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NE 255

# Homework 04

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## Problem 1

(a) Given:

$$A = \begin{bmatrix} 0 & 3 \\ 4 & 4 \end{bmatrix}$$

The eigenvalues and eigenvectors are calculated as follows:

$$A\mathbf{x} = \lambda\mathbf{x}$$
$$\det(A - \lambda\mathbf{x}) = 0$$

$$\det(A - \lambda\mathbf{I}) = \det\left(\begin{bmatrix} 0 & 3 \\ 4 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$
$$\det\left(\begin{bmatrix} -\lambda & 3 \\ 4 & 4-\lambda \end{bmatrix}\right) = -\lambda(4-\lambda) - [(3)(4)] = 0$$
$$\lambda^2 - 4\lambda - 12 = 0 \quad \rightarrow \quad \boxed{\lambda_1 = 6, \lambda_2 = -2}$$

$$A\mathbf{x_1} = \lambda_1\mathbf{x_1}$$
$$(A - \lambda_1\mathbf{x_1}) = 0$$
$$\begin{bmatrix} -\lambda_1 & 3 \\ 4 & 4-\lambda_1 \end{bmatrix}\mathbf{x_1} = 0$$
$$\begin{bmatrix} -6 & 3 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x_{1,1} \\ x_{1,2} \end{bmatrix} = 0$$
$$-6x_{1,1} + 3x_{1,2} = 0$$
$$4x_{1,1} - 2x_{1,2} = 0$$
$$\mathbf{x_1} = c_1\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$A\mathbf{x_2} = \lambda_2\mathbf{x_2}$$
$$(A - \lambda_2\mathbf{x_2}) = 0$$
$$\begin{bmatrix} -\lambda_2 & 3 \\ 4 & 4-\lambda_2 \end{bmatrix}\mathbf{x_2} = 0$$
$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} x_{2,1} \\ x_{2,2} \end{bmatrix} = 0$$
$$2x_{2,1} + 3x_{2,2} = 0$$
$$4x_{2,1} - 6x_{2,2} = 0$$
$$\mathbf{x_2} = c_2\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

where $c_1$ and $c_2$ are arbitrary constants, and where the sign of the eigenvector is also arbitrary as long as the ratio between the two eigenvectors stays constant.

(b)

- The spectral radius of the above matrix is equal to the maximum absolute value of its eigenvalues, $\rho(A) = 6$.
- Spectral radii are important because they can be used to determine characteristics related to convergence for the power sequence of a matrix, such that:

$A \in \mathbf{C}^{nxn}$, $\rho(A)$, then
$\lim_{k\to\infty} A^k = 0 \iff \rho(A) < 1$, and
$\lim_{k\to\infty} ||A^k|| = \infty \iff \rho(A) > 1$

- Considering that the spectral radius is determined by the ratio of the scattering macroscopic cross section and the total macroscopic cross section, $c = \Sigma_s/\Sigma_t$, the largest possible spectral radius for an iteration matrix in nuclear engineering is 1. (This is unlikely, considering other forms of interaction exist besides scattering, but is mathematically plausible.)
- Considering the following systems:

| System | $\Sigma_s[cm^{-1}]$ | $\Sigma_t[cm^{-1}]$ | Spectral Radius, c = $\Sigma_s/\Sigma_t$ |
|---|---|---|---|
| 1 | 0.3 | 1.5 | 0.2 |
| 2 | 0.8 | 1.0 | 0.8 |
| 3 | 5.0 | 10.0 | 0.5 |

System 2 will converge the most slowly, and System 1 will converge the most quickly. (Systems with higher ratios of scattering will converge slower than their counterparts.)

(c) According to the Richard iteration scheme, where $\bar{Q}$ includes all sources and $k$ is the inner iteration index:

$$\mathbf{L}[\psi]_g^{k+1} = \mathbf{MS}[\phi]_g^k + [\bar{Q}]_g \ ,$$
$$[\phi]_g^{k+1} = \mathbf{D}[\psi]_g^{k+1} \ .$$

# Problem 2

See attached Python script.

# Problem 3

(a) Monte Carlo particle histories can run simultaneously because each particle interaction is independent of one another.
(b) Given the following properties:

$$\Sigma_s = 200cm^{-1}$$
$$\Sigma_f = 530cm^{-1}$$
$$\Sigma_a = 150cm^{-1}$$
$$\Sigma_s^{elastic} = 180cm^{-1}$$
$$\Sigma_s^{inelastic} = 20cm^{-1}$$

Probability of a defined collision-type within a material of x [cm] thickness can be found,

$$P_{col,i}(x) = 1 - e^{-\Sigma_i x}$$
$$P_{col,s}(x) = 1 - e^{-(200\ cm^{-1})(x\ cm)}$$
$$P_{col,f}(x) = 1 - e^{-(530\ cm^{-1})(x\ cm)}$$
$$P_s(inelastic) = \frac{P_{inelasic}}{P_s} \rightarrow \frac{20}{200} = 0.10$$
$$P_{col}(elastic) = \frac{P_{elastic}}{P_{col}} \rightarrow \frac{180}{200 + 530 + 150} \approx 0.20$$

# Homework 04

## Sources

### Problem 1

(a) From class notes: 05-eigenvalue/01-eigen
(b) From https://en.wikipedia.org/wiki/Spectral_radius
(b, c) From class notes: 04-operator/02-solution-iter

### Problem 2

From https://gist.github.com/louismullie/3769218

### Problem 3

From class notes: 06-monte-carlo/04-tracking