intro to data science with probability & statistics

CSCI 3022

September 19, 2018

1. More discrete RVs
2. Common distributions

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The Discrete Uniform Distribution

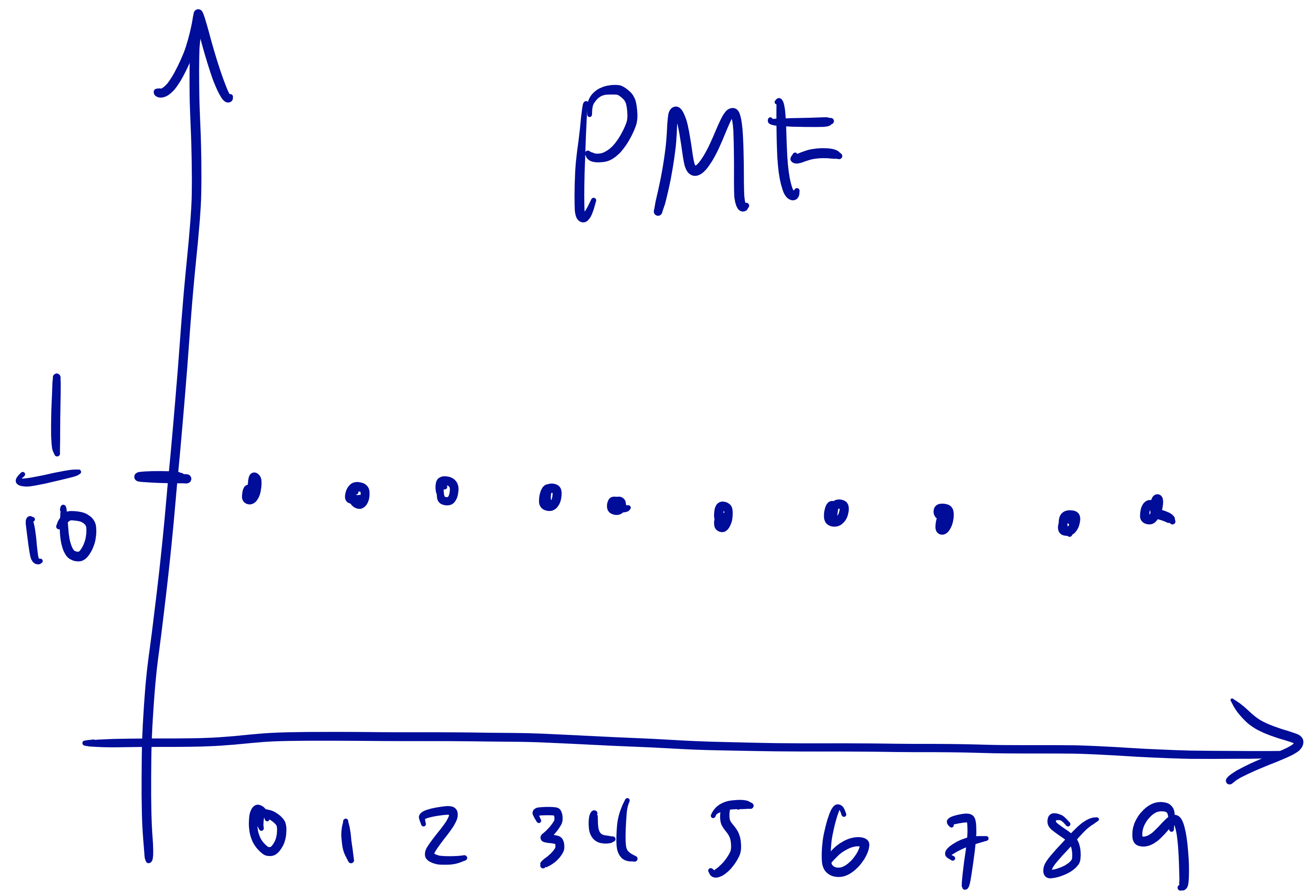
- **Definition**: the discrete *uniform* distribution assigns a probability mass of $\frac{1}{n}$ to each of n values in [a,b]. We write it as *unif(a,b).*

