

Fig. 3.2.2

(a) Fuzzy truth qualification

It claims the degree of truth of a fuzzy proposition.

Expression : It is expressed as X is t. Here t is a fuzzy truth value.

Example : (Water is hot) is NOT VERY true. Here the qualified proposition is (water is hot) and the qualifying fuzzy truth value is "NOT VERY true"

(b) Fuzzy probability qualification

It claims the probability, either numerical or an interval, of fuzzy proposition

Expression : (Water is hot) is likely. Here the qualifying fuzzy probability is "likely".

(c) Fuzzy possibility qualification

It claims the possibility of fuzzy proposition.

Expression : It is expressed as "X is π", where π is a fuzzy possibility and can take values such as possible, quite possible, almost possible.

Example : (Water is hot) is almost impossible.

(d) Fuzzy usuality qualification

The propositions that are usually true or the events that have high probability of occurrence are related by the concept of usuality qualification.

Expression : "Usually (X is F)"

Here the subject X is a variable taking values in a universe of discourse U. Predicate F is a fuzzy subset of U and is interpreted as a usual value of X (denoted by U(X) = F).

Example : Usually Riya is very cheerful.

3.2.3 Formation of Rules :

The general form of natural language expression is,

"IF antecedent THEN consequent."

The above expression is referred to as IF-THEN rules-based form.

There are three general forms for any linguistic variable.

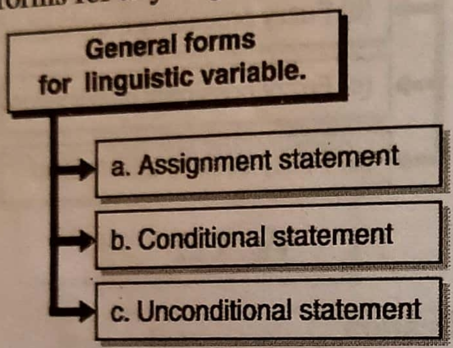

Fig. 3.2.3

➔ a. **Assignment statement**

x = big

Tomato color = Red

Jack is not tall.

➔ b. **Conditional statement**

IF temp is very cold THEN wear sweater. IF A is high THEN B is low ELSE C is low.

➔ c. **Unconditional statement**

Goto sum

Stop

Multiply by 10

Turn the knob to the left

3.2.4 Decomposition of Compound Rules

A collection of many simple rules combined together is called a **compound rule**. It may be required to decompose such compound rules and reduce to a number of simple canonical rule forms.

Methods used for decomposition of rules

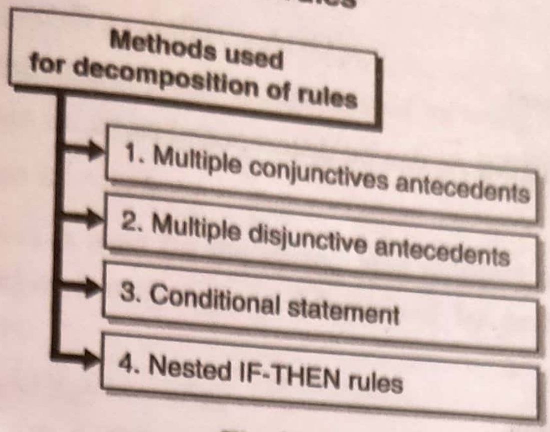

Fig. 3.2.4

1. **Multiple conjunctives antecedents**

IF x is $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n$ THEN y is $\tilde{B}_m$. Assume a new fuzzy subset $\tilde{A}_m$ is defined as,

$$\tilde{A}_m = \tilde{A}_1 \cap \tilde{A}_2 \cap \ldots \cap \tilde{A}_n$$

and expressed as a membership function.

$$\mu_{\tilde{A}_m}(x) = \min \mu_{\tilde{A}_1}(x), \mu_{\tilde{A}_2}(x) \ldots \mu_{\tilde{A}_n}(x)$$

Which is based on the fuzzy intersection operation.

2. **Multiple disjunctive antecedents**

IF x is $\tilde{A}_1$ or x is $\tilde{A}_2$, … OR x is $\tilde{A}_n$ THEN y is $\tilde{B}_m$ can be written as,

IF x is $\tilde{A}_n$ THEN y is $\tilde{B}_m$ where the fuzzy set $\tilde{A}_m$ is defined as,

$$\tilde{A}_m = \tilde{A}_1 \cup \tilde{A}_2 \cup \ldots \cup \tilde{A}_n$$ and expressed as a membership function

$$\mu_{\tilde{A}_m}(x) = \max \left[\mu_{\tilde{A}_1}(x), \mu_{\tilde{A}_2}(x), \ldots \mu_{\tilde{A}_n}(x)\right]$$

Which is based on the fuzzy union operation.

3. **Conditional statement**

The compound statement,

IF $\tilde{A}_1$ THEN ($\tilde{B}_1$ ELSE $\tilde{B}_2$) can be decomposed into two simple canonical rule forms, connected by OR.

IF $\tilde{A}_1$ THEN $\tilde{B}_1$

IF NOT $\tilde{A}_1$ THEN $\tilde{B}_2$

The compound statement,

IF $\tilde{A}_1$ (THEN $\tilde{B}_1$) UNLESS $\tilde{A}_2$ can be decomposed as,

IF $\tilde{A}_1$ THEN $\tilde{B}_1$

OR

IF $\tilde{A}_2$ THEN NOT $\tilde{B}_1$

The compound statement,

IF $\tilde{A}_1$ THEN $\tilde{B}_1$ ELSE IF $\tilde{A}_2$ THEN $\tilde{B}_2$

Can be decomposed as,

IF $\tilde{A}_1$ THEN $\tilde{B}_1$

OR

IF NOT $\tilde{A}_1$ AND IF $\tilde{A}_2$ THEN $\tilde{B}_2$

4. **Nested IF-THEN rules**

The rule,

"If $\tilde{A}_1$ THEN [IF $\tilde{A}_2$ THEN $\tilde{B}_1$]" can be decomposed as,

If $\tilde{A}_1$ AND $\tilde{A}_2$ THEN $\tilde{B}_1$

3.2.5 Aggregation of Fuzzy Rules

When the rule-based system involves more than one rules, the aggregation of these rules is required. The process of obtaining the overall consequents from the individual consequents by each rule is called **aggregation of rules.**

There are two methods used for aggregation of fuzzy rules :

1. Conjunctive system of rules
2. Disjunctive system of rules

1. Conjunctive system of rules

The "and" (conjunctive) connectives is used for the system that requires all the rules to be jointly satisfied. Here the aggregated output y, is determined by performing fuzzy intersection of all individual rule consequents.

$$y = y_1 \cap y_2 \cap y_3 \cap \ldots \cap y_n$$

or

$$y = y_1 \text{ AND } y_2 \text{ AND } \ldots \text{ AND } y_n$$

or aggregated output y can be represented by using membership function

$$\mu_y(y) = \min [\mu_{y_1}(y), \mu_{y_2}(y) \ldots, \mu_{y_n}(y)] \quad \text{For } y \in Y$$

2. **Disjunctive system of rules**

The "or" (disjunctive) is used for the system that requires at least one out of all rules is satisfied. Here, the aggregated output y, is determined by performing fuzzy union of all individual rule consequents.

$$y = y_1 \cup y_2 \cup \ldots \cup y_n$$

$$y = y_1 \text{ OR } y_2 \text{ OR } \ldots \text{ OR } y_n$$

or representing in the form of membership function,

$$\mu_y(y) = \max [\mu_{y_1}(y), \mu_{y_2}(y), \ldots, \mu_{y_n}(y)] \text{ for } y \in Y$$

Syllabus Topic : Fuzzy Reasoning

3.3 Fuzzy Reasoning

- As discussed earlier, fuzzy logic is capable of handling uncertainty and vagueness present in the real-world complex problems.
- Fuzzy reasoning basically deals with reasoning that is approximate rather than fixed or exact.
- In fuzzy logic, both the antecedents and consequents are allowed to be fuzzy propositions.
- There exist four models of fuzzy approximate reasoning.

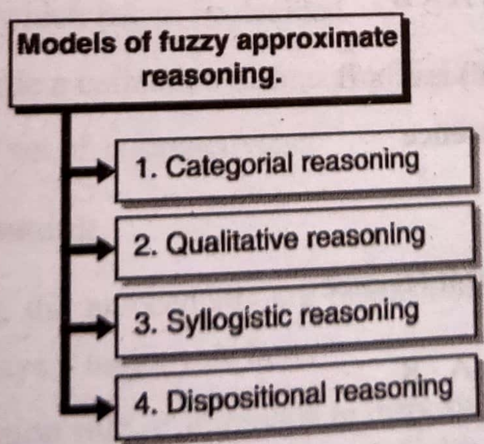

Fig. 3.3.1

1. Categorial Reasoning

In this form of reasoning, the antecedent part of the rule does not contain any fuzzy quantifiers and fuzzy probabilities.

The antecedents are assumed to be in canonical form.

Let $\tilde{X}, \tilde{Y}, \tilde{Z}$ = Fuzzy variables taking in the universe U, V, W

$\tilde{X}, \tilde{B}, \tilde{C}$ = Fuzzy predicates

(a) The projection rule

The projection rule of inference is defined as,

$$\frac{\tilde{X}, \tilde{Y} \text{ is } \tilde{R}}{\tilde{X} \text{ is } [\tilde{R} \downarrow \tilde{X}]}$$

Where $[\tilde{R} \downarrow \tilde{X}]$ denotes the projection of fuzzy relation $\tilde{R}$ on $\tilde{X}$.

(b) The conjunction rule

The conjunction rule of inference is defined as,

$$\tilde{X} \text{ is } \tilde{A}, \tilde{X} \text{ is } \tilde{B} \Rightarrow \tilde{X} \text{ is } \tilde{A} \cap \tilde{B}$$

$$(\tilde{X}, \tilde{Y}) \text{ is } \tilde{A}, \tilde{X} \text{ is } \tilde{B} \Rightarrow (\tilde{X}, \tilde{Y}) \text{ is } \tilde{A} \cap (\tilde{B} \times \tilde{V})$$

$$(\tilde{X}, \tilde{Y}) \text{ is } \tilde{A}, (\tilde{Y}, \tilde{Z}) \text{ is } \tilde{B} \Rightarrow (\tilde{X}, \tilde{Y}, \tilde{Z}) = (\tilde{A} \times \tilde{W}) \cap (\tilde{U} \times \tilde{B})$$

(c) The disjunction rule

The disjunction rule of inference is defined as,

$$\tilde{X} \text{ is } \tilde{A} \text{ OR } \tilde{X} \text{ is } \tilde{B} \Rightarrow \tilde{X} \text{ is } \tilde{A} \times \tilde{B}$$

$$\tilde{X} \text{ is } \tilde{A} \text{ OR } \tilde{Y} \text{ is } \tilde{B} \Rightarrow (\tilde{X}, \tilde{Y}) \text{ is } \tilde{A} \times \tilde{B}$$

(d) The negative rule of inference

$$\text{NOT } (\tilde{X} \text{ is} \tilde{A}) \Rightarrow \tilde{X} \text{ is } \tilde{A}$$

(e) The compositional rule of inference

$$\tilde{X} \text{ is } \tilde{A}, (\tilde{X}, \tilde{Y}) \text{ is } R \Rightarrow \tilde{Y} \text{ is } \tilde{A} \cdot \tilde{R}$$

Where, $\tilde{A} \cdot \tilde{R}$ denotes the max-min composition of fuzzy set $\tilde{A}$ and fuzzy relation R and is given as,

$$\mu_{\tilde{A} \cdot \tilde{R}}(u) = \max_{u} \min \left[\mu_{\tilde{A}}(u), \mu_{\tilde{R}}(u, v)\right]$$

2. Qualitative Reasoning

In this mode of reasoning, the antecedents and consequents have fuzzy linguistic variables. The input-output relationship of a system is expressed as a collection of fuzzy IF-THEN rules. This reasoning is mainly used in control systems.

