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General Properties of Matter Lab
PH29001

EXPERIMENT-7

Measurement of acceleration due to gravity (g) by a compound pendulum

Aim: (i) To determine the acceleration due to gravity (g) by means of a compound pendulum. (ii) To determine the radius of gyration about an axis through the center of gravity for the compound pendulum.

Apparatus: (i) A bar pendulum, (ii) a knife-edge with a platform, (iii) a spirit level, (iv) a precision stopwatch, (v) a meter scale, and (vi) a telescope.

Theory: A simple pendulum consists of a small body called a “bob” (usually a sphere) attached to the end of a string the length of which is great compared with the dimensions of the bob and the mass of which is negligible in comparison with that of the bob. Under these conditions, the mass of the bob may be regarded as concentrated at its center of gravity, and the length of the pendulum is the distance of this point from the axis of suspension. When the dimensions of the suspended body are not negligible in comparison with the distance from the axis of suspension to the center of gravity, the pendulum is called a compound, or physical, pendulum. A rigid body mounted upon a horizontal axis so as to vibrate under the force of gravity is a compound pendulum.In Fig.1 a body of irregular shape is pivoted about a horizontal frictionless axis through P and is displaced from its equilibrium position by an angle θ. In the equilibrium position, the center of gravity G of the body is vertically below P. The distance GP is l and the mass of the body is m.

The restoring torque for an angular displacement θ is

$$\tau = - mg\, l \sin\theta \;\ldots(1)$$

For small amplitudes (θ≈0) -

$$\frac{d^2\theta}{dt^2} =- mgl\theta \;\ldots (2)$$

where I is the moment of inertia of the body through the axis P. Eq-(2) represents a simple harmonic motion and hence the time period of oscillation is given by-

$$T = 2\pi\sqrt{\frac{I}{mgl}} \;\ldots\ldots\ldots (3)$$

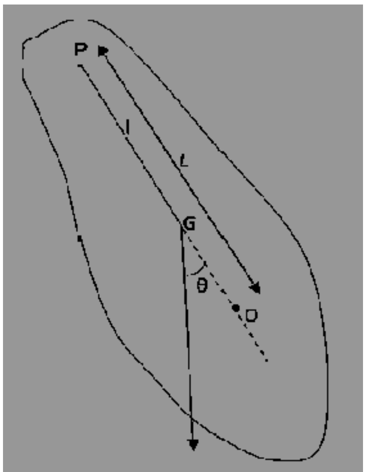


Fig. 1

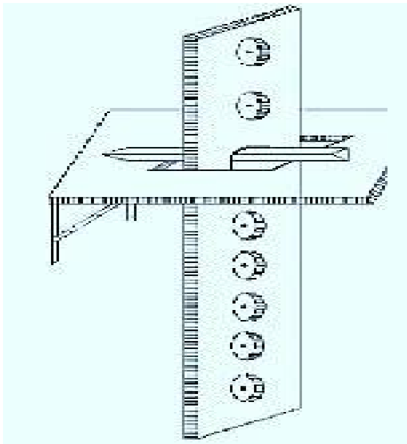


Fig. 2

$I = I_G + ml^2$, where I_G is the moment of inertia of the body about an axis parallel with the axis of oscillation and passing through the center of gravity G.

$$I_G = mK^2 \quad \dots (4)$$

where K is the radius of gyration about the axis passing through G. Thus,

$$T = 2\pi\sqrt{\frac{mK^2+ml^2}{mgl}} = 2\pi\sqrt{\frac{\frac{K^2}{l}+l^2}{g}} \quad \dots (5)$$

The time period of a simple pendulum of length L is given by-

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \dots (6)$$

Comparing with equation (5) we get -

$$L = \frac{K^2}{l} + l \quad \dots (7)$$

This is the length of the "equivalent simple pendulum". If all the mass of the body were concentrated at a point O (See Fig.1) such that, OP $= \frac{K^2}{l} + l$, we would have a simple pendulum with the same time period. Point O is called the 'Centre of Oscillation'. Now from Eq. (7), we get - $l^2 - lL + K^2 = 0 \quad \dots (8)$ and the roots of equation (8) are l_1 *and* l_2 such that $l_1 + l_2 = L$ and $l_1 l_2 = K^2 \quad \dots (9)$

