CSCI 3022

intro to data science with probability & statistics

September 19, 2018

1. More discrete RVs
2. Common distributions

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The Discrete Uniform Distribution

- **Definition**: the discrete *uniform* distribution assigns a probability mass of $\frac{1}{n}$ to each of n values in [a,b]. We write it as *unif(a,b).*

- **Ponder**: can you think of an example of a random experiment whose outcome is a discrete uniform distribution? What are *a* and *b*?

- **Plot**: the probability mass function for unif[0,9], n=10. Then plot the CDF.

The Bernoulli Distribution

- **Definition**: a discrete RV X has a *Bernoulli distribution* with parameter p, where $0 \leq p \leq 1$, if its probability mass function (PMF) is given by:

$$f(1) = P(X=1) = p \quad \text{and} \quad f(0) = P(X=0) = 1\text{-}p$$

- We denote this distribution by *Ber*(p)
- Look closely—what does this PMF remind you of?
- What is the CDF of the Bernoulli distribution?

Counting

- Let's set aside the Bernoulli distribution for a moment and turn to counting.
- Believe it or not, *counting* comes up all over the place in probability, and therefore in CS, math, physics, engineering, data science, etc.
- Some counting is easy: how many integers are there in the interval [0,9]?
- We're interested in counting that requires math though: Ancsa, Brendan, Caterina, David, and Elly are standing in line. How many different orders could they stand in?
- If there are 10 problems on an exam and you need 7 or more correct to pass, how many different ways are there to pass the test?

Counting

- We'll talk about two key kinds of counting problems today:

- Counting **permutations** means counting the number of ways a set of objects can be ordered (or, more precisely, permuted).

Ancsa, Brendan, Caterina, David, and Elly are standing in line. How many different orders could they stand in?

- Counting **combinations** means counting the number of ways that a set of objects can combined into subsets.

If there are 10 problems on an exam and you need 7 or more correct to pass, how many different ways are there to pass the test?

Counting - Permutations

- How many ways are there of ordering 1 object?
- How many ways are there of ordering 2 objects?
- What about 3 objects?
- What is the formula for the number of possible permutations of *n* objects?

Counting - Permutations 2

- Say there are 10 people in a race. How many ways are there of awarding the gold, silver, and bronze?

Counting - Combinations

- We could approach the previous problem another way…

Counting - Combinations

- The previous slide shows us how to use permutations to get to combinations.

- If I have *n* objects and I choose *k* of them, I can do this in *n* choose *k* ways.

- Various notations: $\binom{n}{k}$ $\quad nCk \quad$ $\dfrac{n!}{k!(n-k)!}$

Counting - Combinations

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- Various notations: $\binom{n}{k}$ nCk $\frac{n!}{k!(n-k)!}$ $C_{n,k}$

- So, how many ways are there to choose 3 runners from a total of 10?

Counting - Combinations

- Let's connect some dots.
- Let's compute: $\binom{1}{1}$
- $\binom{2}{1}$, $\binom{2}{2}$
- $\binom{3}{1}$, $\binom{3}{2}$, $\binom{3}{3}$
- Now $\binom{1}{0}$, $\binom{2}{0}$, $\binom{3}{0}$ and $\binom{0}{0}$
- What do you notice?

Combinations applied!

- If there are 10 problems on an exam and you need 7 or more correct to pass, how many different ways are there to pass the test?

The Binomial Distribution

- Suppose I sum 5 Bernoulli random variables. This sum is a RV too!
- It takes on values in the interval [0,5].
- What is the PMF of the sum of 5 Bernoulli RVs? Let’s build it up!

The Binomial Distribution

- The sum of Bernoulli RVs is the **Binomial Distribution** (see top of slide).
- It is the distribution of the number of “heads” you’ll get when flipping a coin n times.
- Note that it is parameterized by the number of flips n and the Bernoulli parameter p. So we call this *Bin(n,p)*.

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$$f(k) = P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

- **Practice**: what's the probability that a biased coin with *p*=0.8 comes up heads 7 times in 10 flips?