Let $\tilde{A}$ and $\tilde{B}$ be the fuzzy input variables and $\tilde{C}$ be the fuzzy output variable.

The relation between $\tilde{A}$ $\tilde{B}$ and $\tilde{C}$ may be expressed as,

If $\tilde{A}$ is x_1 AND $\tilde{B}$ is y_1, THEN $\tilde{C}$ is z_1

If $\tilde{A}$ is x_2 AND $\tilde{B}$ is y_2, THEN $\tilde{C}$ is z_2

⋮

If $\tilde{A}$ is x_n AND $\tilde{B}$ is y_n, THEN $\tilde{C}$ is z_n .

3. Syllogistic Reasoning

In this model, antecedents with fuzzy quantifiers are related to inference rules

A fuzzy syllogism can be expressed as,

$$\begin{array}{l} x = s_1 \text{ A's are B's} \\ \underline{y = s_2 \text{ C's are D's}} \\ z = s_3 \text{ E's are F's} \end{array}$$

Here, A, B, C, D, E, F are fuzzy predicates .

- S_1 and S_2 are given fuzzy quantifiers.
- S_3 is the fuzzy quantifier which has to be decided.
- All fuzzy predicates provide a collection of fuzzy syllogism.

These syllogisms create a set of inference rules.

4. Dispositional Reasoning

In this type of reasoning, the antecedents are dispositions and may contain the fuzzy quantifier "usually". Usuality plays a major role here.

For Example : The projection rule of inference in dispositional reasoning can be written as,

Usually ((X, Y) is R) $\Rightarrow$ usually (X is [R $\downarrow$ X])

Where, [R $\downarrow$ X] is the projection of fuzzy relation R or X.

Syllabus Topic : Fuzzy Inference System

3.4 Fuzzy Inference System

Fuzzy Inference System is the key unit of a fuzzy logic system. Fuzzy inference (reasoning) is the actual process of mapping from a given input to an output using fuzzy logic. It uses the "IF…THEN" rules along with connectors "OR" or "AND" for drawing essential decision rules.

3.4.1 Construction and Working Principle of FIS

Fig. 3.4.1(a) shows the block diagram of general fuzzy inference system.

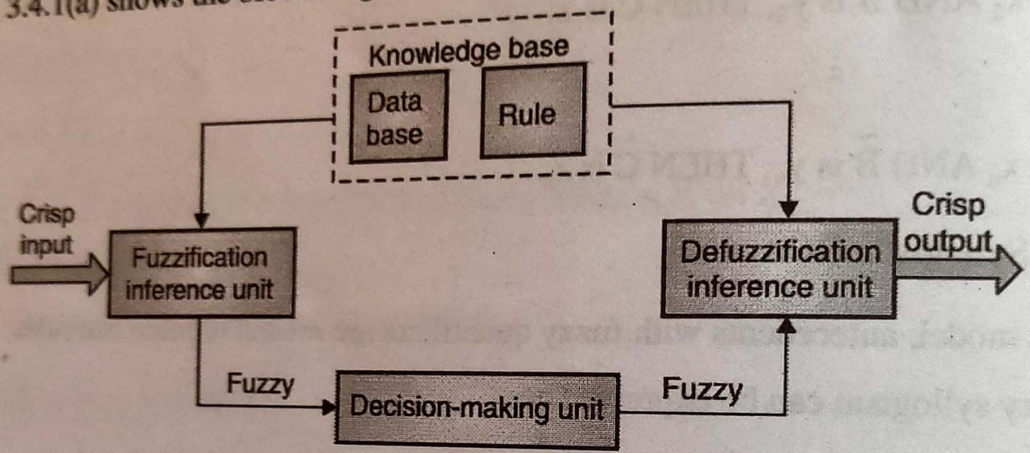

Fig. 3.4.1 (a) : Block diagram : Fuzzy inference system

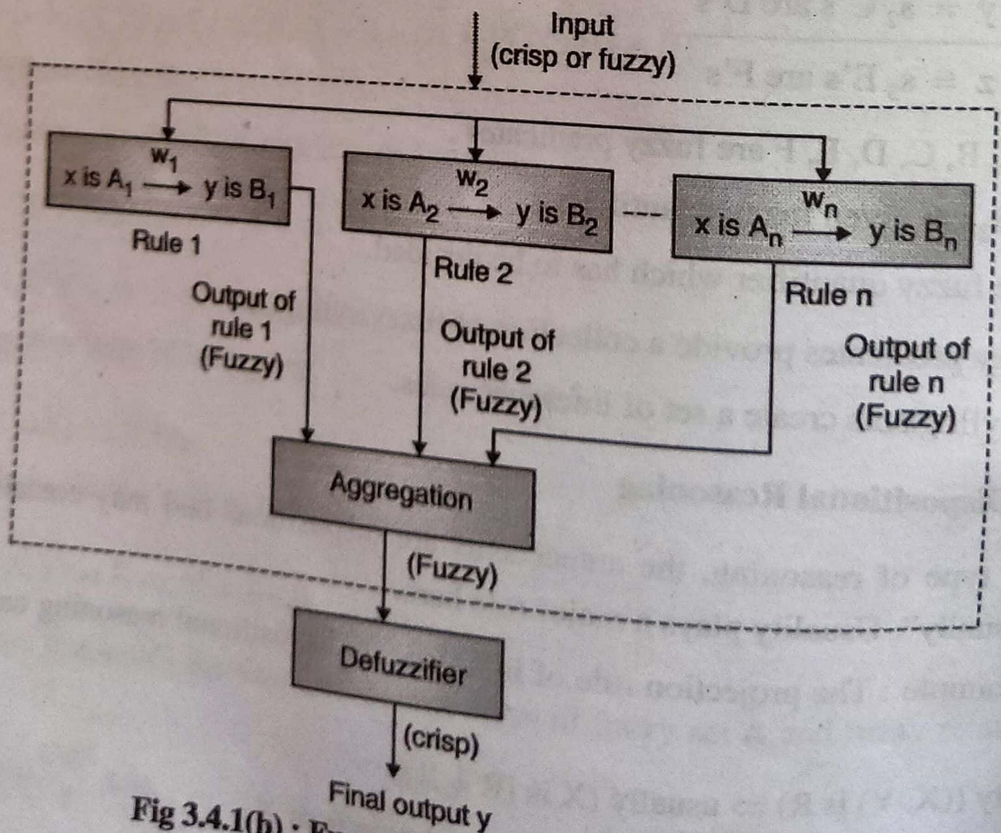

Fig 3.4.1(b) : Fuzzy Inference using If-Then rules

- As shown in Fig. 3.4.1(a), FIS involves five important modules.
  1. Fuzzification Inference Unit (FU)
  2. Decision making / Inferencing Unit
  3. Rule Base
  4. Data Base
  5. Defuzzification Inference Unit (DU)

Fuzzification Inference Unit

This block performs a fuzzification which converts a crisp input in to a fuzzy set. Here we need to decide the proper fuzzification strategy.

Decision making/Inferencing Unit

- The basic function of the inference unit is to compute the overall value of the control output variable based on the individual contribution of each rule in the rule base.
- The output of the fuzzification module representing the crisp input is matched to each rule-antecedent.
- The degree of match of each rule is established. Based on this degree of match, the value of the control output variable in the rule-consequent is modified. The result is, we get the "clipped" fuzzy set representing the control output variable.
- The set of all clipped control output values of the matched rules represent the overall fuzzy value of control output.

Defuzzification Unit

It performs defuzzification which converts the overall control output into a single crisp value.

The **rule base** and the **database** are jointly referred to as the **knowledge base.**

A database

- **Data Base** defines the membership functions of the fuzzy sets used in the fuzzy rules.
- The information in the database includes:
  - Fuzzy Membership Functions for the input and output control variables
  - The physical domains of the actual problems and their normalized values along with the scaling factors.

- **A rule base**

It contains a number of fuzzy IF-THEN rules;

- **Working**

The input to the FIS may be a Fuzzy or crisp value.

1. Fuzzification Unit converts the crisp input into fuzzy input by using any of the fuzzification methods.
2. The next, rule base is formed. Database and rule base are collectively called knowledgebase.
3. Finally, defuzzification process is carried out to produce crisp output.

3.4.2 Methods of FIS

- The most important two types of fuzzy inference method are :

1) Mamdani FIS
2) Sugeno FIS

- Mamdani fuzzy inference is the most commonly seen inference method. This method was introduced by Mamdani and Assilian (1975).
- Another well-known inference method is the so- called Sugeno or Takagi–Sugeno–Kang method of fuzzy inference process. This method was introduced by Sugeno (1985). This method is also called as TS method.
- The main difference between the two methods lies in the consequent of fuzzy rules.

3.4.2.1 Mamdani FIS

- Mamdani FIS was proposed by Ebahim Mamdani in the year 1975 to control a steam engine and boiler combination.
- To compute the output of this FIS given the inputs, six steps has to be followed.

1. Determining a set of fuzzy rules.
2. Fuzzifying the inputs using the input membership functions.
3. Combining the fuzzified inputs according to the fuzzy rules to establish a rule strength (Fuzzy Operations).
4. Finding the consequence of the rule by combining the rule strength and the output membership function (implication).

5. Combining the consequences to get an output distribution (aggregation).
6. Defuzzifying the output distribution (this step is only if a crisp output (class) is needed).

Fuzzy Rule Composition in Mamdani Model

- In Mamdani FIS, The fuzzy rules are formed using IF-THEN statements and AND/OR connectives.
- The consequent of the rule can be obtained in two steps.
  - By computing the strength of each rule
  - By clipping the output membership function at the rule strength.
- The outputs of all the fuzzy rules are then combined to obtain the aggregated fuzzy output. Finally, defuzzification is applied on to the aggregated fuzzy output to obtain a crisp output value.
- Consider two inputs, two rule Mamdani fuzzy inference system.
- Assume two inputs are crisp value x and y.
- Assume the following two rules :

  Rule 1 : if x is A_1 and y is B_1 then z is C_1

  Rule 2 : if x is A_2 and y is B_2 then z is C_2
- Fig. 3.4.2 shows Mamdani fuzzy inference system using **min – max** decomposition.
- Fig. 3.4.2 illustrates a procedure of deriving overall output z when presented with two crisp inputs x and y. In the above Mamdani inference system, we have used **min** as T - norm and **max** as T - conorm operators.
- The T-norm operator is used for inferencing antecedent part of the rule. And co-norm operator used to aggregate outputs resulting form of each rule.
- Mamdani model also supports **max - product** composition to derive overall output z. Here the **algebraic product** is used as T-norm operator and **max** is used as T-conorm operator.

