Department of Physics

Indian Institute of Technology Kharagpur
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Subject No. PH41023 (Statistical Physics-I) Monday 26th November, 2018
Duration : 3 Hrs; Total Marks: 50 FN: 9 am to 12 Noon

End-Autumn Semester Examination-2018

§1. Write the expression for fugacity and draw a graph between fugacity versus temperature for FD and BE ideal gas. [2 Marks]

§2. Assume that the electrons inside a metal at zero Kelvin behave like a completely degenerate Fermi gas. Estimate $\frac{dN}{dE}$ and E_{max}.[2 Marks]

§3. N particles are distributed among three states having energies 0, k_BT, 2 k_BT. Estimate the value of N if the total equilibrium energy of the system is 1000 k_BT.[2 Marks]

§4. Find out the number of ways of distributing two identical particles in 3 different energy levels according to MB, BE and FD statistics? [2 Marks]

§5. What is Bose-Einstein (BE) condensation, write the conditions to have BE condensation. [2 Marks]

§6. The energy spectrum for an electron gas in a magnetic field is given by $\epsilon_{\pm}(\vec{k}) = \epsilon(\vec{k}) \pm \mu_B B$. The $\pm$ represent the down and up spin. Draw ϵ versus k at T=0 for B=0 and B>0 and explain. [2 Marks]

§7. The average energy per volume at a frequency ω from a black body is given by $\frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\beta\hbar\omega}-1}$. Estimate the energy flux from the cavity. (Given: $\int_0^\infty dx \, \frac{x^3}{e^x-1} = \frac{\pi^4}{15}$)[2 Marks]

§8. At what energy (in units of k_BT) is the Fermi function within 1 % of the Maxwell-Boltzmann distribution function? What is the probability of occupancy for MB function?[2 Marks]

§9. Write an expression for the electronic contribution and the lattice contribution to the molar heat capacity of a metal at low temperature. Draw a graph between C_V/T versus T^2. [2 Marks]

§10. At a fixed temperature, estimate the pressure for the Fermion- and the Boson gas. [2 Marks]

§11. Show that the equation of state $pV = \frac{2}{3}U$ holds for ideal quantum gas. If the energy spectrum, $\epsilon = c\hbar k$ then it still obey the same equation of state.[5 Marks]

§12. Write the probability distribution $P_i(n_i, T, \mu)$ for finding n_i particles in a given single-particle state for a system of non-interacting indistinguishable particles obeying BE, FD or MB statistics. Find the average occupation number $< n_i >$ and express P_i as a function of n_i and $< n_i >$ also estimate $\left(\frac{\Delta n_i}{<n_i>}\right)$ for BE, FD and MB particles.[5 Marks]

§13. Two non-interacting particles in a potential well give rise to energy spectrum $E(n) = n\epsilon$ because of the orbital motion of each particle. The n^{th} level having a degeneracy $2n+1$. Find the microcanonical partition function of the system at a fixed energy $E = N\epsilon$, for spin 0 (Bosons); 1/2 (Fermions) and any spin s (MB particle). [5 Marks]

§14. Derive van der Waal equation of state using Mayer function ($f(r) = e^{-\beta u(r)} - 1$) at room temperature, where $u(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$, Lenard-Jones potential. [5 Marks]

§15. Derive expression for classical and quantum heat capacity, C_V corresponding to the rotational modes for a diatomic molecules. Comment on your answer. [5 Marks]

§16. The grand potential (Ξ) of conduction electrons in the presence of magnetic field (B_z) is given by $\Xi = -k_BT\frac{m\,V}{h^2}\int_0^\infty dy\; L(y) \;+\; k_BT\frac{m\,V}{h^2}\frac{(\hbar\omega_c)^2}{24}\frac{dL(\mu)}{d\mu}$, where $\omega_c = \frac{eB}{mc}$ and

$L(x) = \int_{-\infty}^{\infty} dk_z ln\left[1 + e^{-\beta(\frac{\hbar^2 k_z^2}{2m} - x)}\right]; x = \mu - (n + \frac{1}{2})\hbar\omega_c.$

Estimate the magnetic susceptibility for the free electron and comment on your answer. [5 marks]