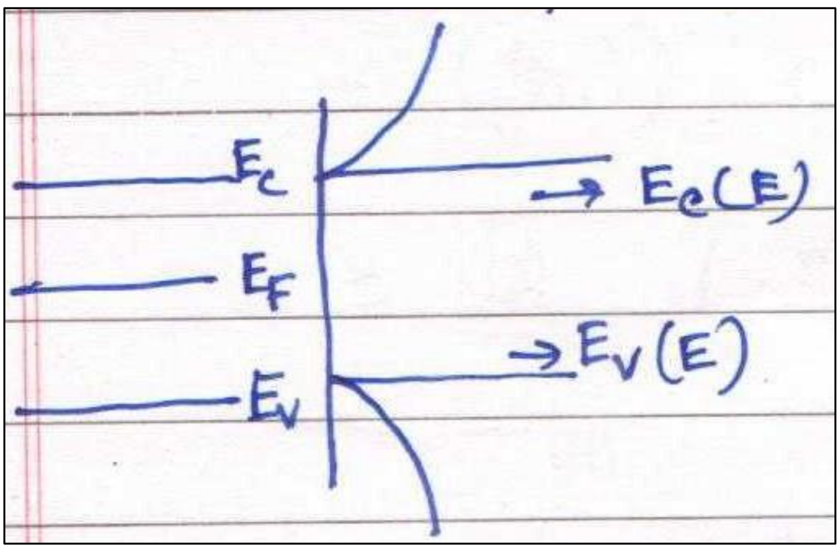
Electrical conducitivity in Intrinsic Semiconductors

$$\overleftrightarrow{\sigma} = -\frac{e^2}{4\pi^3}\int \frac{\tau \mathbf{v}\mathbf{v}}{|\partial E/\partial \mathbf{k}|}\,\frac{\partial f_0}{\partial E} d^2 S\, d E.$$

$$f_0(E) = \frac{1}{1 + \exp[(E - E_F)/k_B T]}$$



Ref. Book – Solid State Properties by M. Dresselhaus, G. Dresselhaus, S. B. Cronin, A. G. S. Filho

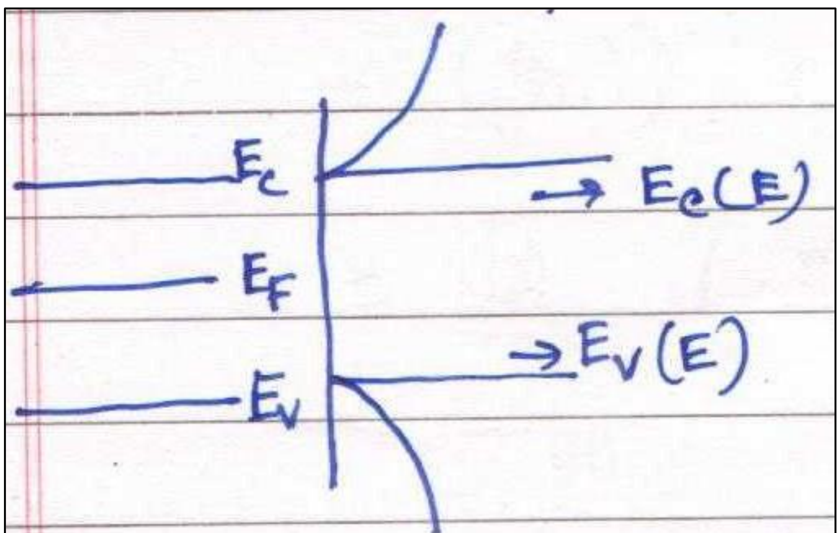
Electrical conducitivity in Semiconductors

Approximation 1

In the case of electron states in intrinsic semiconductors having no donor or acceptor impurities, we have the condition $(E - E_F) \gg k_B T$ since E_F is in the band gap and E is the energy of an electron in the conduction band, as shown in Fig. 7.2. Thus, the first approximation is equivalent to writing

$$f_0(E) = \frac{1}{1 + \exp[(E - E_F)/k_B T]} \simeq \exp[-(E - E_F)/k_B T]$$

which is equivalent to using Maxwell-Boltzmann distribution



E_F is a negative energy as E is usually measured with respect to the bottom of the conduction band.

$$f_0(E) \simeq e^{-|E_F|/KT} e^{-E/KT}$$

So that deriavative of the Fermi function becomes,

$$\frac{\partial f_0(E)}{\partial E} = (-) \frac{e^{-|E_F|/KT}}{KT} e^{-E/KT}$$

the derivative of the Fermi function becomes

$$\frac{\partial f_0(E)}{\partial E} = -\frac{e^{-|E_F|/k_B T}}{k_B T} e^{-E/k_B T}$$

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Approximation 2

We can assume a constant relaxation time τ that is independent of $\vec{k}$ and E for simplicity.

