Assignment - 0

Q1. a) Show that $\lambda_{max} = \frac{b}{T}$

Let the wavelength be λ

Planck energy density is given by →

$$u(\lambda)\,d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{[hc/\lambda K_B T]}-1} d\lambda$$

for planck energy density to be maximum,

$$\frac{du}{d\lambda} = 0$$

$$u = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{[hc/\lambda K_B T]}-1}$$

$$\frac{du}{d\lambda} = 0$$

$$0 = \frac{8\pi hc}{\cancel{\lambda^6}}(-5) \frac{1}{e^{hc/K_B\lambda T}-1} - \frac{8\pi hc}{\cancel{\lambda^5}} \cdot \frac{e^{hc/\lambda K_B T}}{(e^{hc/K_B\lambda T}-1)^2} \cdot \left(\frac{hc}{K_B T}\right)\left(\frac{-1}{\cancel{\lambda^2}}\right)$$

$$5 = \frac{1}{(e^{hc/K_B\lambda T}-1)} \cdot \frac{hc}{K_B T} \cdot \frac{1}{\lambda} \cdot e^{hc/\lambda K_B T}$$

Let $\frac{hc}{\lambda K_B T} = a$

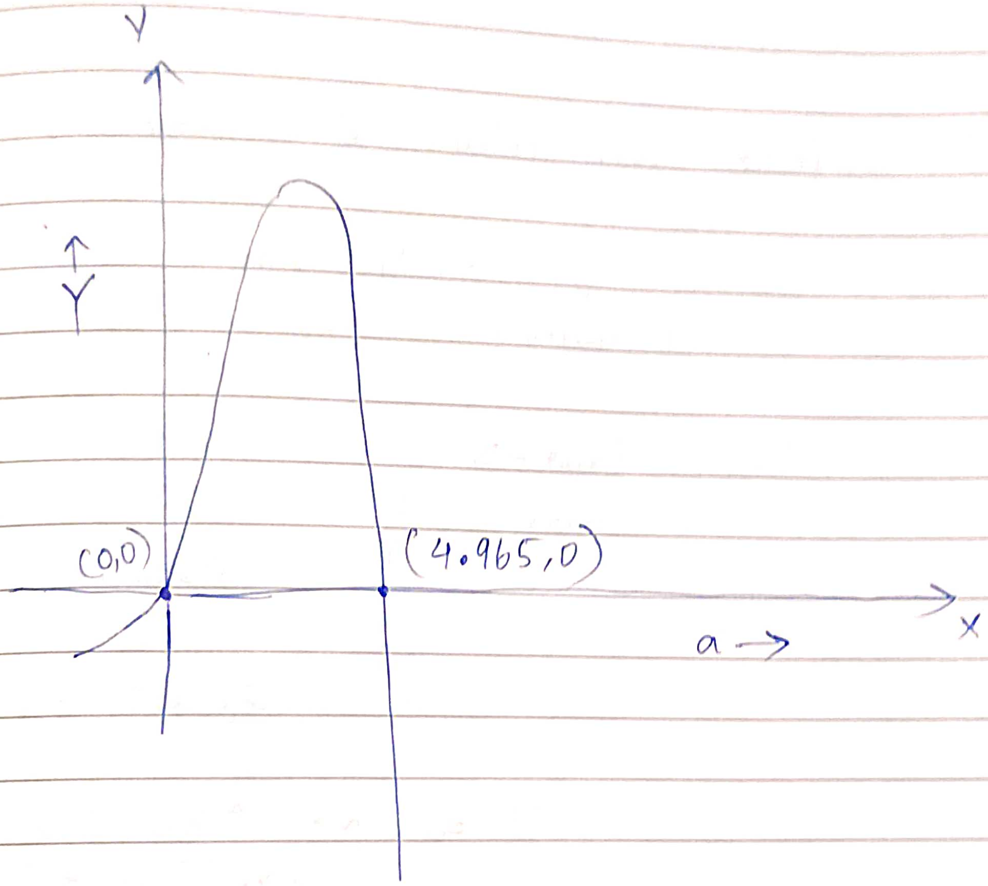
Then, we have →

$$5 = \frac{1}{(e^a - 1)} \cdot a e^a$$

$$5(e^a - 1) = ae^a$$

$$5(e^a - 1) - ae^a = 0$$

On plotting the graph of $Y = 5(e^a - 1) - ae^a$, we get →



Clearly, $Y = 0$ when $a = 0, 4.965$

Since we have to find λ_{max},

$\therefore$, $a \neq 0$ & $a = 4.965$

$$\therefore, \quad a = \frac{hc}{\lambda k_B T}$$

$$\frac{hc}{\lambda_{max} k_B T} = 4.965$$

$$\Rightarrow \lambda_{max} = \frac{hc}{(4.965) k_B} \cdot \frac{1}{T}$$

We can take $b = \frac{hc}{4.965 k_B}$, then →

$$\lambda_{max} = \frac{b}{T}$$

Let $\lambda_{max} = D_{max}$ (as per question)

∴, $D_{max} = \frac{b}{T}$ (hence proved)

b) surface temperature of a star = ?

I_{max} at $\lambda = 446\,nm$

Intensity = ?

