P8130 Final Report (Project 1)

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Read and Clean Data

```
data =
  read_csv("./data.csv") |>
  janitor::clean_names() |>
  mutate(
    gender = factor(case_when(
      gender == "male" ~ 0,
      gender == "female" ~ 1,
      )),
    ethnic_group = factor(case_when(
      ethnic_group == "group A" ~ 0,
      ethnic_group == "group B" ~ 1,
      ethnic_group == "group C" ~ 2,
      ethnic_group == "group D" ~ 3,
      ethnic_group == "group E" ~ 4,
      )),
    parent_educ = factor(case_when(
      parent_educ == "some highschool" ~ 0,
      parent_educ == "some college" ~ 1,
      parent_educ == "associate's degree" ~ 2,
      parent_educ == "bachelor's degree" ~ 3,
      parent_educ == "master's degree" ~ 4,
      )),
    lunch_type = factor(case_when(
      lunch_type == "standard" ~ 0,
      lunch_type == "free/reduced" ~ 1,
      )),
    test_prep = factor(case_when(
      test_prep == "none" ~ 0,
      test_prep == "completed" ~ 1,
      )),
    parent_marital_status = factor(case_when(
      parent_marital_status == "married" ~ 0,
      parent_marital_status == "single" ~ 1,
      parent_marital_status == "widowed" ~ 2,
      parent_marital_status == "divorced" ~ 3,
      )),
```

```
    practice_sport = factor(case_when(
      practice_sport == "never" ~ 0,
      practice_sport == "sometimes" ~ 1,
      practice_sport == "regularly" ~ 2,
      )),
    is_first_child = factor(case_when(
      is_first_child == "no" ~ 0,
      is_first_child == "yes" ~ 1,
      )),
    transport_means = factor(case_when(
      transport_means == "school_bus" ~ 0,
      transport_means == "private" ~ 1,
      )),
    wkly_study_hours = factor(case_when(
      wkly_study_hours == "< 5" ~ 0,
      wkly_study_hours == "10-May" ~ 1,
      wkly_study_hours == "> 10" ~ 2,
      ))
    ) |>
  mutate(nr_siblings = factor(nr_siblings))
```

```
## Rows: 948 Columns: 14
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## chr (10): Gender, EthnicGroup, ParentEduc, LunchType, TestPrep, ParentMarita...
## dbl  (4): NrSiblings, MathScore, ReadingScore, WritingScore
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
'
# Deal with NA -- Calculate the column mean (round to integer) and plug it into NA cell
column_means <- round(colMeans(data, na.rm = TRUE), digits = 0)
for (col in names(data)) {
  data[[col]][is.na(data[[col]])] <- column_means[col]
  }

head(data)
'
```

```
## [1] "\n# Deal with NA -- Calculate the column mean (round to integer) and plug it into NA cell\ncolu
```

```
# Another data set for EDA
data_long <- data |>
  pivot_longer(cols = c(math_score, reading_score, writing_score),
               names_to = "test", values_to = "score")
```

Summary

```r
sum_data_fct =
  data |>
  dplyr::select(1:11) |>
  skimr::skim() |>
  dplyr::select(skim_variable, n_missing, complete_rate, factor.n_unique, factor.top_counts)

colnames(sum_data_fct) = c("Variable", "Missing", "Complete Rate", "Unique", "Top Counts")

knitr::kable(x = sum_data_fct, caption = "Categorical Variables pre-analysis", digits = 1)
```

Table 1: Categorical Variables pre-analysis

Variable	Missing	Complete Rate	Unique	Top Counts
gender	0	1.0	2	1: 488, 0: 460
ethnic_group	59	0.9	5	2: 277, 3: 237, 1: 171, 4: 124
parent_educ	392	0.6	4	1: 199, 2: 198, 3: 104, 4: 55
lunch_type	0	1.0	2	0: 617, 1: 331
test_prep	55	0.9	2	0: 571, 1: 322
parent_marital_status	49	0.9	4	0: 516, 1: 213, 3: 146, 2: 24
practice_sport	16	1.0	3	1: 477, 2: 343, 0: 112
is_first_child	30	1.0	2	1: 604, 0: 314
nr_siblings	46	1.0	8	1: 245, 2: 213, 3: 198, 0: 101
transport_means	102	0.9	2	0: 509, 1: 337
wkly_study_hours	37	1.0	3	1: 508, 0: 253, 2: 150

```r
data =
  data |>
  drop_na()
```

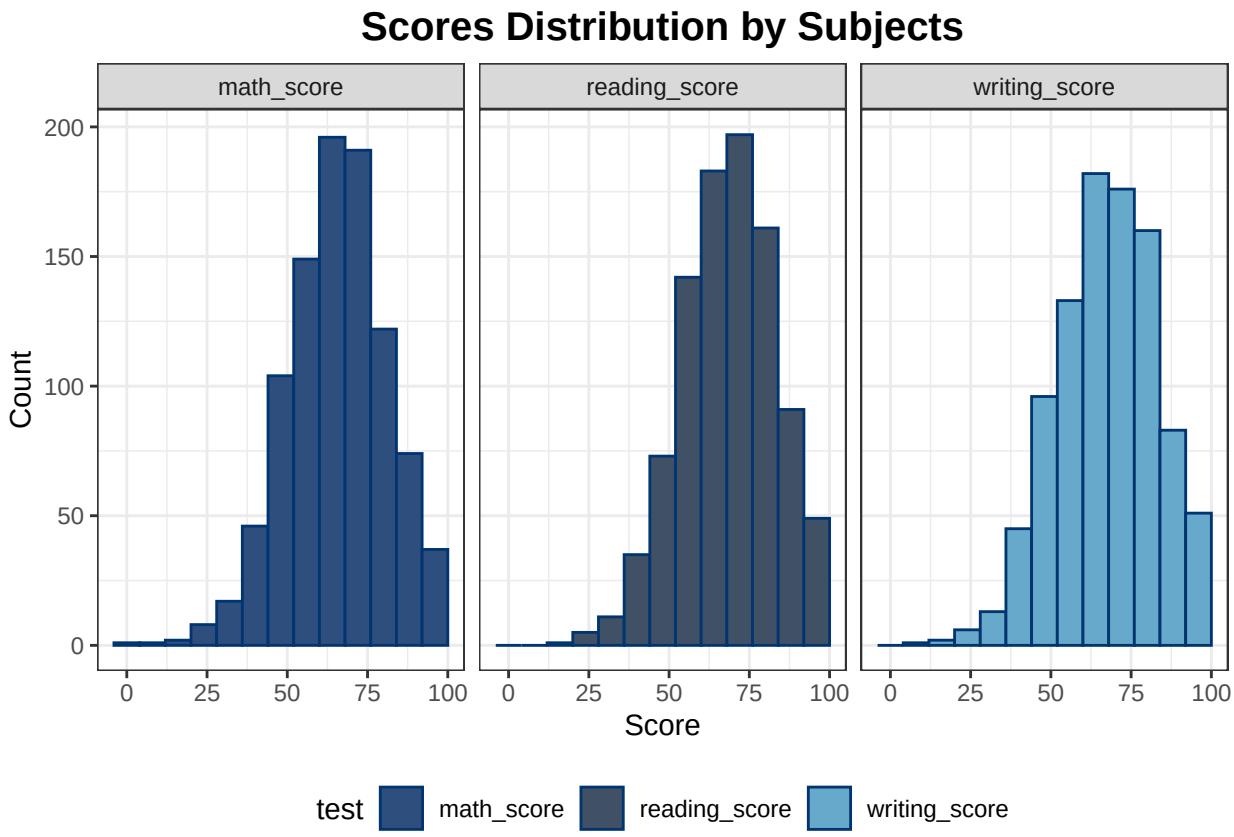
```r
sum_data_score =
  data |>
  dplyr::select(12:14) |>
  skimr::skim() |>
  dplyr::select(skim_variable, numeric.mean, numeric.sd, numeric.p0, numeric.p25, numeric.p50, numeric.

colnames(sum_data_score) = c("Variable", "Mean", "SD", "Min", "Q1", "Median", "Q3", "Max")

knitr::kable(x = sum_data_score, caption = "Continuous Variables pre-analysis", digits = 1)
```

Table 2: Continuous Variables pre-analysis

Variable	Mean	SD	Min	Q1	Median	Q3	Max
math_score	68.7	15.9	18	57	69.0	81	100
reading_score	72.3	14.8	23	61	73.0	84	100
writing_score	72.0	15.2	19	62	72.5	84	100

