Code

Read and Clean Data

```
data =
  read_csv("./data.csv") |>
  janitor::clean_names() |>
  mutate(
    gender = factor(case_when(
      gender == "male" ~ 0,
      gender == "female" ~ 1,
      )),
    ethnic_group = factor(case_when(
      ethnic_group == "group A" ~ 0,
      ethnic_group == "group B" ~ 1,
      ethnic_group == "group C" ~ 2,
      ethnic_group == "group D" ~ 3,
      ethnic_group == "group E" ~ 4,
      )),
    parent_educ = factor(case_when(
      parent_educ == "some highschool" ~ 0,
      parent_educ == "some college" ~ 1,
      parent_educ == "associate's degree" ~ 2,
      parent_educ == "bachelor's degree" ~ 3,
      parent_educ == "master's degree" ~ 4,
      )),
    lunch_type = factor(case_when(
      lunch_type == "standard" ~ 0,
      lunch_type == "free/reduced" ~ 1,
      )),
    test_prep = factor(case_when(
      test_prep == "none" ~ 0,
      test_prep == "completed" ~ 1,
      )),
    parent_marital_status = factor(case_when(
      parent_marital_status == "married" ~ 0,
      parent_marital_status == "single" ~ 1,
      parent_marital_status == "widowed" ~ 2,
      parent_marital_status == "divorced" ~ 3,
      )),
    practice_sport = factor(case_when(
      practice_sport == "never" ~ 0,
      practice_sport == "sometimes" ~ 1,
      practice_sport == "regularly" ~ 2,
      )),
    is_first_child = factor(case_when(
```

```
      is_first_child == "no" ~ 0,
      is_first_child == "yes" ~ 1,
      )),
    transport_means = factor(case_when(
      transport_means == "school_bus" ~ 0,
      transport_means == "private" ~ 1,
      )),
    wkly_study_hours = factor(case_when(
      wkly_study_hours == "< 5" ~ 0,
      wkly_study_hours == "10-May" ~ 1,
      wkly_study_hours == "> 10" ~ 2,
      ))
    ) |>
  mutate(nr_siblings = factor(nr_siblings))
```

```
## Rows: 948 Columns: 14
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## chr (10): Gender, EthnicGroup, ParentEduc, LunchType, TestPrep, ParentMarita...
## dbl  (4): NrSiblings, MathScore, ReadingScore, WritingScore
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
'
data <- read_csv("./data.csv") |>
  janitor::clean_names() |>
  mutate(
    gender = case_when(
      gender == "male" ~ 0,
      gender == "female" ~ 1,
      ),
    ethnic_group = case_when(
      ethnic_group == "group A" ~ 0,
      ethnic_group == "group B" ~ 1,
      ethnic_group == "group C" ~ 2,
      ethnic_group == "group D" ~ 3,
      ethnic_group == "group E" ~ 4,
      ),
    parent_educ = case_when(
      parent_educ == "some highschool" ~ 0,
      parent_educ == "some college" ~ 1,
      parent_educ == "associate" ~ 2,
      parent_educ == "bachelor" ~ 3,
      parent_educ == "master" ~ 4,
      ),
    lunch_type = case_when(
      lunch_type == "standard" ~ 0,
      lunch_type == "free/reduced" ~ 1,
      ),
    test_prep = case_when(
      test_prep == "none" ~ 0,
      test_prep == "completed" ~ 1,
```

```
    ),
  parent_marital_status = case_when(
    parent_marital_status == "married" ~ 0,
    parent_marital_status == "single" ~ 1,
    parent_marital_status == "widowed" ~ 2,
    parent_marital_status == "divorced" ~ 3,
    ),
  practice_sport = case_when(
    practice_sport == "never" ~ 0,
    practice_sport == "sometimes" ~ 1,
    practice_sport == "regularly" ~ 2,
    ),
  is_first_child = case_when(
    is_first_child == "no" ~ 0,
    is_first_child == "yes" ~ 1,
    ),
  transport_means = case_when(
    transport_means == "school_bus" ~ 0,
    transport_means == "private" ~ 1,
    ),
  wkly_study_hours = case_when(
    wkly_study_hours == "< 5" ~ 0,
    wkly_study_hours == "10-May" ~ 1,
    wkly_study_hours == "> 10" ~ 2,
    )
  ) |>
  drop_na()
'
```

```
## [1] "\ndata <- read_csv(\"./data.csv\") |>\n  janitor::clean_names() |>\n  mutate(\n    gender = case
```

```
'
# Deal with NA -- Calculate the column mean (round to integer) and plug it into NA cell
column_means <- round(colMeans(data, na.rm = TRUE), digits = 0)
for (col in names(data)) {
  data[[col]][is.na(data[[col]])] <- column_means[col]
  }

head(data)
'
```

```
## [1] "\n# Deal with NA -- Calculate the column mean (round to integer) and plug it into NA cell\ncolu
```

```
# Another data set for EDA
data_long <- data |>
  pivot_longer(cols = c(math_score, reading_score, writing_score),
               names_to = "test", values_to = "score")
```

Summary

```
'
continuous_vars <- data[, c(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14)]
summary_df <- data.frame(
Min = sapply(continuous_vars, min, na.rm = TRUE),
Q1 = sapply(continuous_vars, function(x) quantile(x, probs = 0.25, na.rm = TRUE)), Median = sapply(conti
Mean = sapply(continuous_vars, mean, na.rm = TRUE),
Q3 = sapply(continuous_vars, function(x) quantile(x, probs = 0.75, na.rm = TRUE)), Max = sapply(continu
)
kable(summary_df, caption = "Summary Statistics of Data", digits = 1)
'
```

```
## [1] "\ncontinuous_vars <- data[, c(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14)]\nsummary_df <- da
```

```
sum_data_fct =
  data |>
  dplyr::select(1:11) |>
  skimr::skim() |>
  dplyr::select(skim_variable, n_missing, complete_rate, factor.n_unique, factor.top_counts)

colnames(sum_data_fct) = c("Variable", "Missing", "Complete Rate", "Unique", "Top Counts")

knitr::kable(x = sum_data_fct, caption = "Categorical Variables pre-analysis", digits = 1)
```

Table 1: Categorical Variables pre-analysis

Variable	Missing	Complete Rate	Unique	Top Counts
gender	0	1.0	2	1: 488, 0: 460
ethnic_group	59	0.9	5	2: 277, 3: 237, 1: 171, 4: 124
parent_educ	392	0.6	4	1: 199, 2: 198, 3: 104, 4: 55
lunch_type	0	1.0	2	0: 617, 1: 331
test_prep	55	0.9	2	0: 571, 1: 322
parent_marital_status	49	0.9	4	0: 516, 1: 213, 3: 146, 2: 24
practice_sport	16	1.0	3	1: 477, 2: 343, 0: 112
is_first_child	30	1.0	2	1: 604, 0: 314
nr_siblings	46	1.0	8	1: 245, 2: 213, 3: 198, 0: 101
transport_means	102	0.9	2	0: 509, 1: 337
wkly_study_hours	37	1.0	3	1: 508, 0: 253, 2: 150

```
data =
  data |>
  drop_na()
```