Wien's law →

$$\lambda_{max} = \frac{b}{T} = \frac{hc}{4.965 k_B} \cdot \frac{1}{T}$$

$$= 2.89777 \times 10^{-3} \cdot \frac{1}{T}$$

$$T = \frac{2.89777 \times 10^{-3}}{446 \times 10^{-9}}$$

$T = 6497.085202\,K$

$T = 6497.085$ Kelvin

Intensity = $R = \sigma T^4$

where $\sigma = 5.67 \times 10^{-8}$

$$R = (5.67 \times 10^{-8})(6497.085)^4$$

$$R = 101.0316 \times 10^6 \; W/m^2$$

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c) $T = 3300k$

$$\lambda = \frac{b}{T} \rightarrow \text{from Wien's law}$$

$$\lambda = \frac{2.8977 \times 10^{-3}}{3300}$$

$$\lambda = 8.78091 \times 10^{-7}\ m$$

Intensity $= R = \sigma T^4$, $\sigma = 5.67 \times 10^{-8}$

$$R = (5.67 \times 10^{-8})(3300)^4$$

$$R = 6.72417207 \times 10^{6}\ w/m^2$$

Q2. $D1 = 80nm$, $D2 = 110nm$

$K.E_1 = E_1 = 11.390eV$, $E_2 = 7.154eV$

a) Find $h = ?$

$$\frac{hc}{\lambda} = K.E + \phi$$

$$K.E = \frac{hc}{\lambda} - \phi$$

$$E_1 = 11.390 \times 1.6 \times 10^{-19} = \frac{hc}{80 \times 10^{-9}} - \phi \quad -①$$

$$E_2 = 7.154 \times 1.6 \times 10^{-19} = \frac{hc}{110 \times 10^{-9}} - \phi \quad -②$$

① − ② $\rightarrow$

$$1.6 \times 10^{-19}[11.390 - 7.154] = hc\left[\frac{10^9}{80} - \frac{10^9}{110}\right]$$

$$= hc\frac{(110-80)}{110 \times 80}$$

$$\Rightarrow h = 6.626986667 \times 10^{-34}\ J\text{-}s$$

b) ϕ - work function = ?
$\nu_0 = ?$, $\lambda_0 = ?$

$$7.154 \times 1.6 \times 10^{-19} - \frac{hc}{110 \times 10^{-9}} = -\phi$$

$$-\phi = 7.154 \times 1.6 \times 10^{-19} - \frac{6.62649867 \times 10^{-34} \times 3 \times 10^{8}}{110 \times 10^{-9}}$$

