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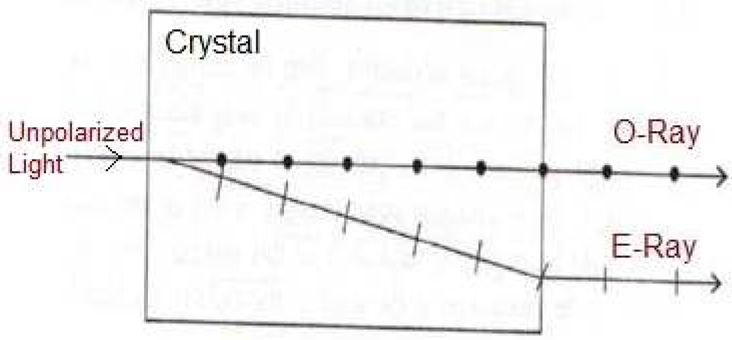
EXPERIMENT-6

Determination of refractive index of Quartz Prism for ordinary, extra-ordinary light

Aim: To determine the refractive indices μ_0 and μ_e of Calcite for ordinary and extraordinary ray using Spectrometer and Sodium light.

Apparatus: Standard Spectrometer, Sodium Light Source, Calcite Prism, Reading lens, Spirit Level.

Theory: Double refraction or Birefringence is a special optical property that is observed in a distinct type of anisotropic crystals such as calcite, quartz, KDP, rutile, etc. Both O-ray and E-ray are associated with this. When a single beam of unpolarized light is incident on such anisotropic crystal (e.g., Calcite), light is split into two beams (viz. o-ray and e-ray) at the crystal surface. Both O-ray and E-ray propagate in two different directions with different velocities. They have mutually orthogonal planes of polarization.



(Double Refraction phenomenon)

1. O-Ray | Ordinary ray
   As shown, the plane of polarization of o-ray is perpendicular to the plane of the figure. It is known as ordinary ray. It follows Snell's law of refraction. The velocity of o-ray i.e, v_o is c/n_o.
2. E-Ray | Extraordinary ray
   As shown, plane of polarization of o-ray is in the same plane as of figure. It is known as extraordinary ray. It does not follow Snell's law of refraction. The velocity of e-ray i.e., v_e is c/n_e.
   The velocities of o-ray and e-ray are different in all the directions except in one particular direction known as optic axis. In this optic-axis, both o-ray and e-ray travel with same velocities. The effective refractive indices no and ne are known as principal refractive indices of the crystal. In the direction perpendicular to the optic-axis there is a maximum difference in the velocity of e-ray and o-ray.

Observations:

Table-1: Vernier constant of the spectrometer

Divisions (say, m) of the vernier scale = Divisions (say, n) of the vernier scale

Value of 1 smallest main scale division (l_1) (Min. or sec.)	Vernier constant $= \frac{1MSD}{Total\ no.of\ VSD}$
$l_1 = 20'$	$\therefore VC = \frac{20'}{60} = 20'' = \delta_s$

Table-2: Angle of Prism

Sr. No.	Vernier	Telescope reading for reflection from 1st surface			Telescope reading for reflection from 2nd surface			Difference $(a \sim b) = 2A$	A	Mean A	Mean value of Angle of prism
		MSR	VSR	Total (a)	MSR	VSR	Total (b)				
1	V1	44°20′	0	44°20′	283°20′	29	283°29′40″	120°50′20″	60°25′10″	60°17′35″= 60.29°	60.19°
	V2	224°20′	0	224°20	104°0′	0	104°	120°20′	60°10′0″		
2	V1	43°0′	0	43°	283°0′	0	283°	120°	60°	60°0′50″ = 60.01°	
	V2	222°40′	55	222°58′20″	102°40′	45	102°55′0″	120°3′20″	60°1′40″		
3	V1	42°40′	58	42°59′20″	282°20′	45	282°35′0″	120°24′20″	60°12′10″	60°4′35″ = 60.26°	
	V2	222°40′	0	222°40′0″	102°40′	18	102°46′0″	119°54′0″	59°57′0″		