Histograms

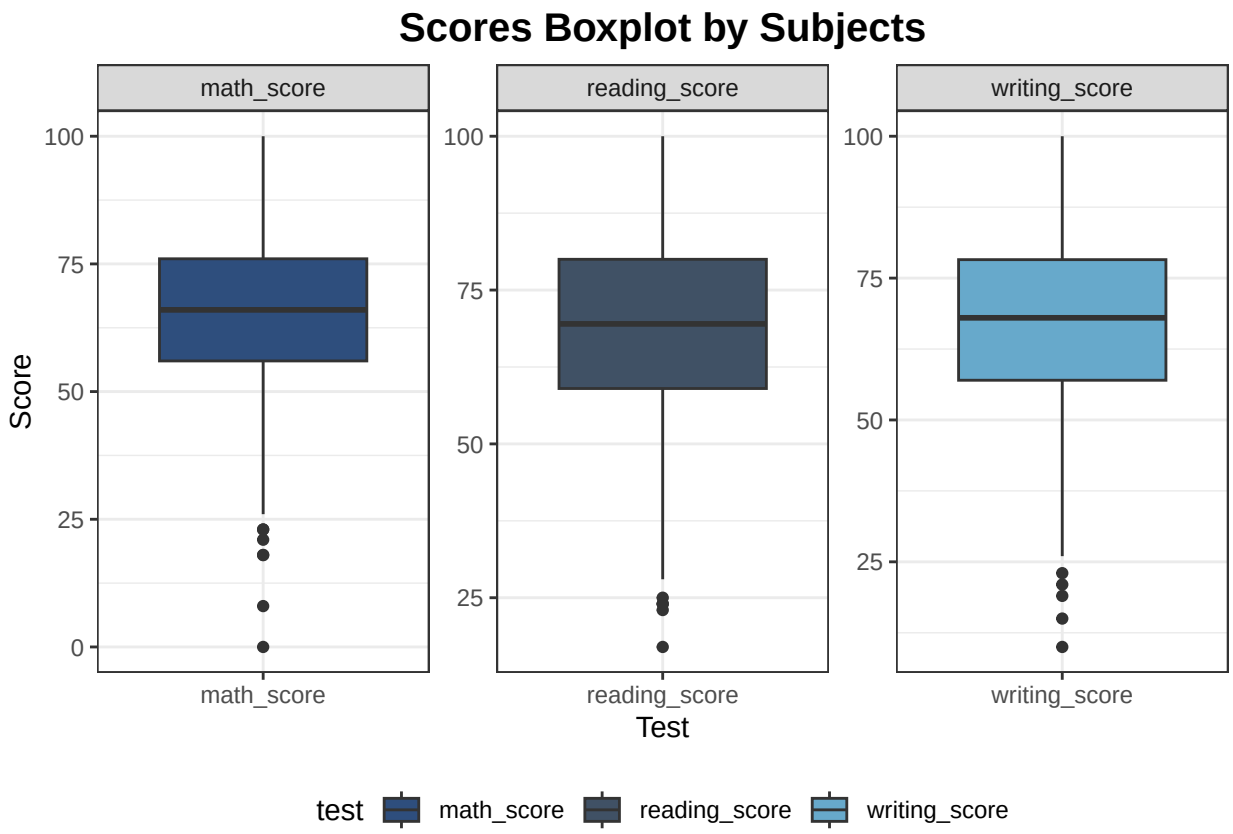
```
data_long |>
  ggplot(aes(x = score, fill = test)) +
  geom_histogram(binwidth = 8, color = "#013571") +
  labs(
    title = "Scores Distribution by Subjects",
    x = "Score",
    y = "Count"
    ) +
  scale_fill_manual(values = c("#2E4E7D", "#405165", "#67A9CB")) +
  facet_grid(~ test) +
  theme_bw() +
  theme(legend.position = "bottom") +
  theme(plot.title = element_text(size = 15, face = "bold", hjust = 0.5))
```



Boxplots

```
data_long |>
  ggplot(aes(x = test, y = score, fill = test)) +
  geom_boxplot() +
  labs(
    title = "Scores Boxplot by Subjects",
```

```
    x = "Test",
    y = "Score"
    ) +
  facet_wrap(~ test, scales = "free") +
  scale_fill_manual(values = c("#2E4E7D", "#405165", "#67A9CB")) +
  theme_bw() +
  theme(legend.position = "bottom") +
  theme(plot.title = element_text(size = 15, face = "bold", hjust = 0.5))
```



Diagnostics

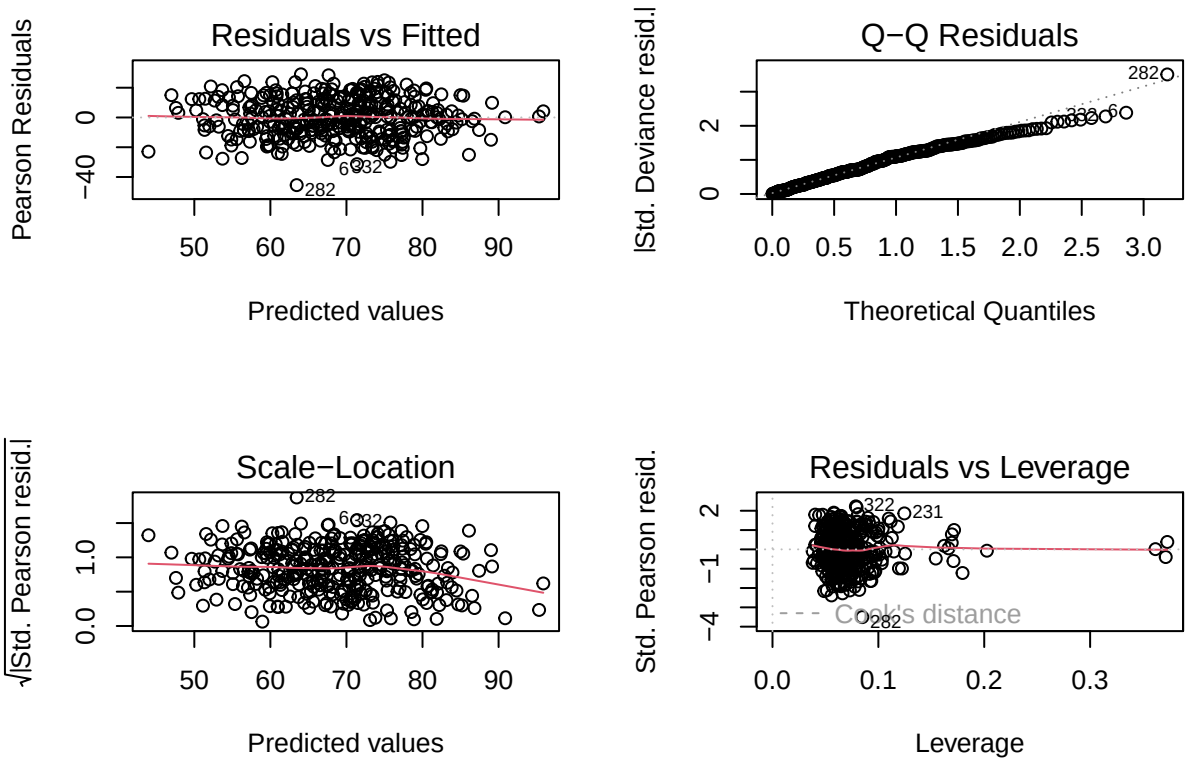
```
# Math
model_math_full = glm(math_score ~ . - reading_score - writing_score, data = data)
model_math_full |> summary()
```

```
##
## Call:
## glm(formula = math_score ~ . - reading_score - writing_score,
##     data = data)
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            62.3523     4.9540  12.586  < 2e-16 ***
```

```
## gender1                 -3.6522     1.4958  -2.442 0.015150 *
## ethnic_group1            1.8120     3.2790   0.553 0.580912
## ethnic_group2           -1.1247     3.1319  -0.359 0.719748
## ethnic_group3            3.0342     3.1826   0.953 0.341109
## ethnic_group4            8.7423     3.3555   2.605 0.009598 **
## parent_educ2             1.8031     1.7975   1.003 0.316545
## parent_educ3             3.1775     2.0927   1.518 0.129886
## parent_educ4             4.0051     2.5782   1.553 0.121282
## lunch_type1            -12.1275     1.5423  -7.863 5.49e-14 ***
## test_prep1               5.7990     1.5706   3.692 0.000260 ***
## parent_marital_status1  -4.2006     1.8079  -2.323 0.020770 *
## parent_marital_status2   7.0930     4.7226   1.502 0.134083
## parent_marital_status3  -4.8362     2.1726  -2.226 0.026694 *
## practice_sport1          3.0566     2.3818   1.283 0.200295
## practice_sport2          3.2296     2.4896   1.297 0.195466
## is_first_child1         -0.3254     1.6378  -0.199 0.842638
## nr_siblings1            -0.1780     2.7665  -0.064 0.948739
## nr_siblings2            -1.1446     2.8721  -0.399 0.690507
## nr_siblings3             3.1546     2.8049   1.125 0.261548
## nr_siblings4             2.8587     3.3920   0.843 0.399963
## nr_siblings5             2.4937     3.9289   0.635 0.526071
## nr_siblings6            14.5158    13.9723   1.039 0.299617
## nr_siblings7             9.5593     8.3433   1.146 0.252735
## transport_means1         1.0585     1.5640   0.677 0.499003
## wkly_study_hours1        6.4822     1.7525   3.699 0.000254 ***
## wkly_study_hours2        4.2523     2.2536   1.887 0.060065 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 183.6931)
##
##     Null deviance: 89074  on 353  degrees of freedom
## Residual deviance: 60068  on 327  degrees of freedom
## AIC: 2878
##
## Number of Fisher Scoring iterations: 2
```

```
par(mfrow = c(2,2))
plot(model_math_full)
```

```
## Warning: not plotting observations with leverage one:
##   186
```