$$\Rightarrow \phi = 6.627200091 \times 10^{-19}\ J$$

$$\phi = 6.6272 \times 10^{-19}\ J$$

$$h\nu_0 = \phi$$

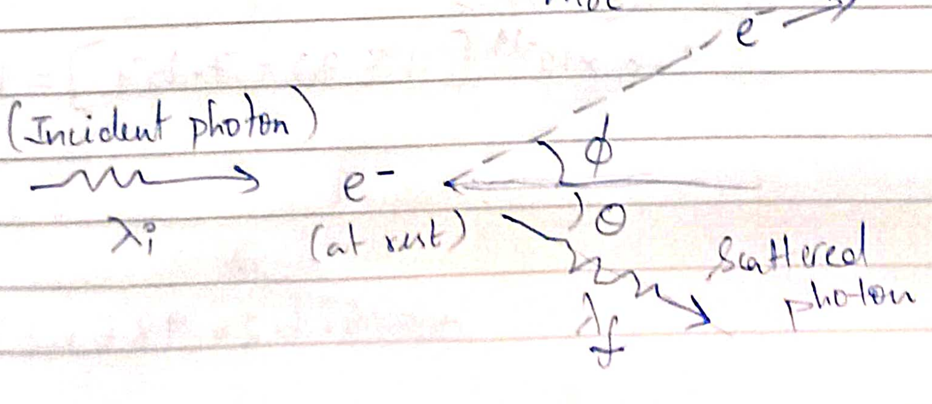
$$\nu_0 = \frac{6.627200091 \times 10^{-19}}{6.62649867 \times 10^{-34}}$$

$$\nu_0 = 10^{15}\ Hz$$

$$\lambda_0 = \frac{c}{\nu_0} = 3 \times 10^{-7}\ m$$

Q3. a) Wavelength shift = ?

$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$



Since the photons are backscattered, $\theta = \pi$

$$\therefore, \lambda_f - \lambda_i = \frac{h}{m_0 c}(1-(-1)) = \frac{2h}{m_0 c}$$

m_0 = rest mass of e^-

$$\lambda_f - \lambda_i = \Delta\lambda = \frac{2 \times 6.626 \times 10^{-34}}{3\times10^{8} \times 9.11\times10^{-31}}$$

$$\Delta\lambda = 4.84888401 \times 10^{-12} \text{ m}$$

$$\Delta\lambda = 4.849 \times 10^{-12} \text{ m}$$

b) Show That → energy of scattered photons is half the rest mass energy of e^- regardless of energy of incident photons.

Let energy of scattered photons be E_2

then, we have → $E_2 = \frac{hc}{\lambda_f}$

where $\lambda_f = \lambda_i + \frac{2h}{m_0 c}$

$$\lambda_f = 4.849 \times 10^{-12} + \lambda_i$$

$$\lambda_f = \lambda_i + \frac{2h}{m_0 c}$$

$$E_2 = \frac{hc}{\lambda_i + \frac{2h}{m_0 c}} \qquad E_1 = \frac{hc}{\lambda_i}$$

$$E_2 = \frac{hc}{\frac{hc}{E_1} + \frac{2h}{m_0 c}} = \frac{c}{\frac{c}{E_1} + \frac{2}{m_0 c}}$$

$$E_2 = \frac{c}{\frac{c}{E_1} + \frac{2}{m_0 c}} = \frac{c\,[m_0 c E_1]}{c^2 m_0 + 2E_1}$$

m_0 = rest mass of e^-

E_1 = rest mass energy of e^-

$$E_2 = \frac{m_0 c^2 E_1}{c^2 m_0 + 2E_1}$$

Given $E >> m_0 c^2$

$$\therefore,\quad E_2 = \frac{m_0 c^2}{2 + c^2 m_0 / E_1} = \frac{m_0 c^2}{2}$$

$$\frac{c^2 m_0}{E_1} \to 0$$

$$E_2 = \frac{m_0 c^2}{2}$$

We know the rest mass energy of e^- is $E_1 = m_0 c^2$

$$\therefore,\quad E_2 = \frac{E_1}{2}$$

$\therefore$, The energy of scattered photons is half the rest mass energy of the e^-.

c) $E_2 = \frac{E_1}{2} = \frac{m_0c^2}{2} = \frac{(9.11\times10^{-31})\times(3\times10^8)^2}{2}$

$E_2 = 256.21875 \times 10^3\ eV$

$E_2 = 0.256\ MeV = 0.256 \times 10^6\ eV$

From conservation of energy → $K_e = E - E_2$ where $E = 150\ MeV$

$E_2 = 0.256\ MeV$

$K_e = (150 - 0.256)\ MeV$

$K_e = 149.744\ MeV$

Q4.