The Binomial Distribution

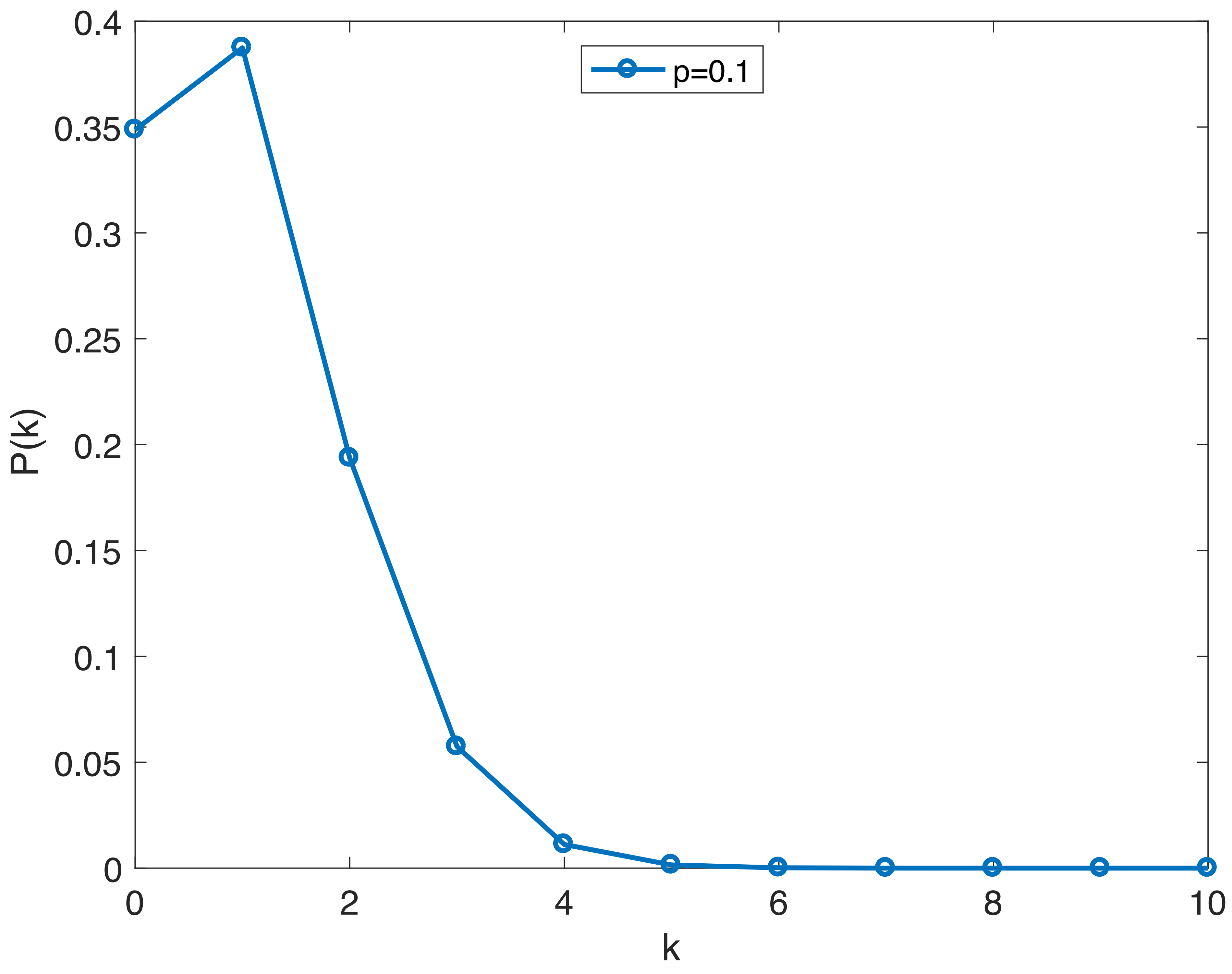
- **Practice**: You are the caretaker of 50 octopuses at the aquarium. 100% of the octopuses are magnificent and intelligent. They *get* you. And you get them. It's a great job, and you feel lucky to have it. However, each octopus has a 12.5% probability of being grumpy on any given day. Today, you choose 20 octopuses at random to put in the Display Habitat.