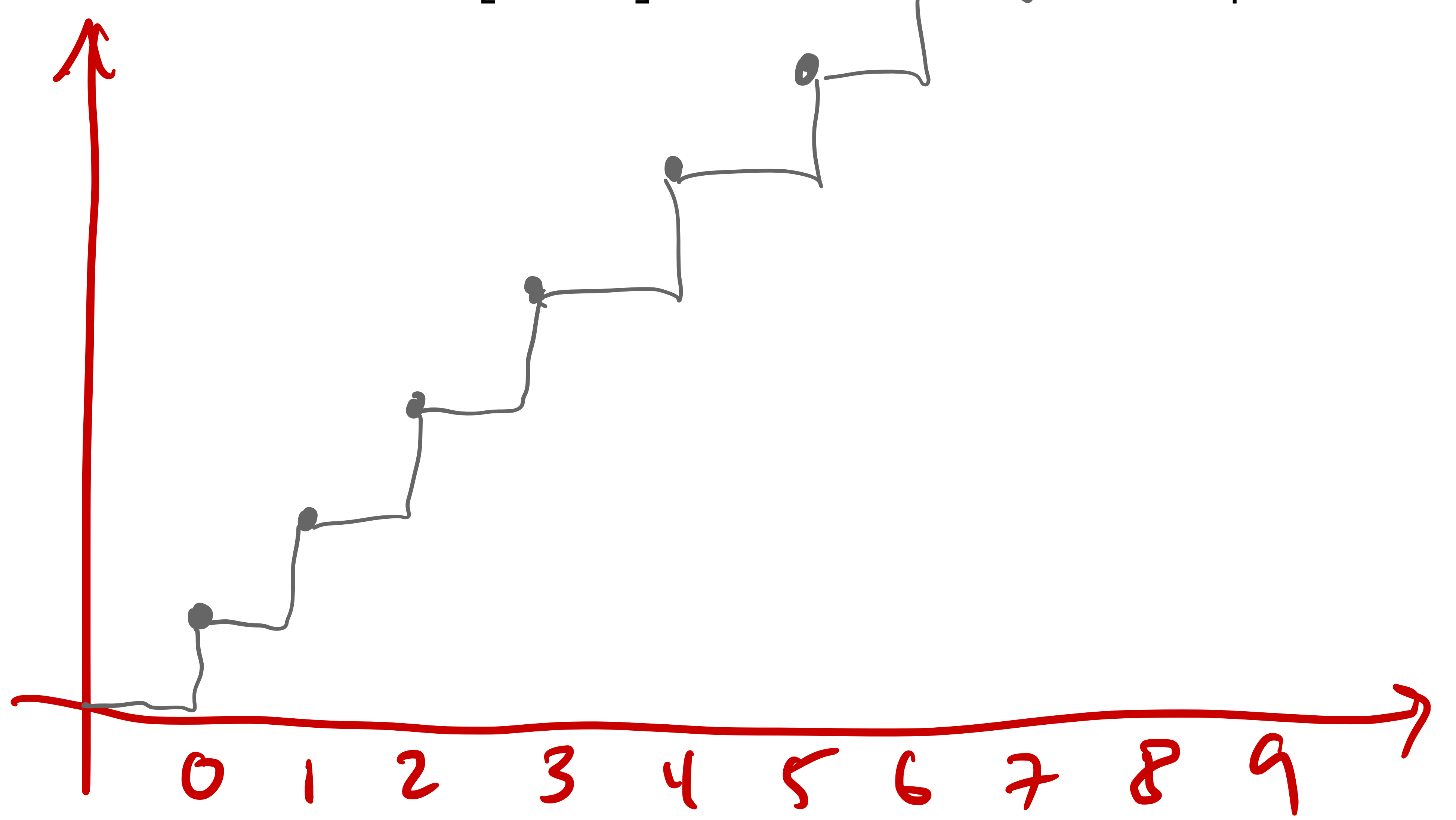
- **Ponder**: can you think of an example of a random experiment whose outcome is a discrete uniform distribution? What are *a* and *b*?

rolling die $a = 1, b = 6, n = 6$

$\frac{10}{10}$

- **Plot**: the probability mass function for unif[0,9], *n*=10. Then plot the CDF.