This approximation is made for simplicity and may not be valid for specific physical situations. Some common scattering mechanisms yield an energy-dependent relaxation time $\tau = \tau_0(E/k_B T)^r$, for which $r = -1/2$ and $r = +3/2$, respectively, for acoustic deformation potential scattering or ionized impurity scattering.

Approximation 3

For carriers near the bottom of conduction band, an isotropic parabolic band can be taken,

$$E = \frac{\hbar^2 k^2}{2m^*}$$

$$\mathbf{v}\mathbf{v} = \frac{1}{3} v^2 \overleftrightarrow{1}$$

$$k^2 = 2m^* E/\hbar^2$$

$$2k\,dk = 2m^* dE/\hbar^2$$

$$v^2 = 2E/m^*$$

$$v = \hbar k/m^*$$

$\overleftrightarrow{1}$ is the unit second rank tensor.

$$d^3k = 4\pi k^2 dk = 4\pi\sqrt{2}(m^*/\hbar^2)^{3/2}\sqrt{E}\,dE$$

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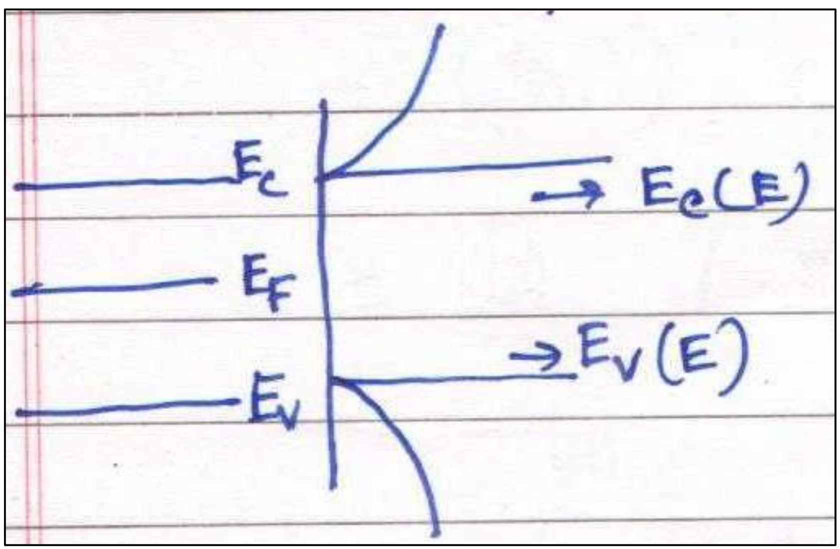
$$\sigma = \frac{e^2\tau}{4\pi^3}\left(\frac{8\sqrt{2}\pi\sqrt{m^*}}{3\hbar^3 k_B T}\right) e^{-|E_F/k_B T} \int_0^\infty E^{3/2} dE\, e^{-E/k_B T}$$



Electrical conducitivity in Semiconductors

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Now, $\int_0^\infty x^p \, dx \, e^{-x} = \Gamma(p+1)$ —.

Γ function has property $\Gamma(p+1) = p\Gamma(p)$

$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$$\sigma = \frac{2e^2\tau}{m^*}\left(\frac{m^* k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-|E_F|/kT}$$

Electrical conducitivity in Semiconductors

$$\sigma = \frac{2e^2\tau}{m^*}\left(\frac{m^* \, k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-E_F/kT}$$

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Using the same approximations

$$n = (4\pi^3)^{-1}\int f_0(E)\, d^3k$$

$$= (4\pi^3)^{-1} e^{-|E_F|/kT}\int e^{-E/kT}\, 4\pi k^2 dk$$

$$= \frac{\sqrt{2}}{\pi^2}\left(\frac{m_*}{\hbar^2}\right)^{3/2} e^{-|E_F|/kT}\int_0^\infty \sqrt{E}\, dE\, e^{-E/kT}$$

Now, $\int_0^\infty \sqrt{E}\, dE\, e^{-E/kT} = \frac{\sqrt{\pi}}{2}(k_B T)^{3/2}$

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Now, $\int_0^\infty \sqrt{E}\, dE\, e^{-E/kT} = \frac{\sqrt{\pi}}{2}(k_B T)^{3/2}$

So, temperature dependent carrier density

$$n = 2\left(\frac{m^* k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-|E_F|/kT}$$

$$\sigma = \frac{ne^2\tau}{m^*}$$

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$$\sigma = \frac{ne^2\tau}{m^*}$$

for a semiconductor with constant τ and isotropic, parabolic dispersion relations.

To find σ for a semiconductor with more than one spherical carrier pocket, the conductivities per carrier pocket are added

$$\sigma = \sum_i \sigma_i$$

Plot of $\ln\sigma$ vs $\frac{1}{T}$ yields activation energy called an Arrhenius plot.