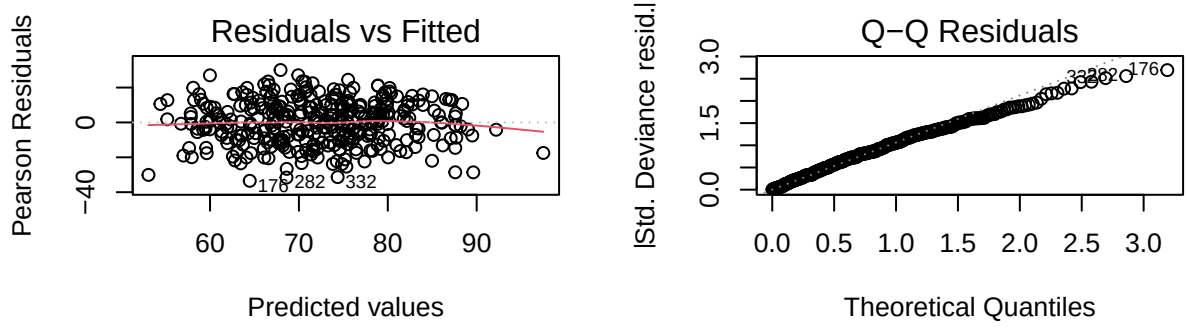

```
# Reading
model_reading_full = glm(reading_score ~ . - math_score - writing_score, data = data)
model_reading_full |> summary()
```

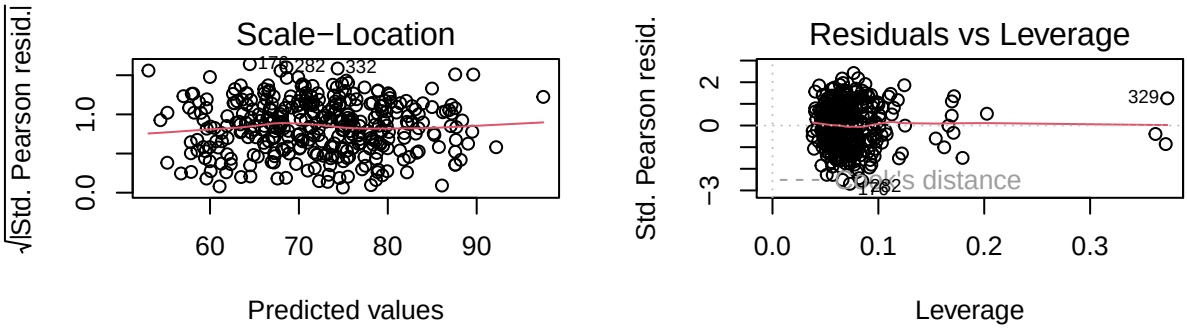
```
##
## Call:
## glm(formula = reading_score ~ . - math_score - writing_score,
##     data = data)
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)               59.3627     4.7169  12.585  < 2e-16 ***
## gender1                    8.2587     1.4242   5.799 1.57e-08 ***
## ethnic_group1              1.4533     3.1220   0.466  0.64188
## ethnic_group2             -0.5044     2.9819  -0.169  0.86578
## ethnic_group3              2.8080     3.0302   0.927  0.35479
## ethnic_group4              4.7359     3.1949   1.482  0.13921
## parent_educ2               2.6502     1.7114   1.549  0.12246
## parent_educ3               4.5816     1.9925   2.299  0.02211 *
## parent_educ4               6.4240     2.4548   2.617  0.00929 **
## lunch_type1               -7.8783     1.4685  -5.365 1.54e-07 ***
## test_prep1                 7.6036     1.4954   5.085 6.21e-07 ***
## parent_marital_status1    -4.6412     1.7214  -2.696  0.00738 **
## parent_marital_status2     4.6364     4.4966   1.031  0.30325
## parent_marital_status3    -4.2660     2.0686  -2.062  0.03997 *
## practice_sport1            1.9156     2.2678   0.845  0.39890
## practice_sport2            1.2989     2.3705   0.548  0.58408
## is_first_child1            0.6384     1.5594   0.409  0.68252
## nr_siblings1               0.4794     2.6341   0.182  0.85569
```

```
## nr_siblings2            -1.4869     2.7347  -0.544  0.58700
## nr_siblings3             1.8958     2.6706   0.710  0.47830
## nr_siblings4             2.3345     3.2296   0.723  0.47028
## nr_siblings5            -1.4797     3.7408  -0.396  0.69269
## nr_siblings6            11.7473    13.3034   0.883  0.37787
## nr_siblings7             7.7275     7.9439   0.973  0.33139
## transport_means1         0.5365     1.4891   0.360  0.71890
## wkly_study_hours1        5.3310     1.6686   3.195  0.00154 **
## wkly_study_hours2        1.1401     2.1458   0.531  0.59557
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 166.5274)
##
##     Null deviance: 77467  on 353  degrees of freedom
## Residual deviance: 54454  on 327  degrees of freedom
## AIC: 2843.3
##
## Number of Fisher Scoring iterations: 2
```

```r
par(mfrow = c(2,2))
plot(model_reading_full)
```

```
## Warning: not plotting observations with leverage one:
##   186
```



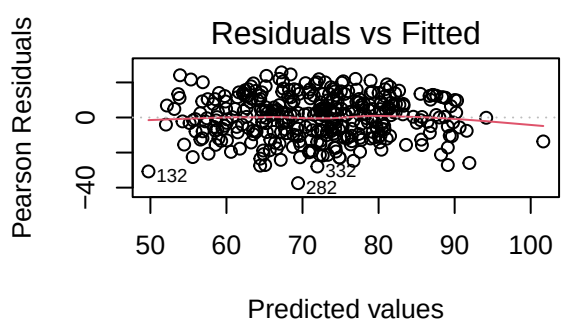


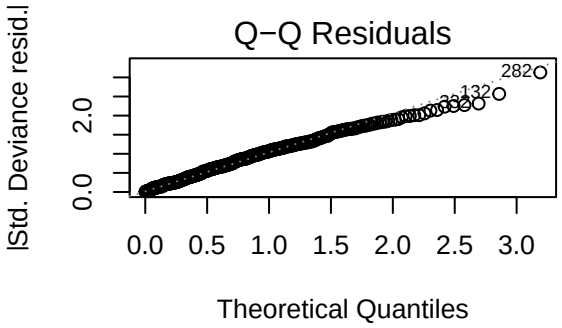

```r
# Writing
model_writing_full = glm(writing_score ~ . - reading_score - math_score, data = data)
model_writing_full |> summary()
```

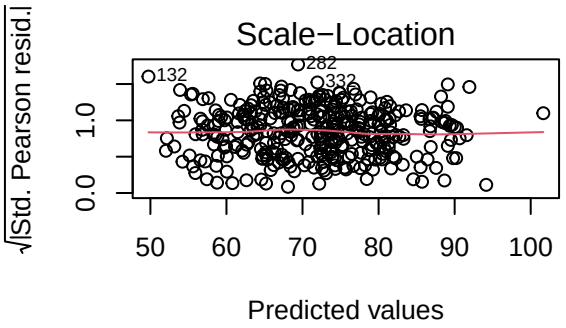
```
##
## Call:
## glm(formula = writing_score ~ . - reading_score - math_score,
##     data = data)
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)              55.1871     4.5675  12.083  < 2e-16 ***
## gender1                  10.0433     1.3791   7.283 2.46e-12 ***
## ethnic_group1             1.7982     3.0232   0.595 0.552382
## ethnic_group2             0.7708     2.8875   0.267 0.789684
## ethnic_group3             5.5577     2.9343   1.894 0.059101 .
## ethnic_group4             5.5666     3.0937   1.799 0.072893 .
## parent_educ2              2.0224     1.6572   1.220 0.223203
## parent_educ3              4.5673     1.9294   2.367 0.018507 *
## parent_educ4              7.5525     2.3771   3.177 0.001629 **
## lunch_type1              -8.9424     1.4220  -6.289 1.03e-09 ***
## test_prep1                9.6428     1.4480   6.659 1.16e-10 ***
## parent_marital_status1   -4.5781     1.6669  -2.747 0.006356 **
## parent_marital_status2    5.2451     4.3542   1.205 0.229221
## parent_marital_status3   -4.4305     2.0031  -2.212 0.027669 *
## practice_sport1           3.3011     2.1960   1.503 0.133746
## practice_sport2           3.0186     2.2954   1.315 0.189415
## is_first_child1          -0.2525     1.5100  -0.167 0.867295
## nr_siblings1              0.3186     2.5507   0.125 0.900665
## nr_siblings2             -1.2993     2.6481  -0.491 0.624008
## nr_siblings3              2.2515     2.5860   0.871 0.384594
## nr_siblings4              2.9536     3.1273   0.944 0.345630
## nr_siblings5             -0.5419     3.6224  -0.150 0.881167
## nr_siblings6             14.3830    12.8821   1.117 0.265024
## nr_siblings7              8.0232     7.6923   1.043 0.297708
## transport_means1          0.9938     1.4420   0.689 0.491208
## wkly_study_hours1         5.4344     1.6157   3.363 0.000861 ***
## wkly_study_hours2         2.0335     2.0778   0.979 0.328454
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 156.147)
##
##     Null deviance: 81858  on 353  degrees of freedom
## Residual deviance: 51060  on 327  degrees of freedom
## AIC: 2820.5
##
## Number of Fisher Scoring iterations: 2
```