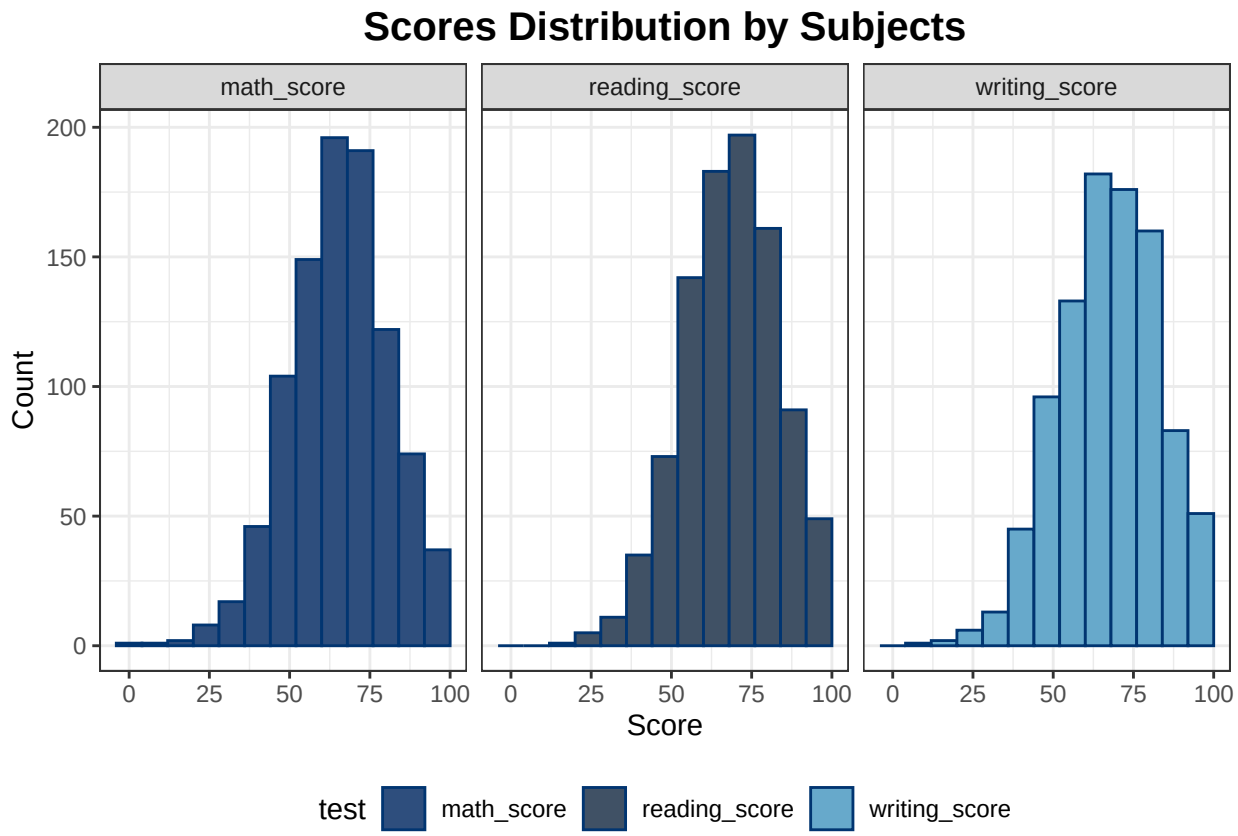
```
sum_data_score =
  data |>
  dplyr::select(12:14) |>
  skimr::skim() |>
  dplyr::select(skim_variable, numeric.mean, numeric.sd, numeric.p0, numeric.p25, numeric.p50, numeric.

colnames(sum_data_score) = c("Variable", "Mean", "SD", "Min", "Q1", "Median", "Q3", "Max")

knitr::kable(x = sum_data_score, caption = "Continuous Variables pre-analysis", digits = 1)
```

Table 2: Continuous Variables pre-analysis

Variable	Mean	SD	Min	Q1	Median	Q3	Max
math_score	68.7	15.9	18	57	69.0	81	100
reading_score	72.3	14.8	23	61	73.0	84	100
writing_score	72.0	15.2	19	62	72.5	84	100

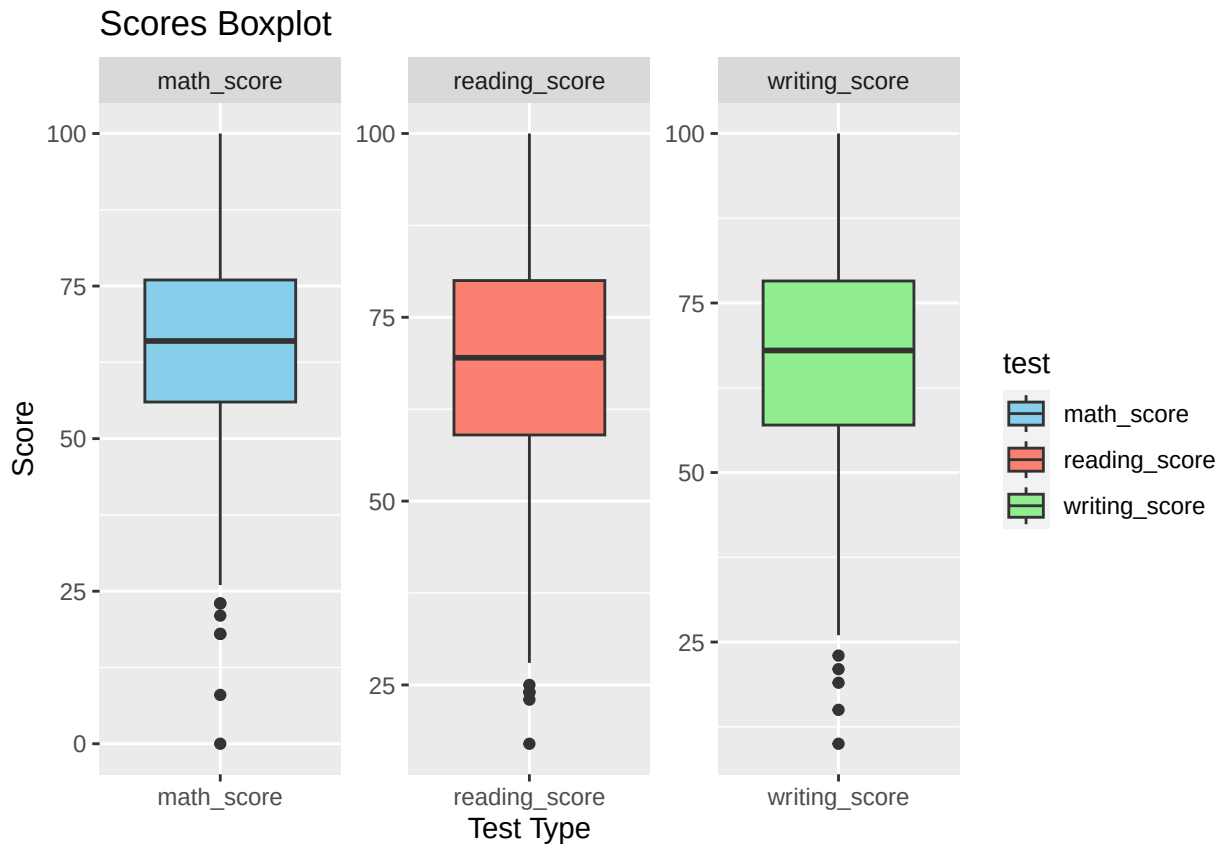
Histograms

```
data_long |>
  ggplot(aes(x = score, fill = test)) +
  geom_histogram(binwidth = 8, color = "#013571") +
  labs(
    title = "Scores Distribution by Subjects",
    x = "Score",
    y = "Count",
  ) +
  scale_fill_manual(values = c("#2E4E7D", "#405165", "#67A9CB")) +
  facet_grid(~ test) +
  theme_bw() +
  theme(legend.position = "bottom") +
  theme(plot.title = element_text(size = 15, face = "bold", hjust = 0.5))
```



Boxplots

```
data_long |>
  ggplot(aes(x = test, y = score, fill = test)) +
  geom_boxplot() +
  labs(title = "Scores Boxplot", x = "Test Type", y = "Score") +
  facet_wrap(~ test, scales = "free") +
  scale_fill_manual(values = c("skyblue", "salmon", "lightgreen"))
```



