Class Test

§1. Let $P(x) = C\, e^{\frac{-x^2}{2a^2}}$ where C and a are constants. Calculate $\langle x \rangle$ and $\langle x^2 \rangle$ given this probability distribution.

§2. Gas molecules of mass m are confined in a cylinder of radius R and height L (with R >> L) kept vertically in the Earth's gravitational field. The average energy of the gas at low temperatures (such that mgL >> k_B T) is given by

§3. The free energy of the gas of N particles in a volume V and at a temperature T is

$F = Nk_BT\, ln[a_0 V(k_BT)^{5/2}/N],$

where a_0 is a constant and Bk denotes the Boltzmann constant. The internal energy of the gas is

§4. Consider an ideal gas whose entropy is given by

$$S = \tfrac{n}{2}\left[\, \sigma + 5Rln\tfrac{U}{n} + 2Rln\tfrac{V}{n}\,\right],$$

where n is the number of moles, σ is a constant, R is the universal gas constant, U is the internal energy and V is the volume of the gas. The specific heat at constant pressure is then given by