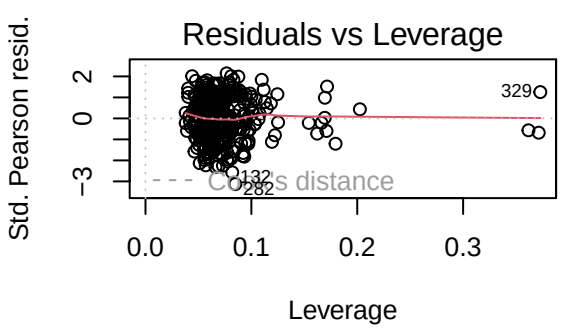
```
par(mfrow = c(2,2))
plot(model_writing_full)
```

```
## Warning: not plotting observations with leverage one:
##   186
```



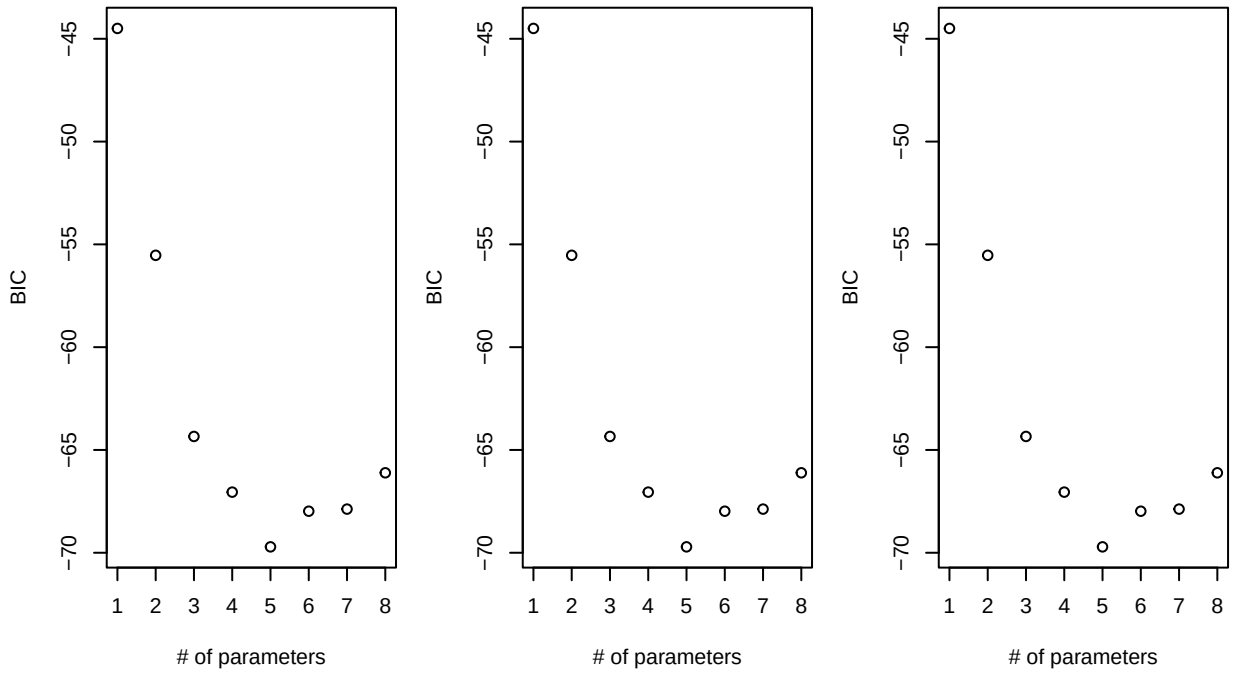






criterion-based Procedures

```
math_c = regsubsets(math_score ~ . - reading_score - writing_score, data = data)
res_math =
  math_c |>
  summary()

reading_c = regsubsets(reading_score ~ . - math_score - writing_score, data = data)
res_reading =
  math_c |>
  summary()

writing_c = regsubsets(writing_score ~ . - math_score - reading_score, data = data)
res_writing =
  math_c |>
  summary()

par(mfrow = c(1, 3), mar = c(8, 4, 4, 1))
plot(1:8, res_math$bic, xlab = "# of parameters", ylab = "BIC")
plot(1:8, res_reading$bic, xlab = "# of parameters", ylab = "BIC")
plot(1:8, res_writing$bic, xlab = "# of parameters", ylab = "BIC")
```




```
res_math$outmat[5,]
```

```
##                gender1          ethnic_group1          ethnic_group2
##                    "*"                    " "                    " "
##          ethnic_group3          ethnic_group4           parent_educ2
##                    " "                    "*"                    " "
##           parent_educ3           parent_educ4            lunch_type1
##                    " "                    " "                    "*"
##             test_prep1 parent_marital_status1 parent_marital_status2
##                    "*"                    " "                    " "
## parent_marital_status3        practice_sport1        practice_sport2
##                    " "                    " "                    " "
##        is_first_child1           nr_siblings1           nr_siblings2
##                    " "                    " "                    " "
##           nr_siblings3           nr_siblings4           nr_siblings5
##                    " "                    " "                    " "
##           nr_siblings6           nr_siblings7       transport_means1
##                    " "                    " "                    " "
##      wkly_study_hours1      wkly_study_hours2
##                    "*"                    " "
```

```
res_reading$outmat[5,]
```

```
##                gender1          ethnic_group1          ethnic_group2
##                    "*"                    " "                    " "
##          ethnic_group3          ethnic_group4           parent_educ2
##                    " "                    "*"                    " "
##           parent_educ3           parent_educ4            lunch_type1
##                    " "                    " "                    "*"
##             test_prep1 parent_marital_status1 parent_marital_status2
##                    "*"                    " "                    " "
```

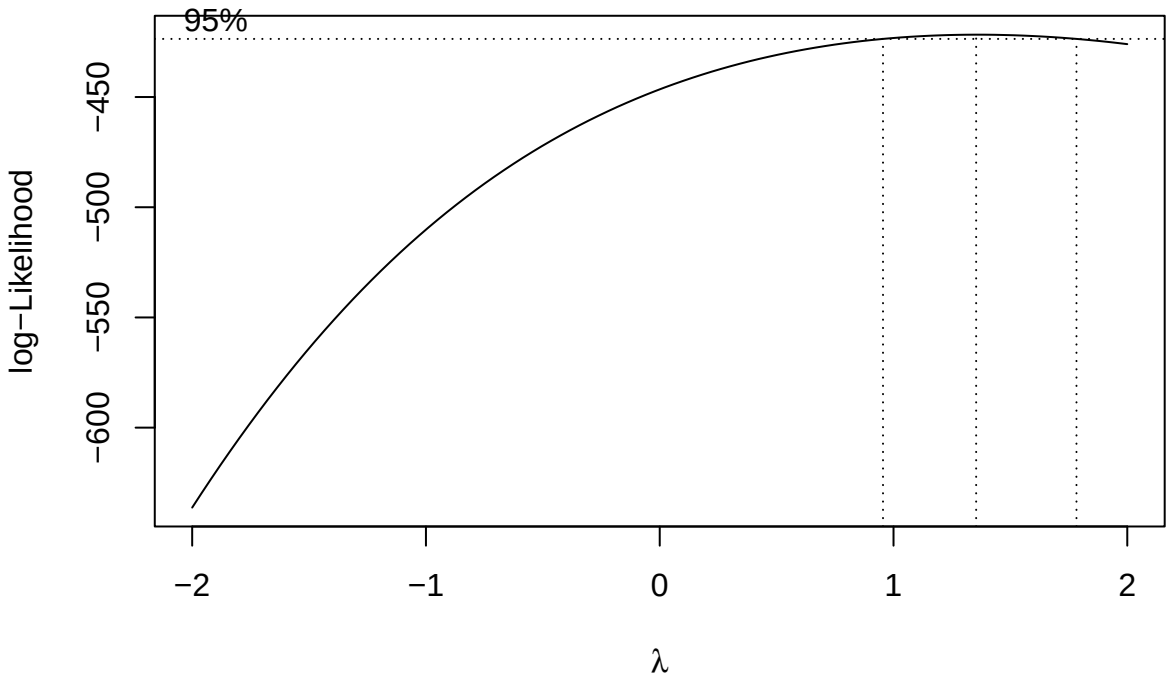
```
## parent_marital_status3        practice_sport1        practice_sport2
##                     " "                   " "                   " "
##        is_first_child1           nr_siblings1           nr_siblings2
##                     " "                   " "                   " "
##           nr_siblings3           nr_siblings4           nr_siblings5
##                     " "                   " "                   " "
##           nr_siblings6           nr_siblings7       transport_means1
##                     " "                   " "                   " "
##      wkly_study_hours1      wkly_study_hours2
##                    "*"                   " "
```

```
res_writing$outmat[5,]
```

```
##                gender1          ethnic_group1          ethnic_group2
##                    "*"                   " "                   " "
##          ethnic_group3          ethnic_group4           parent_educ2
##                     " "                   "*"                   " "
##           parent_educ3           parent_educ4            lunch_type1
##                     " "                   " "                   "*"
##             test_prep1 parent_marital_status1 parent_marital_status2
##                    "*"                   " "                   " "
## parent_marital_status3        practice_sport1        practice_sport2
##                     " "                   " "                   " "
##        is_first_child1           nr_siblings1           nr_siblings2
##                     " "                   " "                   " "
##           nr_siblings3           nr_siblings4           nr_siblings5
##                     " "                   " "                   " "
##           nr_siblings6           nr_siblings7       transport_means1
##                     " "                   " "                   " "
##      wkly_study_hours1      wkly_study_hours2
##                    "*"                   " "
```

