P8130 Final Report (Project 1)

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Abstract

This study used regression modeling techniques to predict academic performance in mathematics, reading, and writing based on multiple variables including personal characteristics and environmental factors. Using stepwise regression and LASSO analyses, we found significant relationships between predictor variables (including gender, race, parent education, lunch type, test preparation, and weekly study time) and test scores. We further found that including another test score as a predictor (e.g., a reading score when predicting a math score) significantly enhanced model performance without destroying the effect of the existing predictor. This comprehensive analysis reveals the complex relationships between predictors and student performance, providing educators and policymakers with actionable insights to tailor interventions and enhance educational programs to achieve overall student progress.

Introduction

The objective of this study is to use regression models to predict academic performance in math, reading, and writing based on various variables, including personal characteristics such as gender, ethnicity, and parental education, as well as environmental factors like lunch type, test preparation, and weekly study hours. Furthermore, the study aims to identify potential correlations and regression model between scores in different subjects. The combination of these analyses is intended to provide educators and policymakers with practical insights for tailoring interventions, improving educational programs, and building strong support structures that promote students' overall academic progress.

Methods

This dataset provides information on public school students, including three test scores and various personal and socioeconomic factors. To facilitate analysis, categorical data have been converted to numerical representations based on their ordinal order or type. We have excluded the missing cells because they are factorial data types.

After processing the data, we created **Table 1**, which presents a summary of the factorial data, including the number of missing data, the number of categories under each variable, and the top counts. For the numeric data (three test scores), we constructed a comprehensive descriptive table (**Table 2**) to provide a snapshot of central tendencies and variability. The distribution of the three response variables (test scores) is presented in **Figure 1** (histogram) and **Figure 2** (boxplot), indicating a normal distribution.

Then we fitted the "full model" using the score of three subjects respectively as the response variables, which consists of all 11 categorical variables as predictors. The model diagnostics are conducted by generating four plot for each model: Residuals vs Fitted, Q-Q Residuals, Scale-Location and Residuals vs Leverage (**Figure 3,4,5**). Next, we use BIC-based procedures to select the appropriate subsets of predictors for three subjects (**Figure 6,7,8,9**).

Based on the full models, we did some tests and calculations:

First, we conducted boxcox method (**Figure 10**) to determine if there's any transformation needed. Second, calculated Cook's distance (**Figure 11**) to check the existence of outliers and influence points. Finally, in order to test the multicollinearity among predictors, we calculated VIF as the criterion of multicollinearity (**Table 3,4,5**).

After all the steps above, we conducted model selection using both stepwise selection method and LASSO method. For stepwise method, the remaining predictors, coefficients and p-values are reported in **Table 6,7,8**.

In the selection procedure using LASSO method, for each subject we used cross-validation to decide the optimal value of method parameter λ, and then fitted LASSO model with this optimal value (**Figure 12,13,14**).

Finally, we tried to figure out if it is possible to leverage one score as the auxiliary information to learn the model for another score (still its model against variables 1-11) better. we plotted the correlation among three score variables (**Figure 15**). Then we refitted the linear models for the scores of three subjects using eleven categorical variables and one other score variable of a different subject as predictors (**Table 9,10,11,12,13,14**). The VIFs are calculated for all six models generated in this step to reveal the potential multicollinearity (**Table 15,16,17,18,19,20**).

Results

Table 1 gives the summary of the factorial data. **Table 2** provides the mean, deviation and quantile information about the continuous data (score variables of three subjects). The distribution of three response score variables are demonstrated by **Figure 1** (histogram) and **Figure 2** (boxplot).

Table 6, **Table 7**, and **Table 8** display the regression models for math, reading, and writing scores using both forward and backward stepwise regression. Moreover, **Figures 3**, **Figures 4**, and **Figures 5** display the diagnostic plots generated by the model.

Figure 6 displays the BIC over number of parameters for models of three subjects. **Figure 7,8,9** shows the BIC consistent with the remaining predictors in models of three subjects.

Figure 10 demonstrates the results of boxcox method: the log-likelihood over boxcox method parameter λ, and **Figure 11** demonstrates the Cook's distance as the result of testing of outliers.

Table 3,4,5 display the result of multicollinearity test: VIFs for three full models.

Figure 12,13,14 are plots of mean cross-validation error over LASSO parameter λ, providing information about optimal values of λ for LASSO models. These optimal values are used for following fitting of LASSO models.

Figure 15 shows the correlation existing among score variables, suggesting the feasibility of using one of the score variables as the auxiliary information to learn the model for another score. **Table 9,10,11,12,13,14** are the results of regression using one subject score as additional predictor for the prediction of another subject score. **Table 15,16,17,18,19,20** shows the VIFs of these six models, indicating the multicollinearity among predictors.

