→ Quest to attain the absolute zero of temperature

→ Kamerlingh Ones at Leiden (Holland).

→ Aim was to liquify helium which, because of the weak attraction between helium atoms, was expected to remain a gas till about 4 K at atmospheric pressure.

→ It was liquified on reaching a record low temperature of 4.2 K at atmospheric pressure (10th July 1908).

→ 1913 Nobel Prize. Citation: "his investigations of the properties of matter at low temperatures, which led, inter alia, to the production of liquid helium".

→ Now once you have access to these low temperatures via liquid helium (ultimate coolant), then it becomes possible to investigate the properties of different substances at such low temperatures.

→ For example, we can study the electrical resistance of metals as a function of temperature and test the existing theories.

→ This is what Kamerlingh Ones did, he examined the behaviour of electrical resistance at liquid helium temperatures.

If we consider the Fermi gas description of metals the electrical conductivity, σ is given by

$$\sigma = \frac{ne^2}{m} \tau,$$

where m is the effective mass of the conduction electrons, $-e$ is the electron charge and τ is the average life time for free motion of the electrons between collisions with impurities and other electrons.

Note that the conductivity is defined by the constitutive equation

$$\vec{j} = \sigma \vec{E},$$

where $\vec{j}$ is the electrical current density which flows in response to the external electric field $\vec{E}$.

The resistivity is given by

$$\vec{E} = \rho \vec{j},$$

so it is the reciprocal of the conductivity $\rho = 1/\sigma$.

Thus, $\rho = \frac{m}{ne^2} \tau^{-1}$.

The resistivity is proportional to the scattering rate, τ^{-1}, of the conduction electrons.

Therefore, the electrical conductivity depends on temperature mainly via the different scattering processes which enter into the mean life time.

If we consider any typical metal, there are three main scattering processes:

i) scattering by impurities;

ii) by electron-electron interactions;

iii) by electron-phonon collisions.

These three are independent processes, so the total effective scattering rate is given by

$$\tau^{-1} = \tau_{imp}^{-1} + \tau_{el\text{-}el}^{-1} + \tau_{el\text{-}ph}^{-1},$$

where τ_{imp}^{-1} is the rate of scattering by impurities, $\tau_{el\text{-}el}^{-1}$ the electron-electron scattering rate, and $\tau_{el\text{-}ph}^{-1}$ the electron-phonon scattering rate. So the resistivity becomes

$$\rho = \frac{m}{ne^2}\left(\tau_{imp}^{-1} + \tau_{el\text{-}el}^{-1} + \tau_{el\text{-}ph}^{-1}\right).$$

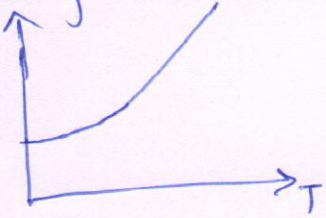
Each of these lifetimes is a characteristic function of temperature.

→ The impurity scattering rate, τ_{imp}^{-1}, is essentially independent of temperature, at least for the case of non-magnetic impurities.

→ The electron-electron scattering rate, $\tau_{el\text{-}el}^{-1}$, is proportional to T^2, where T is the temperature.

→ The electron-phonon scattering rate $\tau_{el\text{-}ph}^{-1}$ at low-temperatures (well below the Debye temperature) is proportional to T^5.

Therefore, the resistivity of a metal at low temperatures is expected to behave as



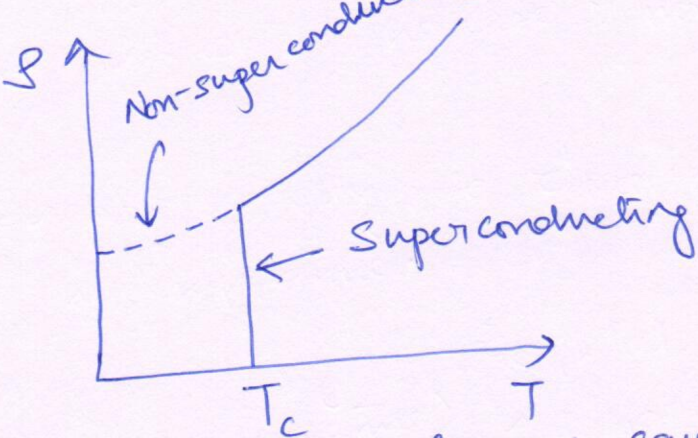
$$\rho = \rho_0 + aT^2 + \dots,$$

wherein the zero temperature part, the residual resistivity ρ_0, depends only on the ~~conduction electrons~~ concentration of impurities.

