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EXPERIMENT-4

Determination of Cauchy's constants

Aim: To determine Cauchy's constants of the given prism using a mercury vapor lamp.

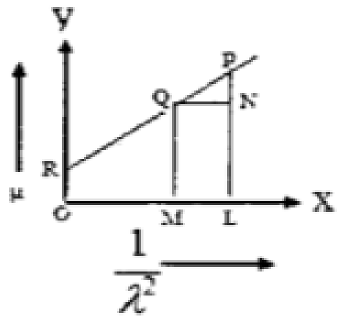
Apparatus: Spectrometer, Prism, Mercury Vapor Lamp, Prism clamp, Magnifying glass.

Theory: The refractive index of a material depends upon wavelength. Cauchy gave an empirical relation between refractive index (μ) and wavelength (λ) which is:

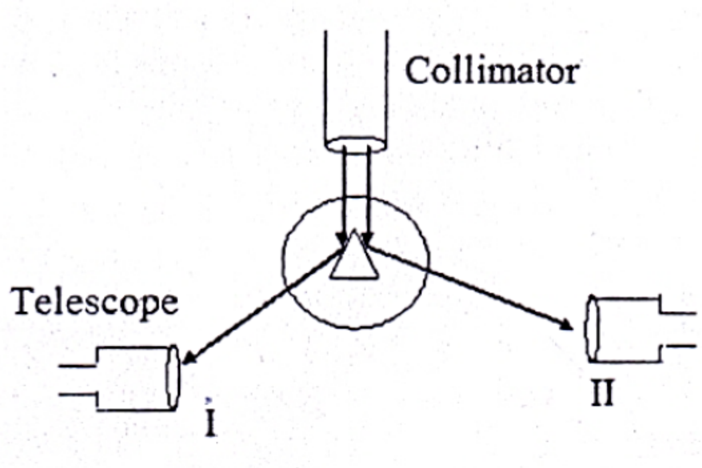
$$\mu = A + \frac{B}{\lambda^2}$$

Here, A and B are known as Cauchy's constants

This equation is of the form: y = mx + c. Hence, plotting a between μ and $\frac{1}{\lambda^2}$ would give us the constants A and B. The value of Cauchy's constant A is given by the intercept on the y-axis and the value of B is given by the slope of the line.



From the graph above, we get - OR = A and B = $tan\theta = \frac{PN}{QN}$



Apparatus set up

Working Formula-

The refractive index of the material at various wavelengths can be determined by the formula shown below:

$$\mu = \frac{(\frac{A+\delta_m}{2})}{sin\frac{A}{2}} \quad (1)$$

- A = Angle of the prism

$$\mu = A + \frac{B}{\lambda^2} \qquad (2)$$

Observations:

Table-1: Vernier constant of the spectrometer

Divisions (say, m) of the vernier scale = Divisions (say, n) of the vernier scale

Value of 1 smallest main scale division (l_1) (Min. or sec.)	Value of 1 vernier division $\left(l_2 = \frac{n}{m} \times l_1\right)$ (Min. or sec.)	Vernier constant $= (l_1 - l_2)$ (Min. or sec.)
$l_1 = 30'$	$l_2 = \frac{59}{60} \times 30' = \left(\frac{59}{2}\right)'$	Vernier constant $= \left(30' - \left(\frac{59}{2}\right)'\right) = 30'' = \delta_s$

Table-2: ANGLE OF PRISM

Vernier no.	Reading at the 1st position of the telescope		Reading at the 2nd position of the telescope						2A	Mean 2A = (a+b)/2 = C	A = C/2
	Main scale (M)	Vernier scale (V)	Total (T) = M + V	Mean T	Main scale (M)	vernier scale (V)	Total (T) = M + V	Mean T = b			
1	31°	35	31°17′30′′	31°14′0″ $(= a_1)$	152°	21	152°10′30″	152°24′30″ $(= a_2)$	121°10′30″ $= a_2 - a_1 = a$	121°17′22.5″	60°38′52.5″ = 60.64°
	31°	21	31°10′30′′		151°	77	151°38′30″				
2	210°	9	210°4′30′′	210°5′45′′ $(= b_1)$	331°	60	331°30′0′′	331°35′15″ $(= b_2)$	121°24′15″ $= b_2 - b_1 = b$		
	211°	14	211°7′0″		331°	81	331°40′30″				

