Quantum Mechanics

Assignment-0

1. (a) Show that the maximum of the Planck energy density occurs for a wavelength of the form $D_{max} = b/T$, where T is the temperature and b is a constant that needs to be estimated.

(b) Estimate the surface temperature of a star if the radiation it emits has a maximum intensity at a wavelength of 446 nm. What is the intensity radiated by the star?

(c) Estimate the wavelength and the intensity of the radiation emitted by a glowing tungsten filament whose surface temperature is 3300 K.

2. When two ultraviolet beams of wavelengths D1 = 80 nm and D2 = 110 nm fall on a lead surface, they produce photoelectrons with maximum energies 11.390 eV and 7.154 eV, respectively.

a) Estimate the numerical value of the Planck constant.

(b) Calculate the work function, the cutoff frequency, and the cutoff wavelength of lead.

3. High energy photons (ϒ -rays) are scattered from electrons initially at rest. Assume the photons are backscatterred and their energies are much larger than the electron's rest-mass energy, $E >> m_ec^2$. (a) Calculate the wavelength shift.

(b) Show that the energy of the scattered photons is half the rest mass energy of the electron, regardless of the energy of the incident photons.

(c) Calculate the electron's recoil kinetic energy if the energy of the incident photons is 150 MeV.

4. Calculate the minimum energy of a photon so that it converts into an electron–positron pair. Find the photon's frequency and wavelength.

5. Calculate the de Broglie wavelength for (a) a proton of kinetic energy 70 MeV kinetic energy and (b) a 100 g bullet moving at 900 m s^{-1}.

6. Estimate the uncertainty in the position of (a) a neutron moving at 5 x 10^6 m s^{-1} and (b) a 50 kg person moving at 2 m s^{-1}.

7. Positronium is the bound state of an electron and a positron; it is a short-lived, hydrogen-like atom where the proton is replaced by a positron.

(a) Calculate the energy and radius expressions, E_n and r_n. (b) Estimate the values of the energies and radii of the three lowest states. (c) Calculate the frequency and wavelength of the electromagnetic radiation that will just ionize the positronium atom when it is in its first excited state.

8. A 45 kW broadcasting antenna emits radio waves at a frequency of 4 MHz.
(a) How many photons are emitted per second?
(b) Is the quantum nature of the electromagnetic radiation important in analyzing the radiation emitted from this antenna?

9. Consider a mass–spring system where a 4 kg mass is attached to a massless spring of constant k = 196 Nm^{-1}; the system is set to oscillate on a frictionless, horizontal table. The mass is

pulled 25 cm away from the equilibrium position and then released.
(a) Use classical mechanics to find the total energy and frequency of oscillations of the system.
(b) Treating the oscillator with quantum theory, find the energy spacing between two consecutive energy levels and the total number of quanta involved. Are the quantum effects important in this system?

10. (a) Estimate the energy of the electrons that we need to use in an electron microscope to resolve a separation of 0.27 nm.
(b) In a scattering of 2 eV protons from a crystal, the fifth maximum of the intensity is observed at an angle of 30^{o}. Estimate the crystal's planar separation.

11. A photon of energy 3 keV collides elastically with an electron initially at rest. If the photon emerges at an angle of 60^{o}, calculate
(a) the kinetic energy of the recoiling electron and
(b) the angle at which the electron recoils.

12. Calculate the group and phase velocities for the wave packet corresponding to a relativistic particle.

13. Use the uncertainty principle to estimate: (a) the ground state radius of the hydrogen atom and
(b) the ground state energy of the hydrogen atom.

14. A neutron is confined in space to 10^{-14} m. Calculate the time its packet will take to spread to
(a) four times its original size,
(b) a size equal to the Earth's diameter, and
(c) a size equal to the distance between the Earth and the Moon.