Diagnostics

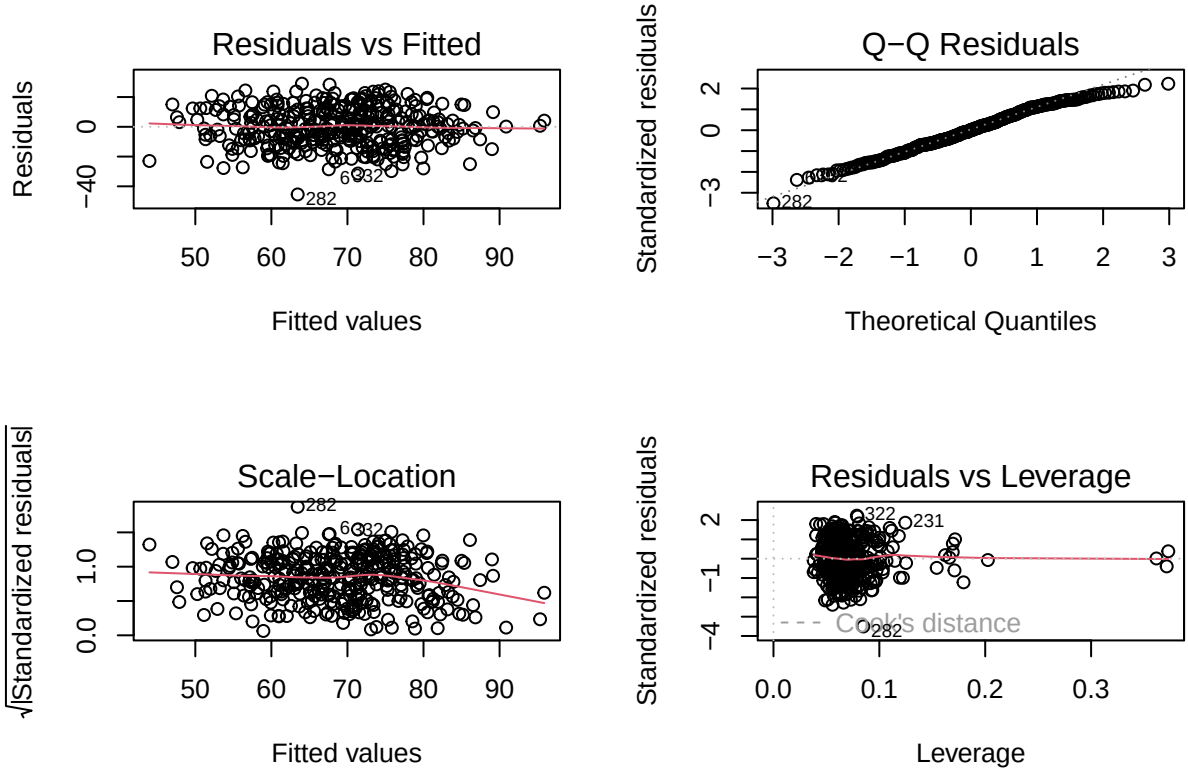
```
# Math
model_math_full = lm(math_score ~ .-reading_score -writing_score, data = data)

summary(model_math_full)
```

```
##
## Call:
## lm(formula = math_score ~ . - reading_score - writing_score,
##     data = data)
##
```

```
## Residuals:
##     Min      1Q  Median      3Q     Max
## -45.458  -8.961   0.089   9.800  28.981
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)              62.3523     4.9540  12.586  < 2e-16 ***
## gender1                  -3.6522     1.4958  -2.442 0.015150 *
## ethnic_group1             1.8120     3.2790   0.553 0.580912
## ethnic_group2            -1.1247     3.1319  -0.359 0.719748
## ethnic_group3             3.0342     3.1826   0.953 0.341109
## ethnic_group4             8.7423     3.3555   2.605 0.009598 **
## parent_educ2              1.8031     1.7975   1.003 0.316545
## parent_educ3              3.1775     2.0927   1.518 0.129886
## parent_educ4              4.0051     2.5782   1.553 0.121282
## lunch_type1             -12.1275     1.5423  -7.863 5.49e-14 ***
## test_prep1                5.7990     1.5706   3.692 0.000260 ***
## parent_marital_status1   -4.2006     1.8079  -2.323 0.020770 *
## parent_marital_status2    7.0930     4.7226   1.502 0.134083
## parent_marital_status3   -4.8362     2.1726  -2.226 0.026694 *
## practice_sport1           3.0566     2.3818   1.283 0.200295
## practice_sport2           3.2296     2.4896   1.297 0.195466
## is_first_child1          -0.3254     1.6378  -0.199 0.842638
## nr_siblings1             -0.1780     2.7665  -0.064 0.948739
## nr_siblings2             -1.1446     2.8721  -0.399 0.690507
## nr_siblings3              3.1546     2.8049   1.125 0.261548
## nr_siblings4              2.8587     3.3920   0.843 0.399963
## nr_siblings5              2.4937     3.9289   0.635 0.526071
## nr_siblings6             14.5158    13.9723   1.039 0.299617
## nr_siblings7              9.5593     8.3433   1.146 0.252735
## transport_means1          1.0585     1.5640   0.677 0.499003
## wkly_study_hours1         6.4822     1.7525   3.699 0.000254 ***
## wkly_study_hours2         4.2523     2.2536   1.887 0.060065 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13.55 on 327 degrees of freedom
## Multiple R-squared:  0.3256, Adjusted R-squared:  0.272
## F-statistic: 6.073 on 26 and 327 DF,  p-value: < 2.2e-16
```

```r
par(mfrow = c(2,2))
plot(model_math_full)
```

```
## Warning:         :
##   186
```



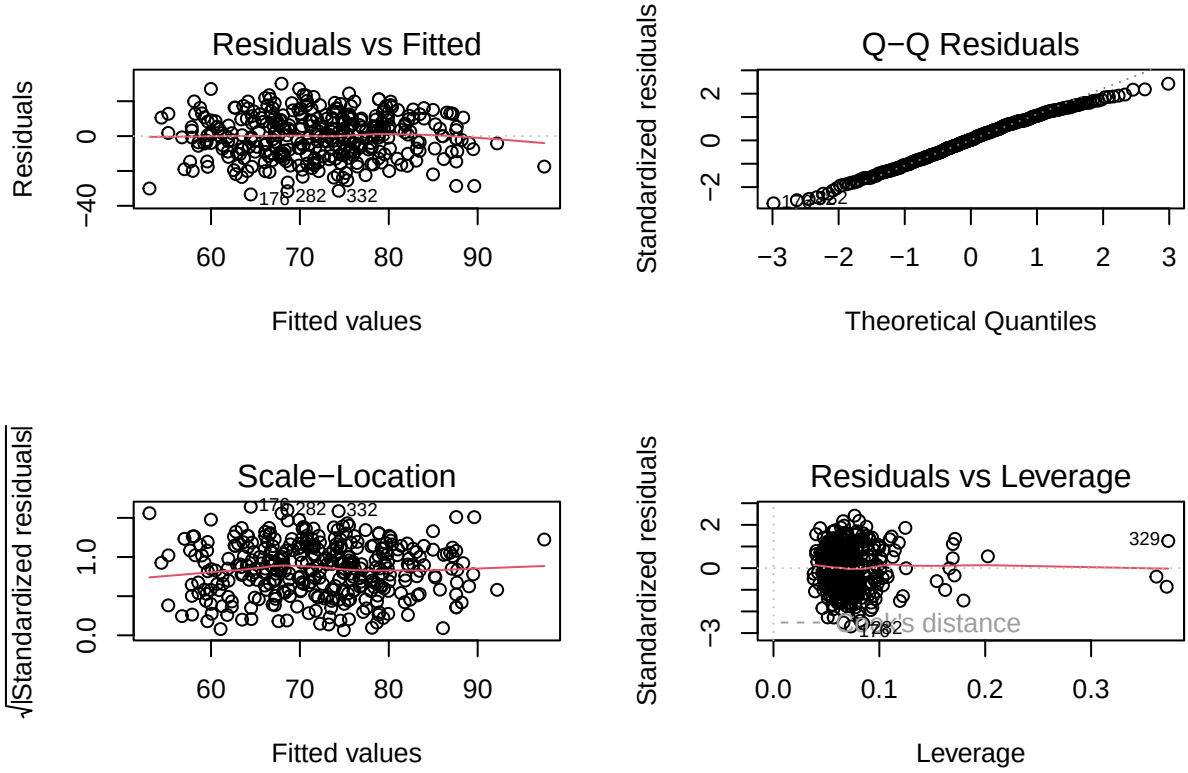

```
# Reading
model_reading_full = lm(reading_score ~ .-math_score -writing_score, data = data)

summary(model_reading_full)
```

```
##
## Call:
## lm(formula = reading_score ~ . - math_score - writing_score,
##     data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -33.470  -8.942   0.403   9.553  30.063
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            59.3627     4.7169  12.585  < 2e-16 ***
## gender1                 8.2587     1.4242   5.799 1.57e-08 ***
## ethnic_group1           1.4533     3.1220   0.466  0.64188
## ethnic_group2          -0.5044     2.9819  -0.169  0.86578
## ethnic_group3           2.8080     3.0302   0.927  0.35479
## ethnic_group4           4.7359     3.1949   1.482  0.13921
## parent_educ2            2.6502     1.7114   1.549  0.12246
## parent_educ3            4.5816     1.9925   2.299  0.02211 *
## parent_educ4            6.4240     2.4548   2.617  0.00929 **
```

```
## lunch_type1             -7.8783     1.4685  -5.365 1.54e-07 ***
## test_prep1               7.6036     1.4954   5.085 6.21e-07 ***
## parent_marital_status1  -4.6412     1.7214  -2.696  0.00738 **
## parent_marital_status2   4.6364     4.4966   1.031  0.30325
## parent_marital_status3  -4.2660     2.0686  -2.062  0.03997 *
## practice_sport1          1.9156     2.2678   0.845  0.39890
## practice_sport2          1.2989     2.3705   0.548  0.58408
## is_first_child1          0.6384     1.5594   0.409  0.68252
## nr_siblings1             0.4794     2.6341   0.182  0.85569
## nr_siblings2            -1.4869     2.7347  -0.544  0.58700
## nr_siblings3             1.8958     2.6706   0.710  0.47830
## nr_siblings4             2.3345     3.2296   0.723  0.47028
## nr_siblings5            -1.4797     3.7408  -0.396  0.69269
## nr_siblings6            11.7473    13.3034   0.883  0.37787
## nr_siblings7             7.7275     7.9439   0.973  0.33139
## transport_means1         0.5365     1.4891   0.360  0.71890
## wkly_study_hours1        5.3310     1.6686   3.195  0.00154 **
## wkly_study_hours2        1.1401     2.1458   0.531  0.59557
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.9 on 327 degrees of freedom
## Multiple R-squared:  0.2971, Adjusted R-squared:  0.2412
## F-statistic: 5.315 on 26 and 327 DF,  p-value: 6.451e-14
```

```
par(mfrow = c(2,2))
plot(model_reading_full)
```

```
## Warning:        :
##   186
```