Conclusions

Both the histogram and boxplot distributions showed a normal distribution. Moreover, the full model diagnosis indicated that there was no need for transformations of the test scores and that there was no multicollinearity among predictors. Criterion-based procedures suggest that employing regression models for three test scores with 4–5 predictors will yield optimal results with a minimum Bayesian information criterion. These approaches guide the construction of effective models for the three test scores.

The stepwise regression and LASSO techniques revealed significant predictors for math, reading, and writing scores. To be specific, gender, ethnic group, lunch type, test preparation, parent marital status, and weekly study hours were found to be significant variables for math scores, with an adjusted R-squared of 0.2742. In addition, LASSO expanded the model by including parental education, time spent in sports, and the number of siblings.

Similarly, the stepwise regression models for reading and writing scores also revealed statistically significant relationships between test scores and predictors, including gender, ethnicity, parental education, lunch type, test preparation, parental marital status, and weekly study hours. These predictors explain the model with an adjusted R-squared of 0.2513 for reading and 0.3298 for writing. Furthermore, the LASSO analysis expands the models by including two variables: whether the student is the first child and the number of siblings.

Furthermore, the inclusion of an additional test score as a predictor resulted in a significant increase in the adjusted R square to greater than 0.8 without introducing collinearity problems or altering the effects of the existing predictors. This emphasizes the interdependence of test scores and their potential as predictive variables for each other, thereby enhancing the models' predictive capacity.

The combination of various analytical approaches not only identified important factors affecting test scores but also established the best model structures, leading to a detailed comprehension of the complex relationship between predictors and student test results. This thorough analysis provides a basis for making informed decisions and implementing targeted interventions to improve academic performance and promote fair educational outcomes.

Contribution

Xiaoting Tang: Method, **Yifei Liu**: Result Display

Longyu Zhang: Interpretation, **Huanyu Chen**: Writing

Appendix

Table

Table 1: Categorical Variables pre-analysis

Variable	Missing	Unique	Top Counts
gender	0	2	1: 488, 0: 460
ethnic_group	59	5	2: 277, 3: 237, 1: 171, 4: 124
parent_educ	392	4	1: 199, 2: 198, 3: 104, 4: 55
lunch_type	0	2	0: 617, 1: 331
test_prep	55	2	0: 571, 1: 322
parent_marital_status	49	4	0: 516, 1: 213, 3: 146, 2: 24
practice_sport	16	3	1: 477, 2: 343, 0: 112
is_first_child	30	2	1: 604, 0: 314
nr_siblings	46	8	1: 245, 2: 213, 3: 198, 0: 101
transport_means	102	2	0: 509, 1: 337
wkly_study_hours	37	3	1: 508, 0: 253, 2: 150

Table 2: Continuous Variables pre-analysis

Variable	Mean	SD	Min	Q1	Median	Q3	Max
math_score	68.7	15.9	18	57	69.0	81	100
reading_score	72.3	14.8	23	61	73.0	84	100
writing_score	72.0	15.2	19	62	72.5	84	100

Table 3: Math Scores Models by Stepwise Regression

Term	Estimate	P Value
gender1	-3.70	0.01
ethnic_group1	2.45	0.45
ethnic_group2	0.30	0.92
ethnic_group3	4.17	0.18
ethnic_group4	10.18	0.00
lunch_type1	-12.38	0.00
test_prep1	6.08	0.00
parent_marital_status1	-4.08	0.02
parent_marital_status2	6.80	0.14
parent_marital_status3	-5.25	0.01
wkly_study_hours1	5.92	0.00
wkly_study_hours2	3.83	0.08

Table 4: Reading Scores Models by Stepwise Regression

Term	Estimate	P Value
gender1	8.18	0.00
ethnic_group1	1.89	0.54
ethnic_group2	0.38	0.90
ethnic_group3	3.38	0.26
ethnic_group4	5.69	0.07
parent_educ2	2.40	0.15
parent_educ3	4.67	0.02
parent_educ4	6.49	0.01
lunch_type1	-8.26	0.00
test_prep1	7.62	0.00
parent_marital_status1	-4.60	0.01
parent_marital_status2	4.18	0.34

Term	Estimate	P Value
parent_marital_status3	-4.30	0.03
wkly_study_hours1	5.16	0.00
wkly_study_hours2	1.05	0.62

Table 5: Writing Scores Models by Stepwise Regression

Term	Estimate	P Value
gender1	10.03	0.00
ethnic_group1	2.21	0.46
ethnic_group2	1.85	0.52
ethnic_group3	6.34	0.03
ethnic_group4	6.62	0.03
parent_educ2	1.79	0.27
parent_educ3	4.60	0.02
parent_educ4	7.21	0.00
lunch_type1	-9.26	0.00
test_prep1	9.61	0.00
parent_marital_status1	-4.42	0.01
parent_marital_status2	4.67	0.28
parent_marital_status3	-4.64	0.02
wkly_study_hours1	5.17	0.00
wkly_study_hours2	1.89	0.36

