Roll No: 20PH20014
Name: Jessica John Britto

Optics Lab
PH29204

EXPERIMENT-3

DISPERSIVE POWER OF A PRISM

Aim: To determine the dispersive power of material of prism using Spectrometer.

Apparatus: Spectrometer, Prism, Mercury Vapor Lamp etc.

Theory: A spectrometer is used to measure the necessary angles. The spectrometer consists of three units: (1) collimator, (2) telescope, and (3) prism table. The prism table, its base and telescope can be independently moved around their common vertical axis. A circular angular scale enables one to read angular displacements (together with two venires located diametrically opposite to each other).
Experiment, we need to produce a parallel beam of rays to be incident on the prism. This is done with the help of a collimator. The collimator has an adjustable rectangular slit at one end and a convex lens at the other end. When the illuminated slit is located at the focus of the lens (See Fig. 1), a parallel beam of rays emerges from the collimator. We can test this point, with the help of a telescope adjusted to receive parallel rays. We first prepare the telescope towards this purpose as follows:

Setting the eyepiece:

- Focus the eyepiece of the telescope on its cross wires (for viewing the cross wires against a white background such as a wall) such that a distinct image of the crosswire is seen by you. In this context, remember that the human eye has an average "least distance of distinct vision" of about 25 cm. When you have completed the above eyepiece adjustment, you have apparently got the image of the crosswire located at a distance comfortable for your eyes. Henceforth do not disturb the eyepiece.

Setting the Telescope:

- Focus the telescope onto a distant (infinity!) object. Focusing is done by changing the separation between the objective and the eyepiece of the telescope. Test for the absence of a parallax between the image of the distant object and the vertical crosswire. Parallax effect (i.e. separation of two things when you move your head across horizontally) exits, if the cross-wire and the image of the distant object are not at the same distance from your eyes. Now the telescope is adjusted for receiving parallel rays. Henceforth do not disturb the telescope focusing adjustment.

Setting the Collimator:

- Use the telescope for viewing the illuminated slit through the collimator and adjust the collimator (changing the separation between its lens and slit) till the image of the slit is brought to the plane of cross wires as judged by the absence of parallax between the image of the slit and cross wires.

Optical leveling of the Prism:

- The prism table would have been nearly leveled before uses have started the experiment. However, for your experiment, you need to do a bit of leveling using reflected rays. For this purpose, place the table with one apex at the center and facing the collimator, with the ground (non-transparent) face perpendicular to the collimator axis and away from collimator. Slightly adjust the prism so that the beam of light from the collimator falls on the two reflecting faces symmetrically (Fig. 2) when you have achieved this lock the prism table in this position. Turn the telescope to one side so as to receive the reflected image of the slit centrally into the field of view. This may be achieved by using one of the leveling screws. The image must be central whichever face is used as the reflecting face. Similarly, repeat this procedure for the other side.

Finding angle of minimum deviation (*Dm*)

- Unlock the prism table for the measurement of the angle of minimum deviation(*Dm*).
- Locate the image of the slit after refraction through the prism as shown in Fig. 3. Keeping the image always in the field of view, rotate the prism table till the position where the deviation of the image of the slit is smallest.
- At this position, the image will go backward, even when you keep rotating the prism table in the same direction. Lock both the telescope and the prism table and to use the fine adjustment screw for finer settings. Note the angular position of the prism.
- In this position the prism is set for minimum deviation. Without disturbing the prism table, remove the prism and turn the telescope (now unlock it) towards the direct rays from the collimator. Note the scale reading of this position. The angle of the minimum angular deviation, i.e., (*Dm*) is the difference between the readings for these last two settings.