The Bernoulli Distribution

- **Definition**: a discrete RV X has a *Bernoulli distribution* with parameter p, where $0 \leq p \leq 1$, if its probability mass function (PMF) is given by:

$$f(1) = P(X=1) = p \quad \text{and} \quad f(0) = P(X=0) = 1\text{-}p$$

- We denote this distribution by *Ber*(p)

best joke (in-class only)

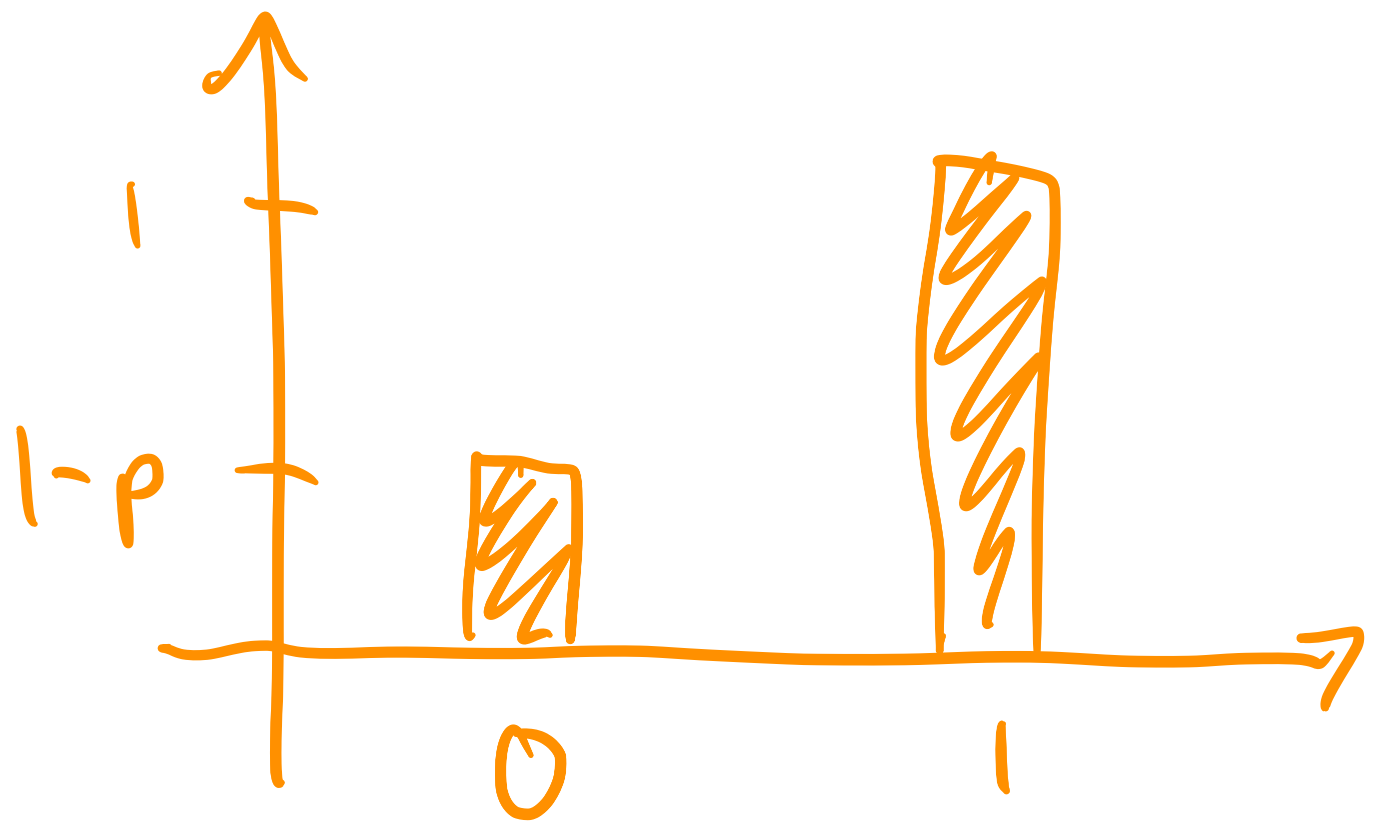
- Look closely—what does this PMF remind you of?