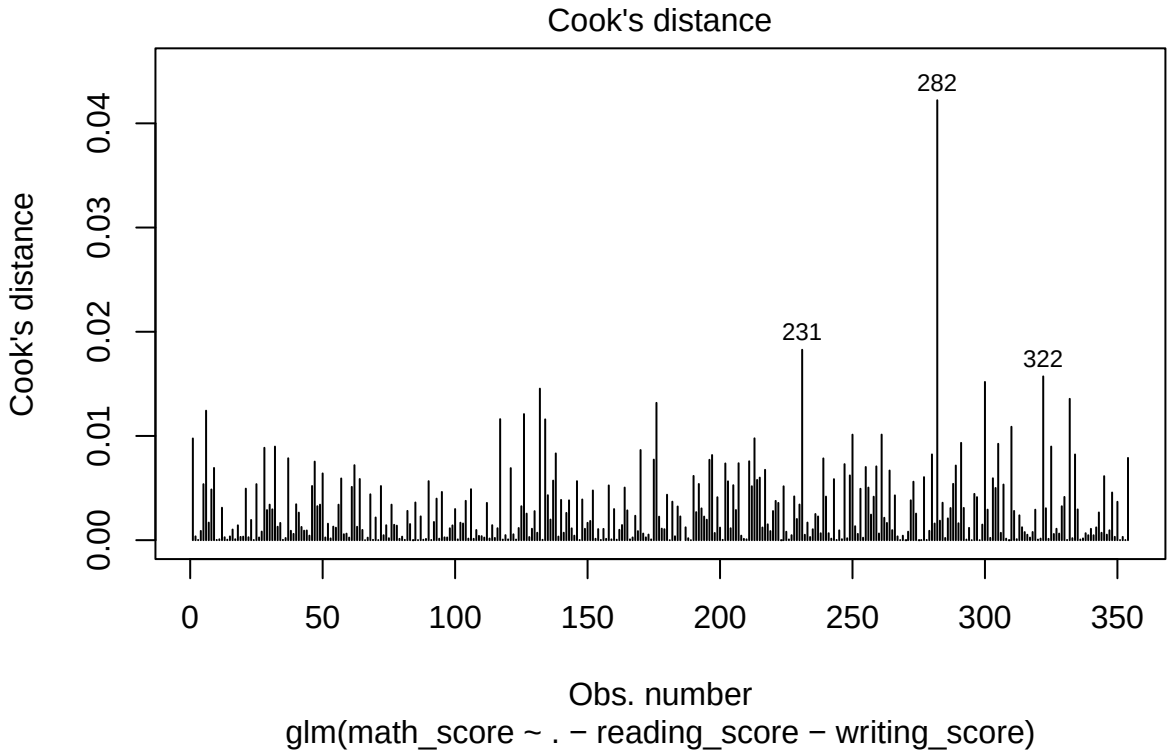
Transformation

```
boxcox(model_reading_full)
```

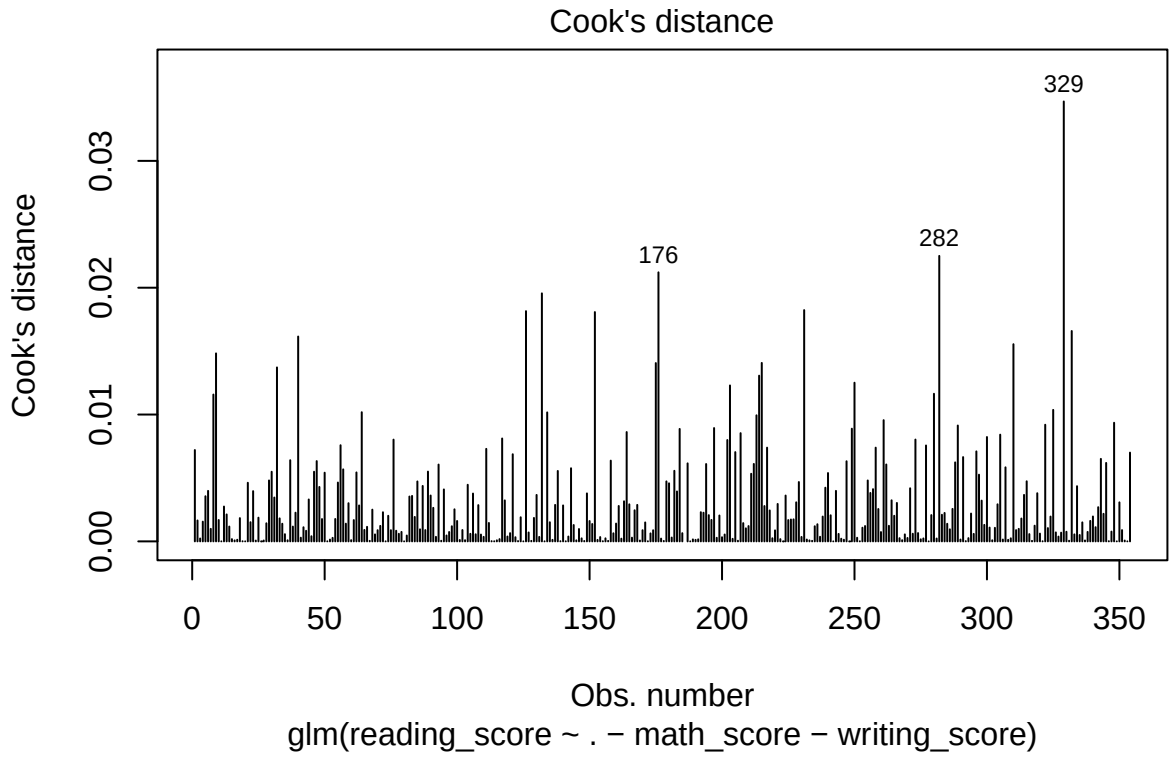


Outlier and influence points

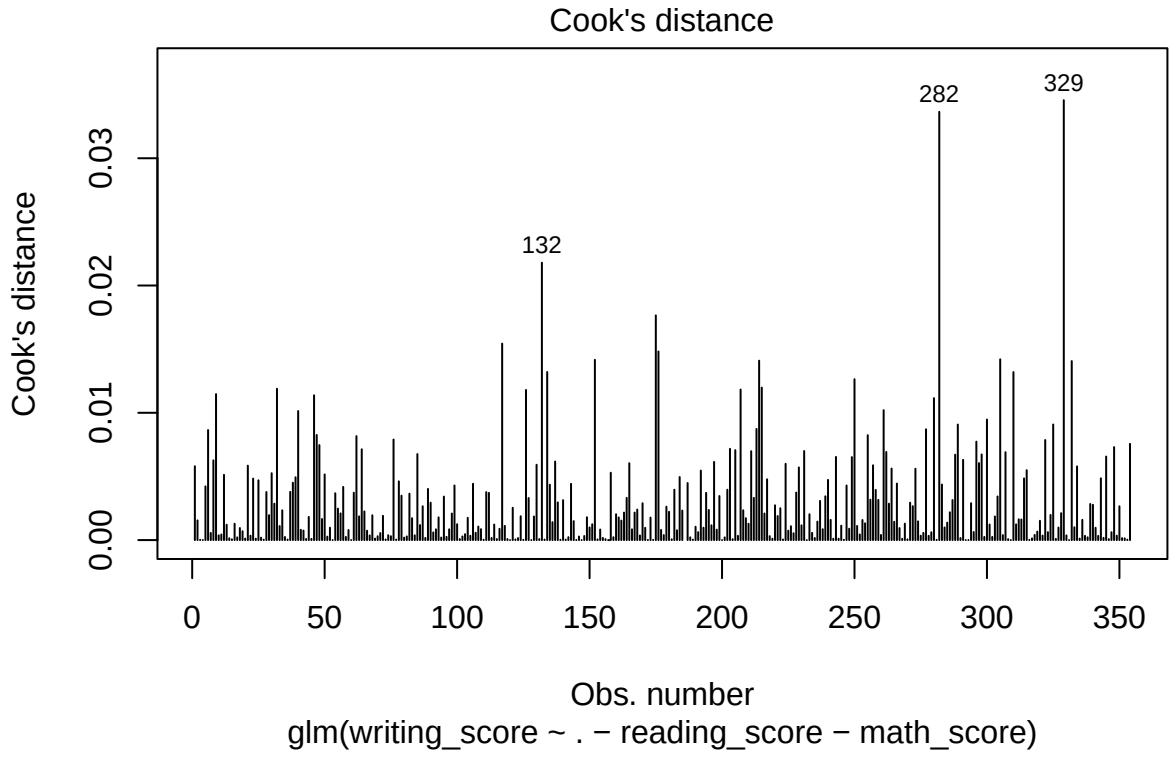
```
plot(model_math_full, which = 4)
```



```r
plot(model_reading_full, which = 4)
```




```r
plot(model_writing_full, which = 4)
```



