Department of Physics
Indian Institute of Technology
Kharagpur-721302, West Bengal, India

Subject No. PH41023 (Statistical Mechanics - 1)

Duration: 3 hrs
Total Marks: 50

End - semester exam paper

Five marks questions : $4 \times 5 = 20$

§1. For the grand canonical ensemble calculate $< N >$ and fluctuations in N, i.e., σ_N^2.

§2. Brownian particles are in contact with a thermal bath of temperature B. They obey the Langevin equation, $m\frac{dv}{dt} = -\zeta v + F(t)$. (a) Calculate the ensemble averaged velocity correlation function. (b) Using it derive the fluctuation dissipation relation, in $t \to \infty$ limit.

§3. Using the Langevin equation in the previous question, (a) calculate the ensemble averaged mean squared displacement, (b) analyse it's behaviour at small and large time limits, and (c) derive the Einstein's expression for the self-diffusion coefficient, in $t \to \infty$ limit.

§4. Active Brownian particles obey the Langevin equations, $\frac{dx}{dt} = v$ and $m\frac{dv}{dt} = -\gamma(v)v - \gamma_0\frac{dU}{dx} + \sqrt{\zeta kT}\,\xi(t)$, derive the corresponding Fokker-Planck equation.

Ten marks questions : $3 \times 10 = 30$

§5. (i) Calculate the internal energy and Helmholtz free energy of a gas composed of particles of spin-1/2 upto terms of order T^4 when the degeneracy is sufficiently high. **[8 Marks]**

(ii) Let the density of states of the electrons in some sample be assumed to be a constant D and the total number of electrons be equal to N. Calculate the Fermi potential at 0 K. **[2 Marks]**

§6. (i) Consider an ideal Bose gas composed of N particles in a volume V, and let N_0 and N' be the number of particles in the lowest state (momentum $p = 0$) and the excited states, respectively. Show that when the temperature falls below a critical temperature T_c, N_0 becomes comparable with the total number N and in this region the chemical potential is equal to zero. **[7 Marks]**

(ii) Bose-Einstein condensation is produced in a vapour of Rubidium-87. The first evidence of condensation appears at a tempearture of 170 nK with $2.5X10^{12}$ atoms/cm^3. Compare the condensation tempearture obtained in this experiment with the critical temperature for an ideal bose gas at the same density. Given : $g_{3/2}(1) \sim 2.61$ and the mass of a nucleon $\sim 1.66X10^{-27}kg$ **[3 Marks]**

§7. (i) Find the fermi momentum and internal energy of an extreme relativistic ideal fermi gas (spin 1/2) which is completely degenerate. **[5 Marks]**

(ii) A Fermi gas of N spin-1/2 particles is initially confined, at zero tempearture, to a compartment of volume V_0 in a thermally isolated container of total volume $V_0 + \Delta V$, the remaining volume ΔV being empty. When the partition separating the two compartments is removed, the gas expands to fill the whole volume, and eventually reaches a new equilibrium state. Estimate the temperature of this final state, assuming $\frac{\Delta V}{V_0} << 1$. **[5 Marks]**