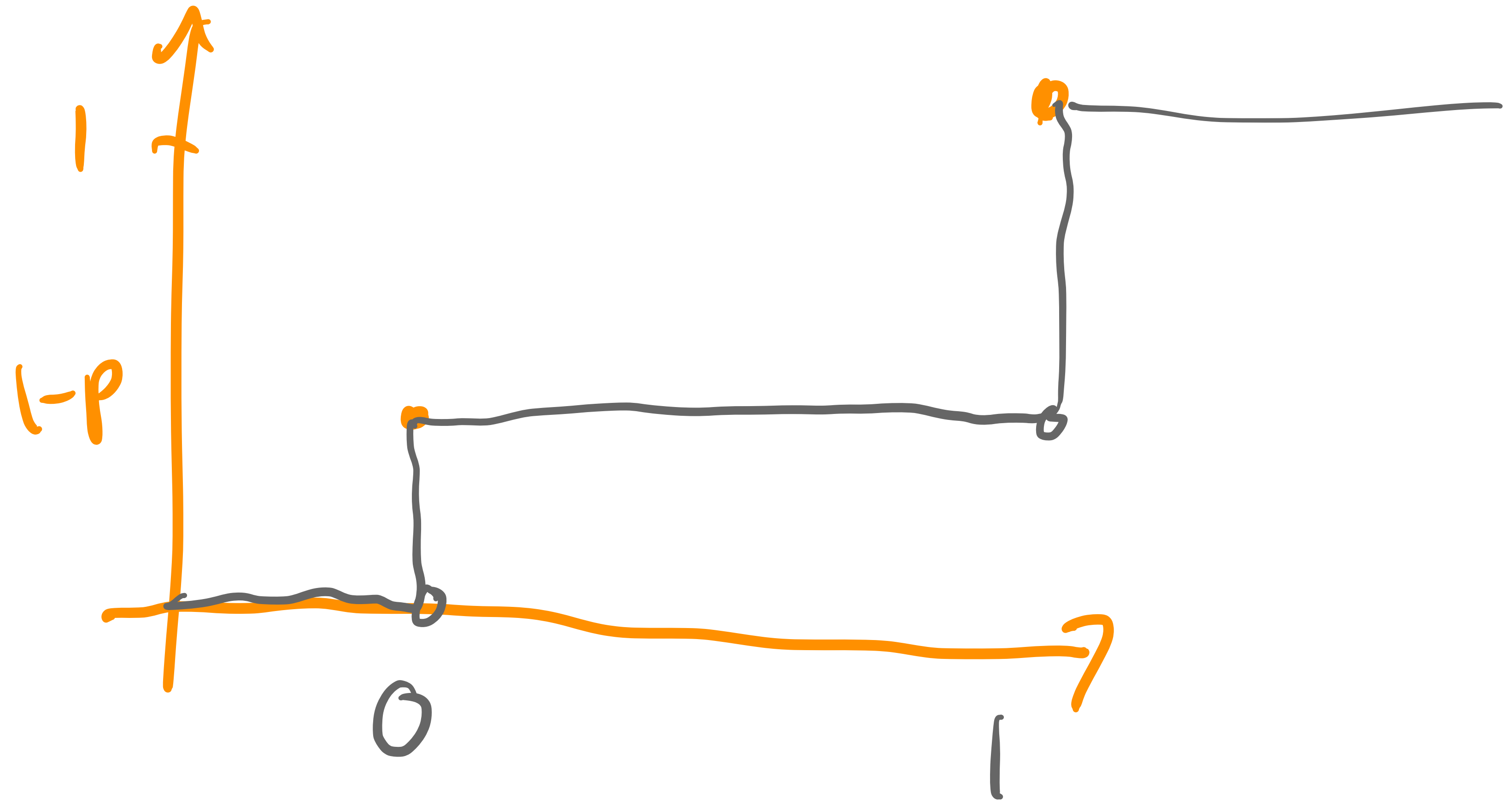
a biased coin

- What is the CDF of the Bernoulli distribution?

$F(0) = 1-p$

$F(1) = 1$





Counting

- Let's set aside the Bernoulli distribution for a moment and turn to counting.
- Believe it or not, *counting* comes up all over the place in probability, and therefore in CS, math, physics, engineering, data science, etc.
- Some counting is easy: how many integers are there in the interval [0,9]?
- We're interested in counting that requires math though: Ancsa, Brendan, Caterina, David, and Elly are standing in line. How many different orders could they stand in?
- If there are 10 problems on an exam and you need 7 or more correct to pass, how many different ways are there to pass the test?

