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# ELECTROMAGNETISM LABORATORY

**EXPERIMENT-5**

**Title: Fresnel's Equations**

**Aim:**

- To determine the refractive index of the material.
- To plot experimental and theoretical amplitude reflection coefficients for different incident angles for both parallel and perpendicular polarizations.

## Apparatus and Accessories:

1. He-Ne laser (1.0 mW, =632.8 nm)
2. Polariser
3. Prism (A=60 degrees) which is made of flint glass
4. Analyzer
5. Photodetector
6. Prism table with angular scale
7. Multirange meter with amplifier

**Theory:**

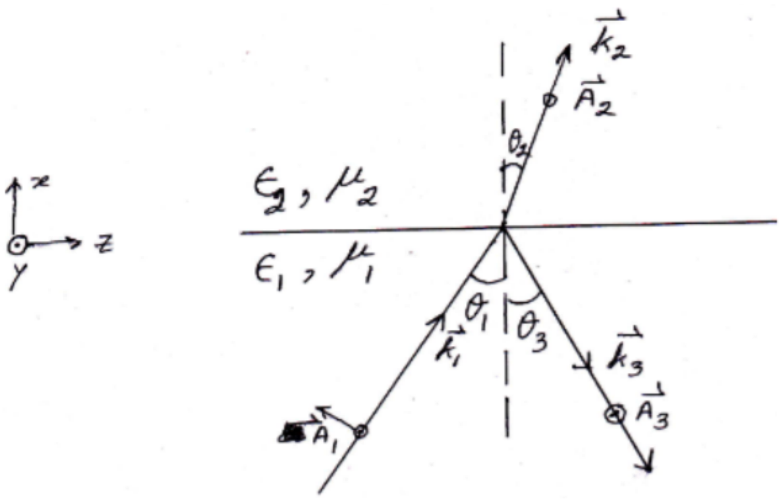



Fig.1

In figure 1, we consider the reflection and refraction of a plane of electromagnetic wave incident at the interface of two dielectrics characterised by $(\epsilon_1, \mu_1)$ and $(\epsilon_2, \mu_2)$.

where$\epsilon_1$ *and* $\epsilon_2$ are the dielectric permittivity

$\mu_1$and $\mu_2$are magnetic permeability.

We assume that the media are non-absorbing, isotropic and homogeneous.

Let -

$$\vec{E_1} = \vec{A_1} \exp\left(i\left(\omega_1 t - \vec{k_1}\cdot\vec{r}\right)\right)$$
$$\vec{E_2} = \vec{A_2} \exp\left(i\left(\omega_2 t - \vec{k_2}\cdot\vec{r}\right)\right)$$
$$\vec{E_3} = \vec{A_3} \exp\left(i\left(\omega_3 t - \vec{k_3}\cdot\vec{r}\right)\right)$$

where $\vec{A}_1$,$\vec{A}_2$and $\vec{A}_3$ are amplitudes

$\vec{k}_1$,$\vec{k}_2$and $\vec{k}_3$ are propagation vectors

$\vec{\theta}_1$,$\vec{\theta}_2$and $\vec{\theta}_3$ are the angles that $\vec{k}_1$,$\vec{k}_2$and $\vec{k}_3$ make with the normal to the interface.

Since these fields satisfy the wave equation, we have the following equations-

$$k_1^2 = \omega_1^2\epsilon_1\mu_1$$
$$k_2^2 = \omega_2^2\epsilon_2\mu_2$$
$$k_3^2 = \omega_3^2\epsilon_3\mu_3$$

<u>Case-1:$\vec{E}$is lying in the plane of incidence-</u>

We resolve $\vec{E}$ along the x-axis and z-axis, and since the z-component is tangential to the surface, we must have $\vec{E}_z$ continuous across the interface.

Thus, $E_{1Z} + E_{3Z} = E_{2Z}$ $\rightarrow$ (1)

$$\left[-\vec{A}_1 \exp\left[i\left(\omega_1 t - \vec{k}_1.\vec{r}\right)\right]\cos\theta_1 - \vec{A}_3 \exp\left[i\left(\omega_3 t - \vec{k}_3.\vec{r}\right)\right]\cos\theta_3\right]_{x=0} = \left[-\vec{A}_2 \exp\left[i\left(\omega_2 t - \vec{k}_2.\vec{r}\right)\right]\cos\theta_2\right]_{x=0}$$

Now, at the surface, i.e, x=0, we have the following-

$$\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z = k_y y + k_z z \, (at\, the\, surface)$$