Table 6: VIF for Math Score

Term	VIF	VIF_CI	Tolerance
gender	1.1	[1, 1.4]	0.9
ethnic_group	1.2	[1.1, 1.4]	0.8
parent_educ	1.2	[1.1, 1.4]	0.8

Term	VIF	VIF_CI	Tolerance
lunch_type	1.1	[1, 1.4]	1.0
test_prep	1.1	[1, 1.3]	0.9
parent_marital_status	1.2	[1.1, 1.4]	0.9
practice_sport	1.2	[1.1, 1.4]	0.9
is_first_child	1.2	[1.1, 1.3]	0.9
nr_siblings	1.5	[1.4, 1.8]	0.6
transport_means	1.1	[1, 1.3]	0.9
wkly_study_hours	1.1	[1.1, 1.3]	0.9

Table 7: VIF for Reading Score

Term	VIF	VIF_CI	Tolerance
gender	1.1	[1, 1.4]	0.9
ethnic_group	1.2	[1.1, 1.4]	0.8
parent_educ	1.2	[1.1, 1.4]	0.8
lunch_type	1.1	[1, 1.4]	1.0
test_prep	1.1	[1, 1.3]	0.9
parent_marital_status	1.2	[1.1, 1.4]	0.9
practice_sport	1.2	[1.1, 1.4]	0.9
is_first_child	1.2	[1.1, 1.3]	0.9
nr_siblings	1.5	[1.4, 1.8]	0.6
transport_means	1.1	[1, 1.3]	0.9
wkly_study_hours	1.1	[1.1, 1.3]	0.9

Table 8: VIF for Reading Score

Term	VIF	VIF_CI	Tolerance
gender	1.1	[1, 1.4]	0.9
ethnic_group	1.2	[1.1, 1.4]	0.8

Term	VIF	VIF_CI	Tolerance
parent_educ	1.2	[1.1, 1.4]	0.8
lunch_type	1.1	[1, 1.4]	1.0
test_prep	1.1	[1, 1.3]	0.9
parent_marital_status	1.2	[1.1, 1.4]	0.9
practice_sport	1.2	[1.1, 1.4]	0.9
is_first_child	1.2	[1.1, 1.3]	0.9
nr_siblings	1.5	[1.4, 1.8]	0.6
transport_means	1.1	[1, 1.3]	0.9
wkly_study_hours	1.1	[1.1, 1.3]	0.9

Table 9: Math Scores Models Using Reading Score as Additional Predictor

Term	Estimate	P Value
gender1	-11.5	0.0
ethnic_group1	0.6	0.7
ethnic_group2	-0.2	0.9
ethnic_group3	0.7	0.6
ethnic_group4	4.6	0.0
lunch_type1	-4.7	0.0
test_prep1	-1.2	0.1
practice_sport1	1.4	0.2
practice_sport2	2.4	0.0
wkly_study_hours1	1.4	0.1
wkly_study_hours2	3.4	0.0
reading_score	0.9	0.0

Table 10: Math Scores Models Using Writing Score as Additional Predictor

Term	Estimate	P Value
gender1	-13.7	0.0
ethnic__group1	0.1	0.9
ethnic__group2	-1.6	0.2
ethnic__group3	-2.2	0.1
ethnic__group4	3.5	0.0
parent__educ2	-0.1	0.9
parent__educ3	-1.4	0.1
parent__educ4	-3.8	0.0
lunch__type1	-3.1	0.0
test__prep1	-3.7	0.0
wkly__study__hours1	1.0	0.2
wkly__study__hours2	2.3	0.0
writing__score	1.0	0.0

Table 11: Reading Scores Models Using Math Score as Additional Predictor

Term	Estimate	P Value
gender1	11.3	0.0
ethnic__group1	-0.2	0.9
ethnic__group2	0.2	0.9
ethnic__group3	-0.1	1.0
ethnic__group4	-2.9	0.0
parent__educ2	1.0	0.2
parent__educ3	2.1	0.0
parent__educ4	3.5	0.0
lunch__type1	2.2	0.0

Term	Estimate	P Value
test_prep1	2.5	0.0
is_first_child1	1.1	0.1
wkly_study_hours1	0.0	1.0
wkly_study_hours2	-2.4	0.0
math_score	0.8	0.0