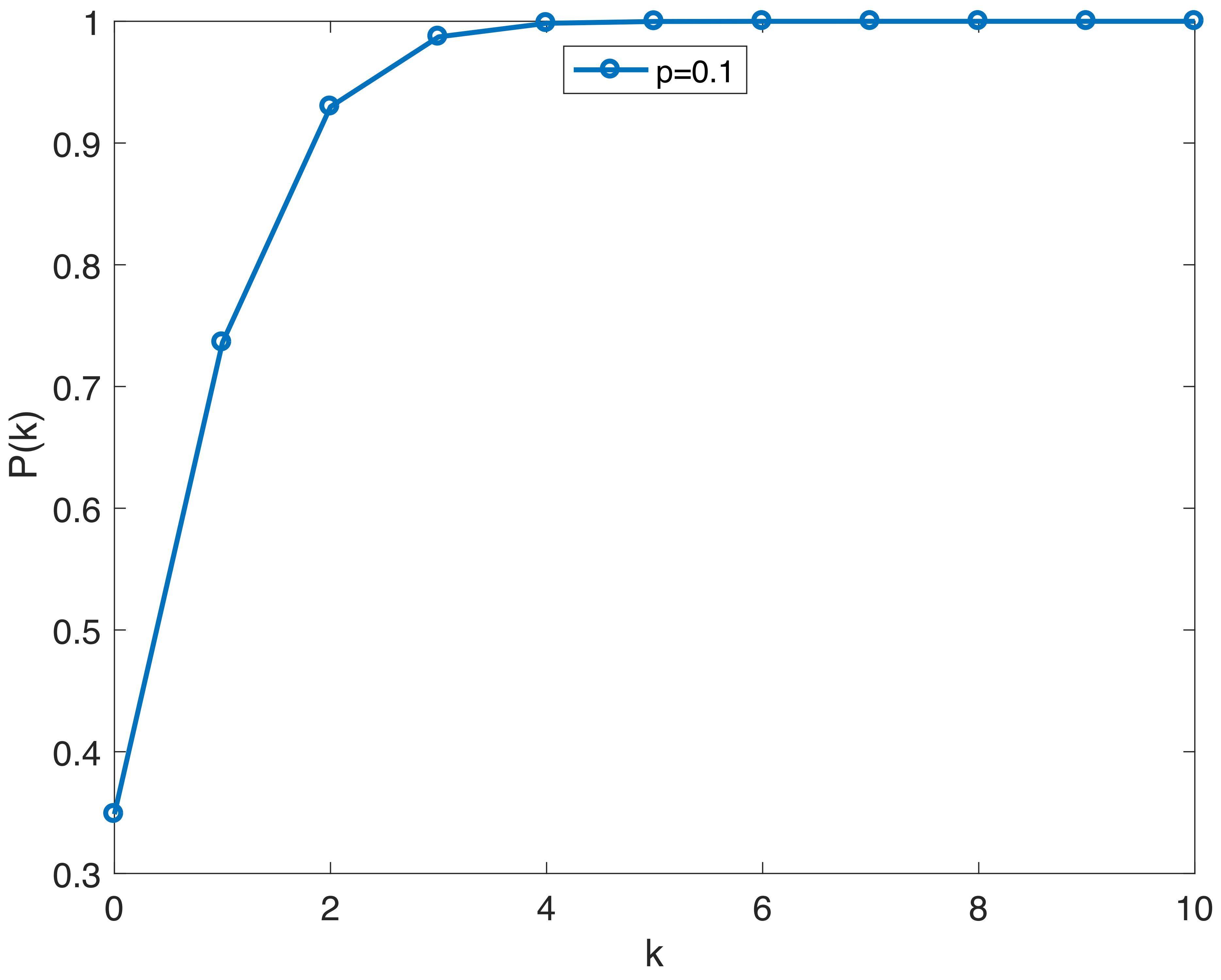
- What is the probability that 5 of the Display Habitat octopuses will be grumpy?

