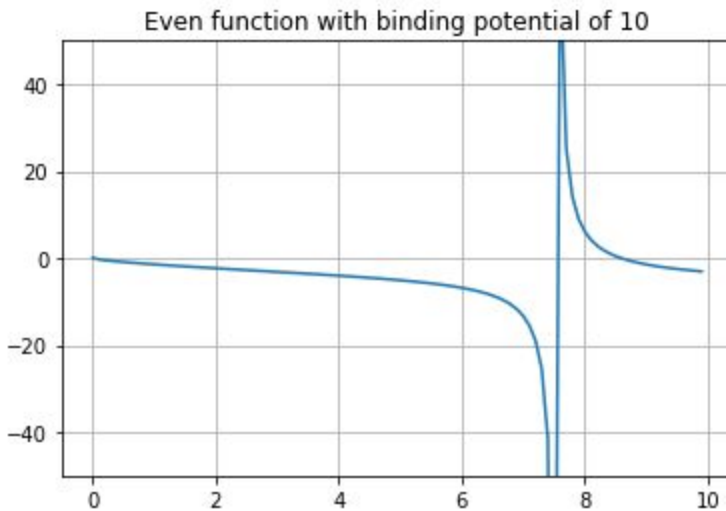


Jessica Hamilton
Computational
Exercise 15
Finding Bound State Energy

TO begin to determine the roots to this function, we must plot it first. Using this graph we can



Determine where the roots reasonably lie and how we can employ trial and error root finding methods to determine the precise $f(0)$ value.

The results of the functions using the Bisection method and the Newton/Raphson method are as follows:

Guess for the Bisection Method: 8.592785275230199

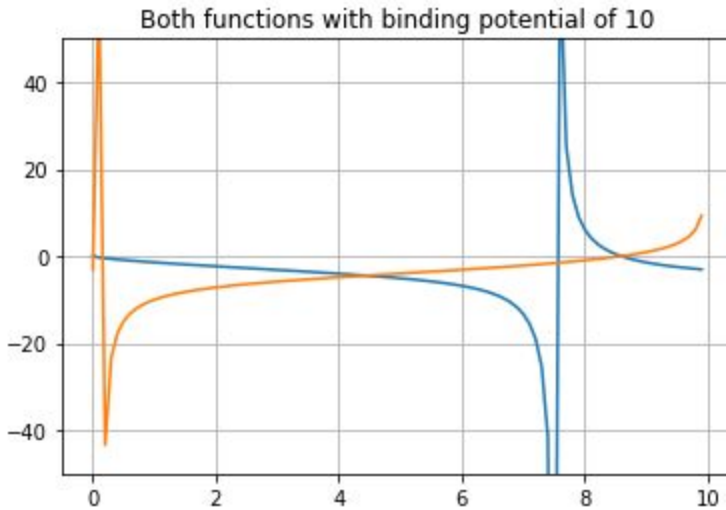
Guess for the Newton/Raphson Method: 8.592785275229831

When looking at the two results, there are quite similar to 10^{-8} decimal place. Then the question would be which is more accurate, both are precise to 10^{-8} . I then determined the precision by computing the function value with the newly determined value, found below.

The $f(E_b)$ values for the Bisection method result and the Newton/Raphson method:

-1.7155166176507919e-12 3.68594044175552e-14

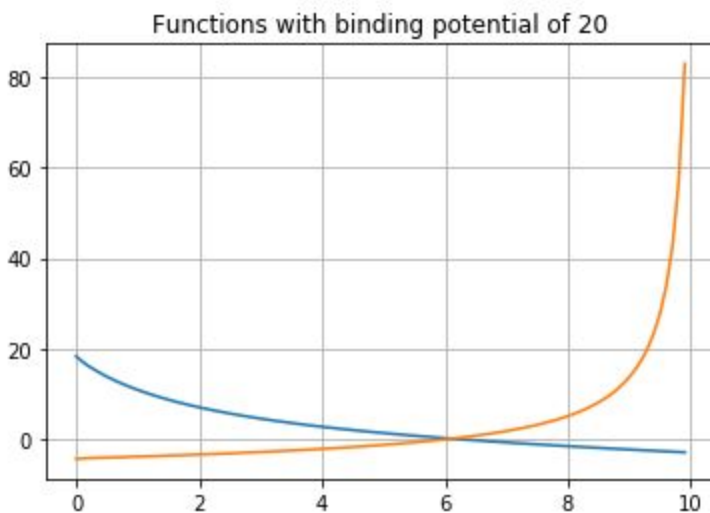
Based on the values determined by the two different methods, the second method, Newton/Raphson is more precise. This would be due to the fact that the result is two orders of magnitude smaller (closer to zero).