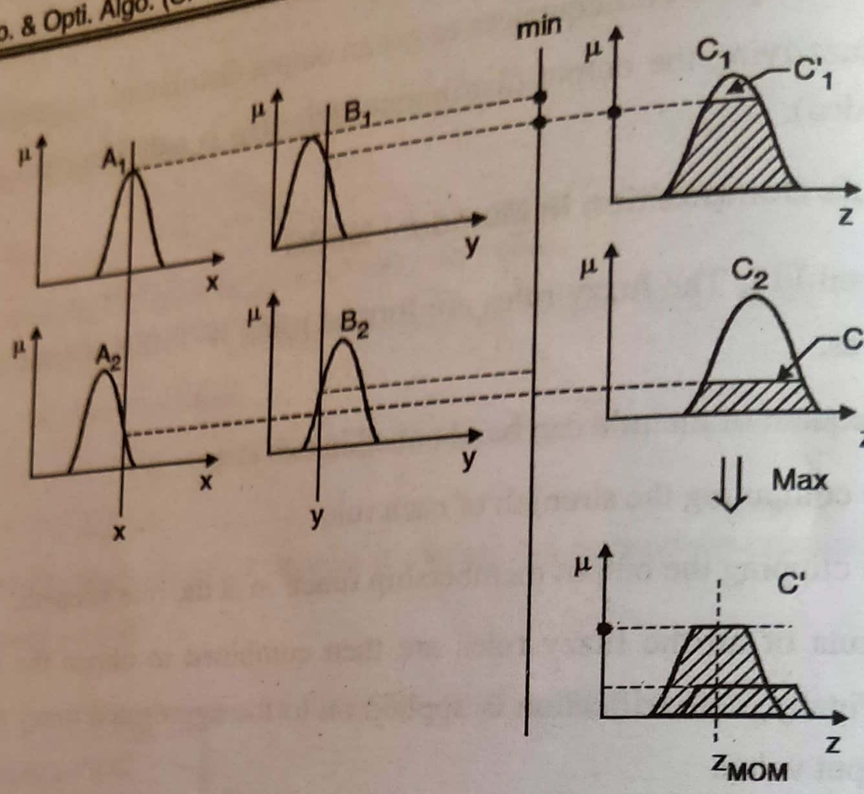

Fig. 3.4.2 : Mamdani fuzzy inference systems using max - min decomposition

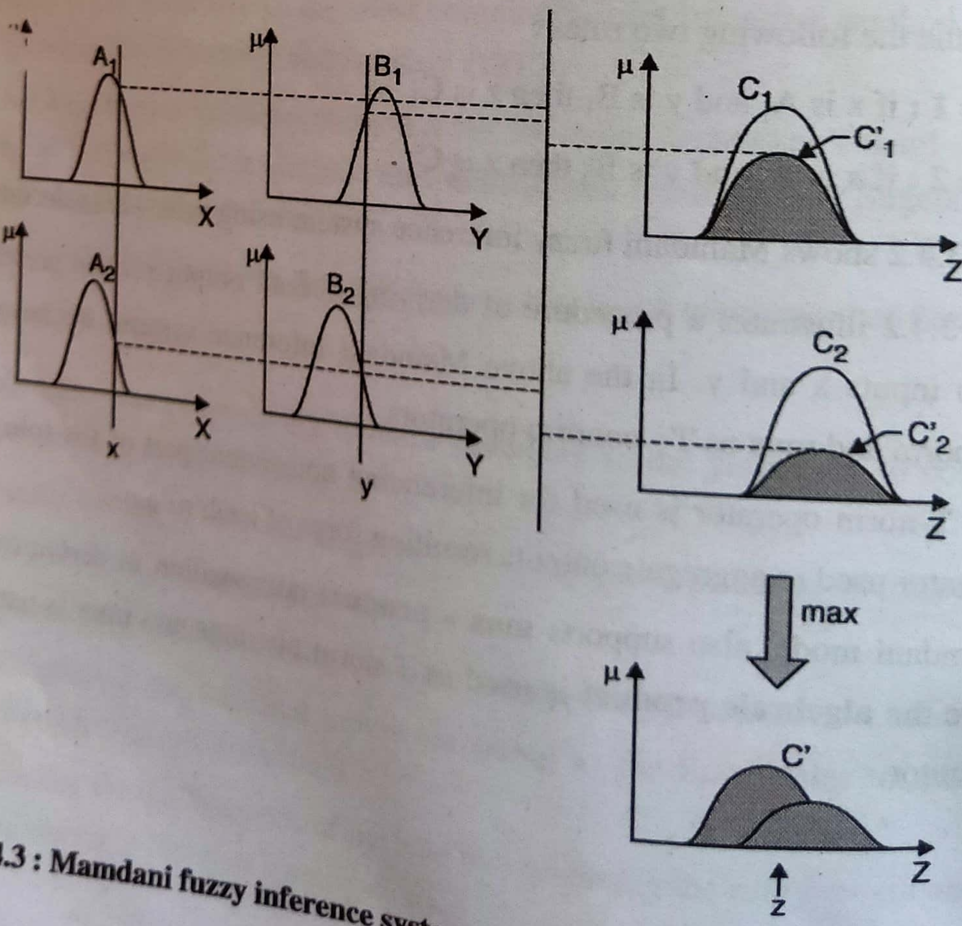

Fig. 3.4.3 : Mamdani fuzzy inference systems using max - product decomposition

3.4.2.2 Takagi-Sugeno-Kang (TSK) FIS

- Takagi - Sugeno FIS was proposed by Takagi, Sugeno and Kang in the year 1985.
- A typical fuzzy rule in TSK model has the form ,

  IF x is A and y is B then z = f(x, y)

  Where,

  x, y and z are linguistic variables.

  A and B are fuzzy sets in the antecedent part of the rule.

  Z = f(x, y) is a crisp function in the consequent part of the rule.

  Usually f(x, y) is a polynomial in the input variables x and y.

☞ Fuzzy Inference Process

The fuzzy inference process under Takagi-Sugeno Fuzzy Model (TS Method) works in the following way :

Step 1 : Fuzzifying the inputs

Here, the inputs of the system are made fuzzy.

Step 2 : Applying the fuzzy operator

In this step, the fuzzy operators must be applied to get the output.

☞ First order Sugeno fuzzy model

When f(x,y) is a first order polynomial (e.g. z = ax+by+c) the resulting FIS is called , first order Sugeno fuzzy model.

☞ Zero Order fuzzy model

In zero order fuzzy model, the output z is a constant (i.e. a=b=0).

The typical form of the rule in zero order FIS is

IF x is A and y is B then z = c

Where c is a constant

In this case the output of each fuzzy rules is a constant and hence the overall output is obtained via weighted average method.

The output level z_i of each rule is weighted by the firing strength w_i of the rule.

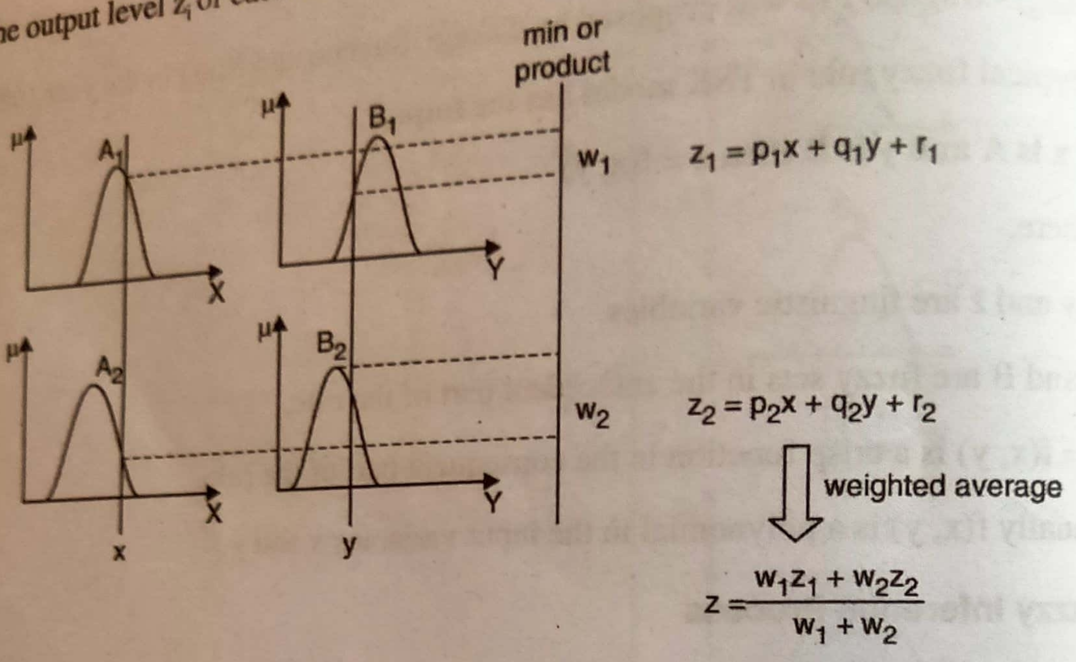

Fig. 3.4.4 : Reasoning in Sugeno FIS

3.4.2.3 Comparison between the Mamdani System and the Sugeno Model

- **Output Membership Function :** The main difference between them is on the basis of output membership function. The Sugeno output membership functions are either linear or constant.
- **Aggregation and Defuzzification Procedure :** The difference between them also lies in the consequence of fuzzy rules and due to the same their aggregation and defuzzification procedure also differs.
- **Mathematical Rules :** More mathematical rules exist for the Sugeno rule than the Mamdani rule.
- **Adjustable Parameters :** The Sugeno controller has more adjustable parameters than the Mamdani controller.

Syllabus Topic : Fuzzy Expert System

3.5 Fuzzy Expert System

- An expert fuzzy system is analogous to a human expert in a specific domain. An expert fuzzy system performs two major functions.
- It deals with uncertainty and incomplete/ vague information

- It posses user-interaction function, which contains an explanation of system intention and desires.
- The fuzzy expert system incorporates fuzzy sets/ fuzzy logic for their reasoning process and knowledge representation scheme.
- The basic block diagram of an expert system is shown in Fig.3.5.1. It contains three main components.

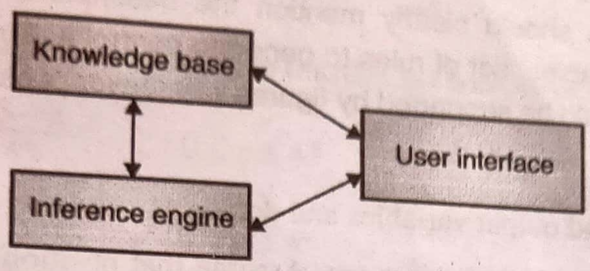

Fig 3.5.1 : Block diagram of an expert system

1. **Knowledge base :** contains the knowledge specific to the domain of application.
2. **Inference Engine :** It makes the use of knowledge present in the knowledge base to perform reasoning for user's queries.
3. **User Interface :** It's an interface where the user interacts with the system.

- An example of an expert system is **MYCIN**, which introduces the concept of certainty factor for dealing with uncertainty. The rule strength in MYCIN is called certainty factor. This factor lies in the interval [0,1].
- When a rule is fired, its pre-state condition is evaluated. And a firing strength, is associated with the pre -state condition. When the rule is fired, the firing strength is compared with the previous mentioned threshold interval. If it is higher, the consequent of the rule is determined and a conclusion is made with a certainty.
- The obtained conclusion and its certainty are the evidence provided by this fire rule for the hypothesis given by user.
- The hypothesis evidence from different rules is combined into belief measure and disbelief measures which are values lying in the [0,1] and [-1,0] respectively. If belief measure lies above a threshold, a hypothesis is believed, otherwise hypothesis is disbelieved.

3.6 Design of Controllers (Solved Problems)

Note : All the problems based on controller design have been solved using Mamdani Fuzzy Inference model and mean of max defuzzification method.

1. Domestic Shower Controller

Ex. 3.6.1 : Design a fuzzy controller to regulate the temperature of a **domestic shower**. Assume that :

(a) The temperature is adjusted by single mixer tap.

(b) The flow of water is constant.

(c) Control variable is the ratio of the hot to the cold water input.

The design should clearly mention the descriptors used for fuzzy sets and control variables, set of rules to generate control action and defuzzification. The design should be supported by figures whereever possible.

Soln. :

Step 1 : Identify input and output variables and decide descriptors for the same.

- Here **input** is the position of mixer tap. Assume that position of mixer tap is measured in degrees (0° to 180°). It represents opening of the mixer tap in degrees. 0° indicates tap is closed and 180° indicate tap is fully opened.
- **Output** is temperature of water according to the position of mixer tap. It is measured in C. We take **five descriptors** for each input and output variables.
- Descriptors for input variable (position of mixer tap) are given below.

