Syllabus - To be discussed

- **Free Electron Theory – Some accomplishments and Limitations**
- Crystal binding, Structure of solids, Symmetry, unit cell, simple crystal structures
- Diffraction – Bragg's law, structure factor, different methods for structure determination
- Perioidic potential in one dimension, electrons in a weak periodic potential, tight-binding approximation, bands, Brillouin zone
- Vibration of lattice – Mono- and di- atomic chains, periodic lattice, phonons, phonon spectrum, heat capacity
- Thermal expansion and resistivity, Boltzman transport theory
- Discussions on magnetism

Physics of solids without considering their microscopic structure

Electronic Properties of Metals: Drude Theory

Three years after J.J. Thomson's discovery of electron (1897), P. Drude suggested a simple model to explain many of the observed properties of metals. Drude model has many shortcomings, however, is still of fundamental importance for the concepts associated with electrical conductivity

Paul Drude (Wikipedia)

Existence of electrons as charge carriers

+

Highly successful kinetic gas theory

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Drude model is based on the following assumptions –

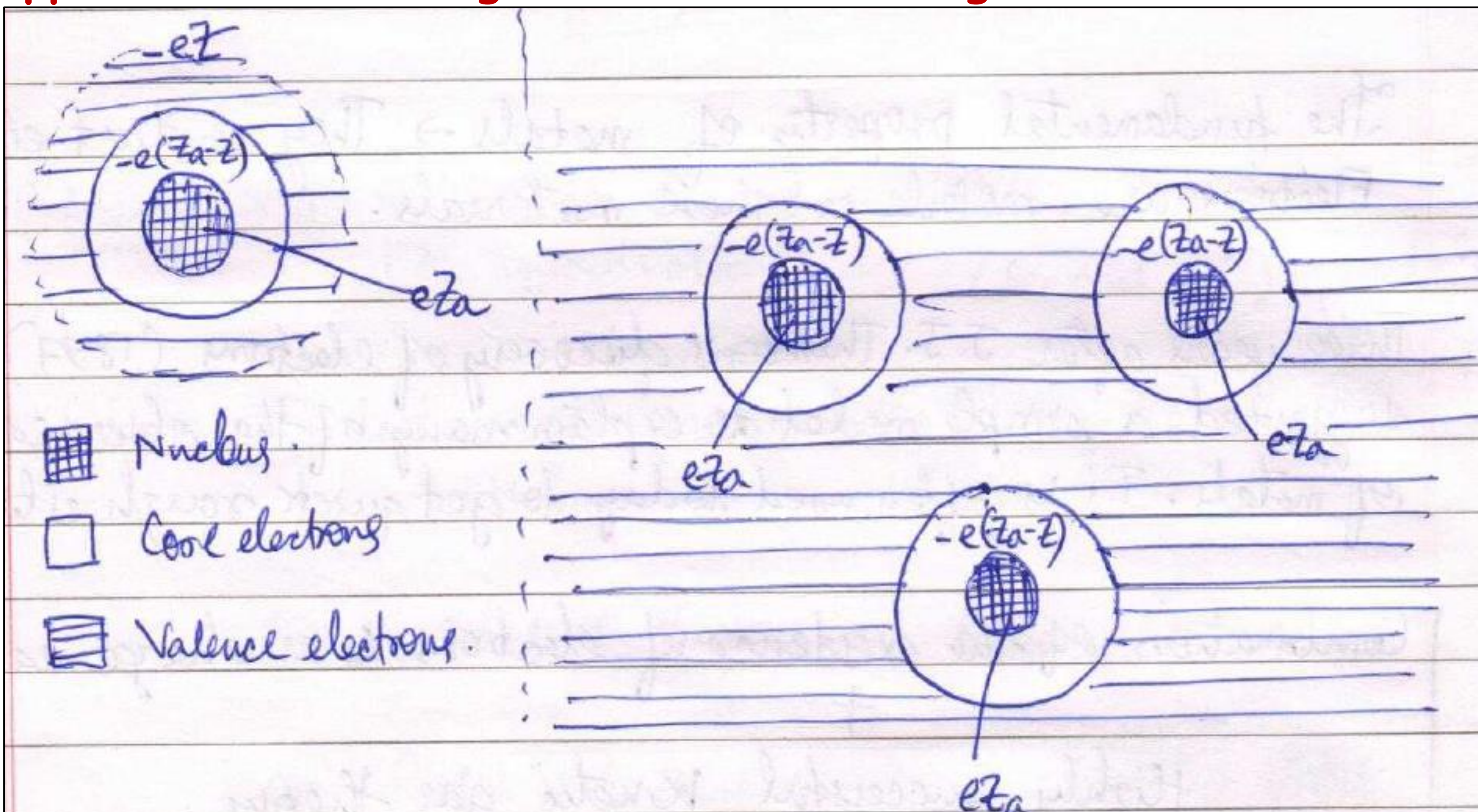
(i) The electrons in a solid do not interact with each other at all

(There is no Coulomb interaction and, as opposed to a classical gas model, they do not collide with each other. This is known as independent electron approximation) -

Somehow surprisingly good

Assumptions of Drude Theory – Contd.