Thus both l_1 *and* l_2 are positive. This means that on one side of C.G there are two positions of the center of suspension about which the time periods are the same. Similarly, there will be a pair of positions of the center of suspension on the other side of the C.G about which the time periods are to be the same. Thus there are four positions of the centers of suspension, two on either side of the C.G, about which the time periods of the pendulum would be the same. The distance between two such positions of the centers of suspension, asymmetrically located on either side of C.G, is the length L of the simple equivalent pendulum. Thus, if the body was supported on a parallel axis through point O (see Fig. 1), it would oscillate with the same time period T as when supported at P. Now it is evident that on either side of G, there are infinite numbers of such pair of points satisfying Eq. (9).

If the body is supported by an axis through G, the time period of oscillation would be infinite. From any other axis in the body, the time period is given by Eq. (5). From Eq. (6) and (9), the values of g and K are given by - $g = 4\pi^2 \frac{L}{T^2}$ ... (10) $and\ K = \sqrt{l_1 l_2}$ ... (11)

By determining $L,\ l_1\ and\ l_2$ graphically for a particular value of T, the acceleration due to gravity g at that place and the radius of gyration K of the compound pendulum can be determined. On plotting a graph of Time Period (T) vs Distance (d), we get the following graph-

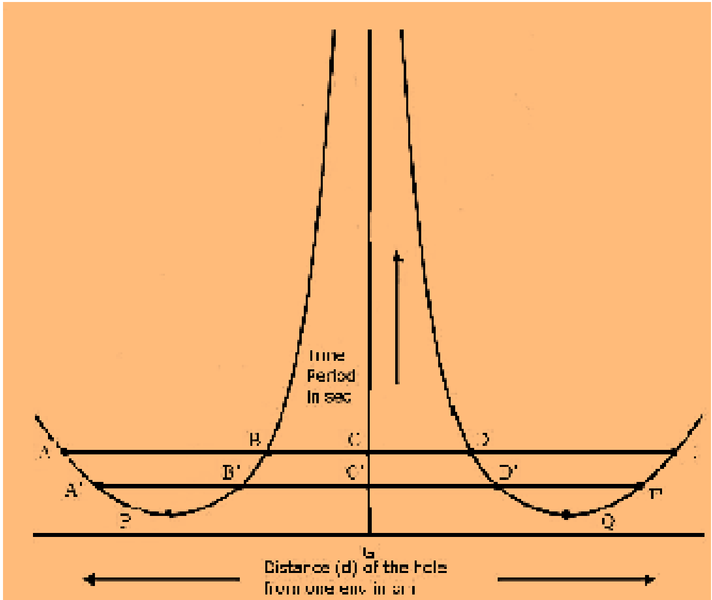


Fig. 3

Observations:

- Least count of meter-scale = 0.1 cm.
- Least count of stopwatch = 0.01 second.
- The distance between two holes is 5cm.