The minimum energy Emin of a photon required to produce an electron-positron pair must be equal to the rest mass energies of the electron & positron, this corresponds to the case where the kinetic energies of the electron & positron are zero.

The K.E of e^+ & e^- will be zero for minimum case.

$\hbar\omega = E_{e^-} + E_{e^+} + E_N$

$= (m_ec^2 + K_{e^-}) + (m_ec^2 + K_{e^+}) + K_N$

$K_{e^-} = K_{e^+} = 0$

$\therefore$ $\hbar\omega_{min} = E_{min} = 2m_ec^2$

$= 2 \times 0.511\ MeV$

$= 1.022\ MeV$

$E_{min} = 1.022\ MeV$

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$E_{min} = h\nu$

$\nu = \dfrac{1.022 \times 10^{6} \times 1.6 \times 10^{-19}}{6.626 \times 10^{-34}}$ $= \cancel{\nu_{min}}$

$\nu = 2.4715 \times 10^{20}\ Hz$

$\nu = 2.4715 \times 10^{20}\ Hz$

$\lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^{8}}{2.4715 \times 10^{20}}$

$\lambda = 1.2138 \times 10^{-12}\ m$

Q5. a) $K.E = 70 MeV \rightarrow$ proton

$\lambda = ?$

$K.E = \dfrac{p^2}{2m_p}$ & $\lambda = \dfrac{h}{p}$

$\Rightarrow p = \dfrac{h}{\lambda}$

$K.E = \dfrac{h^2}{\lambda^2 2m_p}$

$70 \times 10^{6} \times 1.6 \times 10^{-19} = \dfrac{h^2}{\lambda^2 2m_p}$

$\lambda^2 = \dfrac{h^2}{(K.E) \cancel{\times} (2m_p)}$

$\lambda = \dfrac{h}{\sqrt{2m_p (K.E)}} = \dfrac{6.626 \times 10^{-34}}{\sqrt{2m_p (K.E)}}$

$m_p = 1.672 \times 10^{-27}\ kg$

$K.E = 70 \times 10^{6} \times 1.6 \times 10^{-19}\ J$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times (70 \times 10^{6} \times 1.6 \times 10^{-19}) \times (1.672 \times 10^{-27})}}$$

$$\lambda = 3.423807081 \times 10^{-15} m$$

$$\lambda = 3.42381 \times 10^{-15} m$$

b) $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{(100 \times 10^{-3})(900)}$

$$\lambda = 7.362 \times 10^{-36} m$$

Q6. $\Delta x = ?$

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4\pi \Delta p}$$

$$\Delta x = \frac{6.626 \times 10^{-34}}{4\pi \, m_N v}$$

$m_N = 1.65 \times 10^{-27} kg$
$v = 5 \times 10^{6} m/s$

$$\Delta x \simeq \frac{6.626 \times 10^{-34}}{4\pi (1.65 \times 10^{-27} \times 5 \times 10^{6})}$$