The above condition has to be true for all values of y and z (on the plane x=0), and therefore we must have -

$$\omega_1 = \omega_2 = \omega_3 = \omega$$
$$k_{1y} = k_{2y} = k_{3y}$$
$$k_{1z} = k_{2z} = k_{3z}$$

Thus the frequencies associated with the reflected and refracted ray must be the same as that of the incident wave, which is also physically obvious. Therefore, we get the following-

$$k_1 = \omega(\epsilon_1\mu_1)^{1/2} = k_3 = \omega n_1$$
$$k_2 = \omega(\epsilon_2\mu_2)^{1/2} = \omega n_2$$

Now, $k_{1y} = k_{2y} = k_{3y} = 0$, i.e, $\vec{k}_1$, $\vec{k}_2$ and $\vec{k}_3$ will all be parallel to the x-z plane and $k_1 sin\theta_1 = k_2 sin\theta_2 = k_3 sin\theta_3$ and therefore, $\theta_1 = \theta_3$ → angle of incidence equals angle of reflection. Again, we have-

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{k_2}{k_1} = \left(\frac{\epsilon_1\mu_1}{\epsilon_2\mu_2}\right)^{1/2} = \frac{n_2}{n_1}$$

Or, $n_1 sin\theta_1 = n_2 sin\theta_2$ which is Snell's Law of refraction.

Putting the values of $\omega$ and k in equation -1, we get,

$-A_1\cos\theta_1 - A_3\cos\theta_1 = - A_2 cos\theta_2 \rightarrow (3)$

Similarly , since the normal component of the electric displacement vector $\vec{D}(= \vec{D}_x)$ is aso continuous across the interface, we must have-

$D_{1x} + D_{3x} = D_{2x}$ **or** $\epsilon_1 A_1\sin\theta_1 - \epsilon_1 A_3\sin\theta_3 = \epsilon_2 A_2 \sin\theta_2 \rightarrow (2)$

We assume both media are non-magnetic, i.e, $\mu_1 = \mu_2 = \mu_0$

Using eqs (2) and (3), we can get-

$$r_p = \frac{E_3}{E_1} = \frac{-\epsilon_1\sin\theta_1\cos\theta_2 + \epsilon_2\sin\theta_2\cos\theta_1}{\epsilon_2\sin\theta_2\cos\theta_1 + \epsilon_1\sin\theta_1\cos\theta_2} \rightarrow (4)$$

where $r_p$ is the amplitude reflection coefficient.

Whereas, amplitude transmission coefficient $t_p$,

$$t_p = \frac{E_2}{E_1} = \frac{2\epsilon_1\sin\theta_1\cos\theta_1}{\epsilon_2\sin\theta_2\cos\theta_1 + \epsilon_1\sin\theta_1\cos\theta_2} \rightarrow (5)$$

In terms of refractive index, we can rewrite the equations (4) and (5) as -

$$r_p = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2} \quad --(6)$$
$$t_p = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2} \quad --(7)$$

These equations above are the Fresnel's Equations.

$$R_P = |r_p|^2 = \frac{(n_2\cos\theta_1 - n_1\cos\theta_2)^2}{(n_2\cos\theta_1 + n_1\cos\theta_2)^2} = Reflectivity$$

$$T_P = |t_p|^2 = \left(\frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2}\right)^2 = Transmittivity$$

$$|t_p|^2 = \left(\frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2}\right)^2$$

$$= \left(\frac{1}{n_2\cos\theta_1 + n_1\cos\theta_2}\right)^2 4n_1\cos\theta_1 n_2\cos\theta_2 = Transmittivity$$

Assuming $R_P + T_P = 1$, i.e, assuming no absorption.

a) When $n_2 = n_1$, $\theta_2 = \theta_1$, $r_p = 0$, this implies that there is no reflected wave. Thus, if we put a transparent substance (like glass) inside a liquid which has the same refractive index as glass, then the glass will not be visible from outside.

b) $r_p$ can also be written as -

$$r_p = \frac{n_2^2\cos\theta_1\sin\theta_2 - n_1^2\sin\theta_1\cos\theta_2}{n_1^2\sin\theta_1\cos\theta_2 + n_2^2\cos\theta_1\sin\theta_2}$$