  EL - Extreme Left

  L - Left

  C - Centre

  R - Right

  ER - Extreme Right

  { EL, L, C, R, ER }
- Descriptors for output variable (Temperature of water) are given below :

  VCT - Very Cold Temperature

  CT - Cold Temperature

  WT - Warm Temperature

  HT - Hot Temperature

  VHT - Very Hot Temperature

  { VCT, CT, WT, HT, VHT }

Step 2 : Define membership functions for input and output variables.

We use triangular MFs because of its simplicity.

1. Membership functions for input variable - position of mixer tap.

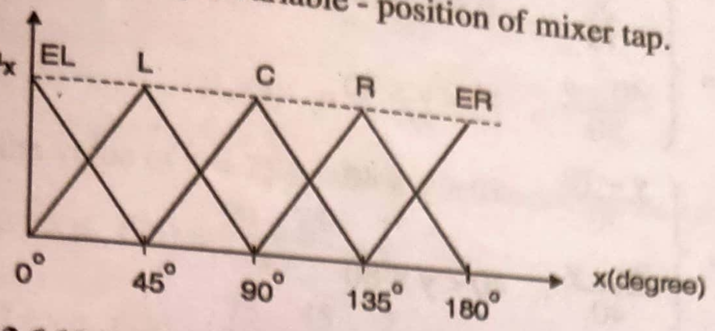

Fig. P. 3.6.1(a) : Membership function for position of mixer tap

$$\mu_{EL}(x) = \frac{45-x}{45}, \quad 0 \le x \le 45$$

$$\mu_L(x) = \begin{cases} \dfrac{x}{45}, & 0 \le x \le 45 \\ \dfrac{90-x}{45}, & 45 < x \le 90 \end{cases}$$

$$\mu_c(x) = \begin{cases} \dfrac{x-45}{45}, & 45 \le x \le 90 \\ \dfrac{135-x}{45}, & 90 < x \le 135 \end{cases}$$

$$\mu_R(x) = \begin{cases} \dfrac{x-90}{45}, & 90 \le x \le 135 \\ \dfrac{180-x}{45}, & 135 < x \le 180 \end{cases}$$

$$\mu_{ER}(x) = \frac{x-135}{45}, \quad 135 \le x \le 180$$

2. Membership functions for output variable - temperature of water.

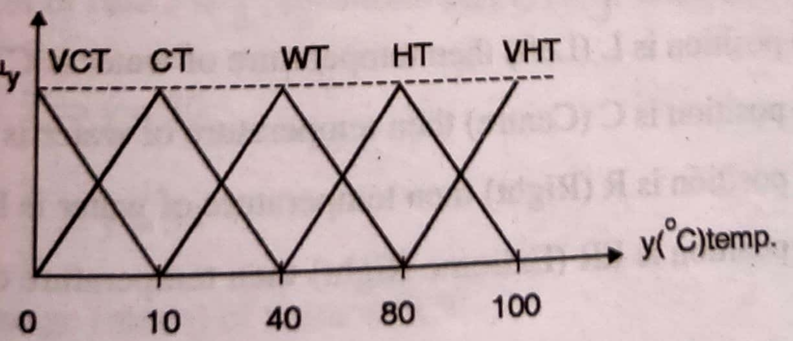

Fig. P. 3.6.1(b) : Membership functions for water temperature

$$\mu_{VCT}(y) = \frac{10-y}{10}, \quad 0 \le y \le 10$$

$$\mu_{CT}(y) = \begin{cases} \frac{y}{10}, & 0 \le y \le 10 \\ \frac{40-y}{30}, & 10 < y \le 40 \end{cases}$$

$$\mu_{WT}(y) = \begin{cases} \frac{y-10}{30}, & 10 \le y \le 40 \\ \frac{80-y}{40}, & 40 < y \le 80 \end{cases}$$

$$\mu_{HT}(y) = \begin{cases} \frac{y-40}{40}, & 40 \le y \le 80 \\ \frac{100-y}{20}, & 80 < y \le 100 \end{cases}$$

$$\mu_{VHT}(y) = \frac{y-80}{20}, \quad 80 \le y \le 100$$

Step 3 : Form a rule base.

Table P. 3.6.1

Input (Mixer tap position)	Output (Temperature of water)
EL	VCT
L	CT
C	WT
R	HT
ER	VHT

We can read the rule base shown in Table P. 3.6.1 in terms of If-then rules.

Rule 1 : If mixer tap position is EL (Extreme left) then temperature of water is VCT (Very cold).

Rule 2 : If mixer tap position is L (Left) then temperature of water is CT (cold).

Rule 3 : If mixer tap position is C (Centre) then temperature of water is WT (Warm)

Rule 4 : If mixer tap position is R (Right) then temperature of water is HT (Hot)

Rule 5 : If mixer tap position is ER (Extreme Right) then temperature of water is VHT (Very hot).

Thus, we have five rules.

Step 4 : Rule Evaluation.

Assume that mixer tap position is 75°. This value x = 75° maps to following two MFs of rule 2 and rule 3 respectively.

Rule 2 : $$\mu_L(x) = \frac{90-x}{45}$$

Rule 3 : $$\mu_c(x) = \frac{x-45}{45}$$

Now, substitute the value of $x = 75$ in above two equations, we get strength of each rule

Strength of rule 2 $\Rightarrow \mu_L(75) = \frac{90-75}{45} = \frac{1}{3}$

Strength of rule 3 $\Rightarrow \mu_c(75) = \frac{75-45}{45} = \frac{2}{3}$

Step 5 : Defuzzification.

We apply **mean of maximum** defuzzification technique.

We find the rule with the maximum strength

$$= \max(\text{Strength of rule 1, strength of rule 2})$$

$$= \max(\mu_L(x), \mu_c(x))$$

$$= \max\left(\frac{1}{3}, \frac{2}{3}\right) = \frac{2}{3}$$

Thus, rule 3 has the maximum strength.

According to rule 3, If mixer tap position is C (center) then water temperature is **Warm.**

So, we use Output MFs of **warm water temperature** for defuzzification .We have following two equations for warm water temperature.

$$\mu_{WT}(y) = \frac{y-10}{30} \quad \text{and}$$

$$\mu_{WT}(y) = \frac{80-y}{40}$$

Since, the strength of rule 3 is $\frac{2}{3}$, substitute $\mu_{WT}(y) = \frac{2}{3}$ in the above two equations.

$$\frac{y-10}{30} = \frac{2}{3} \Rightarrow y = 30$$

$$\frac{80-y}{40} = \frac{2}{3} \Rightarrow y = 53$$

Now take the average (mean) of these values.

...Ans.

$$y^* = \frac{30+53}{2} = \mathbf{41.5°C}$$

2. Washing Machine Controller

Ex. 3.6.2 : Design a controller to determine the wash time of a domestic washing machine. Assume that input is dirt and grease on cloths. Use three descriptors for input variables and five descriptors for output variables. Derive set of rules for controller action and defuzzification. The design should be supported by figures wherever possible. Show that if the cloths are soiled to a larger degree the wash time will be more and vice-versa.

Soln. :

Step 1 : Identify input and output variables and decide descriptors for the same.

- Here inputs are 'dirt' and 'grease'. Assume that they are measured in percentage (%). That is amount of dirt and grease is measured in percentage.
- Output is 'wash time' measured in minutes.
- We use three descriptors for each of the input variables.

Descriptors for dirt are as follows :

SD - Small Dirt

MD - Medium Dirt

LD - Large Dirt

{ SD, MD, LD }

Descriptors for grease are { NG, MG, LG }

NG - No grease

MG - Medium grease

LG - Large grease

We use five descriptors for output variable.

So, descriptors for wash time are {VS, S, M, L, VL}

VS - Very Short

S - Short

M - Medium

L - Large

VL - Very Large

Step 2 : Define membership functions for each of the input and output variables.

We use triangular MFs because of their simplicity.

(1) **Membership functions for dirt**

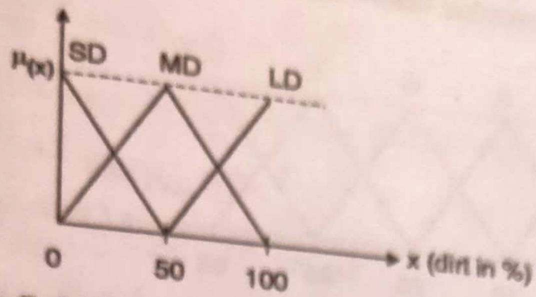

Fig. P. 3.6.2(a) : Membership functions for dirt

$$\mu_{SD}(x) = \frac{50-x}{50} \quad , \quad 0 \le x \le 50$$

$$\mu_{MD}(x) = \begin{cases} \dfrac{x}{50} & , \quad 0 \le x \le 50 \\ \dfrac{100-x}{50} & , \quad 50 < x \le 100 \end{cases}$$

$$\mu_{LD}(x) = \frac{x-50}{50} \quad , \quad 50 \le x \le 100$$

(2) **Membership functions for grease**

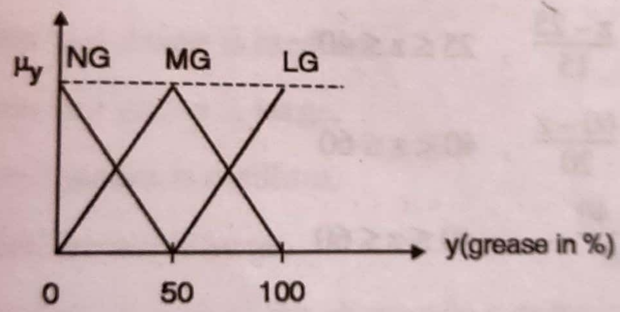

Fig. P. 3.6.2(b) : Membership functions of grease

$$\mu_{NG}(y) = \frac{50-y}{50} \quad , \quad 0 \le y \le 50$$

$$\mu_{MG}(y) = \begin{cases} \dfrac{y}{50} & , \quad 0 \le y \le 50 \\ \dfrac{100-y}{50} & , \quad 50 < y \le 100 \end{cases}$$

$$\mu_{LG}(y) = \frac{y-50}{50} \quad , \quad 50 \le y \le 100$$

(3) **Membership functions for Wash time**

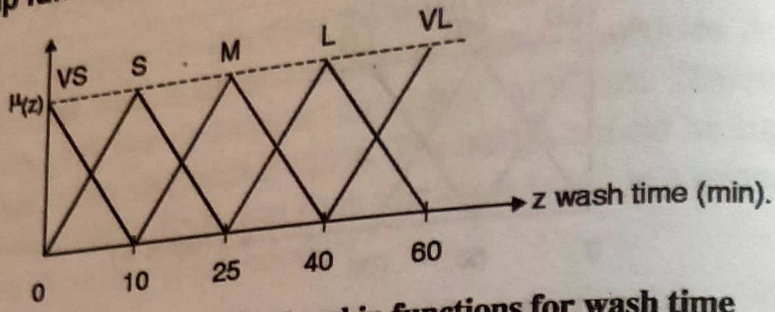

Fig. P. 3.6.2(c) : Membership functions for wash time

$$\mu_{VS}(z) = \frac{10-z}{10}, 0 \le z \le 10$$

$$\mu_S(z) = \begin{cases} \frac{z}{10}, & 0 \le z \le 10 \\ \frac{25-z}{15}, & 10 < z \le 25 \end{cases}$$

$$\mu_M(z) = \begin{cases} \frac{z-10}{15}, & 10 \le z \le 25 \\ \frac{40-z}{15}, & 25 < z \le 40 \end{cases}$$

$$\mu_L(z) = \begin{cases} \frac{z-25}{15}, & 25 \le z \le 40 \\ \frac{60-z}{20}, & 40 < z \le 60 \end{cases}$$

$$\mu_{VL}(z) = \frac{z-40}{20}, \quad 40 \le z \le 60$$

Step 3 : Form a Rule base

x \ y	NG	MG	LG
SD	VS	M	L
MD	S	M	L
LD	M	L	VL

- The above matrix represents in all nine rules. For example, first rule can be "**If dirt is small and no grease then wash time is very short**" similarly all nine rules can be defined using if --- then.