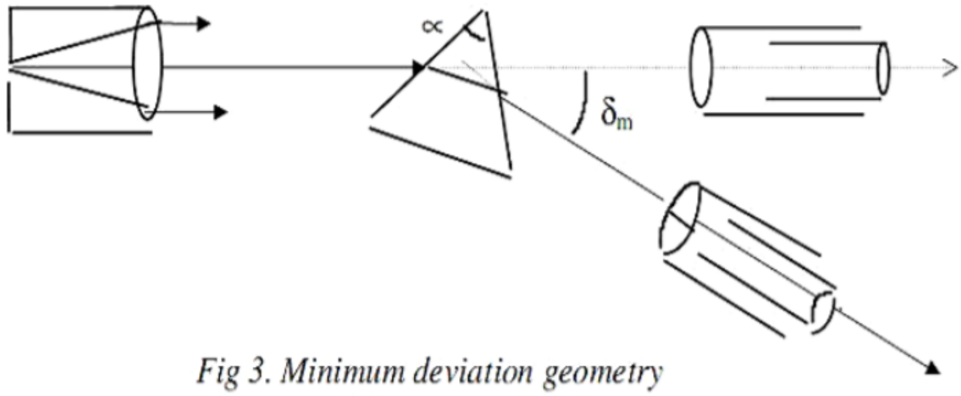

Fig 3. Minimum deviation geometry

Working Formula-

The dispersive power of the prism is given by-

$$\omega = \frac{\mu_1 - \mu_2}{\mu_{ab} - 1}$$

Say, $\mu_1 = \mu_v$ (refractive index of violet color), $\mu_2 = \mu_r$ (refractive index of red color) and μ_y (refractive index of yellow color), where μ_{ab} is-

$$\mu_{ab} = \frac{\mu_1 + \mu_2}{2} = \frac{\mu_v + \mu_r}{2} \approx \mu_y$$

$$\therefore \omega = \frac{\mu_1 - \mu_2}{\mu_{ab} - 1} = \frac{\mu_v - \mu_r}{\mu_{ab} - 1} \approx \frac{\mu_v - \mu_r}{\mu_y - 1}$$

$$\mu_v = \frac{\sin\left(\frac{A + D_v}{2}\right)}{\sin\frac{A}{2}}$$

$$\mu_r = \frac{\sin\left(\frac{A + D_r}{2}\right)}{\sin\frac{A}{2}}$$

$$\mu_y = \frac{\sin\left(\frac{A + D_y}{2}\right)}{\sin\frac{A}{2}}$$

- A = Angle of the prism
- *Dv* = Angle of minimum deviation for violet color
- *Dr* = Angle of minimum deviation for red color
- *Dy* = Angle of minimum deviation for yellow color

Observations:

Table-1: Vernier constant of the spectrometer

Divisions (say, m) of the vernier scale = Divisions (say, n) of the vernier scale

Value of 1 smallest main scale division (l_1) (Min. or sec.)	Value of 1 vernier division $\left(l_2 = \frac{n}{m} \times l_1\right)$ (Min. or sec.)	Vernier constant $= (l_1 - l_2)$ (Min. or sec.)
$l_1 = 30'$	$l_2 = \frac{59}{60} \times 30' = \left(\frac{59}{2}\right)'$	Vernier constant $= \left(30' - \left(\frac{59}{2}\right)'\right) = 30'' = \delta_s$

Table-2: ANGLE OF PRISM

Vernier no.	Reading at the 1st position of the telescope				Reading at the 2nd position of the telescope				2A	Mean 2A = (a+b)/2= C	A = C/2
	Main scale (M)	vernier scale (V)	Total (T) = M+V	Mean T	Main scale (M)	vernier scale (V)	Total (T) = M+V	Mean T = b			
1	1.5°	32	1°46′0″	1°43′45″= a_1	121°	1	121°0′30″	120°59′0″ $= a_2$	119°15′15″ $= a_2 - a_1 =$ a	119°16′45″	59°38′22.5″= 59°64′= 60.06°
	1.5°	23	1°41′30″		120.5°	55	120°57′30″				
2	185.5°	26	181°43′0″	181°44′15″ $= b_1$	301°	4	301°2′00″	301°2′30″ $= b_2$	119°18′15″ $= b_2 - b_1 =$ b		
	185.5°	31	181°45′30″		301°	6	331°3′0″				