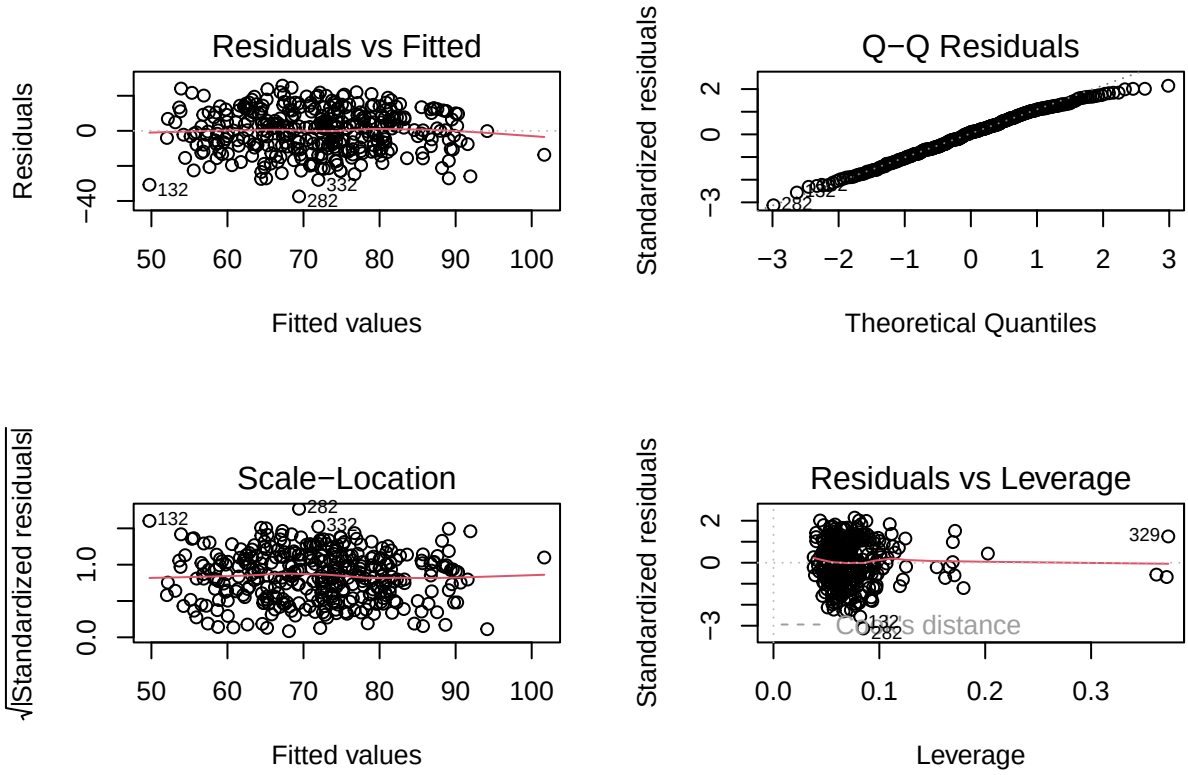

```r
# Writing
model_writing_full = lm(writing_score ~ .-reading_score -math_score, data = data)

summary(model_writing_full)
```

```
##
## Call:
## lm(formula = writing_score ~ . - reading_score - math_score,
##     data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -37.416  -8.131   1.123   9.165  25.765
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            55.1871     4.5675  12.083  < 2e-16 ***
## gender1                10.0433     1.3791   7.283 2.46e-12 ***
## ethnic_group1           1.7982     3.0232   0.595 0.552382
## ethnic_group2           0.7708     2.8875   0.267 0.789684
## ethnic_group3           5.5577     2.9343   1.894 0.059101 .
## ethnic_group4           5.5666     3.0937   1.799 0.072893 .
## parent_educ2            2.0224     1.6572   1.220 0.223203
## parent_educ3            4.5673     1.9294   2.367 0.018507 *
## parent_educ4            7.5525     2.3771   3.177 0.001629 **
```

```
## lunch_type1             -8.9424     1.4220  -6.289 1.03e-09 ***
## test_prep1               9.6428     1.4480   6.659 1.16e-10 ***
## parent_marital_status1  -4.5781     1.6669  -2.747 0.006356 **
## parent_marital_status2   5.2451     4.3542   1.205 0.229221
## parent_marital_status3  -4.4305     2.0031  -2.212 0.027669 *
## practice_sport1          3.3011     2.1960   1.503 0.133746
## practice_sport2          3.0186     2.2954   1.315 0.189415
## is_first_child1         -0.2525     1.5100  -0.167 0.867295
## nr_siblings1             0.3186     2.5507   0.125 0.900665
## nr_siblings2            -1.2993     2.6481  -0.491 0.624008
## nr_siblings3             2.2515     2.5860   0.871 0.384594
## nr_siblings4             2.9536     3.1273   0.944 0.345630
## nr_siblings5            -0.5419     3.6224  -0.150 0.881167
## nr_siblings6            14.3830    12.8821   1.117 0.265024
## nr_siblings7             8.0232     7.6923   1.043 0.297708
## transport_means1         0.9938     1.4420   0.689 0.491208
## wkly_study_hours1        5.4344     1.6157   3.363 0.000861 ***
## wkly_study_hours2        2.0335     2.0778   0.979 0.328454
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.5 on 327 degrees of freedom
## Multiple R-squared:  0.3762, Adjusted R-squared:  0.3266
## F-statistic: 7.586 on 26 and 327 DF,  p-value: < 2.2e-16
```

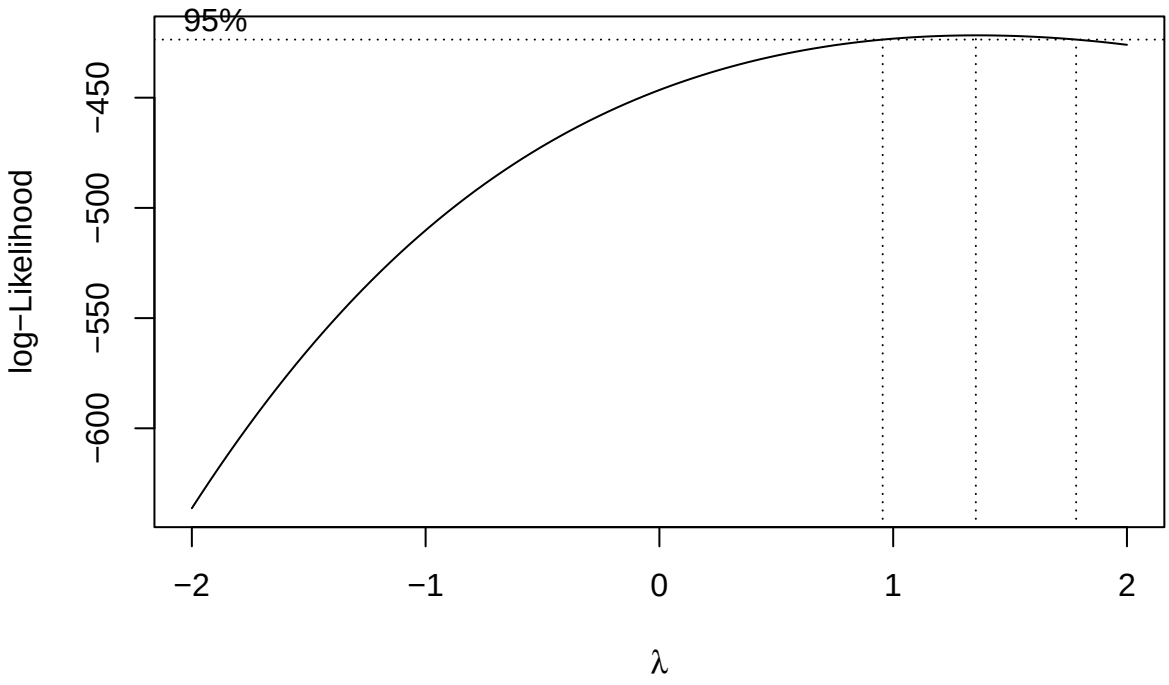
```r
par(mfrow = c(2,2))
plot(model_writing_full)
```

```
## Warning:        :
##   186
```