Multicollinearity

```
# check VIF
vif_math =
  performance::check_collinearity(model_math_full) |>
  as_tibble() |>
  mutate(VIF_CI = str_c("[", round(VIF_CI_low, 1), ", ", round(VIF_CI_high, 1), "]")) |>
  dplyr::select(Term, VIF, VIF_CI, Tolerance)
knitr::kable(x = vif_math, caption = "VIF for Math Score", digits = 1)
```

Table 3: VIF for Math Score

Term	VIF	VIF_CI	Tolerance
gender	1.1	[1, 1.4]	0.9
ethnic_group	1.2	[1.1, 1.4]	0.8
parent_educ	1.2	[1.1, 1.4]	0.8
lunch_type	1.1	[1, 1.4]	1.0
test_prep	1.1	[1, 1.3]	0.9
parent_marital_status	1.2	[1.1, 1.4]	0.9
practice_sport	1.2	[1.1, 1.4]	0.9
is_first_child	1.2	[1.1, 1.3]	0.9
nr_siblings	1.5	[1.4, 1.8]	0.6
transport_means	1.1	[1, 1.3]	0.9
wkly_study_hours	1.1	[1.1, 1.3]	0.9

```
vif_reading =
  performance::check_collinearity(model_reading_full) |>
  as_tibble() |>
  mutate(VIF_CI = str_c("[", round(VIF_CI_low, 1), ", ", round(VIF_CI_high, 1), "]")) |>
  dplyr::select(Term, VIF, VIF_CI, Tolerance)
knitr::kable(x = vif_reading, caption = "VIF for Reading Score", digits = 1)
```

Table 4: VIF for Reading Score

Term	VIF	VIF_CI	Tolerance
gender	1.1	[1, 1.4]	0.9
ethnic_group	1.2	[1.1, 1.4]	0.8
parent_educ	1.2	[1.1, 1.4]	0.8
lunch_type	1.1	[1, 1.4]	1.0
test_prep	1.1	[1, 1.3]	0.9
parent_marital_status	1.2	[1.1, 1.4]	0.9
practice_sport	1.2	[1.1, 1.4]	0.9
is_first_child	1.2	[1.1, 1.3]	0.9
nr_siblings	1.5	[1.4, 1.8]	0.6
transport_means	1.1	[1, 1.3]	0.9
wkly_study_hours	1.1	[1.1, 1.3]	0.9

```
vif_writing =
  performance::check_collinearity(model_writing_full) |>
  as_tibble() |>
  mutate(VIF_CI = str_c("[", round(VIF_CI_low, 1), ", ", round(VIF_CI_high, 1), "]")) |>
  dplyr::select(Term, VIF, VIF_CI, Tolerance)
knitr::kable(x = vif_writing, caption = "VIF for Reading Score", digits = 1)
```

Table 5: VIF for Reading Score

Term	VIF	VIF_CI	Tolerance
gender	1.1	[1, 1.4]	0.9
ethnic_group	1.2	[1.1, 1.4]	0.8
parent_educ	1.2	[1.1, 1.4]	0.8
lunch_type	1.1	[1, 1.4]	1.0
test_prep	1.1	[1, 1.3]	0.9
parent_marital_status	1.2	[1.1, 1.4]	0.9
practice_sport	1.2	[1.1, 1.4]	0.9
is_first_child	1.2	[1.1, 1.3]	0.9
nr_siblings	1.5	[1.4, 1.8]	0.6
transport_means	1.1	[1, 1.3]	0.9
wkly_study_hours	1.1	[1.1, 1.3]	0.9

Model building for math

```
# backward model
step(model_math_full, direction='backward')
```

```
## Start:  AIC=2878.02
## math_score ~ (gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     nr_siblings + transport_means + wkly_study_hours + reading_score +
##     writing_score) - reading_score - writing_score
##
##                         Df Deviance    AIC
## - nr_siblings            7    61456 2872.1
## - parent_educ            3    60735 2875.9
## - practice_sport         2    60412 2876.0
## - is_first_child         1    60075 2876.1
## - transport_means        1    60152 2876.5
## <none>                        60068 2878.0
## - gender                 1    61163 2882.4
## - parent_marital_status  3    62260 2884.7
## - wkly_study_hours       2    62582 2888.5
## - test_prep              1    62572 2890.5
## - ethnic_group           4    63860 2891.7
## - lunch_type             1    71425 2937.3
##
## Step:  AIC=2872.11
## math_score ~ gender + ethnic_group + parent_educ + lunch_type +
```

```
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     transport_means + wkly_study_hours
##
##                         Df Deviance    AIC
## - parent_educ            3    62111 2869.9
## - is_first_child         1    61457 2870.1
## - practice_sport         2    61829 2870.2
## - transport_means        1    61514 2870.4
## <none>                        61456 2872.1
## - gender                 1    62644 2876.9
## - parent_marital_status  3    63819 2879.5
## - wkly_study_hours       2    63807 2881.4
## - test_prep              1    64028 2884.6
## - ethnic_group           4    65559 2887.0
## - lunch_type             1    73858 2935.2
##
## Step:  AIC=2869.86
## math_score ~ gender + ethnic_group + lunch_type + test_prep +
##     parent_marital_status + practice_sport + is_first_child +
##     transport_means + wkly_study_hours
##
##                         Df Deviance    AIC
## - practice_sport         2    62417 2867.6
## - is_first_child         1    62113 2867.9
## - transport_means        1    62142 2868.0
## <none>                        62111 2869.9
## - gender                 1    63275 2874.4
## - parent_marital_status  3    64477 2877.1
## - wkly_study_hours       2    64331 2878.3
## - test_prep              1    64934 2883.6
## - ethnic_group           4    66259 2884.8
## - lunch_type             1    74436 2931.9
##
## Step:  AIC=2867.6
## math_score ~ gender + ethnic_group + lunch_type + test_prep +
##     parent_marital_status + is_first_child + transport_means +
##     wkly_study_hours
##
##                         Df Deviance    AIC
## - is_first_child         1    62425 2865.7
## - transport_means        1    62444 2865.8
## <none>                        62417 2867.6
## - gender                 1    63581 2872.1
## - parent_marital_status  3    64755 2874.6
## - wkly_study_hours       2    64625 2875.9
## - test_prep              1    65248 2881.3
## - ethnic_group           4    66529 2882.2
## - lunch_type             1    74657 2929.0
##
## Step:  AIC=2865.65
## math_score ~ gender + ethnic_group + lunch_type + test_prep +
##     parent_marital_status + transport_means + wkly_study_hours
##
##                         Df Deviance    AIC
```

```
## - transport_means        1    62453 2863.8
## <none>                        62425 2865.7
## - gender                 1    63583 2870.2
## - parent_marital_status  3    64773 2872.7
## - wkly_study_hours       2    64627 2873.9
## - test_prep              1    65251 2879.3
## - ethnic_group           4    66531 2880.2
## - lunch_type             1    74659 2927.0
##
## Step:  AIC=2863.8
## math_score ~ gender + ethnic_group + lunch_type + test_prep +
##     parent_marital_status + wkly_study_hours
##
##                         Df Deviance    AIC
## <none>                       62453 2863.8
## - gender                 1    63614 2868.3
## - parent_marital_status  3    64774 2870.7
## - wkly_study_hours       2    64646 2872.0
## - test_prep              1    65373 2878.0
## - ethnic_group           4    66550 2878.3
## - lunch_type             1    74664 2925.0

##
## Call:  glm(formula = math_score ~ gender + ethnic_group + lunch_type +
##     test_prep + parent_marital_status + wkly_study_hours, data = data)
##
## Coefficients:
##            (Intercept)                 gender1           ethnic_group1
##                67.3260                 -3.7049                  2.4461
##          ethnic_group2           ethnic_group3           ethnic_group4
##                 0.3026                  4.1687                 10.1791
##           lunch_type1              test_prep1  parent_marital_status1
##              -12.3773                  6.0788                 -4.0821
## parent_marital_status2  parent_marital_status3       wkly_study_hours1
##                 6.7982                 -5.2507                  5.9171
##      wkly_study_hours2
##                 3.8301
##
## Degrees of Freedom: 353 Total (i.e. Null);  341 Residual
## Null Deviance:       89070
## Residual Deviance: 62450     AIC: 2864
```

```r
model_math_fit_back = lm(formula = math_score ~ gender + ethnic_group + parent_educ +
    lunch_type + test_prep + parent_marital_status + practice_sport +
    is_first_child + wkly_study_hours, data = data)

summary(model_math_fit_back)
```

```
##
## Call:
## lm(formula = math_score ~ gender + ethnic_group + parent_educ +
##     lunch_type + test_prep + parent_marital_status + practice_sport +
##     is_first_child + wkly_study_hours, data = data)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -42.641  -9.388   0.444  10.841  29.060
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)              63.3058     4.0723  15.545  < 2e-16 ***
## gender1                  -3.7768     1.4786  -2.554 0.011080 *
## ethnic_group1             2.0233     3.2739   0.618 0.536983
## ethnic_group2            -0.1921     3.1097  -0.062 0.950767
## ethnic_group3             3.5985     3.1572   1.140 0.255191
## ethnic_group4             9.8452     3.3254   2.961 0.003289 **
## parent_educ2              1.6680     1.7628   0.946 0.344724
## parent_educ3              3.1571     2.0672   1.527 0.127641
## parent_educ4              3.7243     2.5498   1.461 0.145058
## lunch_type1             -12.4609     1.5198  -8.199 5.22e-15 ***
## test_prep1                5.9501     1.5447   3.852 0.000140 ***
## parent_marital_status1   -4.1882     1.7844  -2.347 0.019505 *
## parent_marital_status2    7.3458     4.7089   1.560 0.119707
## parent_marital_status3   -4.9516     2.1536  -2.299 0.022104 *
## practice_sport1           3.1345     2.3452   1.337 0.182276
## practice_sport2           3.2766     2.4641   1.330 0.184500
## is_first_child1          -0.1431     1.5713  -0.091 0.927481
## wkly_study_hours1         6.1263     1.7189   3.564 0.000418 ***
## wkly_study_hours2         4.2272     2.2378   1.889 0.059752 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13.55 on 335 degrees of freedom
## Multiple R-squared:  0.3094, Adjusted R-squared:  0.2723
## F-statistic: 8.338 on 18 and 335 DF,  p-value: < 2.2e-16
```

```
# lasso model
lambda_seq = 10 ^ seq(-3, 0, by = .1)

cv_object_math = cv.glmnet(as.matrix(data[1:11]), data$math_score,
                           lambda = lambda_seq,
                           nfolds = 5)

model_math_lasso = glmnet(as.matrix(data[1:11]), data$math_score, lambda = cv_object_math$lambda.min, al
coef(model_math_lasso)
```

```
## 12 x 1 sparse Matrix of class "dgCMatrix"
##                                  s0
## (Intercept)            62.7158706
## gender                 -3.4172517
## ethnic_group            2.0740949
## parent_educ             0.9804808
## lunch_type            -11.7678104
## test_prep               5.0255504
## parent_marital_status  -1.0446103
## practice_sport          0.4391390
## is_first_child          .
```

```
## nr_siblings             0.7146589
## transport_means         .
## wkly_study_hours        2.4395500
```

```
model_math_lasso$dev.ratio
```

```
## [1] 0.2622201
```