Table-1-

<u>Data for the Time period(T) versus Distance(d) graph</u>

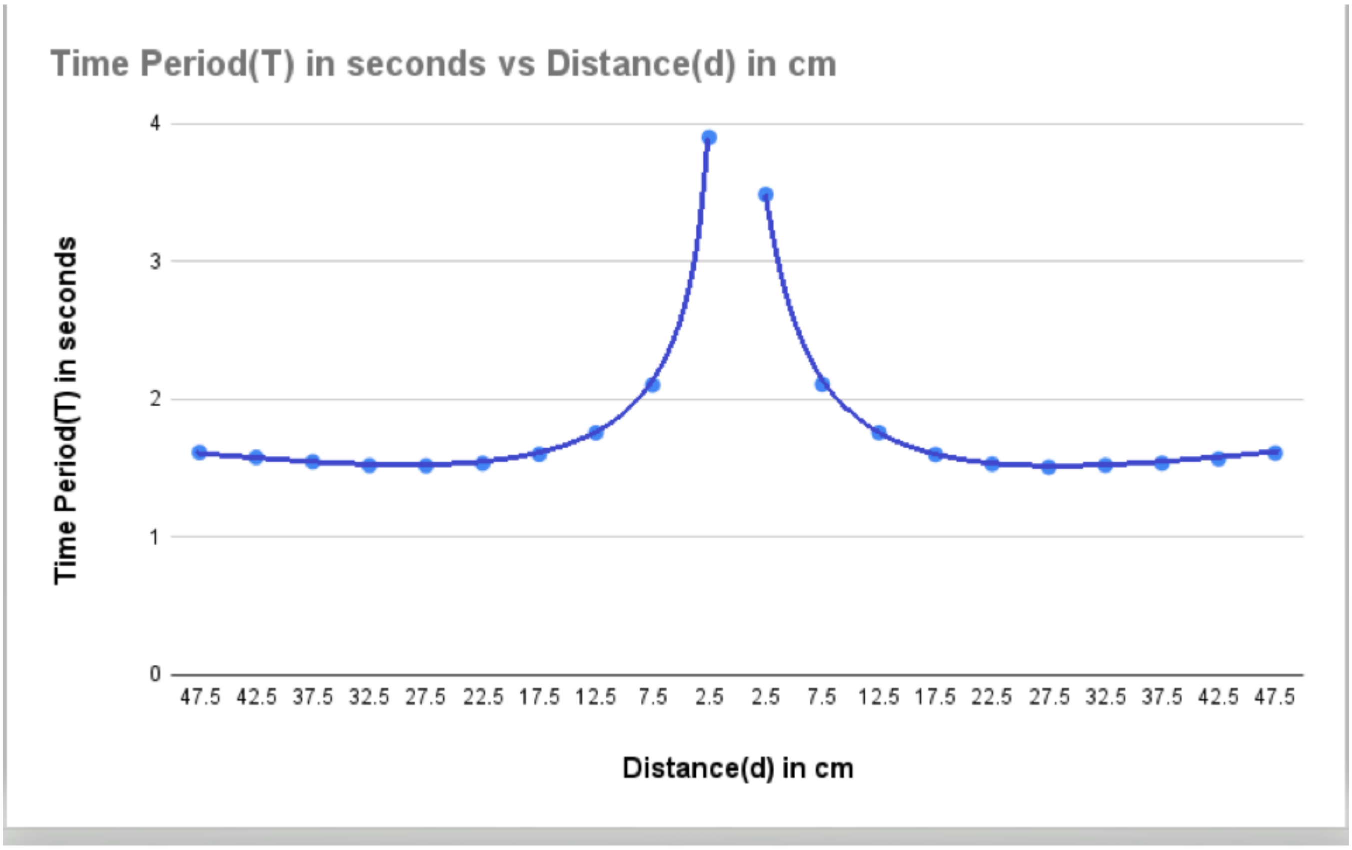
Serial no of holes C.G		**Distance (d) of the hole from C.G (cm)**	**Time for 30 oscillations (sec)**	**Mean time t for 30 oscillations (sec)**	**Time period T = t/30 (sec)**
One side of	01	47.5	(i)48.47 (ii)48.37 (iii)48.53	48.45	1.615
	02	42.5	(i)47.32 (ii)47.44	47.39	1.579