This is indeed the case for most of the metals, eg. Pt, Au, etc.

When this was tested for mercury by Kamerlingh Ones in 1911, the resistance from its melting point (233 K) to 4.2 K, the resistance fell by a factor of 500. ~~Now next each~~ In next some hundredths of a degree a sudden fall in resistance at once to a millionth of its original value at the melting point was observed.

[This unexpected absence of electrical resistance below a certain temperature was termed superconductivity by Ones.]



The question is whatever physical agency causes electrical resistance in mercury above 4.2 K, is also present just below the temperature, then why it suddenly became ineffective?

In superconductors below T_c the resistivity ρ becomes zero and so the conductivity σ appears to become infinite. However, to be consistent with ~~as~~ the constitutive relation, the electric field must be zero

$$\vec{E} = 0,$$

at all points inside ~~a~~ a superconductor. In this way the current $\vec{j}$, can be finite. So we have current flow without electric field.

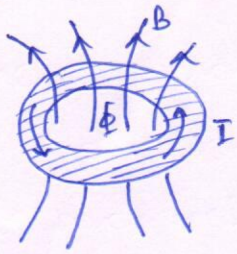
The change from finite to zero resistivity at the superconducting critical temperature T_c represents a thermodynamic phase transition from ~~one~~ the "normal state" to the "superconducting state".

In the normal state, the resistivity and other properties behave similar to what we expect of a normal metal, while in the superconducting state many physical properties including resistivity are quite different.

The key characteristic of the superconducting state is that the resistivity is exactly zero, $\rho = 0$, or the conductivity is infinite.

The most convincing evidence that superconductors really have $\rho = 0$ is the observation of persistent currents.

consider a closed loop of superconducting wire.



We can set up a current, I, circulating in the loop. Because there is no dissipation of energy due to finite resistance, the energy stored in the magnetic field of the ring will remain constant and the current never decays.

The magnetic flux through the center of the superconducting ring is given by

$|ds| \equiv$ elemental area

$$\Phi = \int \vec{B}\cdot d\vec{s},$$

where $d\vec{s}$ is a vector perpendicular to the plane of the ring.

~~Thus~~ We have

$$\frac{d\Phi}{dt} = \int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

$$= -\int \vec{\nabla}\times\vec{E}\cdot d\vec{s}$$

$$= -\oint \vec{E}\cdot d\vec{r},$$

where the line integral is taken around the closed path around the inside of the ring. We can choose this path to be just inside the superconductor, and so that $\vec{E}=0$ everywhere along the path.

Therefore, $\frac{d\Phi}{dt} = 0.$

The magnetic flux through the ring ~~system~~ stays constant as a function of time.

$T > T_c$

normal state.

$T > T_c$

apply an external mag. field B_{ext} which easily passes through the material

cool the system $T < T_c$

flux in the ring is given by $\Phi = \int \vec{B}_{ext} \cdot d\vec{s}$,

$\Downarrow$

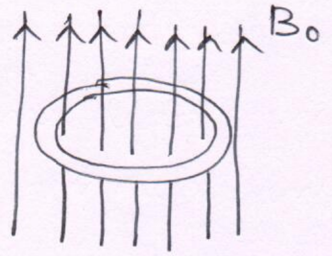
Remains constant, even if we turn off the source of external mag. field. $\vec{B}_{ext} = 0$

The only way the superconductor can keep Φ constant is to generate its own mag. field through the center of the ring, which it must acheive by having a circulating current I around the ring.

The value of I will be exactly the one required to induce a magnetic flux equal to Φ inside the ring. Also, since Φ is constant the current I must also be constant.

Thus we have set up a circulating persistent current in our superconducting ring.

An important consequence of the persistent currents that flow in materials with zero resistance is that the magnetic flux that passes through a continuous loop of such a material remains constant.



Ring area: A

An initial magnetic field $\vec{B}_0$ is applied perpendicular to the plane of the ring when the temperature is above the critical temperature.

Flux through the ring is $B_0 A$.