Model building for reading

```
# backward model
step(model_reading_full, direction='backward')
```

```
## Start:  AIC=2843.29
## reading_score ~ (gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     nr_siblings + transport_means + wkly_study_hours + math_score +
##     writing_score) - math_score - writing_score
##
##                         Df Deviance    AIC
## - nr_siblings            7    55342 2835.0
## - practice_sport         2    54578 2840.1
## - transport_means        1    54476 2841.4
## - is_first_child         1    54482 2841.5
## - ethnic_group           4    55682 2843.2
## <none>                        54454 2843.3
## - parent_educ            3    56013 2847.3
## - parent_marital_status  3    56363 2849.5
## - wkly_study_hours       2    56459 2852.1
## - test_prep              1    58760 2868.2
## - lunch_type             1    59248 2871.2
## - gender                 1    60054 2875.9
##
## Step:  AIC=2835.02
## reading_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     transport_means + wkly_study_hours
##
##                         Df Deviance    AIC
## - practice_sport         2    55488 2831.9
## - transport_means        1    55354 2833.1
## - is_first_child         1    55382 2833.3
## <none>                        55342 2835.0
## - ethnic_group           4    56661 2835.3
## - parent_educ            3    57024 2839.6
## - parent_marital_status  3    57267 2841.1
## - wkly_study_hours       2    57312 2843.4
## - test_prep              1    59565 2859.0
## - lunch_type             1    60780 2866.2
## - gender                 1    61036 2867.7
```

```
##
## Step:  AIC=2831.94
## reading_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + is_first_child + transport_means +
##     wkly_study_hours
##
##                         Df Deviance    AIC
## - transport_means        1    55493 2830.0
## - is_first_child         1    55529 2830.2
## <none>                        55488 2831.9
## - ethnic_group           4    56782 2832.1
## - parent_educ            3    57143 2836.3
## - parent_marital_status  3    57391 2837.9
## - wkly_study_hours       2    57447 2840.2
## - test_prep              1    59804 2856.5
## - lunch_type             1    60909 2862.9
## - gender                 1    61166 2864.4
##
## Step:  AIC=2829.98
## reading_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + is_first_child + wkly_study_hours
##
##                         Df Deviance    AIC
## - is_first_child         1    55533 2828.2
## <none>                        55493 2830.0
## - ethnic_group           4    56789 2830.2
## - parent_educ            3    57143 2834.3
## - parent_marital_status  3    57393 2835.9
## - wkly_study_hours       2    57452 2838.3
## - test_prep              1    59916 2855.1
## - lunch_type             1    60916 2861.0
## - gender                 1    61168 2862.4
##
## Step:  AIC=2828.23
## reading_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + wkly_study_hours
##
##                         Df Deviance    AIC
## <none>                        55533 2828.2
## - ethnic_group           4    56839 2828.5
## - parent_educ            3    57188 2832.6
## - parent_marital_status  3    57432 2834.1
## - wkly_study_hours       2    57508 2836.6
## - test_prep              1    60064 2854.0
## - lunch_type             1    60973 2859.3
## - gender                 1    61177 2860.5


##
## Call:  glm(formula = reading_score ~ gender + ethnic_group + parent_educ +
##     lunch_type + test_prep + parent_marital_status + wkly_study_hours,
##     data = data)
##
## Coefficients:
##            (Intercept)                 gender1           ethnic_group1
```

```
##                61.6474                   8.1816                   1.8945
##          ethnic_group2            ethnic_group3            ethnic_group4
##                 0.3778                   3.3789                   5.6870
##           parent_educ2             parent_educ3             parent_educ4
##                 2.3964                   4.6728                   6.4917
##           lunch_type1               test_prep1  parent_marital_status1
##                -8.2631                   7.6175                  -4.5976
## parent_marital_status2  parent_marital_status3       wkly_study_hours1
##                 4.1841                  -4.3042                   5.1565
##      wkly_study_hours2
##                 1.0458
##
## Degrees of Freedom: 353 Total (i.e. Null);  338 Residual
## Null Deviance:       77470
## Residual Deviance: 55530     AIC: 2828
```

```
model_reading_back = lm(formula = reading_score ~ gender + ethnic_group + parent_educ +
    lunch_type + test_prep + parent_marital_status + is_first_child +
    transport_means + wkly_study_hours, data = data)
summary(model_reading_back)
```

```
##
## Call:
## lm(formula = reading_score ~ gender + ethnic_group + parent_educ +
##     lunch_type + test_prep + parent_marital_status + is_first_child +
##     transport_means + wkly_study_hours, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -32.522  -9.335   0.253   9.491  29.948
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)               61.0959     3.4189  17.870  < 2e-16 ***
## gender1                    8.2151     1.4010   5.864 1.08e-08 ***
## ethnic_group1              1.8440     3.0962   0.596  0.55187
## ethnic_group2              0.3221     2.9318   0.110  0.91257
## ethnic_group3              3.3272     2.9801   1.116  0.26502
## ethnic_group4              5.6186     3.1503   1.784  0.07540 .
## parent_educ2               2.4730     1.6822   1.470  0.14248
## parent_educ3               4.7430     1.9674   2.411  0.01645 *
## parent_educ4               6.4579     2.4012   2.689  0.00751 **
## lunch_type1               -8.2690     1.4432  -5.730 2.24e-08 ***
## test_prep1                 7.5208     1.4711   5.112 5.35e-07 ***
## parent_marital_status1    -4.5595     1.6944  -2.691  0.00748 **
## parent_marital_status2     4.3781     4.4330   0.988  0.32405
## parent_marital_status3    -4.3645     2.0421  -2.137  0.03330 *
## is_first_child1            0.7327     1.4725   0.498  0.61910
## transport_means1           0.2718     1.4551   0.187  0.85195
## wkly_study_hours1          5.1383     1.6296   3.153  0.00176 **
## wkly_study_hours2          1.0442     2.1217   0.492  0.62294
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 12.85 on 336 degrees of freedom
## Multiple R-squared:  0.2837, Adjusted R-squared:  0.2475
## F-statistic: 7.829 on 17 and 336 DF,  p-value: < 2.2e-16
```

```
# lasso model
lambda_seq = 10 ^ seq(-3, 0, by = .1)

cv_object_reading = cv.glmnet(as.matrix(data[1:11]), data$reading_score,
                          lambda = lambda_seq,
                          nfolds = 5)
cv_object_reading$lambda.min
```

```
## [1] 0.5011872
```

```
model_reading_lasso = glmnet(as.matrix(data[1:11]), data$reading_score, lambda = cv_object_reading$lambd
coef(model_reading_lasso)
```

```
## 12 x 1 sparse Matrix of class "dgCMatrix"
##                               s0
## (Intercept)           63.0047330
## gender                 6.8714456
## ethnic_group           1.0191726
## parent_educ            1.6822432
## lunch_type            -7.2445118
## test_prep              6.2890596
## parent_marital_status -0.7735146
## practice_sport         .
## is_first_child         .
## nr_siblings            .
## transport_means        .
## wkly_study_hours       0.4772919
```

```
model_reading_lasso$dev.ratio
```

```
## [1] 0.2302132
```