$r_p$ will be zero when,

$$n_1^2\sin\theta_1\cos\theta_2 = n_2^2\cos\theta_1\sin\theta_2$$

Or $n_1^2\sin\theta_1\cos\theta_2\sin\theta_2 = n_2^2\sin^2\theta_2\cos\theta_1$

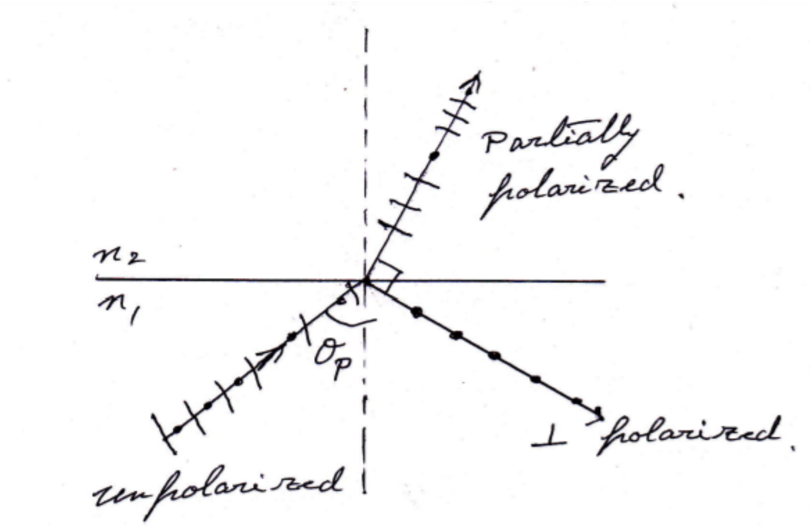
Or $n_1^2\sin\theta_1\cos\theta_2\sin\theta_2 = n_1^2\sin^2\theta_2\cos\theta_1$

$$\sin 2\theta_1 = \sin 2\theta_2$$

Thus, either $\theta_1 = \theta_2$ [case-a] or $-\theta_2 + \frac{\pi}{2} = \theta_1$

Thus, when $\frac{\pi}{2} = \theta_1 + \theta_2$ [i.e, when the reflected and transmitted rays are at right angles to each other], there is no reflection for the parallel complement of $\vec{E}$. The corresponding angle of incidence can readily be found: $\cos\theta_1 = \sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$

or, $\theta_1 = \theta_p = \arctan\left(\frac{n_1}{n_2}\right)$, $r_p = 0$. Thus, if the incident wave is unpolarised, then only the perpendicular component of $\vec{E}$ will be reflected and the reflected light will be linearly polarized. This is Brewster's Law and $\theta_p$ is the Brewster's angle.




Case-2: $\vec{E}$ is lying perpendicular to the plane of incidence-

The incident wave is along the y-axis. The electric field vectors associated with the reflective and transmitted waves will also be perpendicular to the plane of incidence.

$$\vec{E_1} = \vec{A_1} \exp\left(i\left(\omega t - \vec{k_1}\cdot\vec{r}\right)\right)\hat{y}$$
$$\vec{E_2} = \vec{A_2} \exp\left(i\left(\omega t - \vec{k_2}\cdot\vec{r}\right)\right)\hat{y}$$
$$\vec{E_3} = \vec{A_3} \exp\left(i\left(\omega t - \vec{k_3}\cdot\vec{r}\right)\right)\hat{y}$$

The magnetic field is given by -

$$\vec{H} = \frac{\vec{k}\times\vec{E}}{\omega\mu} = \frac{1}{\omega\mu}[\hat{x}(-k_z E_y) + \hat{z}\,(k_x E_y)]$$

[since $k_y = 0$]

Using boundary condition, tangential component of magnetic field is continuous across the interface -

$$\left(\frac{\varepsilon_1}{\mu_1}\right)^{1/2} A_1\cos\theta_1 - \left(\frac{\varepsilon_1}{\mu_1}\right)^{1/2} A_3\cos\theta_1 = \left(\frac{\varepsilon_2}{\mu_2}\right)^{1/2} A_2\cos\theta_2 \quad \rightarrow (1)$$

Also since $\vec{E}$ has only a y-component which is longitudinal to the interface, we have -

$A_1 + A_3 = A_2 \rightarrow (2)$

Solving these two equations, we get -

$$r_s = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

$$t_s = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

From the graph of $r^{\parallel}$ versus incident angle, we will get Brewster's angle ($\theta p$) from which we can calculate the reflective index of prism material (n), n = $\tan\theta_p$.