Table-3: Angle of minimum deviation for ordinary ray (O-Ray)

Sr. no.	Vernier no.	Dispersed image in telescope at minimum deviation position			Telescope reading for direct image			Difference $(a \sim b)$	Mean deviation δ_m degree
		MS	VS	Total (a)	MS	VS	Total (b)		
1	V1	29°40′	10	29°43′20″	348°	0	348°	41°43′20″	41°32′0″ = 41.53°
	V2	208°40′	49	208°56′20″	168°	0	168°	40°56′20″	
2	V1	29°40′	0	29°40′	348°	0	348°	41°40′0″	
	V2	209°20′	47	209°35′40″	168°	0	168°	41°35′40″	
3	V1	29°40′	0	29°40′	348°	0	348°	41°40′0″	
	V2	209°20′	50	209°36′40″	168°	0	168°	41°36′40″	

Table-4: Angle of minimum deviation for extra ordinary ray (E-Ray)

Sr. no.	Vernier no.	Dispersed image in telescope at minimum deviation position			Telescope reading for direct image			Difference $(a \sim b)$	Mean deviation δ_m degree
		MS	VS	Total (a)	MS	VS	Total (b)		
1	V1	28°40′	21	28°47′0″	348°	0	348°	40°47′0″	39°54′27″ = 39.90°
	V2	208°40′	0	208°40′0″	168°20′	0	168°20′0″	40°20′20″	
2	V1	27°40′	0	27°40′	348°	0	348°	39°40′0″	
	V2	207°20′	50	207°36′40″	168°20′	0	168°20′	39°16′40″	
3	V1	27°40′	47	27°55′40″	348°	0	348°	39°55′40″	
	V2	207°40′	21	207°47′0″	168°20′	0	168°20′	39°27′0″	

Sr. no.	Vernier no.	Dispersed image in telescope at minimum deviation position			Telescope reading for direct image			Difference $(a \sim b)$	Mean deviation δ_m degree
		MS	VS	Total (a)	MS	VS	Total (b)		
1	V1	29°40′	10	29°43′20″	348°	0	348°	41°43′20″	41°32′0″ = 41.53°
	V2	208°40′	49	208°56′20″	168°	0	168°	40°56′20″	
2	V1	29°40′	0	29°40′	348°	0	348°	41°40′0″	

	V2	209°20′	47	209°35′40″	168°	0	168°	41°35′40″	
3	V1	29°40′	0	29°40′	348°	0	348°	41°40′0″	
	V2	209°20′	50	209°36′40″	168°	0	168°	41°36′40″	

Sr. no.	Vernier no.	Dispersed image in telescope at minimum deviation position			Telescope reading for direct image			Difference $(a \sim b)$	Mean deviation δ_m degree
		MS	VS	Total (a)	MS	VS	Total (b)		
1	V1	29°40′	10	29°43′20″	348°	0	348°	41°43′20″	41°32′0″ = 41.53°
	V2	208°40′	49	208°56′20″	168°	0	168°	40°56′20″	
2	V1	29°40′	0	29°40′	348°	0	348°	41°40′0″	
	V2	209°20′	47	209°35′40″	168°	0	168°	41°35′40″	
3	V1	29°40′	0	29°40′	348°	0	348°	41°40′0″	
	V2	209°20′	50	209°36′40″	168°	0	168°	41°36′40″	

Calculation:

Refractive index for O-Ray

$$\mu_0 = \frac{\sin sin\left(\frac{A+\delta_m}{2}\right)}{\sin sin\,\frac{A}{2}} = \frac{\sin sin\left(\frac{60.19+41.53}{2}\right)}{\sin sin\,\frac{60.19}{2}} = 1.5467$$

Refractive index for E-Ray

$$\mu_e = \frac{\sin sin\left(\frac{A+\delta_m}{2}\right)}{\sin sin\,\frac{A}{2}} = \frac{\sin sin\left(\frac{60.19+39.90}{2}\right)}{\sin sin\,\frac{60.19}{2}} = 1.5287$$

