Assignment 3 – Deutsch's Algorithm Report

WooHoo2

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Introduction

1. We want to judge a function f(x): $\{0, 1\} \rightarrow \{0, 1\}$ whether it's balanced or constant.

2. By using classical algorithms, at least two measurements are needed to judge it.

3. By using Deutsch's Algorithm, only one measurement is needed.

4. Implement the Deutsch's Algorithm on some IBM Quantum Computer

Detail

1. Use X gate to transfer $|0\rangle$ to $|1\rangle$

2. Use H gate to get $|0\rangle$ and $|1\rangle$

3. Construct oracle function O_f to get the result x , y$\bigoplus$f(x)

4. Input $|+\rangle$ and $|-\rangle$ to oracle function, and get the output x , y$\bigoplus$f(x)

5. Use CNOT Gate (x, y) to simulate balanced functions, the first qubit has been changed by the CNOT Gate

6. Delete the CNOT Gate (x, y) to simulate constant functions

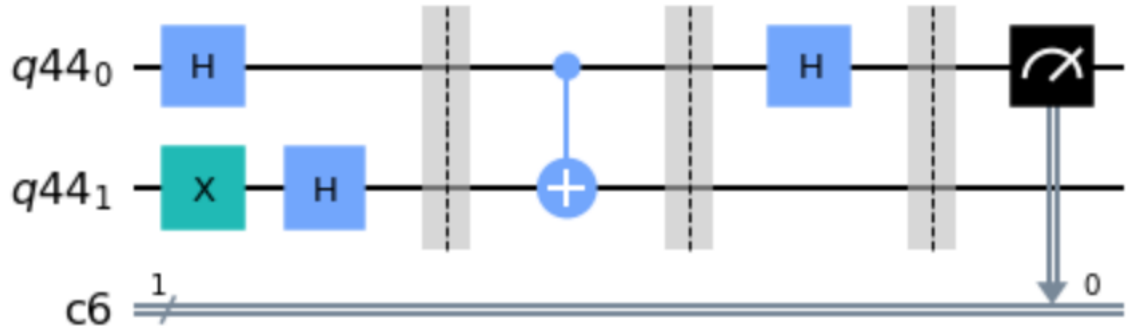
7. Use the X Gate (x) to simulate constant functions which flip the sign of x 8. Apply the Hadamard Gate on the first qubit

9. Measure the first qubit, then we can get the result

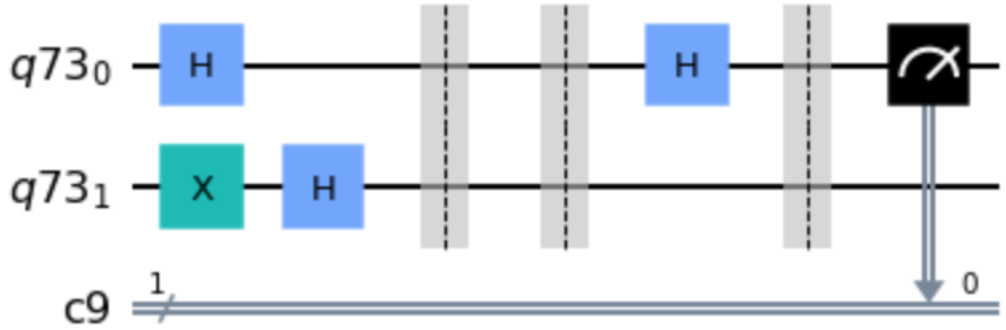
10. Use IBM Quantum resources to implement the algorithm and check the result