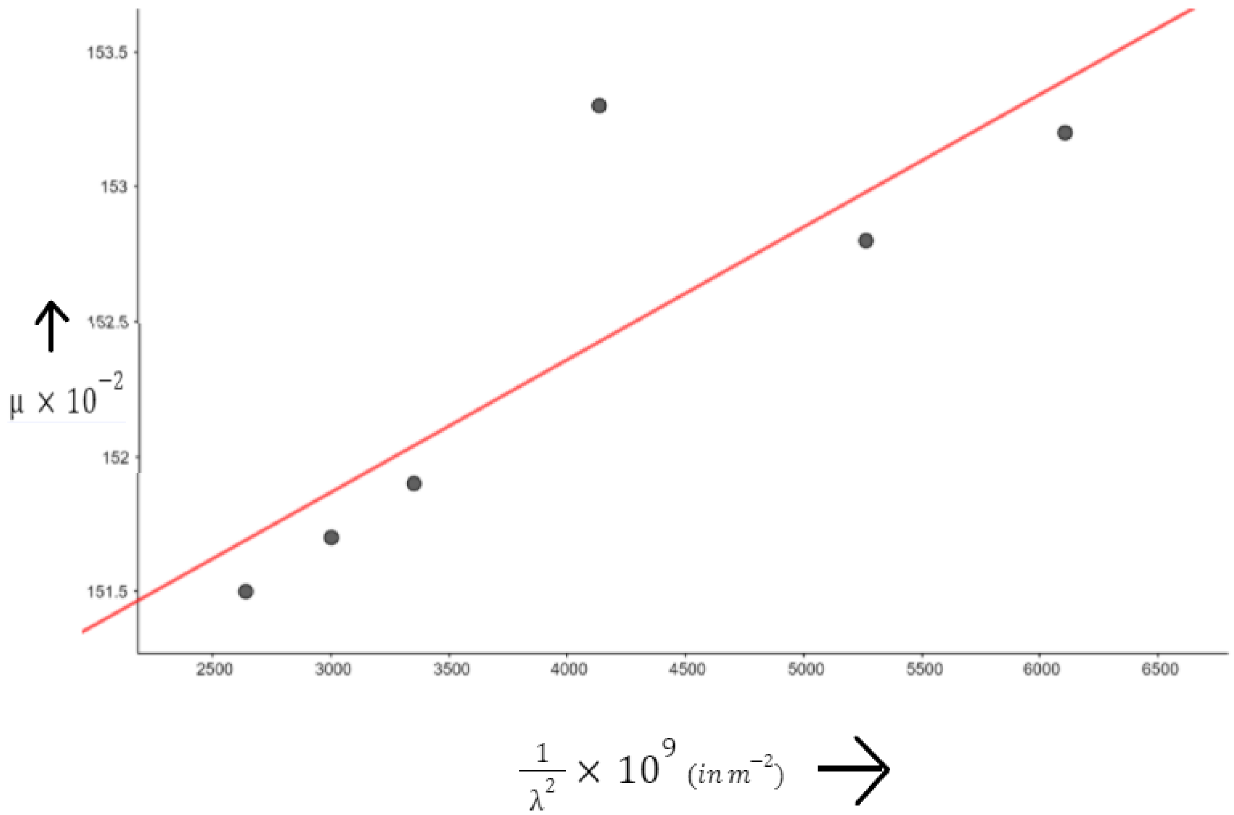
Table-3: Refractive index of the prism for the different spectral lines of mercury source