Table-3: Reading of the telescope at minimum deviation

Color	Vernier no.	No. of obs.	MSR	VSR	Total	Mean reading
Red	1st (d_1)	1	101.5°	41	101°50′30″	101°46′10″
		2	101.5°	32	101°46′0″	
		3	101.5°	36	101°48′0″	
	2nd (d_2)	1	281.5°	26	281°43′0″	281°45′40″
		2	281.5°	31	281°45′30″	
		3	281.5°	37	281°48′30″	
yellow	1st (d_3)	1	102°	37	102°13′30″	102°13′50″
		2	102°	32	102°16′0″	
		3	102°	24	102°12′0″	
	2nd (d_4)	1	282°	20	282°10′0″	282°12′30″

		2	282°	30	282°15′0″	
		3	282°	25	282°12′30″	
Violet	1st (d_5)	1	105°	40	105°20′0″	105°20′40″
		2	105°	41	105°20′30″	
		3	105°	43	105°21′30″	
	2nd (d_6)	1	285°	31	285°15′30″	285°15′20″
		2	285°	28	285°14′0″	
		3	285°	37	285°16′30″	
Direct reading of the telescope						
1st (d_7)		No. of obs.	MSR	VSR	Total	Mean
		1	54°	30	54°15′0″	54°15′40″
		2	54°	35	54°17′30″	
		3	54°	29	54°14′30″	
2nd (d_8)		1	234°	26	234°13′0″	234°14′30″
		2	234°	32	234°16′0″	
		3	234°	29	234°14′30″	

Error Analysis:

Working Formula-

Given the least count is 1V in radians = 1 arc minute = 0.000291 radian.

$$\mu = \frac{\sin\frac{A+d_m}{2}}{\sin\frac{A}{2}}$$

Taking log on both sides of the above equation and then differentiating, we get-

$$\frac{\Delta\mu}{\mu} = \cot\frac{A + d}{2}\left(\frac{\Delta A}{2} + \frac{\Delta d}{2}\right) + \frac{\Delta A}{2}\cot\frac{A}{2}$$

And we know that-

$$\Delta d = 2\,\Delta\theta = 2 \times 1\,V\ in\ radians$$

$$\Delta A = \Delta\theta = 1V\ in\ radians$$

Find refractive index of violet color μ_v -

$$\mu_V = \frac{sin(\frac{60.06+105.6}{2})}{sin(\frac{60.06}{2})} = 1.982$$

$$\frac{\Delta\mu_V}{\mu_V} = cot(\frac{60.06+105.6}{2}) \bullet (\frac{0.000291}{2} + \frac{0.000582}{2}) + cot(\frac{60.06}{2}) \bullet (\frac{0.000291}{2})$$

$$\frac{\Delta\mu_V}{\mu_V} = 0.000306$$

$$\Delta\mu_V = 0.000306 \times \mu_V = 0.000306 \times 1.982 = 0.000607$$

The refractive index of violet color μ_v is 1.982

The error in μ_v is 0.000306

The percentage error in μ_v is 0.0306%

Find refractive index of violet color μ_r -

$$\mu_r = \frac{sin(\frac{60.06+101.7}{2})}{sin(\frac{60.06}{2})} = 1.970$$

$$\frac{\Delta\mu_r}{\mu_r} = cot(\frac{60.06+101.7}{2}) \bullet (\frac{0.000291}{2} + \frac{0.000582}{2}) + cot(\frac{60.06}{2}) \bullet (\frac{0.000291}{2})$$

$$\frac{\Delta\mu_r}{\mu_r} = 0.00032$$

$$\Delta\mu_r = 0.00032 \times \mu_r = 0.00032 \times 1.970 = 0.00063$$

The refractive index of violet color μ_r is 1.970
The error in μ_r is 0.00032
The percentage error in μ_r is 0.063%