Step 4 : Rule Evaluation

Assume that dirt = 60 % and grease = 70%

dirt = 60 % maps to the following two MFs of "dirt" variable

$$\mu_{MD}(x) = \frac{100-x}{50} \text{ and } \mu_{LD}(x) = \frac{x-50}{50}$$

Similarly grease = 70 % maps to the following two MFs of "grease" variable.

$$\mu_{MG}(y) = \frac{100-y}{50} \text{ and } \mu_{LG}(y) = \frac{y-50}{50}$$

Evaluate $\mu_{MD}(x)$ and $\mu_{LD}(x)$ for x = 60, we get

$$\mu_{MD}(60) = \frac{100-60}{50} = \frac{4}{5} \quad \text{...(1)}$$

$$\mu_{LD}(60) = \frac{60-50}{50} = \frac{1}{5} \quad \text{...(2)}$$

Similarly evaluate $\mu_{MG}(y)$ and $\mu_{LG}(y)$ for y = 70, we get

$$\mu_{MG}(70) = \frac{100-70}{50} = \frac{3}{5} \quad \text{... (3)}$$

$$\mu_{LG}(70) = \frac{70-50}{50} = \frac{2}{5} \quad \text{... (4)}$$

The above four equation leads to the following four rules that we are suppose to evaluate.

(1) dirt is **medium** and grease is **medium.**

(2) dirt is **medium** and grease is **large.**

(3) dirt is **large** and grease is **medium.**

(4) dirt is **large** and grease is **large.**

Since the antecedent part of each of the above rule is connected by **and** operator we use min operator to evaluate strength of each rule.

$$\begin{aligned} \text{Strength of rule 1 : } S_1 &= \min(\mu_{MD}(60), \mu_{MG}(70)) \\ &= \min(4/5, 3/5) \\ &= 3/5 \end{aligned}$$

$$\begin{aligned} \text{Strength of rule 2: } S_2 &= \min(\mu_{MD}(60), \mu_{LG}(70)) \\ &= \min(4/5, 2/5) \\ &= 2/5 \end{aligned}$$

$$\begin{aligned} \text{Strength of rule 3: } S_3 &= \min(\mu_{LD}(60), \mu_{MG}(70)) \\ &= \min(1/5, 3/5) \\ &= 1/5 \end{aligned}$$

$$\text{Strength of rule 4: } S_4 = \min(\mu_{LD}(60), \mu_{LG}(70))$$
$$= \min(1/5, 2/5)$$
$$= 1/5$$

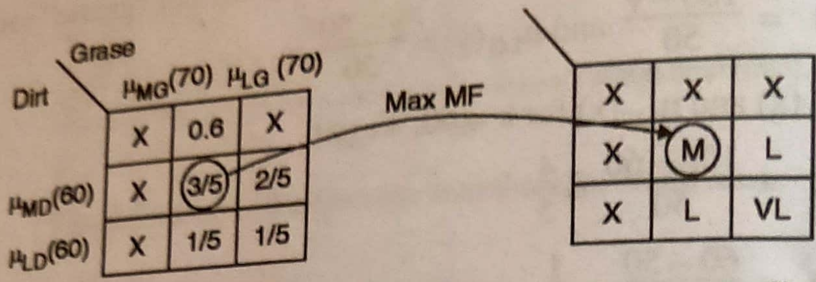

Fig. P. 3.6.2(d) : Rule strength and its mapping to corresponding output MF

Step 5 : Defuzzification

- Since, we use **"Mean of max"** defuzzification technique, we first find the rule with the maximum strength.

$$= \text{Max}(S_1, S_2, S_3, S_4)$$
$$= \text{Max}(3/5, 2/5, 1/5, 1/5)$$
$$= 3/5$$

- This corresponds to rule 1.
- This rule 1 – "dirt is **medium** and grease is **medium**" has maximum strength 3/5.
- The above rule corresponds to the output MF μ_M (z). This mapping is shown in Fig. P. 3.6.2(e).
- To find out the final defuzzified value, we now take average (mean) of μ_M (z).

$$\mu_M(z) = \frac{z-10}{15} \quad \text{and} \qquad \mu_M(z) = \frac{40-2}{15}$$

$$\therefore\ 3/5 = \frac{z-10}{15} \qquad 3/5 = \frac{40-2}{15}$$

$$\therefore\ Z = 19 \qquad z = 31$$

$$\therefore\ z^* = \frac{19+31}{2} = 25 \text{ min}$$

...**Ans.**

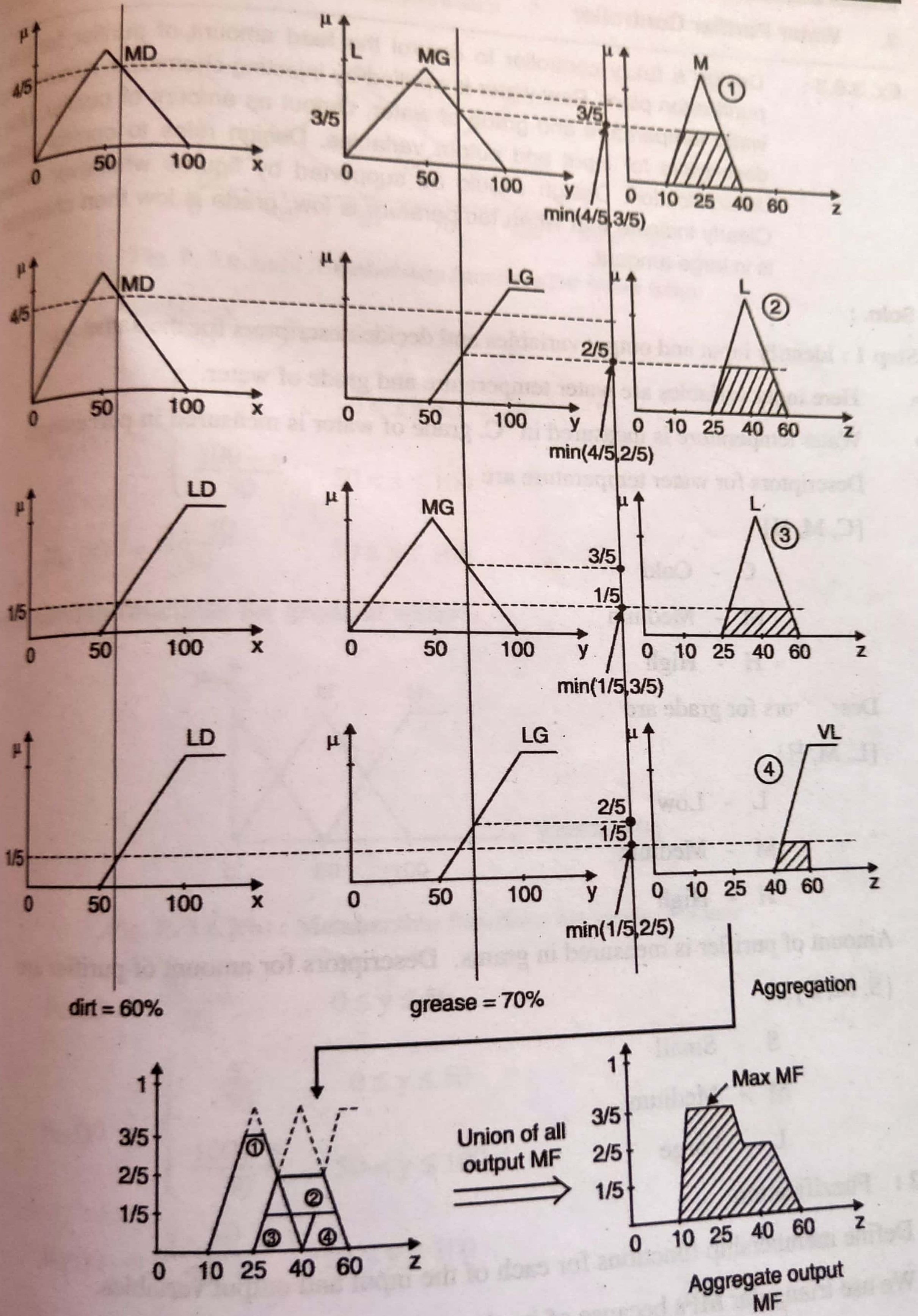

Fig. P. 3.6.2(e) : Process of Rule evaluation and defuzzification

3. Water Purifier Controller

Ex. 3.6.3 : Design a fuzzy controller to control the feed amount of purifier for the water purification plant. Raw water is purified by injecting chemicals. Assume input as water temperature and grade of water. Output as amount of purifier. Use three descriptors for input and output variables. Design rules to control action and defuzzification. Design should be supported by figures whenever necessary. Clearly indicate that when temperature is low, grade is low then chemical used is in large amount.

Soln. :

Step 1 : Identify input and output variables and decide descriptors for the same.

- Here input variables are water temperature and grade of water.
- Water temperature is measured in °C. grade of water is measured in percentage.
- Descriptors for water temperature are

  {C, M, H}

  C - Cold

  M - Medium

  H - High

- Des[illegible]ors for grade are

  {L, M, H}

  L - Low

  M - Medium

  H - High

- Amount of purifier is measured in grams. Descriptors for amount of purifier are {S, M, L}.

  S - Small

  M - Medium

  L - Large

Step 2 : Fuzzification

Define membership functions for each of the input and output variables.

We use triangular MFs because of its simplicity.

(1) **Membership functions for water temperature**

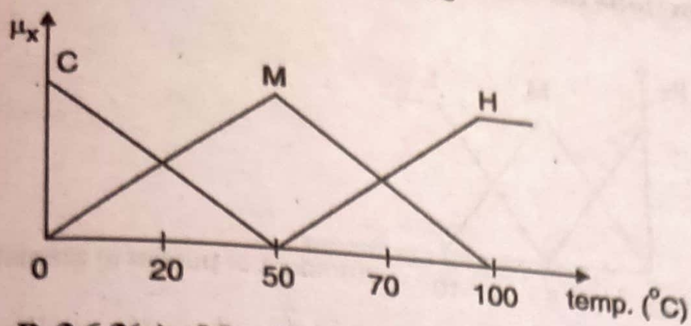

Fig. P. 3.6.3(a) : Membership functions for water temp.

$$\mu_C(x) = \frac{50-x}{50} \quad , \quad 0 \le x \le 50$$

$$\mu_M(x) = \begin{cases} \dfrac{x}{50} & , \quad 0 \le x \le 50 \\ \dfrac{100-x}{50} & , \quad 50 < x \le 100 \end{cases}$$

$$\mu_H(x) = \frac{x-50}{50} \quad , \quad 50 \le x \le 100$$

(2) **Membership functions for grade of water**

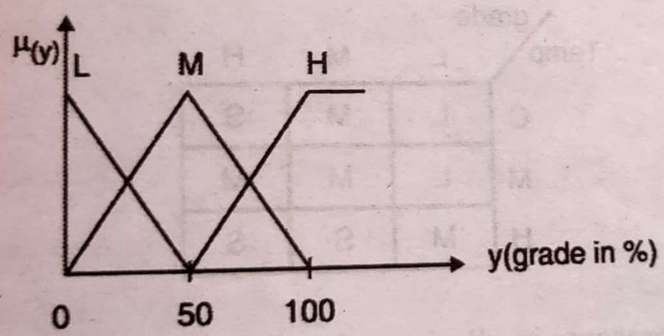

Fig. P. 3.6.3(b) : Membership functions for grade of water

$$\mu_L(y) = \frac{50-y}{50} \quad , \quad 0 \le y \le 50$$

$$\mu_M(y) = \begin{cases} \dfrac{y}{50} & , \quad 0 \le y \le 50 \\ \dfrac{100-y}{50} & , \quad 50 < y \le 100 \end{cases}$$

$$\mu_H(y) = \frac{y-50}{50} \quad , \quad 50 \le y \le 100$$

(3) **Membership functions for amount of purifier**

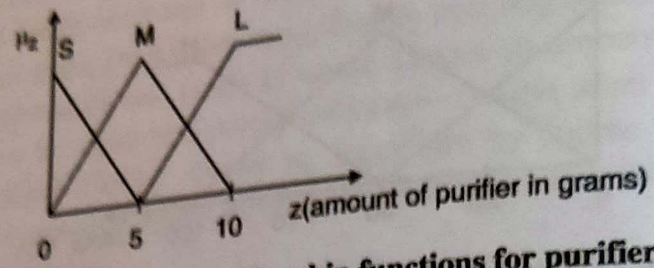

Fig. P. 3.6.3(c) : Membership functions for purifier

$$\mu_S(z) = \frac{5-z}{5} \quad , \ 0 \leq z \leq 5$$

$$\mu_M(z) = \begin{cases} \frac{z}{5} & , \ 0 \leq z \leq 5, \\ \frac{10-z}{5} & , \ 5 < z \leq 10 \end{cases}$$

$$\mu_L(z) = \frac{z-5}{5} \quad , \ 5 < z \leq 10$$

Step 3 : Form a Rule base

Temp \ grade	L	M	H
C	L	M	S
M	L	M	M
H	M	S	S

- The above matrix represents in all nine rules. For example,
- First rule can be, "If temperature is cold and grade is low then amount of purifier required is large."
- Similarly all nine rules can be defined using if-then rules.

