Indain Institute of Technology, Kharagpur

Date FN/AN, Time 3 hrs, Full Marks 100, Dept. Physics
No of Students: 66, End Autumn Semester Examination
Subject Nu. **PH41023** Subject Name : **Statistical Physics I**
2,3,4yr B.Tech, MSc, Instructions: **All questions have equal marks, answer all**

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1. (a.) Model a heavy nucleus of mass number A as a free Fermi gas of an equal number of protons and neutrons confined in a sphere of radius $R = r_0\ A^{1/3}$ where $r_0 = 1.4 \times 10^{-15}$ m. Calculate the Fermi energy and the average energy per nucleon in MeV.($e = 1.e \times 10^{-19}$ C).
(b.)Consider a two dimensional Fermi gas of N electrons of spin 1/2 confined in a box of area A. Calculate the Fermi energy and Fermi momentum. Calculate the internal energy U at $T = 0$ as a function of N and A.

2 A lattice gas consistes of N_0 sites each of which may be occupied by atmost 1 atom. The energy of a site is ϵ if occupied and 0 if empty. The atoms are indistinguishable.
a. Calculate the grand partition function $\mathcal{Z}$ at fugacity z and temperature T.
b. What fraction of the sites are occupied?
c. Find the heat capacity as a function of T at a fixed z

3 a. At what value of $(n\lambda^3)$ does a Bose-Einstein condensate (BEC) form.
b. At fixed n, what is the critical temperature T_c for BEC?
c. For $T < T_c$, what fraction of the particles are in the BEC state. Show this graphically.
d. Show the particle occupation number $n(p)$ as a functin of p for both $T > T_c$ and $T < T_c$.
e. How does P vary with T for $T < T_c$? (give justification)
f. How does C_V vary with T for $T < T_c$? (give justification)

4. Consider N non-interacting, spinless Fermions of mass m in a simple harmonic potential of spring constant $k = \omega^2 m$. We use ψ_n with $n = 0, 1, 2, ...$ to denote the normalised single particle energy eigen-functions.
a. Calculate the normalised ground state wave function for $N = 3$.
b. For arbitrary N and $T = 0$, calculate the Fermi energy ϵ_F as a function of N.
c.At $T = 0$, calculate the internal energy U as a function of ϵ_F.

5. Surface waves on liquid have a dispersion relation given by

$$\epsilon(k) = \hbar\sqrt{\frac{\sigma k^3}{\rho}}$$

where k is the wave number of the surface wave, σ is the surface tension and ρ is the mass density of the liquid. Treating the excitations as bosons with no number conservation, find the internal energy per unit area as a function of temperature.