$$\Delta x \simeq 6.39 \times 10^{-15} m$$

b) $\Delta x \geq \dfrac{\hbar}{2\Delta p}$

$$\Delta x \simeq \frac{\hbar}{2mv} = \frac{h}{4\pi mv}$$

$$\Delta x \simeq \frac{6.626 \times 10^{-34}}{4\pi \times 50 \times 2}$$

$$\Delta x \simeq 5.2728 \times 10^{-37} \, m$$

Q7. a) $E_n = ?$ $r_n = ?$

Reduced mass $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$

for e^- & e^+, $m_1 = m_2 = m_e$

$$\mu = \frac{m_e^2}{2m_e} = \frac{m_e}{2}$$

Bohr's radius $a_0 = \dfrac{4\pi\epsilon_0 \hbar^2}{m_e e^2}$

n^{th} orbit radius $= r_n = a_0 n^2$

for positronium $= a_{0,P} = \dfrac{4\pi\epsilon_0\hbar^2}{\mu e^2}$

$$= \frac{2(4\pi\epsilon_0\hbar^2)}{m_e e^2}$$

$$= 2\frac{(4\pi\epsilon_0\hbar^2)}{m_e e^2}$$

$$= 2a_0$$

$$r_n = 2a_0 n^2$$

$$r_n = \frac{8\pi\epsilon_0 h^2 n^2}{m_e e^2}$$

$$r_n \approx 0.106\, n^2 \text{ nm}$$

$$E_n = -\frac{2\pi^2 m e^4 z^2}{h^2 (4\pi\epsilon_0)^2} \cdot \frac{1}{n^2}$$

$$= -\frac{13.6}{n^2}$$

For positronium →

$$E_n = -\frac{13.6}{2n^2}$$

$$E_n = \frac{\pi^2 m_e e^4 z^2}{h^2 (4\pi\epsilon_0)^2} \cdot \frac{1}{n^2}$$

$$E_n = \frac{-6.8 \text{ eV}}{n^2}$$

$r_p = 2r_e$, $E_p = E_e/2$

b) Values of energies & radii of 3 lowest state

$r_1 = 2a_0 = 0.106\text{nm}$	$E_1 = -6.8\text{eV}$
$r_2 = 2a_0(4) = 8a_0 = 0.424\text{nm}$	$E_2 = -1.7\text{eV}$
$r_3 = 2a_0(9) = 18a_0 = 0.954\text{nm}$	$E_3 = -0.755\text{eV}$

c) $f = ?$

$\lambda = ?$

energy of 1st excited state is E_2

$E_2 = -1.7 eV$

$h\nu = E_\infty - E_2$

$= 0 - (-1.7 eV)$

$= 1.7 \times 1.6 \times 10^{-19}$

$h\nu = 1.7 \times 1.6 \times 10^{-19} J$

$$\nu = \frac{1.7 \times 1.6 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$\nu = 4.105 \times 10^{14} Hz$

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^{8}}{4.105 \times 10^{14}}$$

$\lambda = 7.308 \times 10^{-7} m$

Q8. a) $P = 45 kW$

$f = 4 MHz$

a) no. of photons emitted per second

$$n = \frac{E}{h\nu} = \frac{45 \times 10^{3}}{6.626 \times 10^{-34} \times 4 \times 10^{6}}$$

$n = 1.797 \times 10^{31}$ photons per second

b) As the no. of photons emitted is very large, quantum effects can be ignored. We can ignore the quantum nature of the electromagnetic radiation emitted from the antenna.

Q9. $m = 4\,kg$, $k = 196\,N/m$

$x = 25\,cm = 0.25\,m$

a) $$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{196}{4}} = \frac{7}{2\pi} = 1.1141\,Hz$$

$$E = \frac{1}{2} k x_A^2 = \frac{1}{2}(196)(0.25)^2$$

$$E = 6.125\,J$$

b) energy spacing $\Delta E = h\nu$

$$= (6.626 \times 10^{-34})(1.1141)$$

$$\Delta E = 7.386 \times 10^{-34}\,J$$

$$n = \frac{E}{\Delta E} = \frac{6.125}{7.386 \times 10^{-34}}$$

$$n = 8.4619 \times 10^{33}$$

Since the energy of 1 quanta is very small compared to the total energy, the energy levels of the oscillators can be assumed as continuous

∴, The quantum effects are not important.

Q10.

a) $d = 0.27nm = \lambda$

$$p = \frac{2\pi\hbar}{\lambda}$$

$$E = \frac{p^2}{2m_e} = \frac{4\pi^2\hbar^2}{\lambda^2(2m_e)}$$

$$E = \frac{4\pi^2(1.055 \times 10^{-34})^2}{(0.27 \times 10^{-9})^2(2 \times 9.11 \times 10^{-31})}$$

$$E = 3.308 \times 10^{-18} J$$

$$E = 20.676 eV$$

b) $K.E = 2eV$ protons

5th max intensity observed at 30°, find crystal's plane separation

Bragg's formula →

$$n\lambda = 2d\sin\phi$$

$$5\lambda = 2d\sin 30° = d$$

$$E = \frac{p^2}{2m_p} = \frac{2\pi^2\hbar^2}{m_p\lambda^2}$$

$$m_p = 1.672 \times 10^{-27} kg$$

$$2 \times 1.6 \times 10^{-19} = E = \frac{2\pi^2(1.055 \times 10^{-34})^2 \times 25}{1.672 \times 10^{-27} \times d^2}$$

$$\Rightarrow d = 1.013197732 \times 10^{-10} m$$

Q11. a) Conservation of energy →

$$h\nu + m_ec^2 = h\nu' + (K_e + m_ec^2)$$

ν, ν' → frequencies of initial & scattered photons respectively.