C.G			(iii)47.41		
	03	37.5	(i)46.47 (ii) 46.47 (iii) 46.37	46.436	1.547
	04	32.5	(i) 45.68 (ii) 45.53 (iii) 45.63	45.613	1.520
	05	27.5	(i) 45.68 (ii) 45.28 (iii) 45.68	45.546	1.518
	06	22.5	(i) 46.09 (ii) 46.03 (iii) 46.06	46.060	1.535
	07	17.5	(i) 48 (ii) 47.94 (iii) 48.13	48.023	1.601
	08	12.5	(i) 52.91 (ii) 52.59 (iii) 52.63	52.71	1.757
	09	7.5	(i) 63.09 (ii) 63.35 (iii) 63	63.146	2.104
	10	2.5	(i)117.22 (ii)116.56 (iii)11.32	117	3.900
Other side of C.G	1	47.5	(i) 48.47 (ii) 48.19 (iii) 48.28	48.31	1.610
	2	42.5	(i) 46.26 (ii) 47.45 (iii) 47.16	47.056	1.568
	3	37.5	(i) 46 (ii) 46.13 (iii) 46.18	46.103	1.536
	4	32.5	(i)45.72 (ii)45.59 (iii)45.72	45.676	1.522
	5	27.5	(i) 45.22 (ii) 45.32 (iii) 45.25	45.263	1.508
	6	22.5	(i) 45.82 (ii) 45.91 (iii) 45.97	45.900	1.530
	7	17.5	(i) 47.91 (ii) 47.94	47.993	1.599

			(iii) 48.13		
	8	12.5	(i) 52.53 (ii) 52.56 (iii) 53.18	52.756	1.758
	9	7.5	(i) 63.22 (ii) 63.28 (iii) 63.32	63.273	2.109
	10	2.5	(i)104.53 (ii)104.56 (iii)104.72	104.603	3.486

Graph-

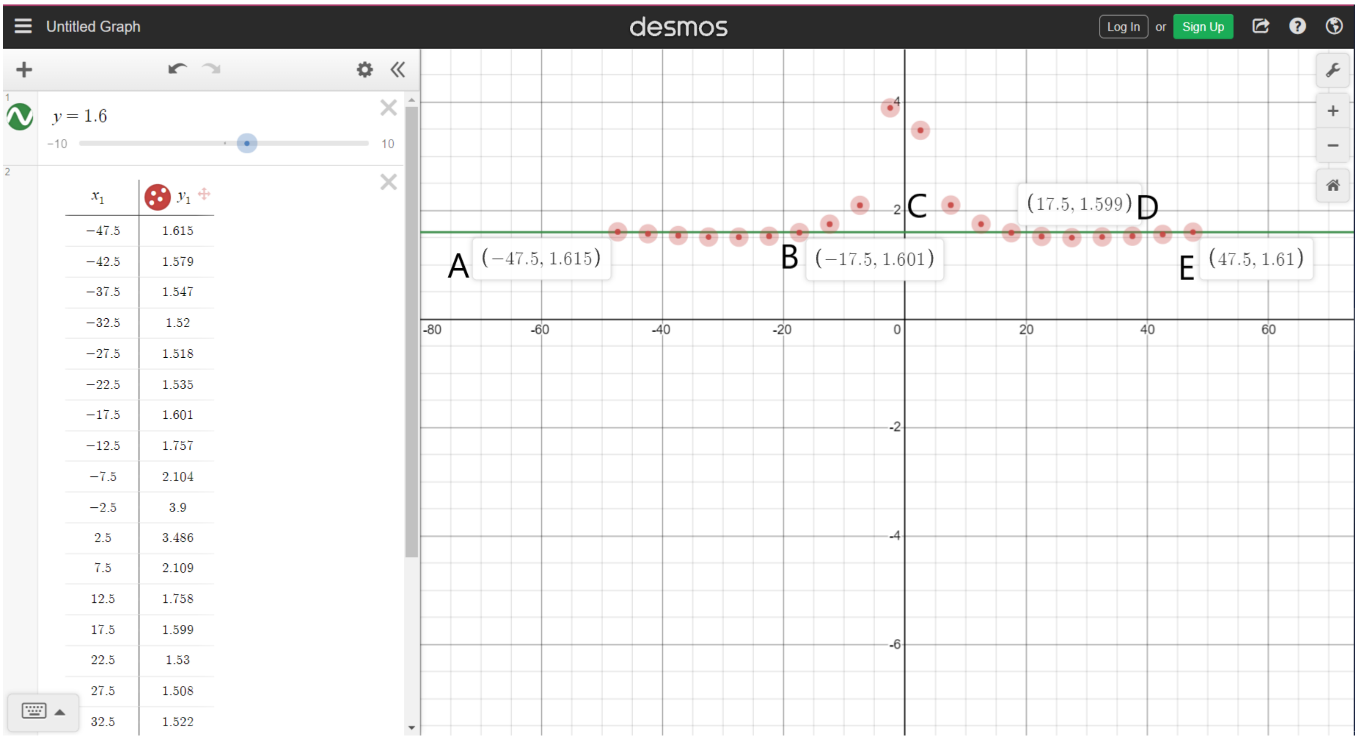
Graph of observation table-1:Data for the Time period(T) versus Distance(d) graph