Quantum Circuit

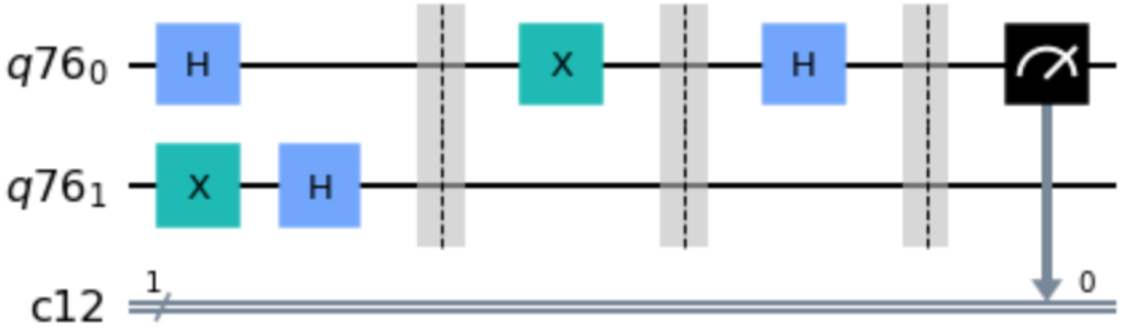
1. Circuit to simulate Balanced Functions



2. Circuit to simulate Constant Functions



3. Circuit to simulate Constant Functions which flip the Sign of x



Analyze

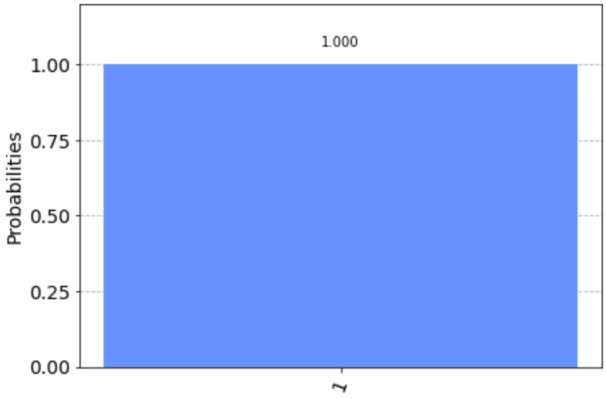
1. We use one X gate and two Hadamard gates to get $|+\rangle$ and $|-\rangle$
2. Apply CNOT gate, then the first qubit has been change to $\pm\frac{|+\rangle}{\sqrt{2}}$ for constant functions and $\pm\frac{|-\rangle}{\sqrt{2}}$ for balanced functions.
3. Apply the Hadamard gate on the first qubit we can get formalized result whether $|0\rangle$ or $|1\rangle$
4. If f(x) is constant, it will keep x in its direction or flip it. Therefore, we get x = $|0\rangle$
5. If f(x) is balanced, we get x = $|1\rangle$

Measurements on "BasicAer"

