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Homework 05

Problem 1

(a) Considering the PDF:

$$p(x) = 3x^2 + x$$

on the interval $0 < x < a$.

$$\begin{aligned}
\int_{-\infty}^{\infty} f(x)dx &= \int_0^a 3x^2 + xdx \\
&= x^3 + \frac{1}{2}x^2|_0^a = 1 \\
a^3 + \frac{1}{2}a^2 - 0 &= 1 \\
a &\approx 0.858
\end{aligned}$$

Since $a > x$, and $x > 0, a$ is positive. Therefore p(x) is always positive, and is a valid PDF when a = 0.858.

(b) Considering the bounding function:

$$g(x) = 5x^{\frac{1}{2}}$$

Efficiency =

$$\begin{aligned}
\frac{\int p(x)dx}{\int g(x)dx} &= \\
\frac{1}{\int_0^{0.858} 5x^{\frac{1}{2}}dx} &= \frac{1}{2.65} \\
&= 0.377
\end{aligned}$$

(c) Rejection sampling is used to generate observations/ independent samples from a probability distribution that is unnormalized. An example is given below, and is shown in Figure 1.

$$\text{Bounding Function} = g(x) = 1.1$$
$$\text{PDF} = p(x) = sin(x), 0 < x < 1.6$$
$$\text{CDF} = F_X(x) = \begin{cases} 0 & x < 0 \\ 0.2 & 0 \leq x < 0.2 \\ 0.4 & 0.2 \leq x < 0.4 \\ 0.6 & 0.4 \leq x < 0.6 \\ 0.7 & 0.6 \leq x < 0.8 \\ 0.84 & 0.8 \leq x < 1.0 \\ 0.93 & 1.0 \leq x < 1.2 \\ 0.98 & 1.2 \leq x < 1.4 \\ 1.0 & 1.4 \leq x < 1.6 \end{cases}$$

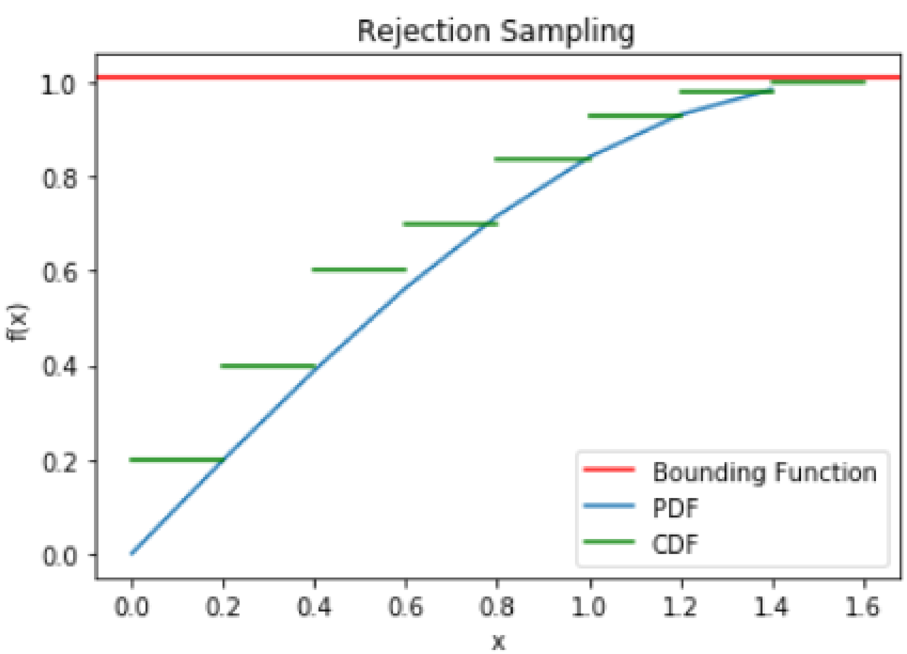

Figure 1: Plotting Bounding Function, PDF, and CDF

Problem 2

(a) Accuracy is defined as how close a measurement lies to the actual value, or "true" value. Precision is the amount that a repeated measurement will yield the same answer, under unchanged conditions. In the sense of a normalized curve, the measure of accuracy is where the peak of the curve lies, and the measure of precision is how wide the curve is. Accuracy can be improved by using better mathematical representations/models. With respect to Monte Carlo methods, precision can be improved by running more histories. Consider,

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i, \quad \lim_{N\to\infty} \bar{x} \to \mu$$

where N is the number of histories run in a simulation, $\bar{x}$ is the measured value, and μ is the actual value. Increasing N will increase precision.

Problem 3

Given the macroscopic cross sections as detailed in the homework description, the likeliness for each interaction can be mapped onto a 1-D space, as shown in Figure 2. Given 100,000 histories,

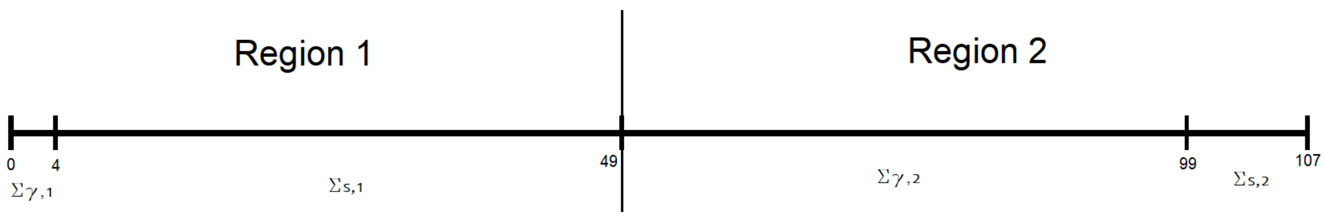

Figure 2: Particle interaction

53,097 underwent capture within region 2, and 18,939 underwent forward scattering within region 1. (See attached Python script.)

Sources

Problem 1

From class notes: 06-monte-carlo/03-sampling

Problem 2

From class notes: 06-monte-carlo/05-scoring-stats