Scale-

- X-axis: 1 unit = 5 cm
- Y-axis: 1 unit = 1 s

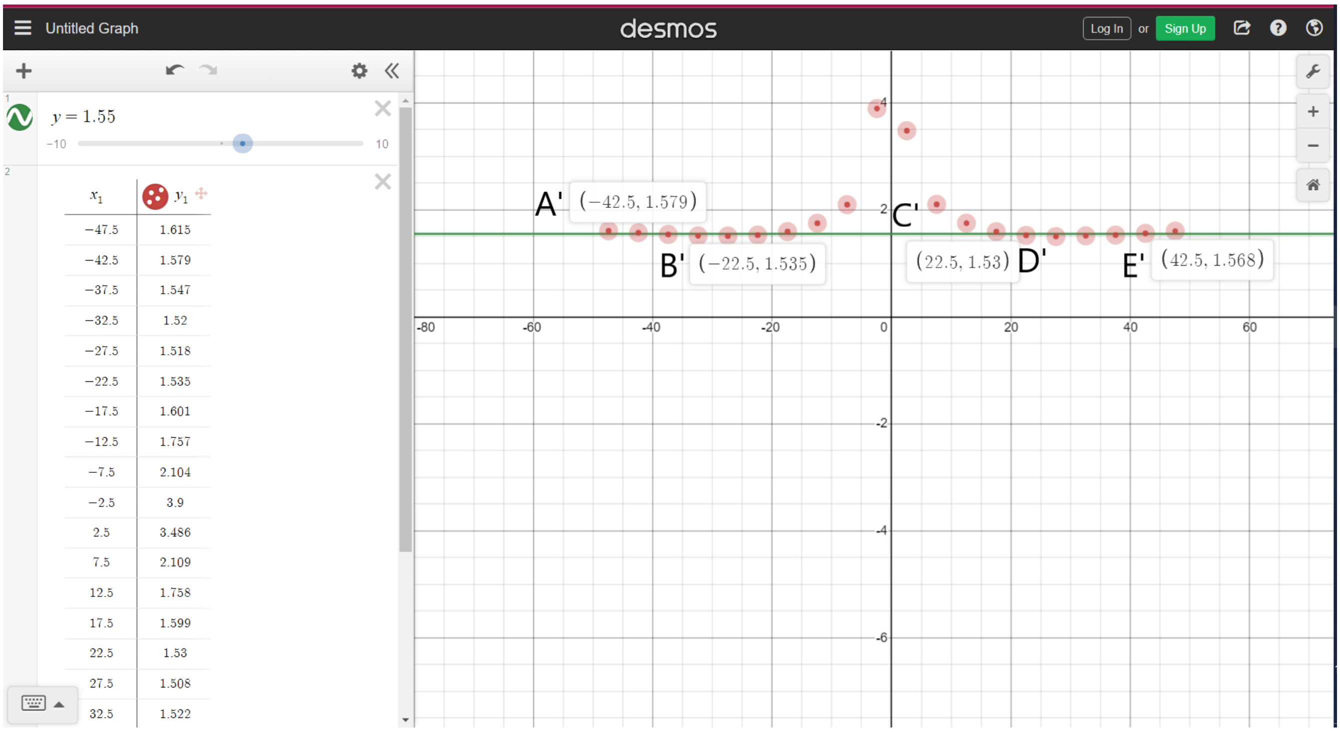
Calculation-



From the graph, it's clear that the $AD = 65.00$ *and* $BE = 65.00$ *when* $t = 1.6$ $(= y)$ *seconds for line* $ABCDE$. Therefore, $L = \frac{AD+BE}{2} = 65.00cm$.