1. Result of Balanced Functions

```
{'1': 1024}
```

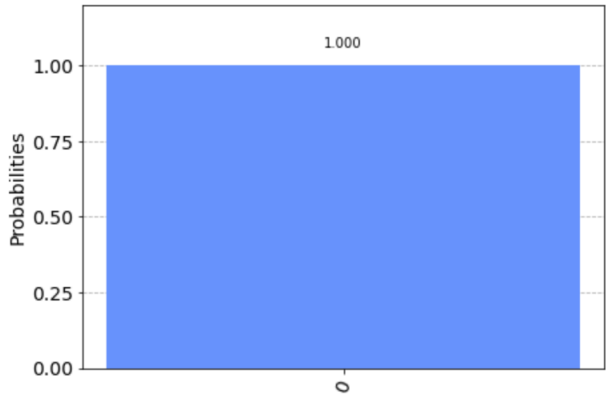
2. Draw Result of Balanced Functions



3. Result of Constant Functions

```
{'0': 1024}
```

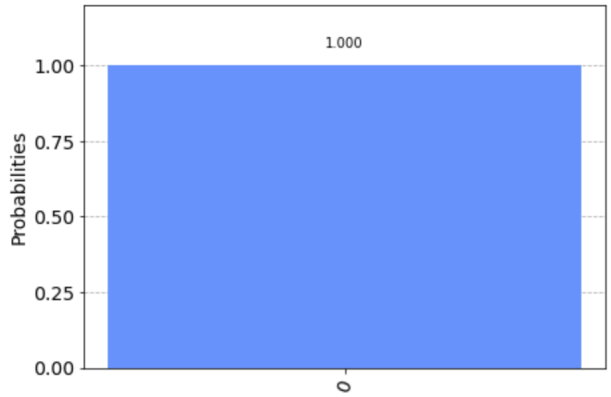
4. Draw Result of Constant Functions



5. Result of Constant Functions which Flip the Sign of x

```
{'0': 1024}
```

6. Draw Result of Constant Functions which Flip the Sign of x

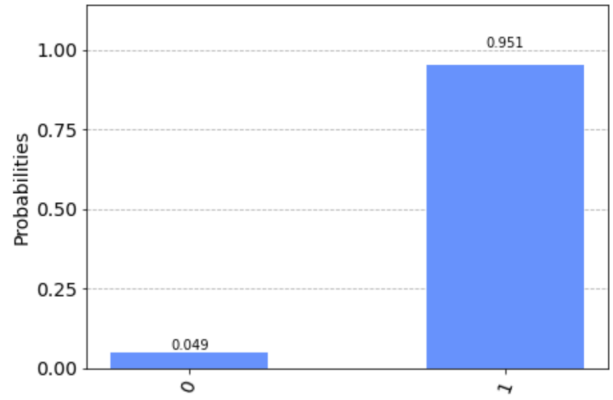


Measurements on "IBM Quantum Computer"

1. Result of Balanced Functions

```
{'0': 195, '1': 3805}
```

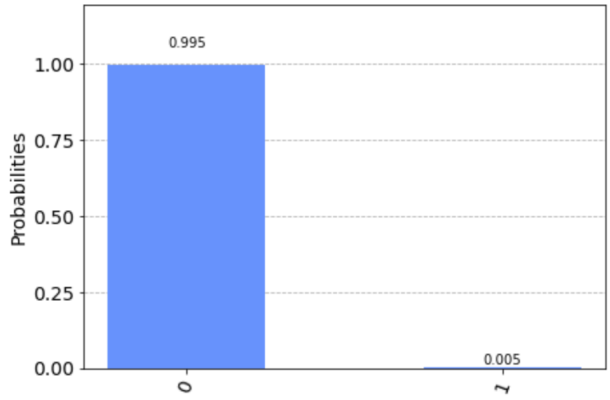
2. Draw Result of Balanced Functions



3. Result of Constant Functions

```
{'0': 3981, '1': 19}
```

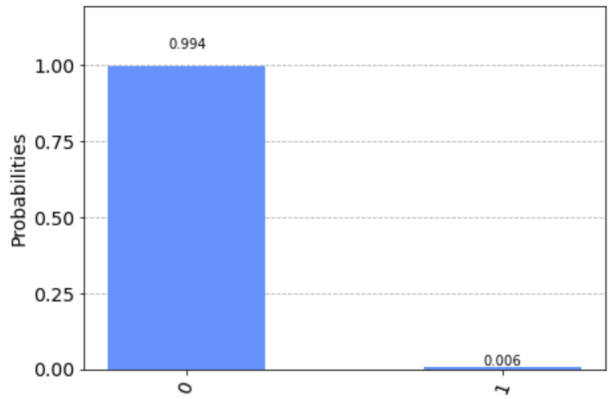
4. Draw Result of Constant Functions



3. Result of Constant Functions which Flip the Sign of x

```
{'0': 3975, '1': 25}
```

4. Draw Result of Constant Functions which Flip the Sign of x



Results Analyze

1. On "BasicAer", all the results of balanced functions are $|1\rangle$ and all results of constant functions are $|0\rangle$, which are as expected.

2. On "IBM Quantum Computer", we use the "ibmq_lima" backend to get more samples.

Most of the results of balanced functions are $|1\rangle$ and most of the results of constant functions are $|0\rangle$.

3. On "IBM Quantum Computer", because there are many samples, the qubit we initialized is not guaranteed to be $|0\rangle$. Therefore, there are some results are $|0\rangle$ for balanced functions and some results are $|0\rangle$ for constant functions.

Reference

1. `https://leimao.github.io/blog/Deutsch-Algorithm/`

2. Lecture Notes "Weeks 7-8 2022.pdf"

3. "program.barrier()" `https://github.com/atilsamancioglu/QX07-DeutschAlgorithm`

4. "use X Gate to simulate constant functions" `https://github.com/MakotoNakai/Deutch-Algorithm/`

blob/master/Deutch_constant.py

5. "use IBMQ" https://fullstackquantumcomputation.tech/blog/post-tutorial-0-Hello-world/