From n and $\theta_1$, using Snell's law we can get $\theta_2$ - $\theta_2 = \sin^{-1}(\frac{\sin\theta_1}{n})$

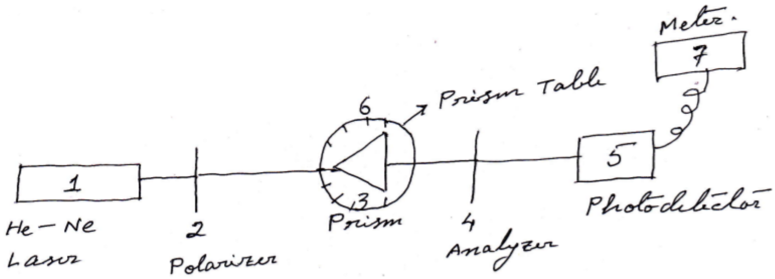
For the refractive index of air is 1, then, we get the following equations -

$r_S = \frac{\cos\theta_1 - n\cos\theta_2}{\cos\theta_1 + n\cos\theta_2}$ [where we have taken $n_2 = n$]

$$r_S = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)} \rightarrow (1)$$

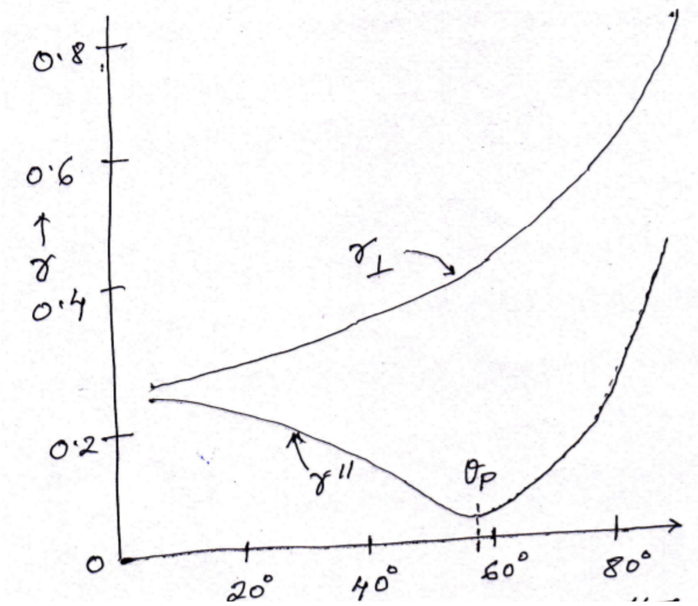
and $r_P = \frac{\cos\theta_2 - n\cos\theta_1}{\cos\theta_2 + n\cos\theta_1}$

$$r_P = -\frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)} \rightarrow (2)$$




Experimental Setup

When we plot $r^{\parallel}$ and $r^{\perp}$ versus $\theta$, we will get the following curves -




**Observations:**

Table-1-

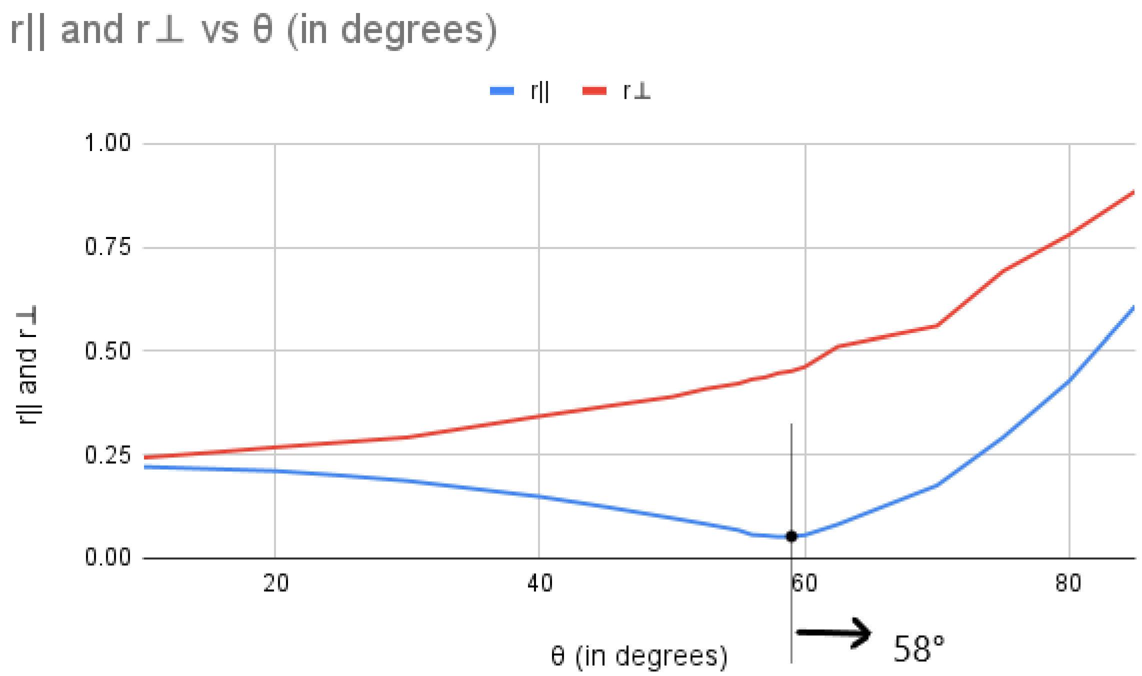
$i_0^{\parallel} = 235\ \mu A$ $i_0^{\perp} = 230\ \mu A$