Table 12: Reading Scores Models Using Writing Score as Additional Predictor

Term	Estimate	P Value
gender1	-1.6	0.0
ethnic_group1	-0.2	0.8
ethnic_group2	-1.3	0.2
ethnic_group3	-2.7	0.0
ethnic_group4	-0.7	0.5
lunch_type1	0.8	0.1
test_prep1	-1.9	0.0
practice_sport1	-1.2	0.1
practice_sport2	-1.5	0.0
is_first_child1	0.8	0.1
writing_score	1.0	0.0

Table 13: Writing Scores Models Using Math Score as Additional Predictor

Term	Estimate	P Value
gender1	13.1	0.0
ethnic_group1	0.3	0.8
ethnic_group2	1.8	0.1

Term	Estimate	P Value
ethnic_group3	2.9	0.0
ethnic_group4	-1.9	0.1
parent_educ2	0.3	0.6
parent_educ3	2.0	0.0
parent_educ4	4.3	0.0
lunch_type1	1.2	0.1
test_prep1	4.6	0.0
wkly_study_hours1	0.0	0.9
wkly_study_hours2	-1.5	0.1
math_score	0.8	0.0

Table 14: Writing Scores Models Using Reading Score as Additional Predictor

Term	Estimate	P Value
gender1	2.3	0.0
ethnic_group1	0.4	0.6
ethnic_group2	1.4	0.1
ethnic_group3	3.0	0.0
ethnic_group4	1.3	0.2
parent_educ2	-0.5	0.4
parent_educ3	0.3	0.7
parent_educ4	1.3	0.1
lunch_type1	-1.6	0.0
test_prep1	2.7	0.0
practice_sport1	1.5	0.0
practice_sport2	1.8	0.0
is_first_child1	-0.8	0.1
reading_score	0.9	0.0

Table 15: VIF for Math Score (include reading score)

Term	VIF	VIF_CI	Tolerance
gender	1.1	[1, 1.3]	0.9
ethnic_group	1.1	[1, 1.3]	0.9
lunch_type	1.1	[1, 1.3]	0.9
test_prep	1.1	[1, 1.3]	0.9
practice_sport	1.0	[1, 1.5]	1.0
wkly_study_hours	1.1	[1, 1.3]	0.9
reading_score	1.3	[1.2, 1.5]	0.8

Table 16: VIF for Math Score (include writing score)

Term	VIF	VIF_CI	Tolerance
gender	1.2	[1.1, 1.4]	0.8
ethnic_group	1.1	[1, 1.3]	0.9
parent_educ	1.1	[1, 1.3]	0.9
lunch_type	1.1	[1.1, 1.4]	0.9
test_prep	1.2	[1.1, 1.4]	0.8
wkly_study_hours	1.1	[1, 1.3]	0.9
writing_score	1.5	[1.3, 1.7]	0.7

Table 17: VIF for Reading Score (include math score)

Term	VIF	VIF_CI	Tolerance
gender	1.1	[1, 1.4]	1.0
ethnic_group	1.1	[1.1, 1.4]	0.9
parent_educ	1.1	[1, 1.3]	0.9
lunch_type	1.2	[1.1, 1.4]	0.8
test_prep	1.1	[1, 1.3]	0.9

Term	VIF	VIF_CI	Tolerance
is_first_child	1.0	[1, 3.7]	1.0
wkly_study_hours	1.1	[1, 1.3]	0.9
math_score	1.4	[1.2, 1.6]	0.7

Table 18: VIF for Reading Score (include writing score)

Term	VIF	VIF_CI	Tolerance
gender	1.2	[1.1, 1.4]	0.9
ethnic_group	1.1	[1, 1.3]	0.9
lunch_type	1.1	[1.1, 1.3]	0.9
test_prep	1.2	[1.1, 1.4]	0.9
practice_sport	1.1	[1, 1.4]	0.9
is_first_child	1.0	[1, 1.6]	1.0
writing_score	1.4	[1.3, 1.6]	0.7

Table 19: VIF for Writing Score (include math score)

Term	VIF	VIF_CI	Tolerance
gender	1.0	[1, 1.5]	1.0
ethnic_group	1.1	[1.1, 1.3]	0.9
parent_educ	1.1	[1, 1.4]	0.9
lunch_type	1.2	[1.1, 1.4]	0.8
test_prep	1.1	[1, 1.3]	0.9
wkly_study_hours	1.1	[1, 1.3]	0.9
math_score	1.4	[1.2, 1.6]	0.7

Table 20: VIF for Writing Score (include reading score)

Term	VIF	VIF_CI	Tolerance
gender	1.1	[1, 1.3]	0.9
ethnic_group	1.1	[1, 1.3]	0.9
parent_educ	1.1	[1, 1.3]	0.9
lunch_type	1.1	[1, 1.3]	0.9
test_prep	1.1	[1, 1.3]	0.9
practice_sport	1.1	[1, 1.3]	0.9
is_first_child	1.0	[1, 1.5]	1.0
reading_score	1.3	[1.2, 1.5]	0.8