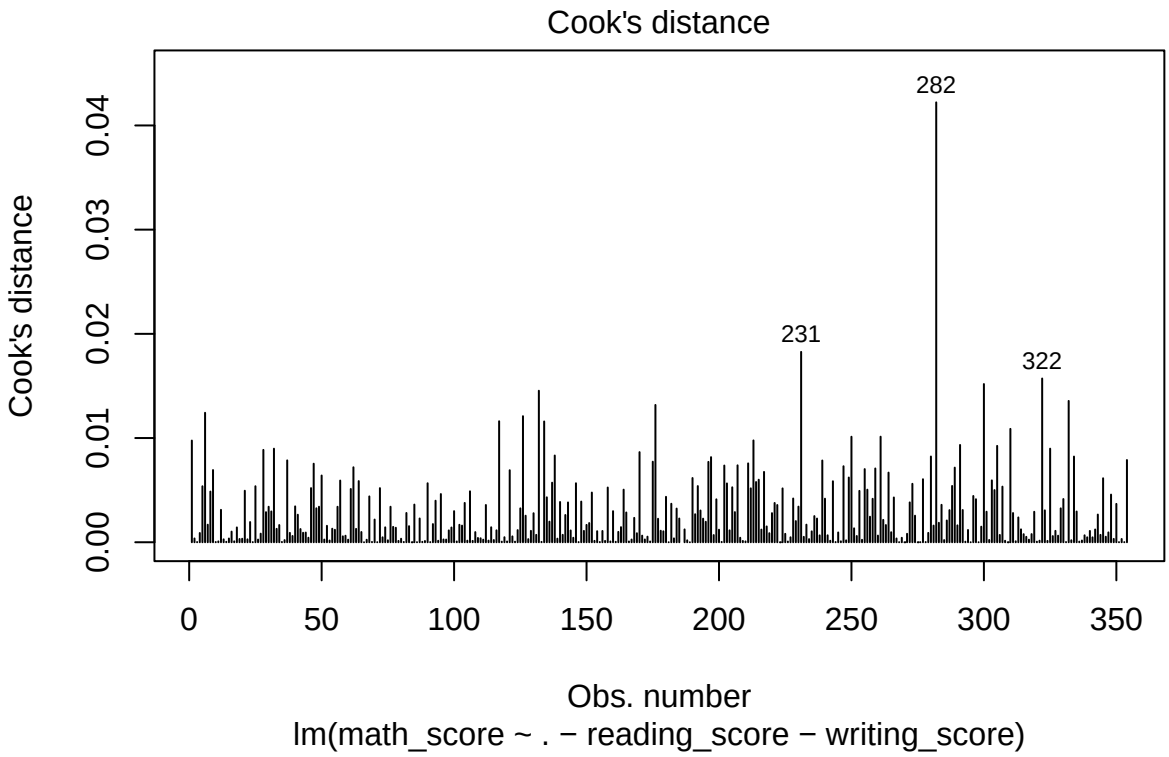
Transformation

```
boxcox(model_reading_full)
```

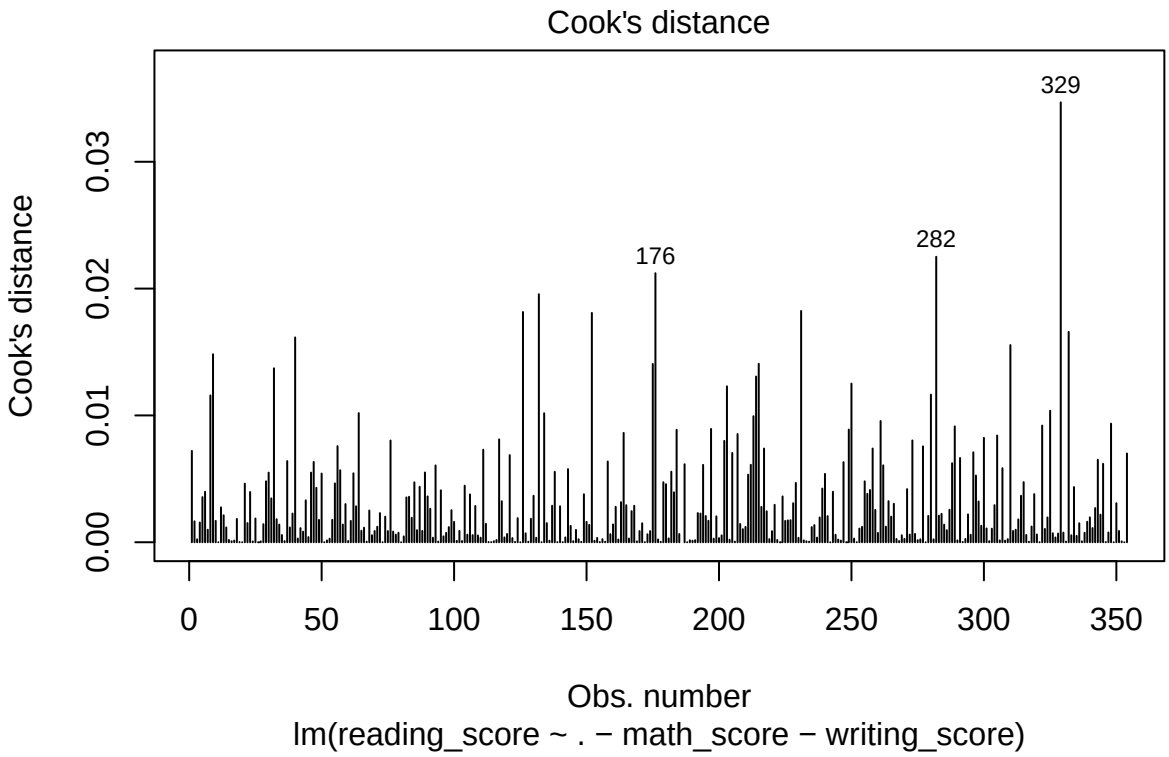


Outlier and influence points

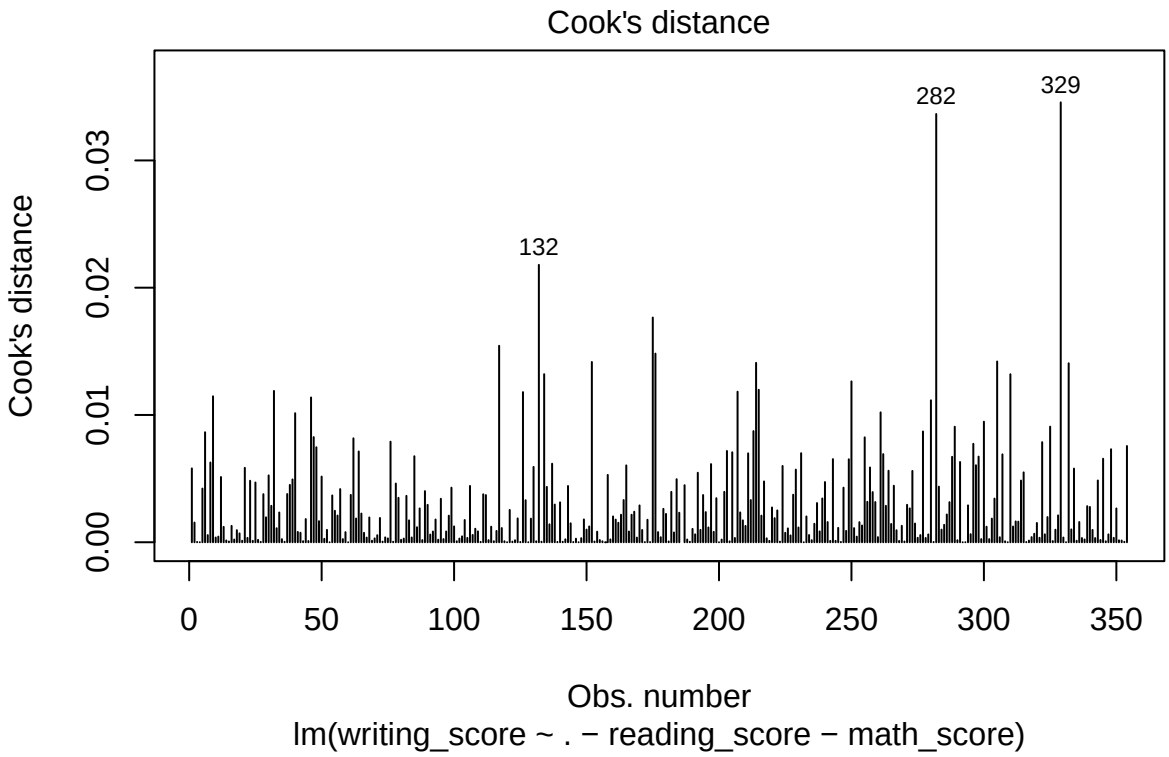
```
plot(model_math_full, which = 4)
```




```
plot(model_reading_full, which = 4)
```




```
plot(model_writing_full, which = 4)
```