| Sl No. | **α** (in degree) | $i^{\parallel}(\mu A)$ | $r^{\parallel}$ | $i^{\perp}(\mu A)$ | $r^{\perp}$ |
|---|---|---|---|---|---|
| 1 | 10 | 11.5 | 0.221 | 13.5 | 0.243 |
| 2 | 15 | 11.0 | 0.216 | 15 | 0.255 |
| 3 | 20 | 10.5 | 0.211 | 16.5 | 0.268 |
| 4 | 25 | 9.4 | 0.200 | 18.0 | 0.280 |
| 5 | 30 | 8.2 | 0.187 | 20.0 | 0.292 |
| 6 | 40 | 5.2 | 0.149 | 27.0 | 0.343 |
| 7 | 45 | 3.6 | 0.124 | 31.0 | 0.367 |
| 8 | 50 | 2.2 | 0.097 | 35.0 | 0.390 |
| 9 | 52.5 | 1.6 | 0.083 | 38.5 | 0.409 |
| 10 | 55 | 1.1 | 0.068 | 41.0 | 0.422 |
| 11 | 56 | 0.77 | 0.057 | 43.0 | 0.432 |
| 12 | 57 | 0.71 | 0.055 | 44.0 | 0.437 |
| 13 | 58 | 0.65 | 0.052 | 46.0 | 0.447 |

| | | | | | |
|---|---|---|---|---|---|
| 14 | 59 | 0.67 | 0.053 | 47.0 | 0.452 |
| 15 | 60 | 0.73 | 0.056 | 49.0 | 0.462 |
| 16 | 62.5 | 1.6 | 0.082 | 60.0 | 0.511 |
| 17 | 70 | 7.3 | 0.176 | 72.0 | 0.560 |
| 18 | 75 | 20.0 | 0.292 | 110.0 | 0.692 |
| 19 | 80 | 43.0 | 0.428 | 140.0 | 0.780 |
| 20 | 85 | 87.0 | 0.608 | 180.0 | 0.885 |

## Graphs:

Graph-1 of Observation Table-1-




<u>Scale of the above graph:</u>

- X-axis- 1 unit = 20°
- Y-axis - 1 unit = 0.25

## Calculations:

The brewster's angle can calculated using the formula - $\tan\theta_P = n$, where n is the refractive index of the material of prism.

Clearly, from the graph, we can get - $\theta_P = 58°$.

Therefore, n = tan(58°) = 1.60033452904 ≈ 1.60

## Error:

We have $n = \tan\theta_P$

$$\therefore \frac{\Delta n}{n} = sec^2(\theta_p)\frac{\Delta\theta_p}{tan(\theta_p)}$$

Here, $\theta_P = 58°, \Delta\theta_p = 0.01°$, n = 1.60

$$\therefore \frac{\Delta n}{n} = sec^2(58°)\frac{0.01°}{tan(58°)}$$

$$\therefore \frac{\Delta n}{n} = 0.0222520388 \approx 0.02$$

$$\therefore \frac{\Delta n}{n} \times 100 = 2.22520388\% \approx 2\%$$

$$\therefore$$

$$\Delta n = 0.0222520388 \times 1.60 = 0.03560326209 \approx 0.03$$

The fractional error in R is 0. 03 and its percentage error is 2%.

**Result:**

1. The value of the refractive index of the material of glass prism is ( $1.60 \pm 0.03$)
2. The fractional error in n is 0. 03 and its percentage error is 2%.

**Precautions:**

1. The laser beam should not penetrate into eyes as this may damage the eyes permanently.
2. The photo detector should be as away from the slit as possible.
3. The laser should be operated at a constant voltage 220V obtained from a stabilizer.
4. Laser should be started at least 15 minutes before starting the experiment.
5. Scale of the vernier should be rotated slowly.
6. Room should be perfectly dark.

**Discussion:**

1. Application of Fresnel equations include improving interface reflection of inclined exposure and developing micro-mirrors for blu-ray DVDs.
2. The Fresnel equations (or Fresnel coefficients) describe the reflection and transmission of light (or electromagnetic radiation in general) when incident on an interface between different optical media.
3. Fresnel Equations can be used to find the refractive index of a material.