Figure

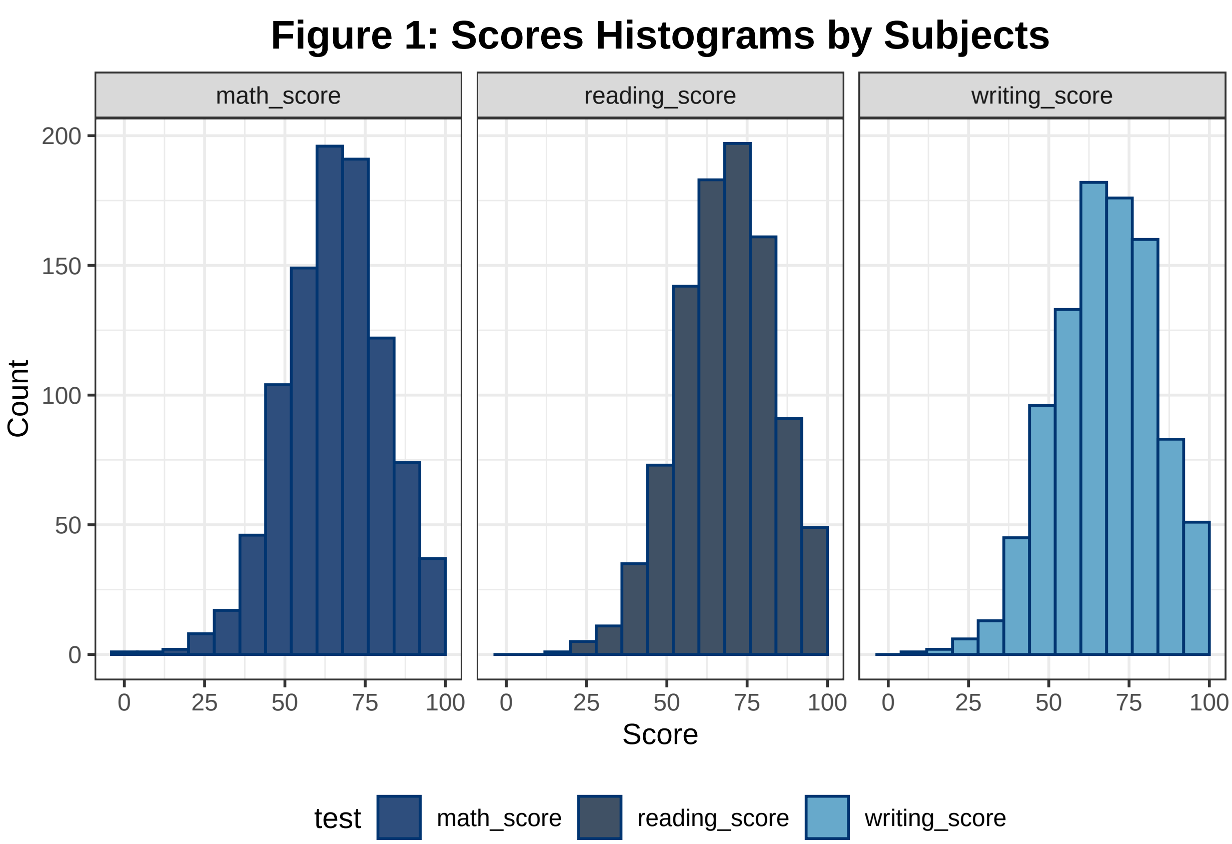

Figure 2: Scores Boxplot by Subjects

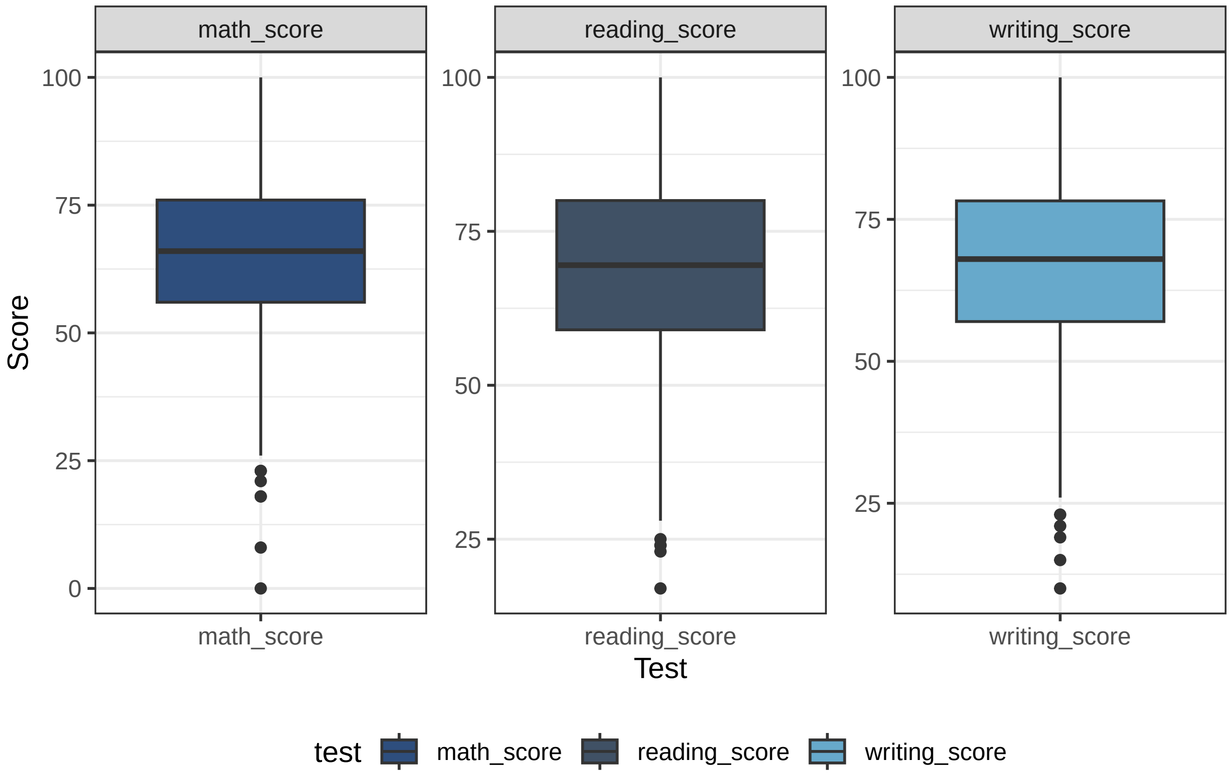