Model building for writing

```
# backward model
step(model_writing_full, direction = "backward", )
```

```
## Start:  AIC=2820.51
## writing_score ~ (gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     nr_siblings + transport_means + wkly_study_hours + math_score +
##     reading_score) - reading_score - math_score
##
##                        Df Deviance    AIC
## - nr_siblings           7    52079 2813.5
```

```
## - is_first_child         1    51064 2818.5
## - practice_sport         2    51421 2819.0
## - transport_means        1    51134 2819.0
## <none>                         51060 2820.5
## - ethnic_group           4    52839 2824.6
## - parent_educ            3    53000 2827.7
## - parent_marital_status  3    53052 2828.1
## - wkly_study_hours       2    52961 2829.4
## - lunch_type             1    57235 2858.9
## - test_prep              1    57985 2863.5
## - gender                 1    59341 2871.7
##
## Step:  AIC=2813.5
## writing_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     transport_means + wkly_study_hours
##
##                          Df Deviance    AIC
## - is_first_child          1    52080 2811.5
## - transport_means         1    52132 2811.9
## - practice_sport          2    52484 2812.2
## <none>                         52079 2813.5
## - ethnic_group            4    53949 2818.0
## - parent_marital_status   3    54107 2821.0
## - parent_educ             3    54148 2821.3
## - wkly_study_hours        2    53910 2821.7
## - test_prep               1    58959 2855.4
## - lunch_type              1    59035 2855.9
## - gender                  1    60523 2864.7
##
## Step:  AIC=2811.51
## writing_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + transport_means +
##     wkly_study_hours
##
##                          Df Deviance    AIC
## - transport_means         1    52133 2809.9
## - practice_sport          2    52489 2810.3
## <none>                         52080 2811.5
## - ethnic_group            4    53950 2816.0
## - parent_marital_status   3    54109 2819.0
## - parent_educ             3    54149 2819.3
## - wkly_study_hours        2    53910 2819.7
## - test_prep               1    58988 2853.6
## - lunch_type              1    59035 2853.9
## - gender                  1    60544 2862.8
##
## Step:  AIC=2809.87
## writing_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + wkly_study_hours
##
##                          Df Deviance    AIC
## - practice_sport          2    52531 2808.6
## <none>                         52133 2809.9
```

```
## - ethnic_group           4    54035 2814.6
## - parent_marital_status  3    54120 2817.1
## - parent_educ            3    54175 2817.5
## - wkly_study_hours       2    53954 2818.0
## - lunch_type             1    59038 2851.9
## - test_prep              1    59324 2853.6
## - gender                 1    60577 2861.0
##
## Step:  AIC=2808.56
## writing_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + wkly_study_hours
##
##                          Df Deviance    AIC
## <none>                         52531 2808.6
## - ethnic_group           4    54482 2813.5
## - parent_educ            3    54457 2815.3
## - parent_marital_status  3    54494 2815.5
## - wkly_study_hours       2    54335 2816.5
## - lunch_type             1    59368 2849.9
## - test_prep              1    59741 2852.1
## - gender                 1    61017 2859.6
```

```
##
## Call:  glm(formula = writing_score ~ gender + ethnic_group + parent_educ +
##     lunch_type + test_prep + parent_marital_status + wkly_study_hours,
##     data = data)
##
## Coefficients:
##            (Intercept)                 gender1           ethnic_group1
##                 58.522                  10.032                  2.213
##          ethnic_group2           ethnic_group3           ethnic_group4
##                  1.850                   6.338                  6.617
##           parent_educ2            parent_educ3            parent_educ4
##                  1.789                   4.598                  7.212
##           lunch_type1              test_prep1  parent_marital_status1
##                 -9.263                   9.609                 -4.417
## parent_marital_status2  parent_marital_status3       wkly_study_hours1
##                  4.668                  -4.644                  5.168
##      wkly_study_hours2
##                  1.893
##
## Degrees of Freedom: 353 Total (i.e. Null);  338 Residual
## Null Deviance:       81860
## Residual Deviance: 52530     AIC: 2809
```

```
model_writing_back = lm(formula = writing_score ~ gender + ethnic_group + parent_educ +
    lunch_type + test_prep + parent_marital_status + practice_sport +
    is_first_child + transport_means + wkly_study_hours, data = data)
summary(model_writing_back)
```

```
##
## Call:
## lm(formula = writing_score ~ gender + ethnic_group + parent_educ +
```

```
##     lunch_type + test_prep + parent_marital_status + practice_sport +
##     is_first_child + transport_means + wkly_study_hours, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -35.016  -8.347   0.861   9.431  25.920
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)               55.6226     3.7915  14.670  < 2e-16 ***
## gender1                   10.0283     1.3627   7.359 1.45e-12 ***
## ethnic_group1              1.9857     3.0171   0.658  0.51089
## ethnic_group2              1.3766     2.8687   0.480  0.63164
## ethnic_group3              5.7836     2.9166   1.983  0.04819 *
## ethnic_group4              6.4017     3.0645   2.089  0.03747 *
## parent_educ2               1.8930     1.6347   1.158  0.24769
## parent_educ3               4.7742     1.9128   2.496  0.01305 *
## parent_educ4               7.5674     2.3506   3.219  0.00141 **
## lunch_type1               -9.3729     1.4034  -6.679 1.01e-10 ***
## test_prep1                 9.5404     1.4363   6.642 1.25e-10 ***
## parent_marital_status1    -4.5162     1.6470  -2.742  0.00643 **
## parent_marital_status2     5.4329     4.3399   1.252  0.21150
## parent_marital_status3    -4.4594     1.9914  -2.239  0.02579 *
## practice_sport1            3.4669     2.1647   1.602  0.11020
## practice_sport2            3.0695     2.2715   1.351  0.17751
## is_first_child1           -0.1246     1.4489  -0.086  0.93152
## transport_means1           0.8261     1.4256   0.580  0.56263
## wkly_study_hours1          5.2430     1.5846   3.309  0.00104 **
## wkly_study_hours2          2.0645     2.0654   1.000  0.31826
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.49 on 334 degrees of freedom
## Multiple R-squared:  0.3638, Adjusted R-squared:  0.3276
## F-statistic: 10.05 on 19 and 334 DF,  p-value: < 2.2e-16
```

```
# lasso model
lambda_seq = 10 ^ seq(-3, 0, by = .1)

cv_object_writing = cv.glmnet(as.matrix(data[1:11]), data$writing_score,
                        lambda = lambda_seq,
                        nfolds = 5)
cv_object_writing$lambda.min
```

```
## [1] 0.5011872
```

```
model_writing_lasso = glmnet(as.matrix(data[1:11]), data$writing_score, lambda = cv_object_writing$lambd
coef(model_writing_lasso)
```

```
## 12 x 1 sparse Matrix of class "dgCMatrix"
##                                s0
## (Intercept)          59.3844759
## gender                8.7384396
```

```
## ethnic_group           1.4955961
## parent_educ            1.8826016
## lunch_type            -8.1037819
## test_prep              8.0886240
## parent_marital_status -0.8123378
## practice_sport         .
## is_first_child         .
## nr_siblings            0.1133873
## transport_means        .
## wkly_study_hours       0.7539334
```

```
model_writing_lasso$dev.ratio
```

```
## [1] 0.3119987
```