Multicollinearity

```
# check VIF
performance::check_collinearity(model_math_full)
```

```
## # Check for Multicollinearity
##
## Low Correlation
##
##                   Term  VIF   VIF 95% CI Increased SE Tolerance
##                 gender 1.06 [1.01, 1.35]         1.03      0.94
##           ethnic_group 1.24 [1.13, 1.43]         1.11      0.81
##            parent_educ 1.22 [1.12, 1.41]         1.10      0.82
##             lunch_type 1.05 [1.01, 1.40]         1.03      0.95
##              test_prep 1.09 [1.03, 1.31]         1.05      0.91
##  parent_marital_status 1.17 [1.08, 1.36]         1.08      0.86
##         practice_sport 1.17 [1.08, 1.36]         1.08      0.86
##         is_first_child 1.15 [1.07, 1.35]         1.07      0.87
##            nr_siblings 1.54 [1.38, 1.78]         1.24      0.65
##        transport_means 1.11 [1.04, 1.32]         1.05      0.90
##       wkly_study_hours 1.14 [1.06, 1.33]         1.07      0.88
##  Tolerance 95% CI
##      [0.74, 0.99]
```

```
##      [0.70, 0.88]
##      [0.71, 0.89]
##      [0.71, 0.99]
##      [0.76, 0.97]
##      [0.74, 0.93]
##      [0.74, 0.93]
##      [0.74, 0.94]
##      [0.56, 0.73]
##      [0.76, 0.97]
##      [0.75, 0.95]
```

```
performance::check_collinearity(model_reading_full)
```

```
## # Check for Multicollinearity
##
## Low Correlation
##
##                   Term  VIF   VIF 95% CI Increased SE Tolerance
##                 gender 1.06 [1.01, 1.35]         1.03      0.94
##           ethnic_group 1.24 [1.13, 1.43]         1.11      0.81
##            parent_educ 1.22 [1.12, 1.41]         1.10      0.82
##             lunch_type 1.05 [1.01, 1.40]         1.03      0.95
##              test_prep 1.09 [1.03, 1.31]         1.05      0.91
##  parent_marital_status 1.17 [1.08, 1.36]         1.08      0.86
##         practice_sport 1.17 [1.08, 1.36]         1.08      0.86
##         is_first_child 1.15 [1.07, 1.35]         1.07      0.87
##            nr_siblings 1.54 [1.38, 1.78]         1.24      0.65
##        transport_means 1.11 [1.04, 1.32]         1.05      0.90
##       wkly_study_hours 1.14 [1.06, 1.33]         1.07      0.88
##  Tolerance 95% CI
##      [0.74, 0.99]
##      [0.70, 0.88]
##      [0.71, 0.89]
##      [0.71, 0.99]
##      [0.76, 0.97]
##      [0.74, 0.93]
##      [0.74, 0.93]
##      [0.74, 0.94]
##      [0.56, 0.73]
##      [0.76, 0.97]
##      [0.75, 0.95]
```

```
performance::check_collinearity(model_writing_full)
```

```
## # Check for Multicollinearity
##
## Low Correlation
##
##                   Term  VIF   VIF 95% CI Increased SE Tolerance
##                 gender 1.06 [1.01, 1.35]         1.03      0.94
##           ethnic_group 1.24 [1.13, 1.43]         1.11      0.81
##            parent_educ 1.22 [1.12, 1.41]         1.10      0.82
##             lunch_type 1.05 [1.01, 1.40]         1.03      0.95
```

```
##              test_prep 1.09 [1.03, 1.31]         1.05      0.91
##  parent_marital_status 1.17 [1.08, 1.36]         1.08      0.86
##         practice_sport 1.17 [1.08, 1.36]         1.08      0.86
##         is_first_child 1.15 [1.07, 1.35]         1.07      0.87
##            nr_siblings 1.54 [1.38, 1.78]         1.24      0.65
##        transport_means 1.11 [1.04, 1.32]         1.05      0.90
##       wkly_study_hours 1.14 [1.06, 1.33]         1.07      0.88
##  Tolerance 95% CI
##      [0.74, 0.99]
##      [0.70, 0.88]
##      [0.71, 0.89]
##      [0.71, 0.99]
##      [0.76, 0.97]
##      [0.74, 0.93]
##      [0.74, 0.93]
##      [0.74, 0.94]
##      [0.56, 0.73]
##      [0.76, 0.97]
##      [0.75, 0.95]
```

Model building for math

```
# backward model
step(model_math_full, direction='backward')
```

```
## Start:  AIC=1871.41
## math_score ~ (gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     nr_siblings + transport_means + wkly_study_hours + reading_score +
##     writing_score) - reading_score - writing_score
##
##                         Df Sum of Sq   RSS    AIC
## - nr_siblings            7    1388.4 61456 1865.5
## - parent_educ            3     667.0 60735 1869.3
## - practice_sport         2     344.1 60412 1869.4
## - is_first_child         1       7.3 60075 1869.5
## - transport_means        1      84.1 60152 1869.9
## <none>                               60068 1871.4
## - gender                 1    1095.1 61163 1875.8
## - parent_marital_status  3    2192.4 62260 1878.1
## - wkly_study_hours       2    2514.1 62582 1881.9
## - test_prep              1    2504.3 62572 1883.9
## - ethnic_group           4    3792.0 63860 1885.1
## - lunch_type             1   11357.8 71425 1930.7
##
## Step:  AIC=1865.5
## math_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     transport_means + wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
```

```
## - parent_educ            3     654.6 62111 1863.2
## - is_first_child         1       0.9 61457 1863.5
## - practice_sport         2     373.0 61829 1863.6
## - transport_means        1      57.9 61514 1863.8
## <none>                                61456 1865.5
## - gender                 1    1188.3 62644 1870.3
## - parent_marital_status  3    2362.9 63819 1872.9
## - wkly_study_hours       2    2350.7 63807 1874.8
## - test_prep              1    2571.8 64028 1878.0
## - ethnic_group           4    4102.7 65559 1880.4
## - lunch_type             1   12401.9 73858 1928.6
##
## Step:  AIC=1863.25
## math_score ~ gender + ethnic_group + lunch_type + test_prep +
##     parent_marital_status + practice_sport + is_first_child +
##     transport_means + wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## - practice_sport         2     306.8 62417 1861.0
## - is_first_child         1       2.1 62113 1861.3
## - transport_means        1      31.6 62142 1861.4
## <none>                                62111 1863.2
## - gender                 1    1164.4 63275 1867.8
## - parent_marital_status  3    2366.7 64477 1870.5
## - wkly_study_hours       2    2220.2 64331 1871.7
## - test_prep              1    2823.2 64934 1877.0
## - ethnic_group           4    4148.6 66259 1878.1
## - lunch_type             1   12325.1 74436 1925.3
##
## Step:  AIC=1861
## math_score ~ gender + ethnic_group + lunch_type + test_prep +
##     parent_marital_status + is_first_child + transport_means +
##     wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## - is_first_child         1       7.6 62425 1859.0
## - transport_means        1      26.3 62444 1859.1
## <none>                                62417 1861.0
## - gender                 1    1163.9 63581 1865.5
## - parent_marital_status  3    2337.1 64755 1868.0
## - wkly_study_hours       2    2207.5 64625 1869.3
## - test_prep              1    2830.8 65248 1874.7
## - ethnic_group           4    4111.9 66529 1875.6
## - lunch_type             1   12239.5 74657 1922.4
##
## Step:  AIC=1859.04
## math_score ~ gender + ethnic_group + lunch_type + test_prep +
##     parent_marital_status + transport_means + wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## - transport_means        1      27.7 62453 1857.2
## <none>                                62425 1859.0
## - gender                 1    1158.1 63583 1863.5
## - parent_marital_status  3    2347.6 64773 1866.1
```

```
## - wkly_study_hours       2    2201.8 64627 1867.3
## - test_prep              1    2826.4 65251 1872.7
## - ethnic_group           4    4105.4 66531 1873.6
## - lunch_type             1   12233.8 74659 1920.4
##
## Step:  AIC=1857.2
## math_score ~ gender + ethnic_group + lunch_type + test_prep +
##     parent_marital_status + wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## <none>                               62453 1857.2
## - gender                 1    1160.8 63614 1861.7
## - parent_marital_status  3    2320.8 64774 1864.1
## - wkly_study_hours       2    2192.8 64646 1865.4
## - test_prep              1    2920.0 65373 1871.4
## - ethnic_group           4    4097.1 66550 1871.7
## - lunch_type             1   12211.5 74664 1918.4
```

```
##
## Call:
## lm(formula = math_score ~ gender + ethnic_group + lunch_type +
##     test_prep + parent_marital_status + wkly_study_hours, data = data)
##
## Coefficients:
##            (Intercept)                 gender1           ethnic_group1
##                67.3260                 -3.7049                  2.4461
##          ethnic_group2           ethnic_group3           ethnic_group4
##                 0.3026                  4.1687                 10.1791
##           lunch_type1              test_prep1  parent_marital_status1
##              -12.3773                  6.0788                 -4.0821
## parent_marital_status2  parent_marital_status3       wkly_study_hours1
##                 6.7982                 -5.2507                  5.9171
##      wkly_study_hours2
##                 3.8301
```

```
model_math_fit_back = lm(formula = math_score ~ gender + ethnic_group + parent_educ +
    lunch_type + test_prep + parent_marital_status + practice_sport +
    is_first_child + wkly_study_hours, data = data)

summary(model_math_fit_back)
```

```
##
## Call:
## lm(formula = math_score ~ gender + ethnic_group + parent_educ +
##     lunch_type + test_prep + parent_marital_status + practice_sport +
##     is_first_child + wkly_study_hours, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -42.641  -9.388   0.444  10.841  29.060
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)             63.3058     4.0723  15.545  < 2e-16 ***
## gender1                 -3.7768     1.4786  -2.554 0.011080 *
## ethnic_group1            2.0233     3.2739   0.618 0.536983
## ethnic_group2           -0.1921     3.1097  -0.062 0.950767
## ethnic_group3            3.5985     3.1572   1.140 0.255191
## ethnic_group4            9.8452     3.3254   2.961 0.003289 **
## parent_educ2             1.6680     1.7628   0.946 0.344724
## parent_educ3             3.1571     2.0672   1.527 0.127641
## parent_educ4             3.7243     2.5498   1.461 0.145058
## lunch_type1            -12.4609     1.5198  -8.199 5.22e-15 ***
## test_prep1               5.9501     1.5447   3.852 0.000140 ***
## parent_marital_status1  -4.1882     1.7844  -2.347 0.019505 *
## parent_marital_status2   7.3458     4.7089   1.560 0.119707
## parent_marital_status3  -4.9516     2.1536  -2.299 0.022104 *
## practice_sport1          3.1345     2.3452   1.337 0.182276
## practice_sport2          3.2766     2.4641   1.330 0.184500
## is_first_child1         -0.1431     1.5713  -0.091 0.927481
## wkly_study_hours1        6.1263     1.7189   3.564 0.000418 ***
## wkly_study_hours2        4.2272     2.2378   1.889 0.059752 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13.55 on 335 degrees of freedom
## Multiple R-squared:  0.3094, Adjusted R-squared:  0.2723
## F-statistic: 8.338 on 18 and 335 DF,  p-value: < 2.2e-16
```

```r
# lasso model
lambda_seq = 10 ^ seq(-3, 0, by = .1)

cv_object_math = cv.glmnet(as.matrix(data[1:11]), data$math_score,
                           lambda = lambda_seq,
                           nfolds = 5)

model_math_lasso = glmnet(as.matrix(data[1:11]), data$math_score, lambda = cv_object_math$lambda.min, al
coef(model_math_lasso)
```

```
## 12 x 1 sparse Matrix of class "dgCMatrix"
##                                s0
## (Intercept)            62.7158706
## gender                 -3.4172517
## ethnic_group            2.0740949
## parent_educ             0.9804808
## lunch_type            -11.7678104
## test_prep               5.0255504
## parent_marital_status  -1.0446103
## practice_sport          0.4391390
## is_first_child          .
## nr_siblings             0.7146589
## transport_means         .
## wkly_study_hours        2.4395500
```

```r
model_math_lasso$dev.ratio
```

```
## [1] 0.2622201
```