(ii) The positive charge is located on immobile ion cores. The electrons can collide with the ion cores. These collisions instantaneously change their velocity. However, in between collisions, the electrons do not interact with the ions either. This is known as the free electron approximation. This approximation is not good. Some scattering mechanism exists



Assumptions of Drude Theory - Contd.

(iii) The electrons reach thermal equilibrium with the lattice through the collisions with the ions. According to equipartition theorem, their mean kinetic energy is -

$$\frac{1}{2} m_e v_t^2 = \frac{3}{2} k_B T$$

(At room temperature this results in average speed $v_t \sim 10^5$ m/s, We shall see later that mean velocity is 10 times larger)

Immediately after each collision, an electron is taken to emerge with a velocity that is not related with its velocity just before the collision, but randomly directed and with a speed appropriate to the temperature prevailing at the place where the collision occurred

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(iv) In between collisions, the electrons move freely. The mean length of this free movement is called the mean free path λ. Given the average speed v_t , the mean free path also corresponds to a mean time between the collisions, given by τ, called the relaxation time, i.e. $\lambda = v_t \tau$

Estimate of radius per electron

A metallic element contains 0.6022×10^{24} atoms per mole.

contains $\frac{\rho_m}{A}$ moles per cm^3

(ρ_m is the mass density, A is the atomic mass of element)

Since each atom contains Z electrons, the no. of electrons/cm^3

$$n = \frac{N}{V} = \left[0.6022 \times 10^{24} \times \frac{\rho_m}{A} \right] \times Z$$

$$\frac{V}{N} = \frac{1}{n} = \frac{4\pi r_s^3}{3}$$

$r_s \rightarrow$ radius of sphere whose volume is equal to the volume per conduction electron

$$\therefore r_s = \left(\frac{3}{4\pi n} \right)^{1/3}$$

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Element	Z	$n\ (10^{22}/cm^3)$	$r_s\ (\mathring{A})$
Li	1	4.70	1.72
Na	1	2.65	2.08

DC electrical conductivity – Drude model

In presence of electric field $\vec{\mathcal{E}}$

$$m_e \frac{d\vec{v}}{dt} = (-)e\vec{\mathcal{E}}$$

$$\hookrightarrow v(t) = (-)\frac{e\vec{\mathcal{E}}\, t}{m_e}$$

Accelerated drift motion in the direction opposite to the field

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Accelerated drift motion in the direction opposite to the field

If we assume that the drift motion is destroyed in a collision with the ions, on average the time for collision free drift is τ

$$\bar{v} = \frac{(-)e\mathcal{E}\tau}{m}$$

Order of magnitude for $\bar{v}$:-

$\mathcal{E} \approx 10\,\mathrm{Vm^{-1}}$ gives, $|\bar{v}| = 10^{-2}\,\mathrm{m\,sec^{-1}}$

$\hookrightarrow$ [very slow compared to v_t]

DC electrical conductivity – Drude model

Considering an area 'A' perpendicular to the electric field

The number of electrons passing through the area per unit time –

$n|\bar{v}|A$

Charge crossing through the area per unit time →

$(-)\, e\, n|\bar{v}|A$.

So the current density → $j = (-)\, e n \bar{v}$

vectors

$$\vec{j} = (-)\, e n \frac{e\vec{\mathcal{E}}\tau}{m_e}$$

$$\vec{j} = \sigma \vec{\mathcal{E}}$$

$$\vec{j} = \frac{ne^2\tau}{m_e}\vec{\mathcal{E}}$$

DC electrical conductivity – Drude model

$$\vec{j} = \frac{ne^2\tau}{m_e}\vec{\mathcal{E}}$$

So, the current density is in the direction of the electric field and proportional to the field strength. → Ohm's law
(Qualitative explanation)

$$\sigma = \frac{ne^2\tau}{m_e} = \frac{1}{\rho}$$

$\left[\begin{array}{l}\sigma \rightarrow \text{conductivity} \\ \rho \rightarrow \text{resistivity}\end{array}\right]$.

Mobility of the electrons $\mu = \frac{e\tau}{m_e} = \frac{|\bar{v}|}{|\mathcal{E}|}$

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$$\sigma = \frac{ne^2\tau}{m_e} = \frac{1}{\rho}$$ [$\sigma \rightarrow$ conductivity, $\rho \rightarrow$ resistivity].

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Room temperature resistivities are typically ~ μohm-cm

$$\tau = \frac{m}{\rho n e^2} \sim 10^{-14} \text{ to } 10^{-15} \text{ seconds.}$$