- What is the probability that all the Display Habitat octopuses will not be grumpy?

- What are we assuming when we answer these Qs using the binomial distr.?

Properties of the Binomial PMF





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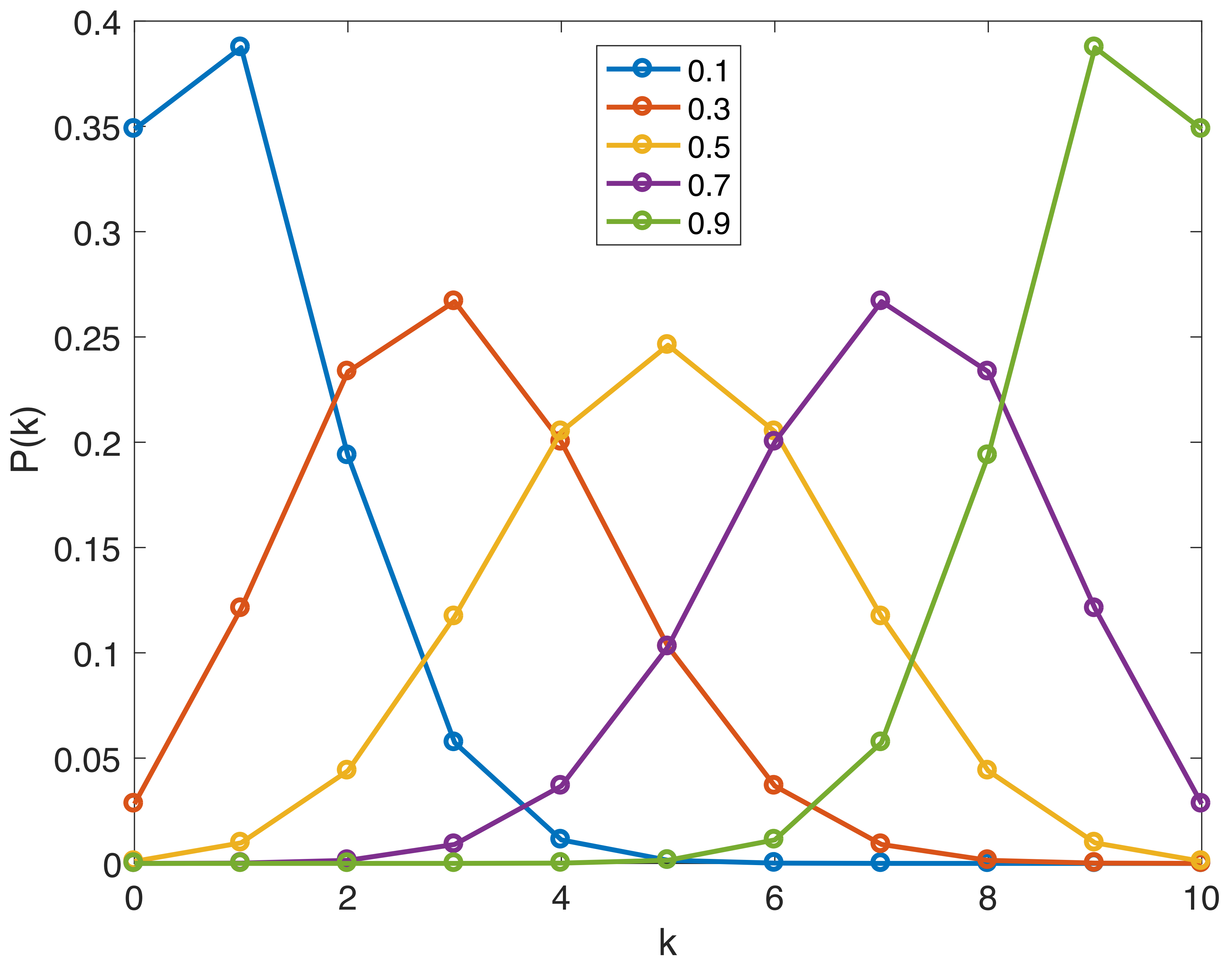