Now if the ring is cooled below ~~its~~ critical temperature while in this applied field, then the flux passing through it is unchanged.

If we now change the applied field, then a current will be induced in the ring, and according to Lenz's law the direction of this current will be such that the magnetic flux it generates compensates for the flux change due to the change in the applied field.

The induced emf (Faraday's law) is

$$-\frac{d\Phi}{dt} = -A\frac{d(B-B_0)}{dt},$$

and this generates an induced current given by

$$L\frac{dI}{dt} = -A\frac{dB}{dt},$$

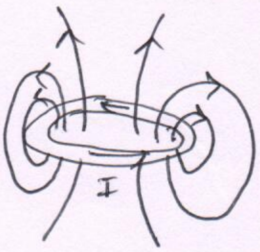
where L is the self-inductance. Notice the absence of ohmic term on the LHS (IR), because we have assumed $R=0$. This yields

$$LI + BA = \text{constant}.$$

LI is the amount of flux passing through the ring generated by the current I flowing in the ring – this is just the definition of the self-inductance L – so $(LI + BA)$ is the total magnetic flux through the ring.

The total flux threading a circuit with zero resistance must therefore remain constant – it can not change. If the applied magnetic field is changed an induced current is set up that creates a flux to compensate exactly for the change in the flux from the applied magnetic field. Because the circuit has no resistance, the induced current can flow indefinitely, and the original amount of flux through the ring can be maintained indefinitely.

This is true even if the external field is removed altogether; the flux through the ring ~~can~~ is maintained by a persistent induced current.



Note that the constant flux through the ring does not mean that the magnetic field is unchanged.

We had started with a uniform external field within the ring, whereas the field produced by ~~the~~ a current in the ring will be much larger close to the ring than at its center.

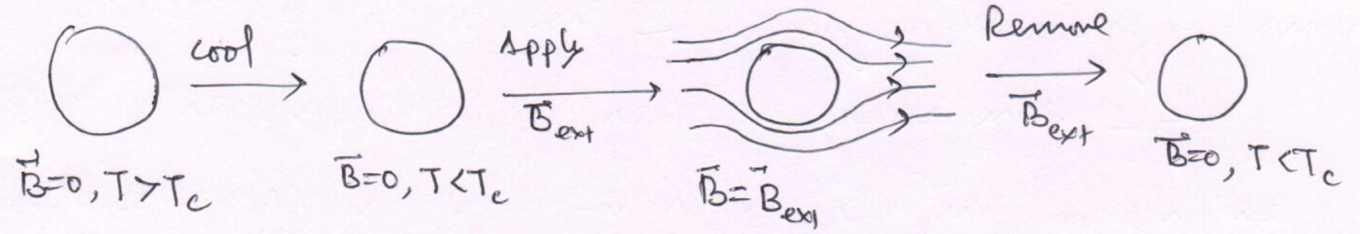
Superconductors are more than perfect conductors of electricity. 1933 Walter Meissner and Robert Ochsenfeld discovered that a superconductor completely excludes an external magnetic field.

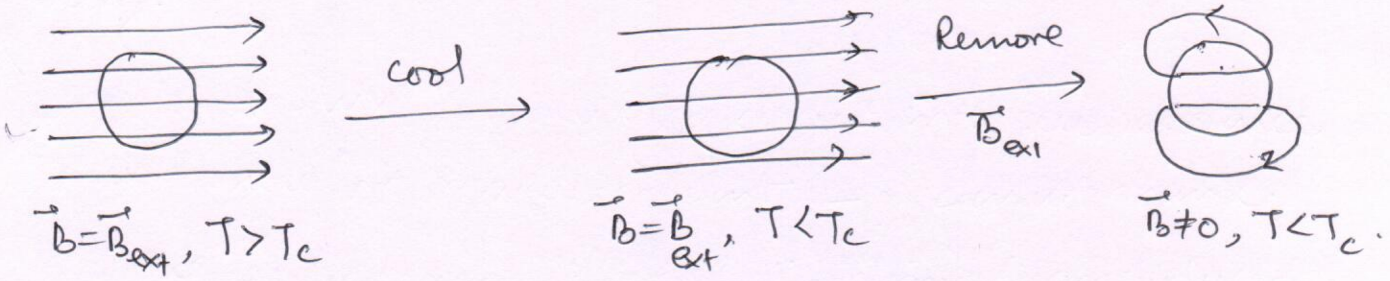
They measured the magnetic flux distribution outside tin and lead specimens, cooled below their superconducting transition temperatures while in a magnetic field.
[Their results could not be explained by the assumption that a superconductor is just a resistanceless metal.]