Figure 3: Diagnostic Plots for Math Test Score (Stepwise)

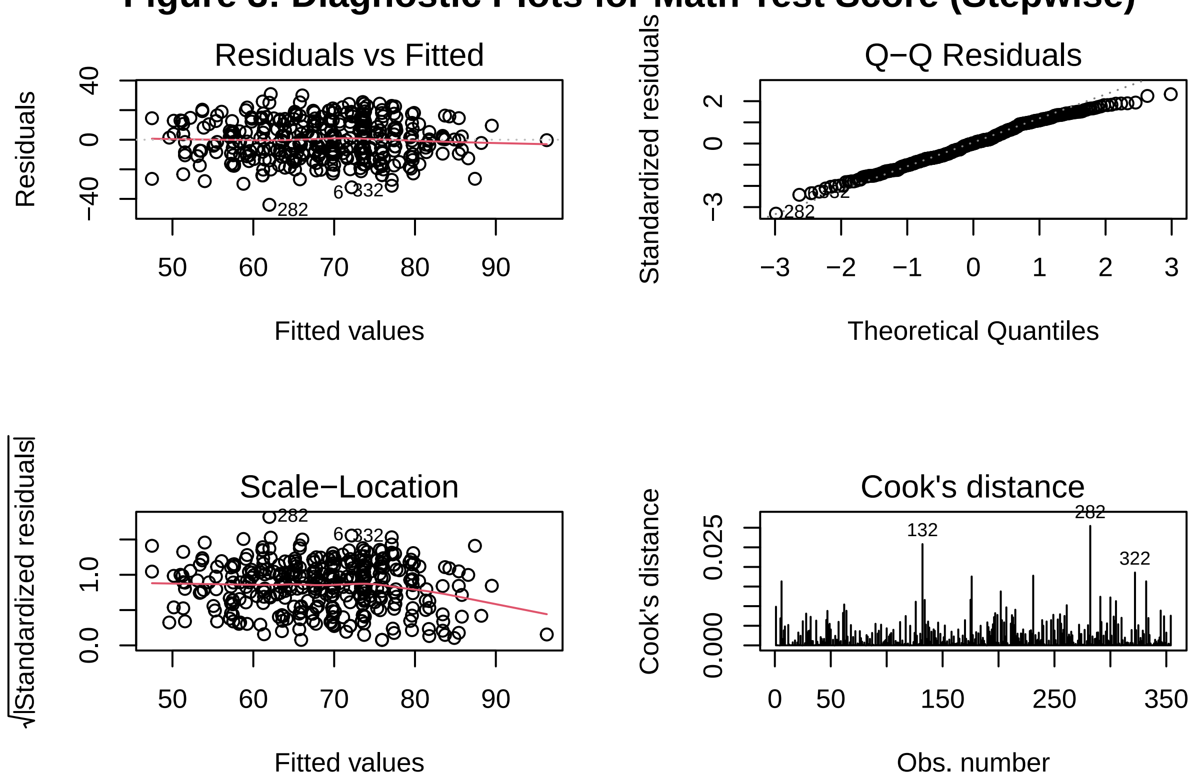

Figure 4: Diagnostic Plots for Reading Test Score (Stepwise)