Step 4 : Rule Evaluation

- Assume water temperature = 5° and grade = 30
- Water temperature = 5° maps to the following two MFs of "temperature" variable.

$$\mu_c(x) = \frac{50-x}{50} \text{ and } \mu_M(x) = \frac{x}{50}$$

Similarly, grade = 30 maps to the following two MFs of "grade" variable.

- $$\mu_L(y) = \frac{50-y}{50} \text{ and } \mu_M(y) = \frac{y}{50}$$

Evaluate $\mu_c(x)$ and $\mu_M(x)$ for $x = 5°$

- We get,

$$\mu c(5) = \frac{50-5}{50} = \frac{9}{10} = 0.9 \quad ...(1)$$

$$\mu_M(5) = \frac{5}{50} = \frac{1}{5} = 0.1 \quad ...(2)$$

Evaluate $\mu_L(y)$ and $\mu_M(y)$ for $y = 30$

- $$\mu_L(30) = \frac{50-30}{50} = \frac{2}{5} = 0.4 \quad ...(3)$$

$$\mu_M(30) = \frac{30}{50} = \frac{3}{5} = 0.6 \quad ...(4)$$

- The above four equation represents following for rules that we need to evaluate.

1. If temperature is **cold** and grade is **low**.
2. If temperature is **cold** and grade is **medium**.
3. If temperature is **medium** and grade is **low**.
4. If temperature is **medium** and grade is **medium**.

- Since the antecedent part of each rule is connected by ***and*** operator we use ***min***

Operator to evaluate strength of each rule

$$\begin{aligned}
\text{Strength of rule1} : S_1 &= \min(\mu_c(5), \mu_L(30)) \\
&= \min(0.9, 0.4) \\
&= 0.4 \\
\text{Strength of rule2} : S_2 &= \min(\mu_c(5), \mu_M(30)) \\
&= \min(0.9, 0.6) \\
&= 0.6 \\
\text{Strength of rule3} : S_3 &= \min(\mu_M(5), \mu_L(30)) \\
&= \min(0.1, 0.4) \\
&= 0.1 \\
\text{Strength of rule4} : S_4 &= \min(\mu_M(5), \mu_M(30)) \\
&= \min(0.1, 0.6) \\
&= 0.1
\end{aligned}$$

Temp \ Grade	$\mu_L(30)$	$\mu_M(30)$	
$\mu_C(5)$	0.4	0.6	X
$\mu_M(5)$	0.1	0.1	X
	X	X	X

(i) Rule strength table

Max MF

Temp \ Grade		
L	M	S
L	M	M
M	S	S

(ii) Rule base table

Fig. P. 3.6.3(d) : Rule strength and its mapping to corresponding output MF

Step 5 : Defuzzification

Since, we use "mean of max" defuzzification technique, we first find the rule with maximum strength.

$$= \max (S_1, S_2, S_3, S_4)$$

$$= \max (0.4, 0.6, 0.1, 0.1)$$

$$= 0.6$$

- This corresponds to rule 2.

  Thus rule 2 : "Temperature is cold and grade is medium" has maximum strength 0.6.
- The above rule corresponds to the output MF $\mu_M(z)$. This is show is shown in Fig. P. 3.6.3(d).
- To find out final defuzzified value, we now take average (i.e. mean) of $\mu_M(z)$.

$$\mu_M(z) = \frac{10-z}{5} \quad \text{and} \quad \mu_M(z) = \frac{z}{5}$$

$$0.6 = \frac{10-z}{5} \qquad \therefore 0.6 = \frac{z}{5}$$

$$\therefore \quad z = 13 \qquad \therefore z = 3$$

$$\therefore z^* = \frac{13+3}{2} = 8 \text{ gms}$$

...Ans.

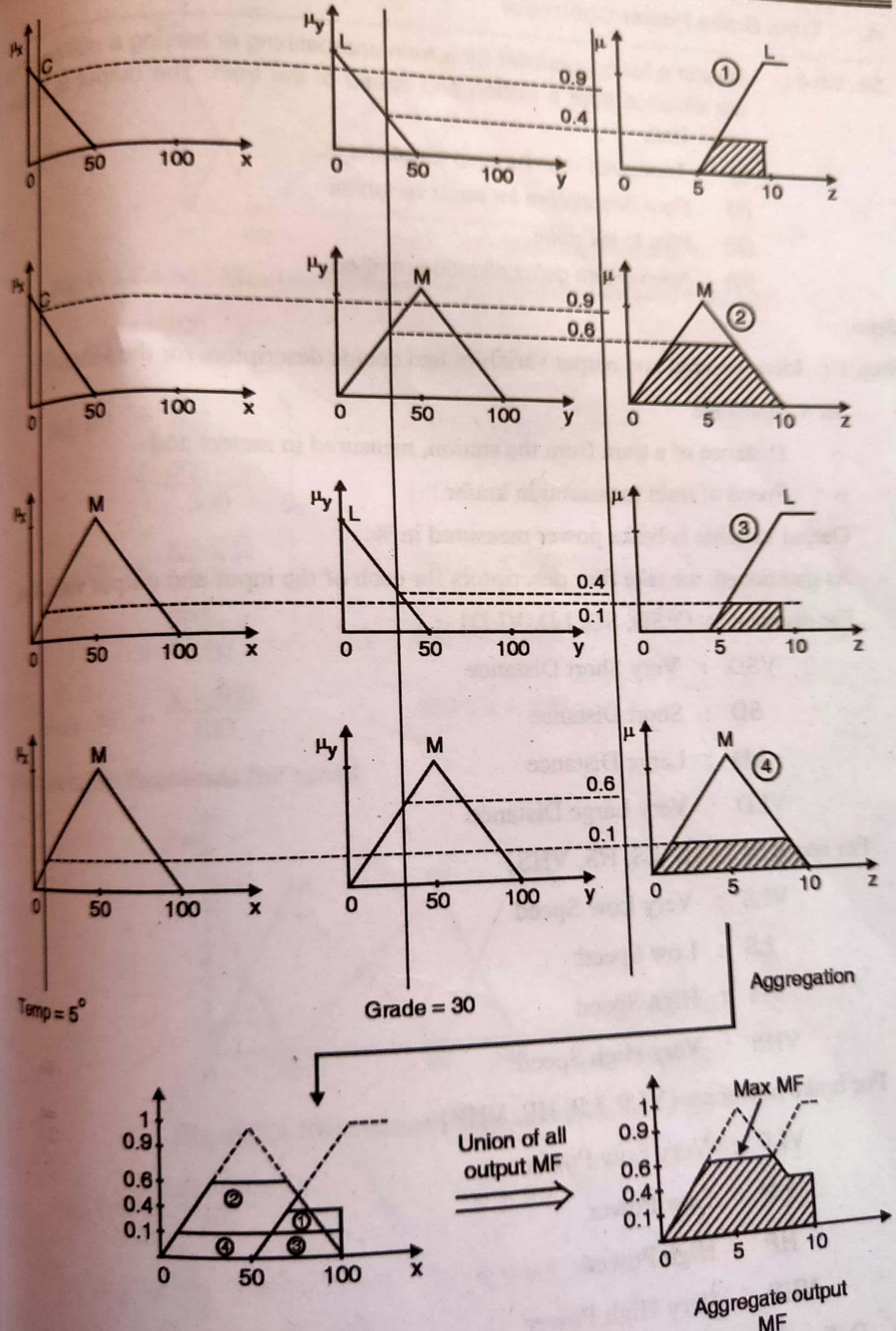

Fig. P. 3.6.3(c) : Process of rule evaluation and defuzzification

4. Train Brake Power Controller

Ex. 3.6.4 : Design a fuzzy controller for a train approaching or leaving a station. The inputs are distance from a station and speed of the train. The output is brake power used. Use,

(i) Triangular membership functions

(ii) Four descriptors for each variables

(iii) Five to six rules.

(iv) Appropriate defuzzification method.

Soln. :

Step 1 : Identify input and output variables and decide descriptors for the same.

- Here inputs are
  - Distance of a train from the station, measured in meters and
  - Speed of train measured in km/hr.
- Output variable is brake power measured in %.
- As mentioned, we take four descriptors for each of the input and output variables.
- For distance ⇒ {VSD, SD, LD, VLD}

  VSD : Very Short Distance

  SD : Short Distance

  LD : Large Distance

  VLD : Very Large Distance
- For speed ⇒ {VLS, LS, HS, VHS}

  VLS : Very Low Speed

  LS : Low Speed

  HS : High Speed

  VHS : Very High Speed
- For brake power ⇒ {VLP, LP, HP, VHP}

  VLP : Very Low Power

  LP : Low Power

  HP : High Power

  VHP : Very High Power

Step 2 : Define membership functions for each of the input and output variables.

- As mentioned, we use triangular membership functions.