The exclusion of the magnetic field from a superconductor occurs regardless of whether the sample becomes superconducting before or after the external magnetic field is applied. In the steady state, the external magnetic field is cancelled in the interior of the superconductor by opposing magnetic fields produced by a steady screening current that flows on the surface of the superconductor.

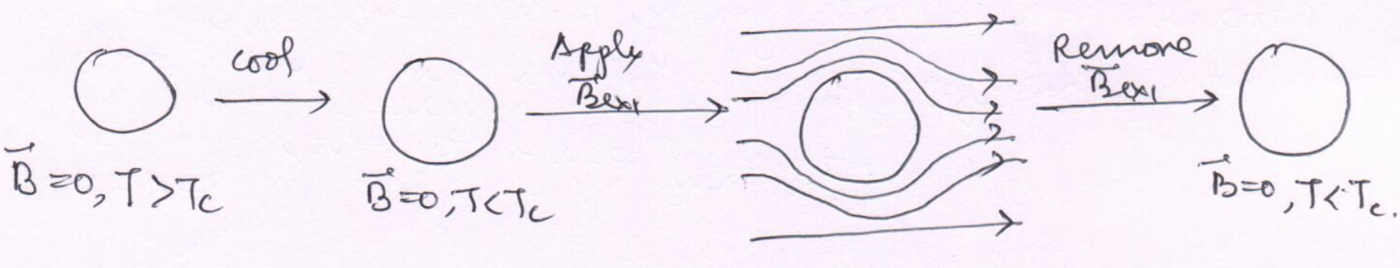
Note that the exclusion of the magnetic field from inside a superconductor cannot be predicted by applying Maxwell's equations to a material that has zero electrical resistance.

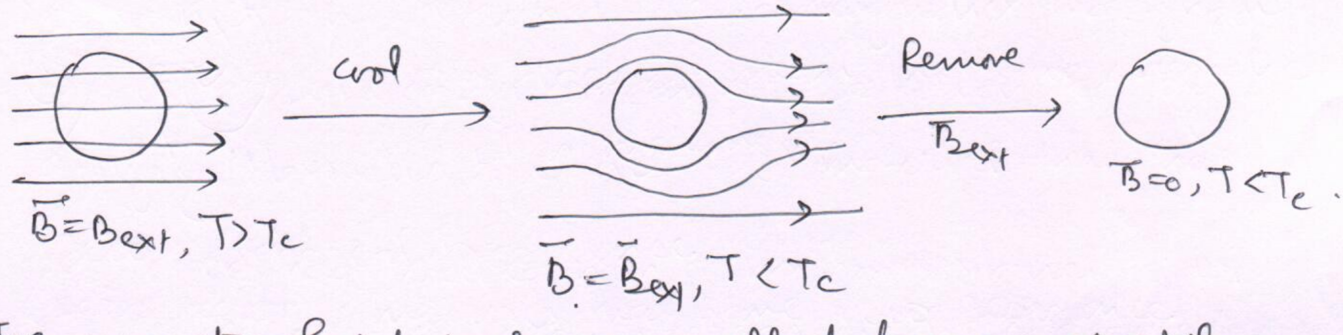
(Let's refer to materials that have zero resistance but do not exhibit Meissner effect as a perfect conductor.).





The flux enclosed by a continuous path through a zero resistance material – a perfect conductor – remains constant and this must be true for any path within the material, whatever its size or orientation. This means that the magnetic field throughout the material must remain constant, that is, $\partial \vec{B}/\partial t = 0$.





The magnetic field is always expelled from a ~~material~~ superconductor. This is achieved spontaneously by producing currents on the surface of the superconductor.

The direction of currents is such as to create a magnetic field that exactly cancels the applied field in the superconductor. It is this active exclusion of magnetic field – the Meissner effect – that distinguishes a superconductor from a perfect conductor, a material that merely has zero resistance.

Thus we can regard zero resistance and zero magnetic fields as two key characteristics of superconductivity.

Perfect Diamagnetism