m_ec^2 → rest mass energy of the initial electron

K_e → recoil kinetic energy.

$$K_e = h(\nu - \nu')$$
$$= hc\left(\frac{\lambda' - \lambda}{\lambda \cdot \lambda'}\right) = \frac{hc}{\lambda}\left(\frac{\lambda' - \lambda}{\lambda'}\right)$$

$$K_e = \frac{h\nu\,\Delta\lambda}{\lambda'}$$

wavelength shift = $\Delta\lambda = \lambda' - \lambda$

$$= \frac{h}{m_ec}(1 - \cos\theta)$$

$$= 2\lambda_c \sin^2\left(\frac{\theta}{2}\right)$$

$$\Delta\lambda = \frac{2\pi\hbar c}{m_e c^2}(1 - \cos\theta)$$

$$\Delta\lambda = 2\pi \times \frac{h}{2\pi} \cdot \frac{1}{m_ec}(1 - \cos 60^\circ)$$

$$m_e = 9.11 \times 10^{-31}\ kg$$

$$\Delta\lambda = \frac{2\pi \times 6.626 \times 10^{-34}}{2\pi} \cdot \frac{(1 - \cos 60^\circ)}{9.11 \times 10^{-31} \times 3 \times 10^{6}}$$

$$\Delta\lambda = 0.0012\,nm$$

$$\lambda' = \Delta\lambda + \lambda = \Delta\lambda + \frac{2\pi\hbar c}{h\nu}$$

$\lambda' = 0.0012 + 0.414$
$\lambda' = 0.4152 nm$

$$k_e = (h\nu)\left(\frac{0.0012}{0.4152}\right)$$

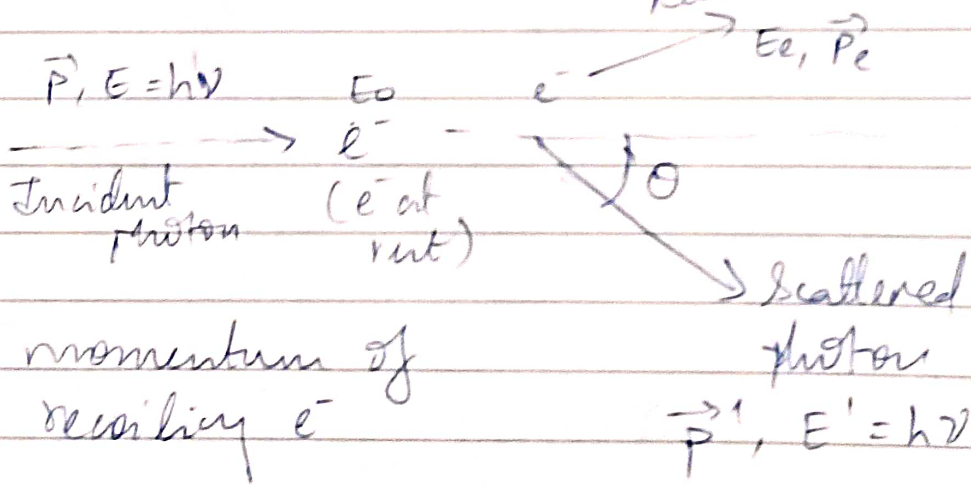
$$k_e = (3\times10^3)\left(\frac{0.0012}{0.4152}\right)$$

$$k_e = 8.671 eV$$

b) conservation of total momentum along x & y-axis

p, p' → initial & final momentum of photons respectively

$p = p_e\cos\phi + p'\cos\theta$ - ①
$0 = p_e\sin\phi - p'\sin\theta$ - ②



p_e → momentum of recoiling e^-

$p_e\cos\phi = p - p'\cos\theta$
$p_e\sin\phi = p'\sin\theta$

$$\tan\phi = \frac{\sin\theta}{\frac{p}{p'} - \cos\theta} = \frac{\sin\theta}{\frac{\lambda'}{\lambda} - \cos\theta}$$