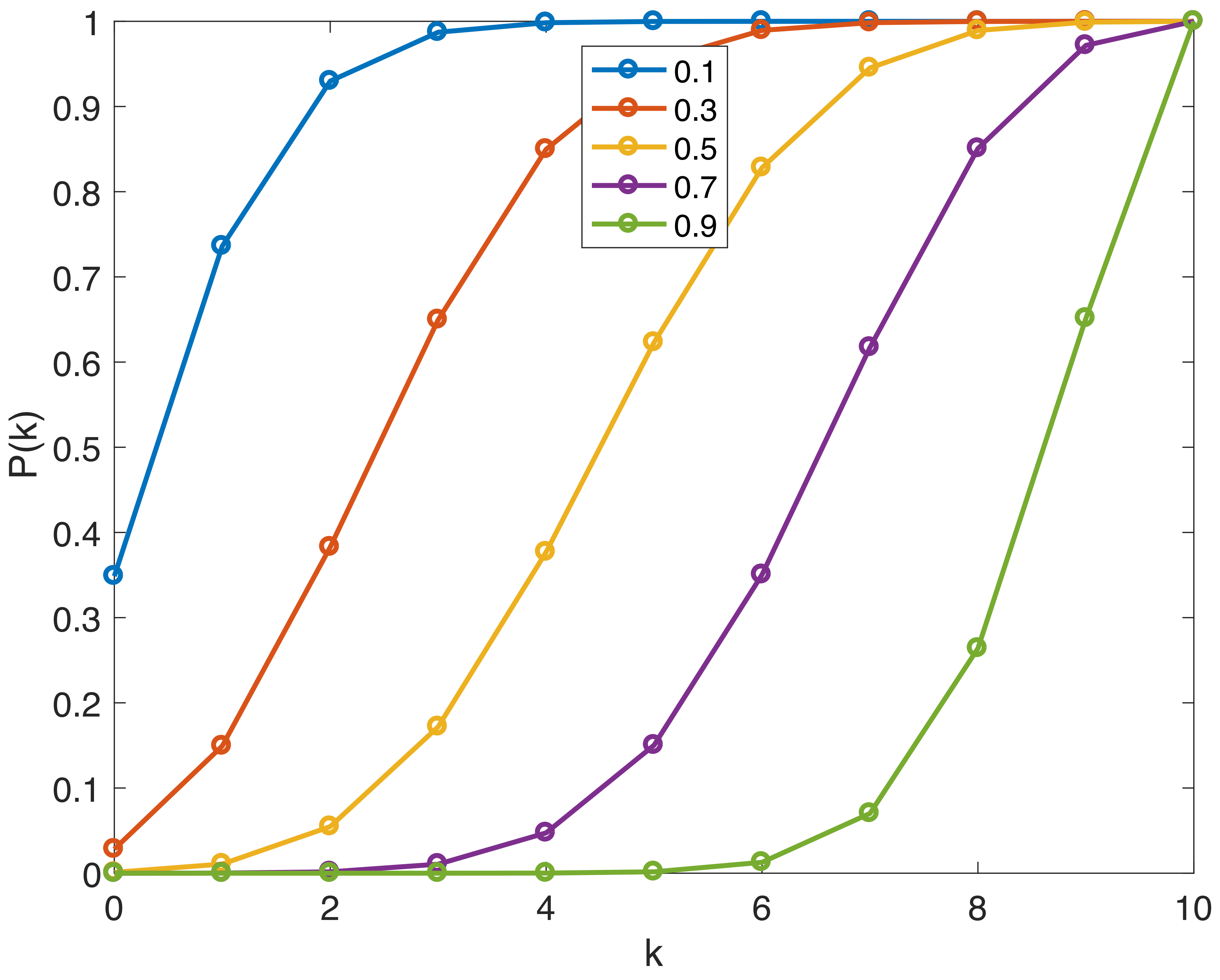

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