**Indian Institute of Technology Kharagpur**
**Mathematical Methods I (Spring 2021-22)**
**Assignment-1**

**Curvilinear Coordinates and Tensors**

**Q.1.** Consider the distance-squared $ds^2$ in a coordinate system $(u_1, u_2, u_3)$ given as:

$$ds^2 = h_1^2 du_1^2 + h_2^2 du_2^2 + h_3^2 du_3^2$$

where $h_{1,2,3}$ are the scale factors which depend on the coordinates.
**(a)(3 points)** Obtain general expressions for the gradient, divergence, curl and Laplacian.
**(b)(3 points)** Use the general expressions obtained above and write down the corresponding ones (i.e.gradient, divergence, curl and Laplacian) for spherical polar and cylindrical coordinates.
**Q.2.(4 points)** The Navier-Stokes equations of hydrodynamics contain a nonlinear term

$$\nabla \times [\vec{v} \times (\nabla \times \vec{v})]$$

where $\vec{v}$ is the fluid velocity. Evaluate this term in cylindrical coordinates assuming fluid is flowing in the z direction.
**Q.3. (4 points)** A single current loop in the x y-plane has a vector potential $\vec{A}$ that is a function only of r and $\theta$ , is entirely in the $\vec{e_\phi}$ direction and is related to the current density $\vec{J}$ by the equation

$$\mu_0 \vec{J} = \nabla \times \vec{B} = \nabla \times (\nabla \times \vec{e_\phi} A_\phi)$$

Evaluate the RHS of this equation in spherical polar coordinate.
**Q.4.(3 points)** The cartesian components of the acceleration vector are

$$a_x = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}, a_y = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}, a_z = \frac{\mathrm{d}^2 z}{\mathrm{d}t^2}$$

Find the components of the acceleration vector in the spherical polar coordinates.
**Q.5.(3 points)** If $A_k^{ij}$ and $B_q^p$ are tensors, show that $A_k^{ij} B_q^i$ is not a tensor.
**Q.6.(4 points)** Prove that central forces are irrotation.