$p = h/\lambda,\ p' = h/\lambda'$
$\lambda = 0.414 nm$
$\lambda' = 0.4152 nm$

$$\phi = \tan^{-1}\left(\frac{\sin\theta}{\frac{\lambda'}{\lambda} - \cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sin 60^\circ}{\frac{0.4152}{0.414} - \cos 60^\circ}\right)$$

$$\phi = 59.86^\circ$$

Q12. $E = mc^2 = \dfrac{m_0c^2}{\sqrt{1 - v^2/c^2}}$

$$p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

$$E = c\sqrt{p^2 + m_0^2c^2}$$

$$c^2\left(p^2 + m_0^2c^2\right) = \frac{m_0^2 c^4}{1 - \frac{v^2}{c^2}}$$

We know →

group velocity $v_g = \dfrac{dE(p)}{dp}$

$$v_g = \frac{1}{2} \cdot \frac{1}{\sqrt{p^2 + m_0^2c^2}} (c)(2p)$$

$$v_g = \frac{pc}{\sqrt{p^2 + m_0^2c^2}} = v$$

Phase velocity $v_p = \dfrac{E(p)}{p} = c\sqrt{1 + \dfrac{m_0^2c^2}{p^2}} = \dfrac{c^2}{v}$

Q3.
a) According to uncertainty principle →

$r p \sim \hbar$ where $r \to$ radius of the $\bar{e}$'s orbit
$p \to \bar{e}$'s momentum

$$p \sim \frac{\hbar}{r}$$

To find ground state radius, minimize electron - proton energy →

$$E(r) = \frac{p^2}{2m_e} - \frac{e^2}{4\pi\epsilon_0 r}$$

$$= \frac{\hbar^2}{2m_e r^2} - \frac{e^2}{4\pi\epsilon_0 r}$$

$$\frac{dE(r)}{dr} = 0$$

$$-\frac{\hbar^2}{m_e r_0^3} + \frac{e^2}{4\pi\epsilon_0 r_0^2} = 0$$

$$\Rightarrow r_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 0.053 nm$$

b) $$E(r_0) = \frac{\hbar^2}{2m r_0^2} - \frac{e^2}{4\pi\epsilon_0 r_0}$$

$$= -\frac{m_e}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 = -13.6 eV$$

Q14. Rest mass energy $= m_N c^2 = 939.6\,MeV$

$\Delta x_0 = 10^{-14}\ m$

Time constant $\tau = \dfrac{2m_N^2 (\Delta x_0^2)}{\hbar c^2}$

$$\tau = \frac{2 \times (1.674 \times 10^{-27}) \times 10^{-28} \times 2\pi}{6.626 \times 10^{-34}}$$

$\tau = 3.174 \times 10^{-21}\ s$

Time taken by a neutron packet to grow from an initial width Δx_0 to a final size $\Delta x(t)$ is $\Rightarrow$

$$t = \tau \sqrt{\left(\frac{\Delta x(t)}{\Delta x_0}\right)^2 - 1}$$

$$t = (3.17 \times 10^{-21}) \sqrt{\left(\frac{\Delta x(t)}{\Delta x_0}\right)^2 - 1}$$

a) 4 times its original size $(\Delta x(t) = 4x_0)$

$t = 3.174 \times 10^{-21} \sqrt{4^2 - 1}$

$t = 3.174 \times 10^{-21} \sqrt{15}$

$t = 12.29 \times 10^{-21}$

$t = 1.229 \times 10^{-20}\ s$

b) size equal to earth's diameter
(diameter $= 12.7 \times 10^6$)

$$t = 3.174 \times 10^{-21} \sqrt{\left(\frac{12.7 \times 10^6}{10^{-14}}\right)^2 - 1}$$

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$t = 4.18$

c) Size equal to the distance between earth & moon (3.84×10^{8})

$$t = 3.174 \times 10^{-21} \sqrt{\left(\frac{3.84 \times 10^{8}}{10^{-14}}\right)^2 - 1}$$

$t = 12.38$