Model building for reading

```r
# backward model
step(model_reading_full, direction='backward')
```

```
## Start:  AIC=1836.68
## reading_score ~ (gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     nr_siblings + transport_means + wkly_study_hours + math_score +
##     writing_score) - math_score - writing_score
##
##                         Df Sum of Sq   RSS    AIC
## - nr_siblings            7     887.9 55342 1828.4
## - practice_sport         2     123.8 54578 1833.5
## - transport_means        1      21.6 54476 1834.8
## - is_first_child         1      27.9 54482 1834.9
## - ethnic_group           4    1227.5 55682 1836.6
## <none>                               54454 1836.7
## - parent_educ            3    1558.4 56013 1840.7
## - parent_marital_status  3    1908.7 56363 1842.9
## - wkly_study_hours       2    2004.0 56459 1845.5
## - test_prep              1    4305.6 58760 1861.6
## - lunch_type             1    4793.1 59248 1864.5
## - gender                 1    5599.8 60054 1869.3
##
## Step:  AIC=1828.41
## reading_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     transport_means + wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## - practice_sport         2     145.3 55488 1825.3
## - transport_means        1      11.4 55354 1826.5
## - is_first_child         1      40.1 55382 1826.7
## <none>                               55342 1828.4
## - ethnic_group           4    1318.8 56661 1828.7
## - parent_educ            3    1681.4 57024 1833.0
## - parent_marital_status  3    1924.1 57267 1834.5
## - wkly_study_hours       2    1969.7 57312 1836.8
## - test_prep              1    4222.4 59565 1852.4
## - lunch_type             1    5437.8 60780 1859.6
## - gender                 1    5693.8 61036 1861.1
##
## Step:  AIC=1825.34
## reading_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + is_first_child + transport_means +
##     wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## - transport_means        1       5.8 55493 1823.4
## - is_first_child         1      40.9 55529 1823.6
## <none>                               55488 1825.3
```

```
## - ethnic_group           4    1294.8 56782 1825.5
## - parent_educ            3    1654.8 57143 1829.7
## - parent_marital_status  3    1902.9 57391 1831.3
## - wkly_study_hours       2    1959.0 57447 1833.6
## - test_prep              1    4316.3 59804 1849.8
## - lunch_type             1    5421.7 60909 1856.3
## - gender                 1    5678.5 61166 1857.8
##
## Step:  AIC=1823.37
## reading_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + is_first_child + wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## - is_first_child         1      39.4 55533 1821.6
## <none>                               55493 1823.4
## - ethnic_group           4    1295.8 56789 1823.5
## - parent_educ            3    1649.4 57143 1827.7
## - parent_marital_status  3    1899.1 57393 1829.3
## - wkly_study_hours       2    1958.5 57452 1831.7
## - test_prep              1    4422.7 59916 1848.5
## - lunch_type             1    5422.5 60916 1854.4
## - gender                 1    5674.9 61168 1855.8
##
## Step:  AIC=1821.62
## reading_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## <none>                               55533 1821.6
## - ethnic_group           4    1305.9 56839 1821.8
## - parent_educ            3    1654.8 57188 1826.0
## - parent_marital_status  3    1899.5 57432 1827.5
## - wkly_study_hours       2    1974.9 57508 1830.0
## - test_prep              1    4531.6 60064 1847.4
## - lunch_type             1    5440.2 60973 1852.7
## - gender                 1    5644.2 61177 1853.9


##
## Call:
## lm(formula = reading_score ~ gender + ethnic_group + parent_educ +
##     lunch_type + test_prep + parent_marital_status + wkly_study_hours,
##     data = data)
##
## Coefficients:
##            (Intercept)                 gender1           ethnic_group1
##                61.6474                  8.1816                  1.8945
##          ethnic_group2           ethnic_group3           ethnic_group4
##                 0.3778                  3.3789                  5.6870
##           parent_educ2            parent_educ3            parent_educ4
##                 2.3964                  4.6728                  6.4917
##           lunch_type1              test_prep1  parent_marital_status1
##                -8.2631                  7.6175                 -4.5976
## parent_marital_status2  parent_marital_status3       wkly_study_hours1
##                 4.1841                 -4.3042                  5.1565
```

```
##      wkly_study_hours2
##                 1.0458
```

```
model_reading_back = lm(formula = reading_score ~ gender + ethnic_group + parent_educ +
    lunch_type + test_prep + parent_marital_status + is_first_child +
    transport_means + wkly_study_hours, data = data)
summary(model_reading_back)
```

```
##
## Call:
## lm(formula = reading_score ~ gender + ethnic_group + parent_educ +
##     lunch_type + test_prep + parent_marital_status + is_first_child +
##     transport_means + wkly_study_hours, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -32.522  -9.335   0.253   9.491  29.948
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)               61.0959     3.4189  17.870  < 2e-16 ***
## gender1                    8.2151     1.4010   5.864 1.08e-08 ***
## ethnic_group1              1.8440     3.0962   0.596  0.55187
## ethnic_group2              0.3221     2.9318   0.110  0.91257
## ethnic_group3              3.3272     2.9801   1.116  0.26502
## ethnic_group4              5.6186     3.1503   1.784  0.07540 .
## parent_educ2               2.4730     1.6822   1.470  0.14248
## parent_educ3               4.7430     1.9674   2.411  0.01645 *
## parent_educ4               6.4579     2.4012   2.689  0.00751 **
## lunch_type1               -8.2690     1.4432  -5.730 2.24e-08 ***
## test_prep1                 7.5208     1.4711   5.112 5.35e-07 ***
## parent_marital_status1    -4.5595     1.6944  -2.691  0.00748 **
## parent_marital_status2     4.3781     4.4330   0.988  0.32405
## parent_marital_status3    -4.3645     2.0421  -2.137  0.03330 *
## is_first_child1            0.7327     1.4725   0.498  0.61910
## transport_means1           0.2718     1.4551   0.187  0.85195
## wkly_study_hours1          5.1383     1.6296   3.153  0.00176 **
## wkly_study_hours2          1.0442     2.1217   0.492  0.62294
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.85 on 336 degrees of freedom
## Multiple R-squared:  0.2837, Adjusted R-squared:  0.2475
## F-statistic: 7.829 on 17 and 336 DF,  p-value: < 2.2e-16
```

```
# lasso model
lambda_seq = 10 ^ seq(-3, 0, by = .1)

cv_object_reading = cv.glmnet(as.matrix(data[1:11]), data$reading_score,
                        lambda = lambda_seq,
                        nfolds = 5)
cv_object_reading$lambda.min
```

```
## [1] 0.5011872
```

```
model_reading_lasso = glmnet(as.matrix(data[1:11]), data$reading_score, lambda = cv_object_reading$lambd
coef(model_reading_lasso)
```

```
## 12 x 1 sparse Matrix of class "dgCMatrix"
##                               s0
## (Intercept)           63.0047330
## gender                 6.8714456
## ethnic_group           1.0191726
## parent_educ            1.6822432
## lunch_type            -7.2445118
## test_prep              6.2890596
## parent_marital_status -0.7735146
## practice_sport         .
## is_first_child         .
## nr_siblings            .
## transport_means        .
## wkly_study_hours       0.4772919
```

```
model_reading_lasso$dev.ratio
```

```
## [1] 0.2302132
```