Also, $l_1 = BC = CD = 17.5$ *and* $l_2 = AC = CE = 47.5, L = l_2 + l_1 = 65.00cm$ and

$K = \sqrt{l_1 l_2} = \sqrt{831.25} = 28.83cm$



From the graph, it's clear that the

$A'D' = 65.00$ and $B'E' = 65.00$ $when$ $t = 1.55$ $(= y)$ $seconds$ for $line$ $A'B'C'D'E'$.

Therefore, $L = \frac{A'D'+B'E'}{2} = 65.00cm.$

Also, $l_1 = B'C' = C'D' = 22.5$ and $l_2 = A'C' = C'E' = 42.5, L = l_2 + l_1 = 65.00cm$ and

$K = \sqrt{l_1 l_2} = \sqrt{956.25} = 30.92cm$

Table-2-

The value of g and K from the Time period(T) vs Distance(d) graph as shown in the fig.-03

We know, $l_1 + l_2 = L$ and $K = \sqrt{l_1 l_2}$

No. of Observations	L(cm)	T(seconds)	$g = 4\pi^2 \frac{L}{T^2}$ (cm/sec^2)	Mean g (cm/sec^2)	K (cm)	Mean K (cm)
1. For line ABCDE	65.0	1.60	1002.38	1035.24	28.83	29.88
2. For line A'B'C'D'E'	65.0	1.55	1068.09		30.92	

Error Analysis-

Finding error in g-

L = 65.00 cm, t = 1.60 seconds, g =1002.38 cm/sec^2 , K = 28.83 cm.

$$g = 4\pi^2 \frac{L}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

As $l_1 + l_2 = L$, so $\frac{\Delta L}{L} = \frac{\Delta l_1}{l_1} + \frac{\Delta l_2}{l_2} = \frac{2\Delta l}{l}$ where $l = 65.0cm$

$$\frac{\Delta g}{g} = \frac{2\Delta l}{l} + \frac{2\Delta T}{T} = \frac{2\times0.1}{65.0} + \frac{2\times0.01}{1.6} = 0.0155769 \approx 0.0156$$

$$\frac{\Delta g}{g} \times 100 = 1.56\%$$

$\Delta g = g \times 0.0155769 = 15.61399615 \approx 15.61$ $(cm\ sec^{-2})$

Therefore, the maximum percentage error in g is 1.56%.

Finding error in K-

L = 65.00 cm, t = 1.60 seconds, g =1002.38 cm/sec^2 , K = 28.83 cm.

$K = \sqrt{l_1 l_2}$

$$\frac{\Delta K}{K} = \frac{\Delta l_1}{2l_1} + \frac{\Delta l_2}{2l_2} = \frac{0.1}{2\times42.5} + \frac{0.1}{2\times22.5} = 3.39869281 \times 10^{-3}$$

$$\frac{\Delta K}{K} \times 100 = 0.339\% \approx 0.34\%$$

$\Delta K = K \times 3.39869281 \times 10^{-3} = 0.097984313 \approx 0.10\ (cm)$

Therefore, the maximum percentage error in K is 0.34%.

Result-

1. The acceleration due to gravity from the experiment is (g) = $(1035.24 \pm 15.61)\ cm\ second^{-2}$.
2. The maximum percentage error in g is 1.56%.
3. The radius of gyration about an axis through the center of gravity for the compound pendulum is (K) =$(29.88 \pm 0.10)\ cm$.
4. The maximum percentage error in K is 0.34%.

Precautions-

1. Ensure that the pendulum oscillates in a vertical plane and that there is no rotational motion of the pendulum.
2. The amplitude of oscillation should remain within 40 of arc.
3. Use a precision stop-watch and note the time accurately as far as possible.
4. Make sure that there is no air current in the vicinity of the pendulum.
5. To reduce statistical error in measurements, at least 3-5 readings must be taken.
6. Zero error must be noted in the measuring instruments.
7. Parallax and back-lash errors during measurement must be avoided.