Find refractive index of violet color μ_y -

$$\mu_y = \frac{sin(\frac{60.06+102.6}{2})}{sin(\frac{60.06}{2})} = 1.975$$

$$\frac{\Delta\mu_y}{\mu_y} = cot(\frac{60.06+102.6}{2}) \bullet (\frac{0.000291}{2} + \frac{0.000582}{2}) + cot(\frac{60.06}{2}) \bullet (\frac{0.000291}{2})$$

$$\frac{\Delta\mu_y}{\mu_y} = 0.000318$$

$$\Delta\mu_y = 0.000318 \times \mu_y = 0.000318 \times 1.975 = 0.000628$$

The refractive index of violet color μ_y is 1.970
The error in μ_y is 0.00032
The percentage error in μ_y is 0.0628%

Dispersive Power-

$$\omega = \frac{\mu_1 - \mu_2}{\mu_{ab} - 1}$$

where μ_{ab} is-

$$\mu_{ab} = \frac{\mu_1 + \mu_2}{2} = \frac{\mu_v + \mu_r}{2} \approx \mu_y$$

As $\mu_1 = \mu_v$ (refractive index of violet color), $\mu_2 = \mu_r$ (refractive index of red color) and μ_y (refractive index of yellow color), then we get ω as given below-

$$\omega = \frac{\mu_V - \mu_r}{\mu_y - 1}$$

$$ln\omega = ln(\mu_V - \mu_r) - ln(\mu_y - 1)$$

Differentiating the above equation, we get-

$$\frac{\Delta\omega}{\omega} = \frac{\Delta(\mu_V - \mu_r)}{\mu_V - \mu_r} - \frac{\Delta(\mu_y - 1)}{\mu_y - 1}$$

For maximum error-

$$\frac{\Delta\omega}{\omega} = \frac{\Delta\mu_V + \Delta\mu_r}{\mu_V - \mu_r} + \frac{\Delta\mu_y}{\mu_y - 1}$$

We have the following values-

$\Delta\mu_V = 0.000607, \Delta\mu_r = 0.00063$ *and* $\Delta\mu_y = 0.000628$

$\mu_V = 1.982,\ \mu_r = 1.970$ *and* $\mu_y = 1.975$

$$\omega = \frac{\mu_V - \mu_r}{\mu_y - 1} = \frac{1.982 - 1.970}{1.975 - 1} = 0.012307692$$

$$\omega = 0.012307692 \approx 0.0123$$

$$\frac{\Delta\omega}{\omega} = \frac{0.000607 + 0.00063}{1.982 - 1.970} + \frac{0.000628}{1.975 - 1} = 0.103727435$$

$$\Delta\omega = 0.103727435 \times \omega = 0.103727435 \times 0.012307692$$

$$\Delta\omega = 1.2766 \times 10^{-3} \approx 0.0012$$

The error in ω is 0.103727435

$$\frac{\Delta\omega}{\omega} \times 100 = 10.3727435\%$$

∴, the percentage error in ω is 10.3727435%

∴, The dispersive power of material of prism using Spectrometer is (0.0123 ± 0.0012)

Results:

1. The dispersive power of material of prism using Spectrometer is (0.0123 ± 0.0012)
2. The percentage error in ω is 10.3727435%
3. The refractive index of violet color μ_v is 1.982
4. The error in μ_v is 0.000306
5. The percentage error in μ_v is 0.0306%
6. The refractive index of violet color μ_r is 1.970
7. The error in μ_r is 0.00032
8. The percentage error in μ_r is 0.063%
9. The refractive index of violet color μ_y is 1.970
10. The error in μ_y is 0.00032
11. The percentage error in μ_y is 0.0628%

Precaution:

1. The slit should be as narrow as possible but the knife edges of the slit should not touch each other.
2. Zero error must be noted in the measuring instruments.
3. To reduce statistical error in measurements, at least 3-5 readings must be taken.
4. Parallax and back-lash errors during measurement must be avoided.
5. The telescope and the collimator should be separately set for parallel rays.
6. The height of the prism table should be so adjusted that the maximum light must fall on the

entire surface of the prism.

7. While taking observations the telescope and the prism table must be clamped.
8. The reading lens should be used for taking readings on both the verniers.