Error Analysis:

The angle of the prism (A) is measured from the difference of the readings for two positions of the telescope and dividing the difference by 2. So the error in the measurement of A is only one vernier constant i.e., $\Delta A=\delta_s$, where δ_s is the value of one vernier constant. Also, the minimum deviation δ_m is measured from the difference of the readings for two positions of the telescope. Hence the error in measurement of δ_m is the two vernier constant i.e., $\Delta\delta_m=2\delta_s$

The refractive index μ_0 of the material of the prism for ordinary ray.

$$\mu_0 = \frac{\sin sin\left(\frac{A+\delta_m}{2}\right)}{\sin sin\,\frac{A}{2}}$$

$$\ln ln\,\mu_0 = \ln ln\ \sin sin\left(\frac{A+\delta_m}{2}\right) + \ln ln\ \sin sin\left(\frac{A}{2}\right)$$ (Considering the maximize error)

$$\frac{\Delta\mu_0}{\mu_0} = \cot cot\left(\frac{A+\delta_m}{2}\right) \times \left(\frac{\Delta A}{2} + \frac{\Delta\delta_m}{2}\right) + \cot cot\,\frac{A}{2} \times \frac{\Delta A}{2}$$

Or, $\frac{\Delta\mu_0}{\mu_0} = \frac{1}{2}\cot cot\left(\frac{A+\delta_m}{2}\right) \times (\Delta A + 2\Delta A) + \cot\frac{1}{2}cot\,\frac{A}{2} \times \Delta A$

Since, $\left|\Delta\delta_m\right| = 2|\Delta A| = 2\delta_s$

And, $\delta_s = 20 = 9.691\times 10^{-5}\,rad$

$$\frac{\Delta\mu_0}{\mu_0} = \frac{3}{2}\cot cot\left(\frac{A+\delta_m}{2}\right) \times \delta_s + \cot\frac{1}{2}cot\ \frac{A}{2}\ \times \delta_s$$

$$or, \Delta\mu_0 = \cot\frac{3}{2}cot\left(\frac{A+D_r}{2}\right) \times \delta_s \times \mu_0 + \frac{1}{2}\cot cot\ \frac{A}{2}\ \times \delta_s \times \mu_0$$

For red light:

$$\Delta\mu_0 = \cot\frac{3}{2}cot\left(\frac{60.19+41.53}{2}\right)\times 9.691\times 10^{-5}\times 1.5467 + \cot\frac{1}{2}cot\ \frac{60.19°}{2}\ \times 9.691\times 10^{-5}\times 1.5467$$

$$= 3.107\times 10^{-4}$$

Similarly,

$$\Delta\mu_e = \cot\frac{3}{2}cot\left(\frac{60.19+39.90}{2}\right)\times 9.691\times 10^{-5}\times 1.5287 + \cot\frac{1}{2}cot\ \frac{60.19°}{2}\ \times 9.691\times 10^{-5}\times 1.5287$$

$$= 3.139\times 10^{-4}$$

Results:

1. The refractive index for ordinary ray (n_o) = 1.5467 $\pm$ 3.107×10^{-4}
2. The refractive index for extra-ordinary ray (n_e) = 1.5287 $\pm$ 3.13910^{-4}
3. The difference [(n_e) - (n_o)] = 0.018 $\pm$ 0.03210^{-4}

Precaution:

1. The telescope and collimator should be individually set for parallel rays by Schuster's method.
2. Slit should be made as narrow as possible in order to avoid diffraction.
3. Both windows should be read.
4. Prism should be properly placed on the prism table for measurement of the angle of the prism as well as for the angle of minimum deviation.
5. Zero error must be noted in the measuring instruments.
6. To reduce statistical error in measurements, at least 3-5 readings must be taken.
7. Parallax and back-lash errors during measurement must be avoided.