Direct reading of the image for left vernier $D_1 = 90°20'$

Direct reading of the image for right vernier $D_2 = 270°20'30''$

Colour of spectral line	Wavelength λ $(in\ A°)$	$\frac{1}{\lambda^2}$ $(in\ m^{-2})$	Vernier V_1 Telescope readings: Reading for min. deviation position (T_1)	D_1	Min. dev. $\delta_1 = T_1 - D_1$	Vernier V_2 Telescope readings: Reading for min. deviation position (T_1)	D_2	Min. dev. $\delta_2 = T_2 - D_2$	Mean dev. $\delta_m = \frac{\delta_1 + \delta_2}{2}$	$\mu = \frac{\sin\frac{(A+\delta_m)}{2}}{\sin\frac{A}{2}}$
Violet	4046.8	6106277224653.8	131°3′	90°20′	40°43′0″	311°4′	270°20′30″	40°43′30″	40°43′15″	1.532
Blue	4358.3	5264604538138.7	130°44′	90°20′	40°24′0″	310°45′	270°20′30″	40°14′30″	40°19′15″	1.528
Blue green	4916.6	4136854500550.8	130°7′	90°20′	39°47′0″	310°8′	270°20′30″	39°47′30″	40°47′15″	1.533
Green	5460.7	3353539021873.5	129°52′	90°20′	39°32′0″	309°51′	270°20′30″	39°30′30″	39°31′15″	1.533
Yellow	5769.9	3003747534598	129°41′	90°20′	39°21′0″	309°40′	270°20′30″	39°19′30″	39°20′15″	1.517
Orange	6152.0	2642210088254	129°32′	90°20′	39°12′0″	309°35′	270°20′30″	39°14′30″	39°13′15″	1.515

Graph:



<u>Scale:</u>

X-axis: 1 unit = 10^9 m^{-2}

Y-axis: 1 unit = 10^{-2}

<u>Calculations:</u>

Using the least squared method, we can find the best-fit curve-

$$\text{Slope (m)} = \frac{\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i\right)}{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$\text{Intercept (b)} = \frac{\sum_{i=1}^{n} y_i - m\sum_{i=1}^{n} x_i}{n}$$

where n = number of datapoints

Upon substituting the values of the datapoints in the above equations, we get the slope (m) as 4.917×10^{-15} and the intercept (b) as 1.5039 which are the same as those obtained from the graph.

Therefore, from the graph, the values of Cauchy's constants are-

$$A = \text{intercept (b)} = 1.5039$$
$$B = \text{slope (m)} = 4.917 \times 10^{-15}\ \text{m}^2$$

These constants can also be calculated considering the two refractive indices μ_1 and μ_2 of the prism and corresponding wavelengths λ_1 and λ_2 respectively.

Now,

$$\mu_1 = A + \frac{B}{\lambda_1^2}$$
$$\mu_2 = A + \frac{B}{\lambda_2^2}$$

From these equations, we get-

$$B = \frac{(\mu_1 - \mu_2)\lambda_1^2\lambda_2^2}{\lambda_2^2 - \lambda_1^2}$$
$$A = \mu_1 - \frac{(\mu_1 - \mu_2)\lambda_2^2}{\lambda_2^2 - \lambda_1^2} = \mu_2 - \frac{(\mu_1 - \mu_2)\lambda_1^2}{\lambda_2^2 - \lambda_1^2}$$

Therefore, using the readings 1 and 2 (violet and blue) from table-3, we get -

A = 1.5112

B = 0.0065346078 x 10^{-12} m^2

Error Analysis:

Working Formula-

The values of Cauchy's constants can be calculated using the following formula-

$$\mu = A + \frac{B}{\lambda^2}$$

Differentiating the above equation, we get-

$$\Delta\mu = \Delta A + \frac{\Delta B}{\lambda^2}$$ (Since, λ is given)

