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Computational Physics Lab Report-4

Aim:

Q1.

*** Gauss-Seidel method:** Consider the set of algebraic linear equations,

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

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$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

Where the coefficients and constants are given by

A =[-6 2 1 2 1;
3 8 -4 1 0;
-1 1 4 10 1;
3 -4 1 9 2;
2 0 1 3 10]

And the coefficient matrix is given by b = [3; 4; -2; 12;1].

a) Write a code to see is the matrix A is diagonally dominant.

b) Write a code for solving this equation using Gauss-Seidel method in which the convergence is achieved if error limit in successive iteration is within 0.001.

Q2.

***Linear interpolation 1:** Given the three data points (x, y) = (1.0, 8.0), (2.1, 20.6) and (5.0, 13.7), write a program to return the value of y for any arbitrary x in the range [1.0, 5.0] using two-point linear interpolation.

Q3.

***Linear interpolation 2:** Write a code for two-point segment linear interpolation for the dataset given in file points.txt (attached)

Tools Used: Jupyter Notebook, Python, NumPy, Pandas, Matplotlib.

Theory:

1. Gauss Siedel Method is an iterative method used to solve a system of linear equations.

2. It can be applied to any matrix with non-zero elements on the diagonals, convergence is only guaranteed if the matrix is either strictly diagonally dominant, or symmetric and positive definite.

$$\mathbf{x}^{(k+1)} = L_*^{-1}\left(\mathbf{b} - U\mathbf{x}^{(k)}\right)$$

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)}\right), \quad i = 1, 2, \ldots, n.$$

Observations:

For problem-1:

a) Not strictly diagonally dominant.

b)

```
Iteration 0: [0 0 0 0 0]
Iteration 1: [-0.5         0.6875     -0.796875    1.89409722 -0.28854167]
Iteration 2: [ 0.17962963 -0.20256076 -5.06756004  1.81061238  0.02764636]
Iteration 3: [-0.80396841 -1.95861842 -4.74478004  1.25187987  0.35970772]
Iteration 4: [-1.4664249  -1.47896567 -3.71649143  1.497832    0.31558452]
Iteration 5: [-1.06052904 -1.14777632 -4.30166431  1.58455301  0.26690634]
Iteration 6: [-1.02686743 -1.46382599 -4.41886947  1.45670612  0.3102486 ]
Iteration 7: [-1.18714344 -1.44634421 -4.15452727  1.47889817  0.30921196]
Iteration 8: [-1.13003456 -1.33836294 -4.22246632  1.5156327   0.29356373]
Iteration 9: [-1.09572718 -1.38978956 -4.28895709  1.49220588  0.30037938]
Iteration 10: [-1.13062418 -1.40702022 -4.23651054  1.48883816  0.30312444]
Iteration 11: [-1.1282917  -1.38125065 -4.23463677  1.4986956   0.29951334]
Iteration 12: [-1.11670559 -1.38589074 -4.25432104  1.49576091  0.30004495]
Iteration 13: [-1.12242263 -1.39322215 -4.24671363  1.49344478  0.30112245]
Iteration 14: [-1.12419099 -1.38846579 -4.24282386  1.49547653  0.30047762]
Iteration 15: [-1.12138746 -1.3878262  -4.24720104  1.49545593  0.30036082]
Iteration 16: [-1.12193013 -1.38980871 -4.24676039  1.4947327   0.30064225]
Iteration 17: [-1.12271169 -1.38920489 -4.24536901  1.49504444  0.30056591]
Iteration 18: [-1.12218733 -1.38874481 -4.24611322  1.4951738   0.30049665]

array([-1.12218733, -1.38874481, -4.24611322,  1.4951738 ,  0.30049665])
```

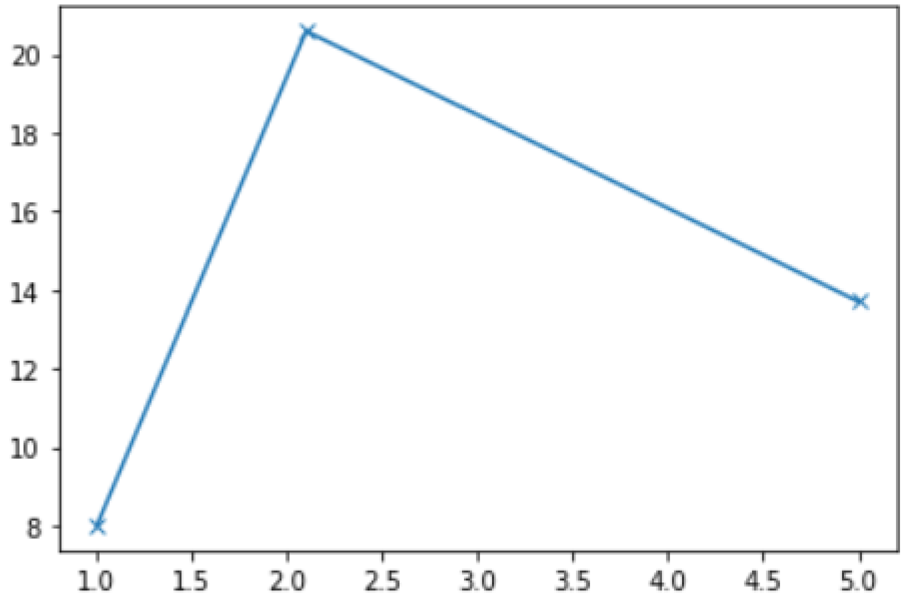
For problem-2:

If two points are given, then the linear interpolant is the straight line between these points, and is given below -

$$\begin{aligned} y &= y_0\left(1 - \frac{x - x_0}{x_1 - x_0}\right) + y_1\left(1 - \frac{x_1 - x}{x_1 - x_0}\right) \\ &= y_0\left(1 - \frac{x - x_0}{x_1 - x_0}\right) + y_1\left(\frac{x - x_0}{x_1 - x_0}\right) \\ &= y_0\left(\frac{x_1 - x}{x_1 - x_0}\right) + y_1\left(\frac{x - x_0}{x_1 - x_0}\right) \end{aligned}$$

Graphs-

For the given three points -



<u>For problem-3:</u>

Graphs-

For the given set of points -