Membership functions for distance

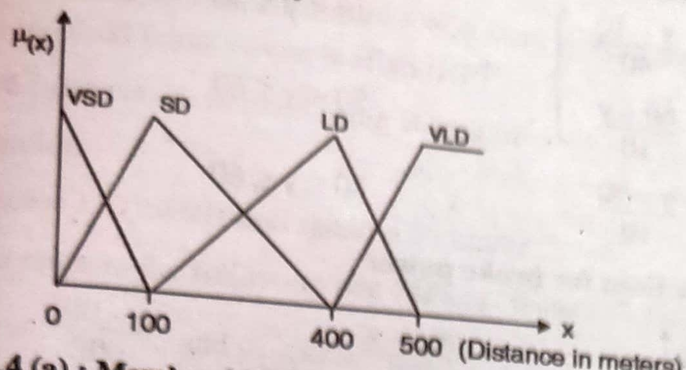

Fig. P. 3.6.4 (a) : Membership functions for distance (distance in meters)

$$\mu_{VSD}(x) = \frac{100-x}{100} \quad , \quad 0 \le x \le 100$$

$$\mu_{SD}(x) = \left.\begin{array}{ll} \frac{x}{100} & , \quad 0 \le x \le 100 \\ \frac{400-x}{300} & , \quad 100 < x \le 400 \end{array}\right\}$$

$$\mu_{LD}(x) = \left.\begin{array}{ll} \frac{x-100}{300} & , \quad 100 \le x \le 400 \\ \frac{500-x}{100} & , \quad 400 < x \le 500 \end{array}\right\}$$

$$\mu_{VLD}(x) = \frac{x-400}{100} \quad , \quad 400 \le x \le 500$$

Membership functions for speed

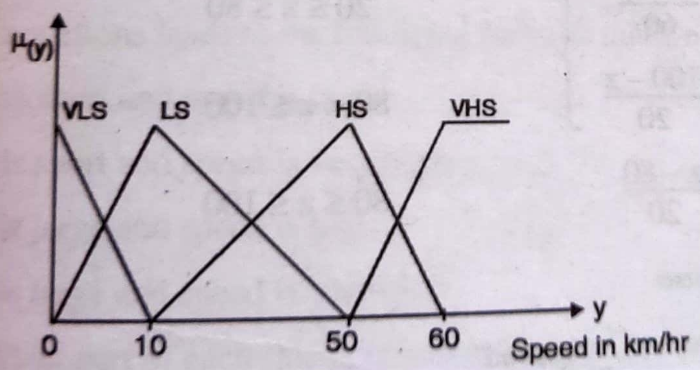

Fig. P. 3.6.4(b) : Membership functions for speed

$$\mu_{VLS}(y) = \frac{10-y}{10} \quad , \quad 0 \le y \le 10$$

$$\mu_{LS} = \left.\begin{array}{ll} \frac{y}{10} & , \quad 0 \le y \le 10 \\ \frac{50-y}{40} & , \quad 10 < y \le 50 \end{array}\right\}$$

$$\mu_{HS}(y) = \begin{cases} \frac{y-10}{40}, & 10 \le y \le 50 \\ \frac{60-y}{10}, & 50 < y \le 60 \end{cases}$$

$$\mu_{VHS}(y) = \frac{y-50}{10}, \quad 50 \le y \le 60$$

3. **Membership functions for brake power**

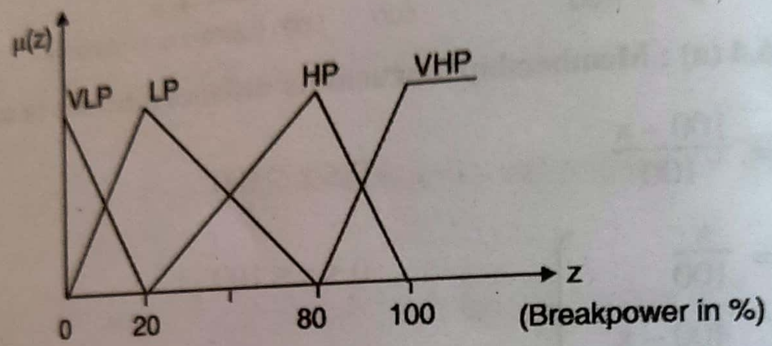

Fig. P. 3.6.4(c) : Membership functions for brake power

$$\mu_{VLP}(z) = \frac{20-z}{20}, \quad 0 \le z \le 20$$

$$\mu_{LP}(z) = \begin{cases} \frac{z}{20}, & 0 \le z \le 20 \\ \frac{80-z}{60}, & 20 < z \le 80 \end{cases}$$

$$\mu_{HP}(z) = \begin{cases} \frac{z-20}{60}, & 20 \le z \le 80 \\ \frac{100-z}{20}, & 80 < z \le 100 \end{cases}$$

$$\mu_{VHP}(z) = \frac{z-80}{20}, \quad 80 \le z \le 100$$

Step 3 : Form a Rule base

Dist \ Speed	VLS	LS	HS	VHS
VSD	HP	HP	VHP	VHP
SD	LP	LP	HP	VHP
LD	VLP	VLP	LP	HP
VLD	VLP	VLP	LP	LP

The above matrix represents in all 16 rules.

For example, First rule can be "If distance of a train is very short (VSD) and speed is very low (VLS) then required brake power is High (HP)".

Similarly all 16 rules can be defined using If then rules.

Step 4 : Rule Evaluation

Assume distance = 110 meters and speed = 52 km/hr

- Distance = 110 maps to the following two MFs of "distance" variable.

$$\mu_{SD}(x) = \frac{400 - x}{300} \text{ and } \mu_{LD}(x) = \frac{x - 100}{300}$$

- Similarly speed = 52 maps to the following two MFs of "speed" variable.

$$\mu_{HS}(y) = \frac{60 - y}{10} \text{ and } \mu_{VHS}(y) = \frac{y - 50}{10}$$

- Evaluate $\mu_{SD}(x)$ and $\mu_{LD}(x)$ for x = 110, we get,

$$\mu_{SD}(110) = \frac{400 - 110}{300} = 0.96 \quad ...(1)$$

$$\mu_{LD}(110) = \frac{110 - 100}{300} = 0.033 \quad ...(2)$$

- Similarly evaluate $\mu_{HS}(y)$ and $\mu_{VHS}(y)$ for y = 52, we get,

$$\mu_{HS}(52) = \frac{60 - 52}{10} = 0.8 \quad ...(3)$$

$$\mu_{VHS}(52) = \frac{52 - 50}{10} = 0.2 \quad ...(4)$$

- The above four equations leads to the following for rules that we needs to evaluate.
  1. Distance is short and speed is high
  2. Distance is short and speed is very high
  3. Distance is large and speed is high
  4. Distance is large and speed is very high
- Since the antecedent part of each rule is connected by ***and*** operator we use ***min***.

  Operator to evaluate strength of each rule

$$\begin{aligned}
\text{Strength of rule1: } S_1 &= \min(\mu_{SD}(110), \mu_{HS}(52)) \\
&= \min(0.96, 0.8) \\
&= 0.8 \\
\text{Strength of rule2 : } S_2 &= \min(\mu_{SD}(110), \mu_{VHS}(52)) \\
&= \min(0.96, 0.2) = 0.2 \\
\text{Strength of rule3 : } S_3 &= \min(\mu_{LD}(110), \mu_{HS}(52))
\end{aligned}$$

$$= \min(0.033, 0.8)$$

$$= 0.033$$

$$\text{Strength of rule: } S_4 = \min(\mu_{LD}(110), \mu_{VHS}(52))$$

$$= \min(0.033, 0.2)$$

$$= 0.033$$

Distance \ Speed	VLS	LS	HS	VHS
VSD	X	X	X	X
SD	X	X	0.8	0.2
LD	X	X	0.03	0.03
VLD	X	X	X	X

(i) Rule strength table

	VLS	LS	HS	VHS
VSD				
SD			HP	VHP
LD			LP	HP
VLD				

(ii) Rule base table

Fig. P. 3.6.4(d) : Rule strength table and its mapping to corresponding output MF

Step 5 : Defuzzification

- We use "mean of max" defuzzification technique.
- We first find the rule with maximum strength

$$= \max(S_1, S_2, S_3, S_4)$$

$$= \max(0.8, 0.2, 0.33, 0.2) = 0.8$$

- This corresponds to rule 1.

  Thus rule 1 – "If dist is short and speed is high" has maximum strength 0.8.
- The above rule corresponds to the output MF $\mu_{HP}(z)$. This mapping is shown in Fig. P. 3.6.4(d).
- To compute the final defuzzified value, we take average of $\mu_{HP}(z)$.

$$\mu_{HP}(z) = \frac{z-20}{60}$$

$$\therefore 0.8 = \frac{z-20}{60}$$

$$\therefore z = 68$$

$$\mu_{HP}(z) = \frac{100-z}{20}$$

$$0.8 = \frac{100-z}{20}$$

$$\therefore z = 84$$

$$\therefore z^* = \frac{68+84}{2} = 76\%$$

...Ans.

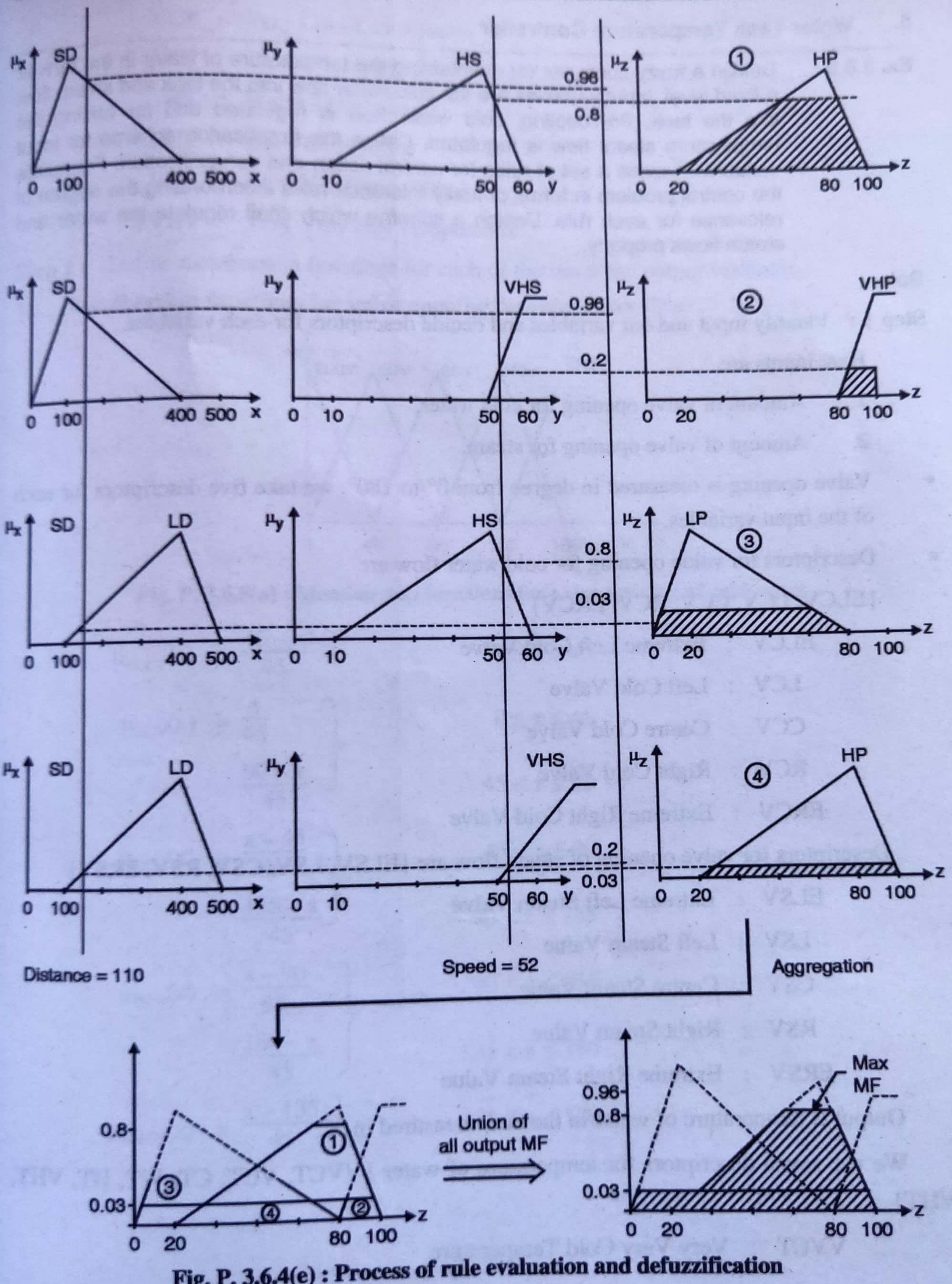

Fig. P. 3.6.4(e) : Process of rule evaluation and defuzzification

5. Water Tank Temperature Controller

Ex. 3.6.5 : Design a fuzzy controller for maintaining the temperature of water in the tank at a fixed level. Input variables are the cold water flow into the tank and steam flow into the tank. For cooling, cold water flow is regulated and for raising the temperature steam flow is regulated. Define the fuzzification scheme for input variables. Device a set of rules for control action and defuzzification. Formulate the control problem in terms of fuzzy inference rules incorporating the degree of relevance for each rule. Design a scheme which shall regulate the water and steam flows properly.