ΔA to be order of $\Delta\mu$ and ΔB is the order of $\Delta\mu\lambda^2$

$$\ln\mu = \ln\sin\left(\frac{\theta+\delta_m}{2}\right) + \ln\sin\frac{\theta}{2}$$
$$\frac{\Delta\mu}{\mu} = \cot\left(\frac{\theta+\delta_m}{2}\right)\cdot\left(\frac{\Delta\theta}{2} + \frac{\Delta\delta_m}{2}\right) + \cot\frac{\theta}{2}\cdot\frac{\Delta\theta}{2}$$
$$\frac{\Delta\mu}{\mu} = \frac{1}{2}\cot\left(\frac{\theta+\delta_m}{2}\right)\cdot(\Delta\theta + 2\Delta\theta) + \frac{1}{2}\cot\left(\frac{\theta}{2}\right)\cdot\Delta\theta$$

Since $|\Delta\delta_m| = 2|\Delta\theta|$

And $|\Delta\theta| = 30'' = 1.454\,X\,10^{-4} rad$

$$\therefore \frac{\Delta\mu}{\mu} = \frac{3}{2}\cot\left(\frac{\theta+\delta_m}{2}\right)\cdot\Delta\theta + \frac{1}{2}\cot\left(\frac{\theta}{2}\right)\cdot\Delta\theta$$
$$\Delta\mu = \frac{3}{2}\cot\left(\frac{\theta+\delta_m}{2}\right)\cdot\Delta\theta\cdot\mu + \frac{1}{2}\cot\left(\frac{\theta}{2}\right)\cdot\Delta\theta\cdot\mu \quad \text{— (i)}$$

Given the least count is 1V in radians = 1 arc minute = 0.000291 radian.

$$\mu = \frac{\sin\frac{A+d_m}{2}}{\sin\frac{A}{2}}$$

Taking log on both sides of the above equation and then differentiating, we get-

$$\frac{\Delta\mu}{\mu} = \cot\frac{A + d}{2}\left(\frac{\Delta A}{2} + \frac{\Delta d}{2}\right) + \frac{\Delta A}{2}\cot\frac{A}{2}$$

And we know that-

$\Delta d = 2\,\Delta\theta = 2 \times 1\,V\ in\ radians$

$\Delta A = \Delta\theta = 1V\ in\ radians$

<u>For Violet Light,</u>

On substituting the values in equation (i), we get-

$\Delta\mu = 0.000468533$

$\frac{\Delta\mu}{\mu} = 3.05831 \times 10^{-4}$

Therefore, the percentage error in μ is 3.05831×10^{-2}
The error in μ is 0.000468533
The refractive index of violet color is 1.532

Therefore, the error in the Cauchy's constants are -
$\Delta A \approx 0.000468533$ and $\Delta B = 0.000468533 \times (4046.8 \times 10^{-10})^2 \approx 7.627 \times 10^{-17} m^2$

Similarly, ΔA and ΔB can be calculated for all other spectral lines.

Results:

1. The values of Cauchy's constants are -
   a. A = 1.5039 with a percentage error of 0.0306% , i.e, A = 1.5039 $\pm$ 0.0004
   b. $B = 4.917 \times 10^{-15}\ m^2$ with an error of $0.076 \times 10^{-15}\ m^2$,
   i.e,

$B = (4.917 \pm 0.076) \times 10^{-15}\ m^2$

2. The angle of prism is 60.64°

Precaution:

1. Spectrometer levelling and adjustments should be properly done.
2. The slit should be sharp and vertical.
3. The position of angle of minimum deviation should be accurately determined.
4. The refracting surfaces of the prism should not be touched with fingers.
5. Zero error must be noted in the measuring instruments.
6. To reduce statistical error in measurements, at least 3-5 readings must be taken.
7. Parallax and back-lash errors during measurement must be avoided.
8. The telescope and the collimator should be separately set for parallel rays.