Counting

- We'll talk about two key kinds of counting problems today:

- Counting **permutations** means counting the number of ways a set of objects can be ordered (or, more precisely, permuted).

Ancsa, Brendan, Caterina, David, and Elly are standing in line. How many different orders could they stand in?

- Counting **combinations** means counting the number of ways that a set of objects can combined into subsets.

If there are 10 problems on an exam and you need 7 or more correct to pass, how many different ways are there to pass the test?

Counting - Permutations

- How many ways are there of ordering 1 object? 1
- How many ways are there of ordering 2 objects? A,B or B,A → 2
- What about 3 objects?
  ABC BAC CAB
  ACB BCA CBA
  6
- What is the formula for the number of possible permutations of *n* objects?

n= 5 objects

$$\boxed{\#\text{perms} = n!}$$

#1 5 possible, #2 4 possible, #3 3 possible, #4 2 possible, #5 1 choice

$$5 \times 4 \times 3 \times 2 \times 1 = 5!$$

$$\boxed{\text{define: } 0! = 1}$$

Counting - Permutations 2

- Say there are 10 people in a race. How many ways are there of awarding the gold, silver, and bronze?

place G S B 4 5 6 7 8 9 10

$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 10!$

(1) $10 \cdot 9 \cdot 8 = 720$

(2) 10! total perms, but 7! I don't care about.

$$\frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 10 \cdot 9 \cdot 8 = 720$$

Counting - Combinations

- We could approach the previous problem another way…

10 people… ask: how many ways could I possibly choose a subset of 3 of them?

10 choose 3

Note: $3! = 6$ ways of ordering those 3.

# of possible GSB = $[10 \text{ choose } 3] \times 3! = \frac{10!}{7!}$

$[10 \text{ choose } 3] = \frac{10!}{7!\,3!}$

$$n \text{ choose } k = \frac{n!}{(n-k)!\,k!}$$

Counting - Combinations

- The previous slide shows us how to use permutations to get to combinations.

- If I have *n* objects and I choose *k* of them, I can do this in *n* choose *k* ways.

- Various notations: $\binom{n}{k}$ $\cancel{nCk}$ $\frac{n!}{k!(n-k)!}$ $C_{n,k}$

:)

formula

{n \choose k}

Counting - Combinations

- The previous slide shows us how to use permutations to get to combinations.

- If I have *n* objects and I choose *k* of them, I can do this in *n* choose *k* ways.

- Various notations: $\binom{n}{k}$ $\quad nCk \quad$ $\frac{n!}{k!(n-k)!}$ $\quad C_{n,k}$

- So, how many ways are there to choose 3 runners from a total of 10?

$$\binom{10}{3} = \frac{10!}{(10-3)!\,3!} = \frac{10!}{7!\,3!} = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = \frac{720}{6} = 120$$

Counting - Combinations

- Let's connect some dots.
- Let's compute: $\binom{1}{1}$ $\frac{1!}{(1-1)!\,1!} = 1$
- $\binom{2}{1}$, $\binom{2}{2}$ $\frac{2!}{(2-1)!\,1!} = 2$ $\frac{2!}{(2-2)!\,2!} = 1$
- $\binom{3}{1}$, $\binom{3}{2}$, $\binom{3}{3}$
- Now $\binom{1}{0}$, $\binom{2}{0}$, $\binom{3}{0}$ and $\binom{0}{0}$
- What do you notice?

$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$
$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

Pascal's Triangle

$$1$$
$$1 \quad 1$$
$$1 \quad 2 \quad 1$$
$$1 \quad 3 \quad 3 \quad 1$$
$$1 \quad 4 \quad 6 \quad 4 \quad 1$$

Binomial Coefficients

Combinations applied!

- If there are 10 problems on an exam and you need 7 or more correct to pass, how many different ways are there to pass the test?

Pass: 7 or 8 or 9 or 10

$$\binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10} = \boxed{176}$$

$$120 \qquad \frac{10!}{(10-8)!\,8!} = \frac{5\cancel{10}\cdot 9}{\cancel{2}\cdot 1} = 45 \qquad 10 \qquad 1$$

$$\frac{10!}{(10-9)!\,9!} = \frac{10!}{(10-1)!\,1!} = \binom{10}{1}$$

wow!!