Soln. :

Step 1 : Identify input and out variables and decide descriptors for each variables.

Here inputs are,

1. Amount of valve opening for cold water.
2. Amount of valve opening for steam.

- Valve opening is measured in degree from 0° to 180°. we take five descriptors for each of the input variables.
- Descriptors for value opening for cold water flow are

{ELCV, LCV, CCV, RCV, ERCV}

ELCV : Extreme Left Cold Valve

LCV : Left Cold Valve

CCV : Centre Cold Valve

RCV : Right Cold Valve

ERCV : Extreme Right Cold Valve

Descriptors for valve opening of steam flow are {ELSV, LSV, CSV, RSV, ERSV}

ELSV : Extreme Left Steam Valve

LSV : Left Steam Value

CSV : Centre Steam Value

RSV : Right Steam Value

ERSV : Extreme Right Steam Value

- Output is temperature of water in the tank measured in °C.

We use seven descriptors for temperature of water {VVCT, VCT, CT, WT, HT, VHT, VVHT}

VVCT : Very Very Cold Temperature

VCT : Very Cold Temperature

CT : Cold Temperature

WT : Warm Temperature

HT : Hot Temperature

VHT : Very Hot Temperature

VVHT : Very Very Hot Temperature

Step 2 : Define membership functions for each of the input and output variables.

1. **Membership functions for valve opening for cold water flow**

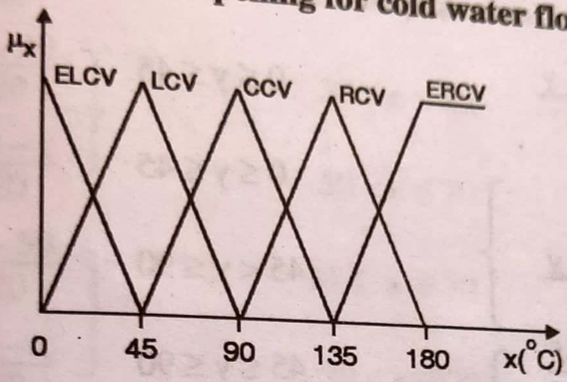

Fig. P. 3.6.5(a) : Membership functions for valve opening of cold water

$$\mu_{ELCV}(x) = \frac{45-x}{45}, \quad 0 \le x \le 45$$

$$\mu_{LCV}(x) = \begin{cases} \dfrac{x}{45}, & 0 \le x \le 45 \\ \dfrac{90-x}{45}, & 45 < x \le 90 \end{cases}$$

$$\mu_{CCV}(x) = \begin{cases} \dfrac{x-45}{45}, & 45 \le x \le 90 \\ \dfrac{135-x}{45}, & 90 < x \le 135 \end{cases}$$

$$\mu_{RCV}(x) = \begin{cases} \dfrac{x-90}{45}, & 90 \le x \le 135 \\ \dfrac{180-x}{45}, & 135 < x \le 180 \end{cases}$$

$$\mu_{ERCV}(x) = \frac{x-135}{45}, \quad 135 \le x \le 180$$

2. **Membership functions for valve opening for steam flow**

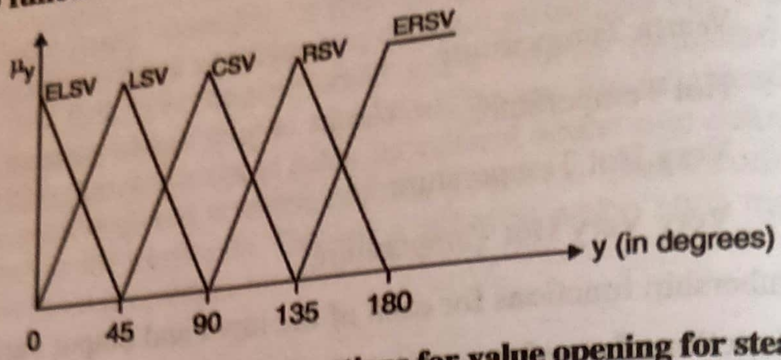

Fig. P. 3.6.5(b) : Membership functions for value opening for steam flow

$$\mu_{ELSV}(y) = \frac{45-y}{45}, \quad 0 \le y \le 45$$

$$\mu_{LSV}(y) = \left.\begin{array}{ll} \frac{y}{45} & , \quad 0 \le y \le 45 \\ \frac{90-y}{45} & , \quad 45 < y \le 90 \end{array}\right\}$$

$$\mu_{ESV}(y) = \left.\begin{array}{ll} \frac{y-45}{45} & , \quad 45 \le y \le 90 \\ \frac{135-y}{45} & , \quad 90 < y \le 135 \end{array}\right\}$$

$$\mu_{RSV}(y) = \left.\begin{array}{ll} \frac{y-90}{45} & , \quad 90 \le y \le 135 \\ \frac{180-y}{45} & , \quad 135 < y \le 180 \end{array}\right\}$$

$$\mu_{ERSV}(y) = \frac{y-135}{45}, \quad 135 \le y \le 180$$

3. **Membership functions for temperature of a water in tank**

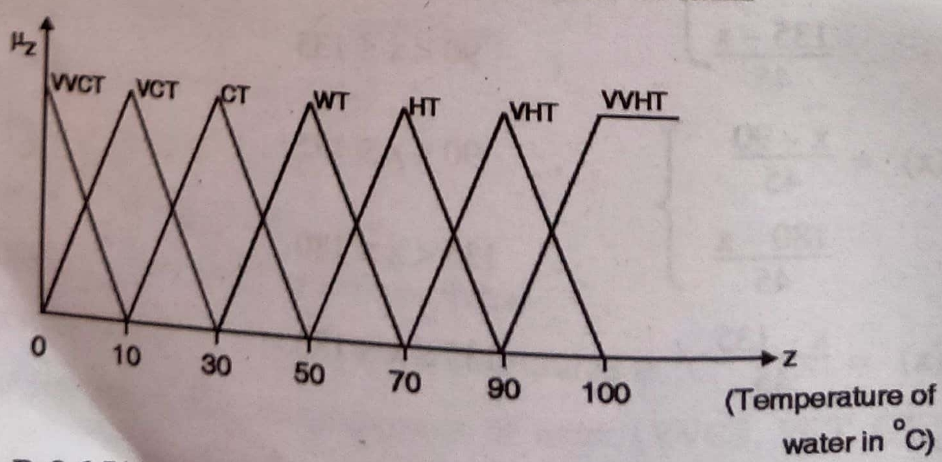

Fig. P. 3.6.5(c) : Membership functions for temperature of water in tank

$$\mu_{VVCT}(z) = \frac{10-z}{10}, \quad 0 \le z \le 10$$

$$\mu_{VCT}(z) = \left.\begin{array}{ll} \frac{z}{10}, & 0 \le z \le 10 \\ \frac{30-z}{20}, & 10 < z \le 30 \end{array}\right\}$$

$$\mu_{CT}(z) = \left.\begin{array}{ll} \frac{z-10}{20}, & 10 \le z \le 30 \\ \frac{50-z}{20}, & 30 < z \le 50 \end{array}\right\}$$

$$\mu_{WT}(z) = \left.\begin{array}{ll} \frac{z-30}{20}, & 30 \le z \le 50 \\ \frac{70-z}{20}, & 50 < z \le 70 \end{array}\right\}$$

$$\mu_{HT}(z) = \left.\begin{array}{ll} \frac{z-50}{20}, & 50 \le z \le 70 \\ \frac{90-z}{20}, & 70 < z \le 90 \end{array}\right\}$$

$$\mu_{VHT}(z) = \left.\begin{array}{ll} \frac{z-70}{20}, & 70 \le z \le 90 \\ \frac{100-z}{10}, & 90 < z \le 100 \end{array}\right\}$$

$$\mu_{VVHT}(z) = \frac{z-90}{10}, \quad 90 \le z \le 100$$

Step 3 : Form a rule base,

X \ Y	ELSV	LSV	CSV	RSV	ERSV
ELCV	WT	WT	HT	VHT	VVHT
LCV	CT	WT	HT	VHT	VVHT
CCV	VCT	CT	WT	HT	VHT
RCV	VCT	VCT	CT	WT	HT
ERCV	VVCT	VVCT	VCT	CT	WT

Step 4 : Rule evaluation

Assume that x = 95° (valve opening of cold water)

y = 50° (valve opening for steam flow)

- Here x = 95° maps to the following two MFs of variable x :

$$\mu_{CCV}(x) = \frac{135 - x}{45} \text{ and } \mu_{RCV}(x) = \frac{x - 90}{45}$$

- Similarly y = 50° maps to the following two MFs of variable y :

$$\mu_{LSV}(y) = \frac{90 - y}{45} \text{ and } \mu_{CSV}(y) = \frac{y - 45}{45}$$

- Evaluate $\mu_{CCV}(x)$ and $\mu_{RCV}(x)$ for x = 95° we get,

$$\mu_{CCV}(95) = \frac{135 - 95}{45} = 0.88 \quad ...(1)$$

$$\mu_{RCV}(95) = \frac{95 - 90}{45} = 0.11 \quad ...(2)$$

- Evaluate $\mu_{LSV}(y)$ and $\mu_{CSV}(y)$ for y = 50°, we get

$$\mu_{LSV}(50) = \frac{90 - 50}{45} = 0.88 \quad ...(3)$$

$$\mu_{CSV}(50) = \frac{50 - 45}{45} = 0.11 \quad ...(4)$$

Above four equations lead to the following four rules that we need to evaluate.

1. Cold water valve is in **center** and steam flow valve is in **left.**
2. Cold water valve is in **center** and steam flow valve is in **center.**
3. Cold water valve is in **right** and steam flow valve is in **left.**
4. Cold water valve is in **right** and steam flow valve is in **center.**

Table P.3.6.5 shows rule strength table.

Table P. 3.6.5 : Rule strength table

	μ_{ELSV}	μ_{LSV}	μ_{CSV}	μ_{RSV}	μ_{ERSV}
μ_{ELSV}	X	X	X	X	X
μ_{LSV}	X	X	X	X	X
μ_{CSV}	X	0.88	0.11	X	X
μ_{RSV}	X	0.11	0.11	X	X
μ_{ERSV}	X	X	X	X	X

Step 5 : Defuzzification

- We find the rule with maximum strength

$$= \max (0.88, 0.11, 0.11, 0.11)$$

$$= 0.88$$

Which corresponds to rule 1.

- The rule 1 corresponds to the output MF $\mu_{CT}(z)$. To compute final defuzzified value we take average of $\mu_{CT}(z)$.

$$\mu_{CT}(z) = \frac{z-10}{20} \Rightarrow 0.88 = \frac{z-10}{20} \Rightarrow z = 27.7$$

$$\mu_{CT}(z) = \frac{50-z}{20} \Rightarrow 0.88 = \frac{50-z}{20} \Rightarrow z = 32.3$$

$$\therefore z^* = \frac{27.7+32.3}{2} = 30^\circ C$$

...Ans.

Review Questions

Q. 1 Explain the basic structure of fuzzy controller. And also explain the steps involved in designing the fuzzy controller. *(Ans. : Refer Sections 3.1 and 3.1.1)*

Q. 2 Explain different types of fyzzy propositions. *(Ans. : Refer Section 3.2.2)*

Q. 3 Explain how the formation and decomposition of compound rules occurs in Fuzzy logic. *(Ans. : Refer Sections 3.2.3 and 3.2.4)*

Q. 4 Write a note on different types of fuzzy approximate reasoning. *(Ans. : Refer Section 3.3)*

Q. 5 With a neat skech/diagram , explain construction and working of FIS. *(Ans. : Refer Section 3.4.1)*

Q. 6 How Sugeno model of FIS differ from Mamdani model ? Explain working of Mamdani FIS. *(Ans. : Refer Sections 3.4.2.3 and 3.4.2.1)*

❑❑❑

Chapter Ends...