Assignment 2: (Course: Quantum Information)

Topic: Density Operator/ Matrix

(1) Show that density operator is a positive operator.

(2) Show that the sets $|\Psi_i\rangle$ and $|\Phi_j\rangle$ generate the same density matrix if and only if $|\Psi_i\rangle = U_{ij}|\Phi_j\rangle$,

We 'pad' whichever set of vectors $|\Psi_i\rangle$ or $|\Phi_j\rangle$ is smaller with additional null vectors so that the two sets have the same number of elements.

(3) Show that $\varrho = p_i|\psi_i\rangle\langle\psi_i| = q_j\,|\phi_j\rangle\langle\phi_j|$ for normalized states $|\psi_i\rangle$, $|\phi_j\rangle$ and probability distributions p_i and q_j if and only if

$$\sqrt{p_i}\,|\Psi_i\rangle = \sum_j u_{ij}\sqrt{q_i}\,|\Psi_j\rangle$$

(4) Show that an arbitrary density matrix for a mixed state qubit may be written as

$$\rho = \frac{1}{2}(I + r\,.\,\sigma)$$

where **r** is a real three-dimensional vector such that $\|\mathbf{r}\| \leq 1$
(known as the Bloch vector for the state ϱ).

(5) What is the Bloch vector representation for the state $\varrho = I/2$?

(6). Show that a state ϱ is pure if and only if $\|\mathbf{r}\| = 1$.

(7) Show that for pure states the description of the Bloch vector for the density operator given above coincides with one qubit state vector.

(8) For each of the four Bell states, find the reduced density operator for each qubit. Check whether they correspond to pure or mixed states.

(9) Suppose a composite of systems A and B is in the state |a|b, where |a is a pure state of system A, and |b is a pure state of system B. Show that the reduced density operator of system A alone is a pure state.