Model building for writing

```
# backward model
step(model_writing_full, direction = "backward", )
```

```
## Start:  AIC=1813.9
## writing_score ~ (gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     nr_siblings + transport_means + wkly_study_hours + math_score +
##     reading_score) - reading_score - math_score
##
##                         Df Sum of Sq   RSS    AIC
## - nr_siblings            7    1019.1 52079 1806.9
## - is_first_child         1       4.4 51064 1811.9
## - practice_sport         2     361.2 51421 1812.4
## - transport_means        1      74.2 51134 1812.4
## <none>                               51060 1813.9
## - ethnic_group           4    1779.1 52839 1818.0
## - parent_educ            3    1940.3 53000 1821.1
## - parent_marital_status  3    1991.7 53052 1821.4
## - wkly_study_hours       2    1901.4 52961 1822.8
## - lunch_type             1    6175.3 57235 1852.3
## - test_prep              1    6924.6 57985 1856.9
## - gender                 1    8281.3 59341 1865.1
##
## Step:  AIC=1806.89
## writing_score ~ gender + ethnic_group + parent_educ + lunch_type +
```

```
##     test_prep + parent_marital_status + practice_sport + is_first_child +
##     transport_means + wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## - is_first_child         1       1.2 52080 1804.9
## - transport_means        1      52.4 52132 1805.2
## - practice_sport         2     404.8 52484 1805.6
## <none>                               52079 1806.9
## - ethnic_group           4    1870.2 53949 1811.4
## - parent_marital_status  3    2027.5 54107 1814.4
## - parent_educ            3    2069.1 54148 1814.7
## - wkly_study_hours       2    1830.3 53910 1815.1
## - test_prep              1    6879.5 58959 1848.8
## - lunch_type             1    6955.3 59035 1849.3
## - gender                 1    8444.0 60523 1858.1
##
## Step:  AIC=1804.9
## writing_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + transport_means +
##     wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## - transport_means        1      53.0 52133 1803.3
## - practice_sport         2     408.3 52489 1803.7
## <none>                               52080 1804.9
## - ethnic_group           4    1869.4 53950 1809.4
## - parent_marital_status  3    2028.9 54109 1812.4
## - parent_educ            3    2068.7 54149 1812.7
## - wkly_study_hours       2    1829.2 53910 1813.1
## - test_prep              1    6907.4 58988 1847.0
## - lunch_type             1    6954.4 59035 1847.3
## - gender                 1    8463.2 60544 1856.2
##
## Step:  AIC=1803.26
## writing_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + practice_sport + wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## - practice_sport         2     397.6 52531 1802.0
## <none>                               52133 1803.3
## - ethnic_group           4    1901.9 54035 1808.0
## - parent_marital_status  3    1986.8 54120 1810.5
## - parent_educ            3    2041.4 54175 1810.9
## - wkly_study_hours       2    1821.0 53954 1811.4
## - lunch_type             1    6905.0 59038 1845.3
## - test_prep              1    7190.9 59324 1847.0
## - gender                 1    8443.2 60577 1854.4
##
## Step:  AIC=1801.95
## writing_score ~ gender + ethnic_group + parent_educ + lunch_type +
##     test_prep + parent_marital_status + wkly_study_hours
##
##                         Df Sum of Sq   RSS    AIC
## <none>                               52531 1802.0
```

```
## - ethnic_group           4    1950.7 54482 1806.9
## - parent_educ            3    1925.8 54457 1808.7
## - parent_marital_status  3    1962.6 54494 1808.9
## - wkly_study_hours       2    1804.0 54335 1809.9
## - lunch_type             1    6837.1 59368 1843.3
## - test_prep              1    7210.3 59741 1845.5
## - gender                 1    8486.0 61017 1853.0

##
## Call:
## lm(formula = writing_score ~ gender + ethnic_group + parent_educ +
##     lunch_type + test_prep + parent_marital_status + wkly_study_hours,
##     data = data)
##
## Coefficients:
##            (Intercept)                 gender1           ethnic_group1
##                 58.522                  10.032                  2.213
##          ethnic_group2           ethnic_group3           ethnic_group4
##                  1.850                   6.338                  6.617
##           parent_educ2            parent_educ3            parent_educ4
##                  1.789                   4.598                  7.212
##           lunch_type1              test_prep1  parent_marital_status1
##                -9.263                   9.609                 -4.417
## parent_marital_status2  parent_marital_status3       wkly_study_hours1
##                  4.668                  -4.644                  5.168
##      wkly_study_hours2
##                  1.893
```

```
model_writing_back = lm(formula = writing_score ~ gender + ethnic_group + parent_educ +
    lunch_type + test_prep + parent_marital_status + practice_sport +
    is_first_child + transport_means + wkly_study_hours, data = data)
summary(model_writing_back)
```

```
##
## Call:
## lm(formula = writing_score ~ gender + ethnic_group + parent_educ +
##     lunch_type + test_prep + parent_marital_status + practice_sport +
##     is_first_child + transport_means + wkly_study_hours, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -35.016  -8.347   0.861   9.431  25.920
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            55.6226     3.7915  14.670  < 2e-16 ***
## gender1                10.0283     1.3627   7.359 1.45e-12 ***
## ethnic_group1           1.9857     3.0171   0.658  0.51089
## ethnic_group2           1.3766     2.8687   0.480  0.63164
## ethnic_group3           5.7836     2.9166   1.983  0.04819 *
## ethnic_group4           6.4017     3.0645   2.089  0.03747 *
## parent_educ2            1.8930     1.6347   1.158  0.24769
## parent_educ3            4.7742     1.9128   2.496  0.01305 *
```

```
## parent_educ4             7.5674     2.3506   3.219  0.00141 **
## lunch_type1             -9.3729     1.4034  -6.679 1.01e-10 ***
## test_prep1               9.5404     1.4363   6.642 1.25e-10 ***
## parent_marital_status1  -4.5162     1.6470  -2.742  0.00643 **
## parent_marital_status2   5.4329     4.3399   1.252  0.21150
## parent_marital_status3  -4.4594     1.9914  -2.239  0.02579 *
## practice_sport1          3.4669     2.1647   1.602  0.11020
## practice_sport2          3.0695     2.2715   1.351  0.17751
## is_first_child1         -0.1246     1.4489  -0.086  0.93152
## transport_means1         0.8261     1.4256   0.580  0.56263
## wkly_study_hours1        5.2430     1.5846   3.309  0.00104 **
## wkly_study_hours2        2.0645     2.0654   1.000  0.31826
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.49 on 334 degrees of freedom
## Multiple R-squared:  0.3638, Adjusted R-squared:  0.3276
## F-statistic: 10.05 on 19 and 334 DF,  p-value: < 2.2e-16
```

```
# lasso model
lambda_seq = 10 ^ seq(-3, 0, by = .1)

cv_object_writing = cv.glmnet(as.matrix(data[1:11]), data$writing_score,
                        lambda = lambda_seq,
                        nfolds = 5)
cv_object_writing$lambda.min
```

```
## [1] 0.5011872
```

```
model_writing_lasso = glmnet(as.matrix(data[1:11]), data$writing_score, lambda = cv_object_writing$lambd
coef(model_writing_lasso)
```

```
## 12 x 1 sparse Matrix of class "dgCMatrix"
##                                s0
## (Intercept)           59.3844759
## gender                 8.7384396
## ethnic_group           1.4955961
## parent_educ            1.8826016
## lunch_type            -8.1037819
## test_prep              8.0886240
## parent_marital_status -0.8123378
## practice_sport         .
## is_first_child         .
## nr_siblings            0.1133873
## transport_means        .
## wkly_study_hours       0.7539334
```

```
model_writing_lasso$dev.ratio
```

```
## [1] 0.3119987
```