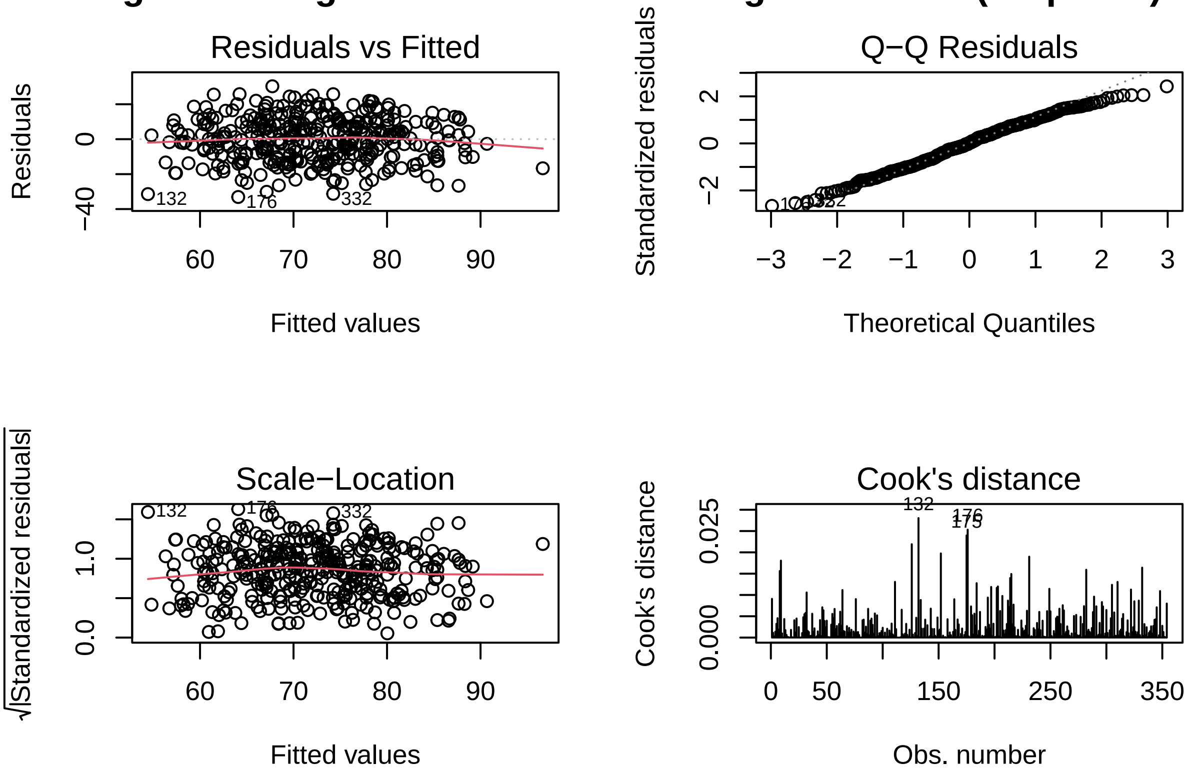

Figure 5: Diagnostic Plots for Writing Test Score (Stepwise)

