Department of Physics

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Assignment # 8

§1. The number of ways in which N identical bosons can be distributed in two energy levels is

§2. A system of particles on N lattice sites is in equilibrium at temperature T and chemical potential μ. Multiple occupancy of a sites is forbidden. The binding energy of a particle at each site is -ϵ. The probability of not occupying a site is

§3. Consider a gas of Cs atoms at a number density of 10^{12} atoms/cc. When the typical inter-particle distance is equal to the thermal de Broglie wavelength of the particles, the temperature of the gas is nearest to (Take the mass of a Cs atom to be 22.7×10^{-26}Kg)

§4. Consider the system having three energy level with energy 0, 2ϵ and 3ϵ with respective degeneracy of 2, 2 and 3. Four boson of spin 0 have to be in their level such that total energy of the system is 10ϵ. The number of ways it can be done is

§5. Derive expression for classical and quantum heat capacity, C_V corresponding to the vibrational modes for a polyatomic gas.[**Hint:** take equal mass of atoms in the molecule and consider them as simple oscillator. Classical $H = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$; Eigenvalue of $\mathbf{H}$ for quantum oscillator, $H_{vib} = (n + \frac{1}{2})\hbar\omega$; and find the partition function, Q_{vib}]

§6. Derive expression for classical and quantum heat capacity, C_V corresponding to the rotational modes for a polyatomic gas.[**Hint:** take equal moment of inertia of atoms in the molecule. Classical $H_{rot} = \frac{1}{2I}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right)$; Eigenvalue of **H** for quantum oscillator, $H_{rot} = \frac{L^2}{2I} = \hbar^2 l(l+1)/2I$; and find the partition function, Q_{rot}]

§7. Consider an electron in a magnetic field $\vec{B}$, with intrinsic spin $\frac{\hbar}{2}\hat{\sigma}$, where $\hat{\sigma}$ pauli spin operator, for $\vec{B} = B\hat{z}$, find the density matrix in canonical ensemble and also obtain $\langle\sigma_x\rangle$.

§8. Consider an ideal quantum gas of Fermi particles at a temperature T. Write the probability p(n) that there are n particles in a given single particle state as a function of the mean occupation number, $\langle n\rangle$.

§9. Maximize the entropy S $= \sum_i P_i ln P_i$, where P_i is the probability of the ith level being occupied, subject to the constraints that $\sum_i P_i = 1$, $\sum_i P_i E_i = U$ and $\sum P_i N_i = N$ to re-derive the grand canonical ensemble.

§10. Show that the entropy of a pure state is zero. How can you maximize the entropy?

§11. Consider a three-state system and think of $\langle\psi_1\rangle$ as a column matrix $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$,and hence $\langle\psi_1|$ as a row vector $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$, and similarly for $\langle\psi_2\rangle$, $\langle\psi_2|$, $\langle\psi_3\rangle$ and $\langle\psi_3|$. Find the density matrix in these basis.