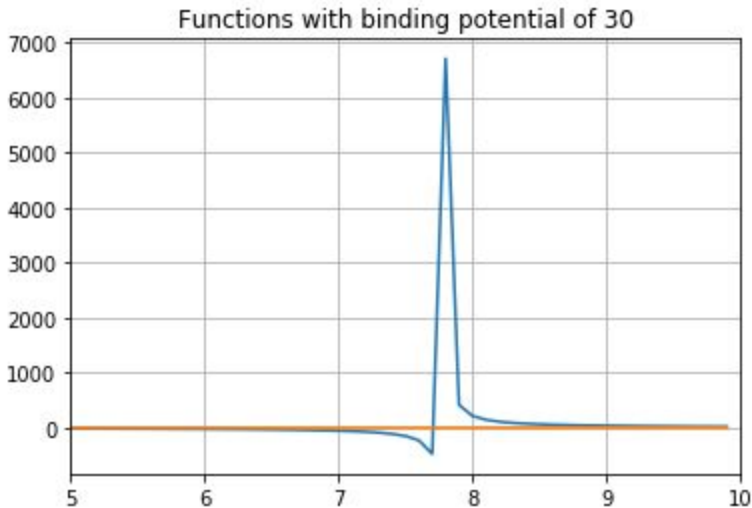


The graph above is showing the two different equations provided and how they are kind of like a mirror image of one another. This equation also does not extend deeply into negative values. I believe that the root is equal or relatively close to being equal to the original equation. The roots determined by both methods are below.

Guess for the Bisection and Newton/Raphson methods for equation two: 8.592785275230199
8.592785275229838

To explore the variation in the binding potential for 10 to 20 and even 30 we see the mirror effect of each equation. The binding potential of 20 is a better visual than the binding potential of 30. Once you change the value to 30, the even function actually goes to zero and the other function significantly peaks at around 8. Graphs and results are below.





Guess for bisection and Newton Methods with binding potential set to 20: 6.1084670175459905
6.10846701754763

Guess for bisection and Newton Methods with binding potential set to 30: 6.1084670175459905
6.10846701754763

The y values for the function with binding potential of 20: $1.6577850203702837e-12$
 $-2.220446049250313e-15$

The y values for the function with binding potential of 30: $-7.30526750203353e-13$
 $-1.3322676295501878e-15$

When looking at the root values for the function with the two different binding potentials, it seems the binding potential at 30 is more unrealistic and the values of the root does not make too much sense. I would say the best binding potential would be between 10 and 20. The values for the function are very close again, this time to the 10^{-10} . When looking at the values for $f(E_b)$, the bisection method seems to be better for the binding potential set to 20, since it is positive. The function with binding potential set to 30 seems to be fairly off due to the fact that the $f(E_b)$ values are both negative for both methods. With that being said over all (All functions) the Newton/Raphson method seems to be more precise.

Trying a different method in determining the roots of the equation provided, I decided to use the Ridder method. It seems the precision for the bisection and the ridder are on the same scale and the precision for the Newton/Raphson is better. This makes sense with the similarities between the bisection method and the ridder method. But the ridder tends to run faster than the bisection. This method uses the false position method a trial and error method but tends to have a better constraint on the tolerance. This method also determines that convergence is guaranteed such as the bisection method.

This is the guess for the Ridder Method: 8.592785275228836

again the bisection and Newtons/Raphson: 8.592785275230199 8.592785275229831

Respective precisions for Bisection, Newton/Raphson, Ridder: $-1.7155166176507919 \times 10^{-12}$
 $3.68594044175552 \times 10^{-14}$ $4.778399897986674 \times 10^{-12}$