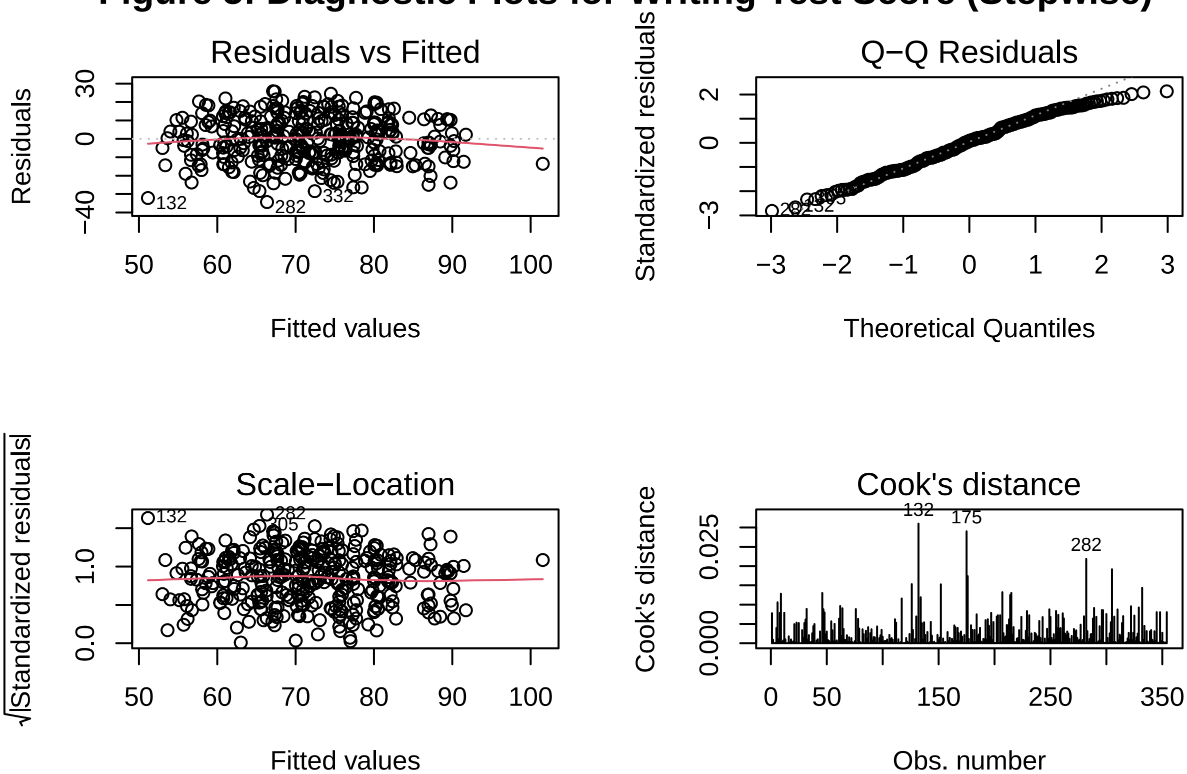

Figure 10: Boxcox Method

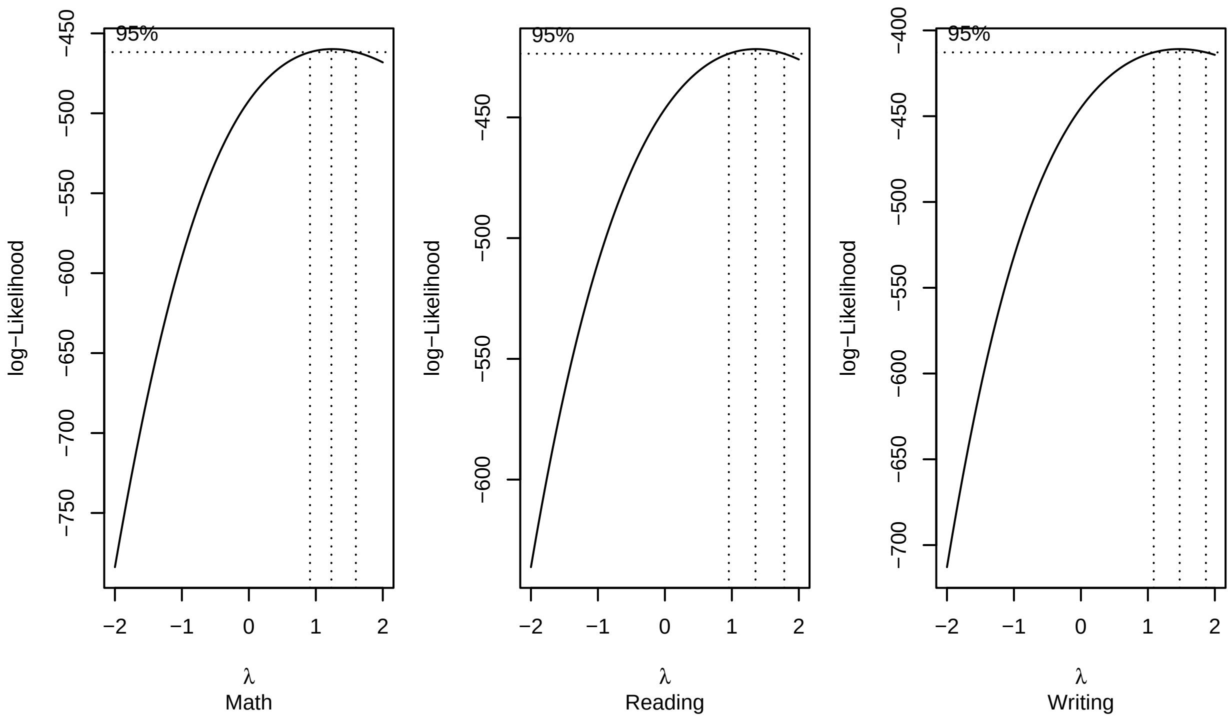

Figure 11: Cook's Distance