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General Properties of Matter Lab
PH29001

EXPERIMENT-6
Viscous Flow in a Capillary Tube

Aim: To determine the coefficient of viscosity of a liquid in a capillary tube

Apparatus: Manostat, capillary tube, water supply vessel, drain hose with pinch clamp, volumetric beaker, ruler, wooden dowel rod, stopwatch, dial thermometer, level.

Theory: The dynamics of fluid can be understood in terms of Bernoulli's equation:

$$\left(\frac{1}{2}\rho v^2 + \rho g h + P_0\right) = constant$$

Bernoulli's equation does not take into account the viscosity of the fluid although it can be used in various situations.

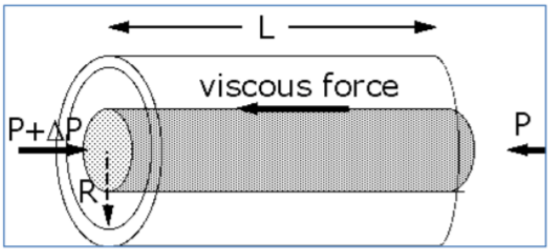
The term "viscosity" refers to the internal frictional force appear due to relative motion of different layers of moving fluid. Different layers move at different velocities. Since work must be done in overcoming these frictional forces as the fluid moves through the tube, a pressure difference must be maintained between the ends of the tube if the flow of the fluid is to continue. The effects of viscosity are not negligible when considering fluid flow through a long tube with a small diameter.

For a cylindrical tube,cylindrical layers of fluid of different radii will have different velocities, with the velocity increasing as the radius decreases, until the maximum velocity isreached at the center of the tube. The velocity of the fluid will thus vary from zero at the wall of the tube to a maximum value at the center.

For a given fluid, the strength of the internal frictional force is given by the coefficient of viscosity, h, which is the viscous force F per unit surface area Aof the moving layer of fluid, divided by the rate of change of velocity v with distance perpendicular r to the surface of the moving layer.

$$\eta = \left(-\frac{F}{A}\right) / \left(\frac{dv}{dr}\right)$$

Poiseuille's Law: Consider a solid cylinder of fluid, of radius r inside a hollow cylindrical pipe of radius R.



The driving force on the cylinder due to the pressure difference is:

$$F_{pressure} = \Delta P\left(\pi r^2\right)$$

The viscous drag force opposing motion depends on the *surface area of the cylinder*(length L and radius r):

$$F_{viscosity} = -\eta\left(2\pi r L\right)\frac{dv}{dr}$$

In equilibrium condition of constant speed, where the net force goes to zero.

$$F_{pressure} + F_{viscosity} = 0$$

$$\Delta P\left(\pi r^2\right) = \eta\left(2\pi rL\right)\frac{dv}{dr}$$

so

$$\frac{dv}{dr} = \frac{\Delta P\left(\pi r^2\right)}{\eta\left(2\pi rL\right)} = \left(\frac{\Delta P}{2\eta L}\right)\cdot r$$

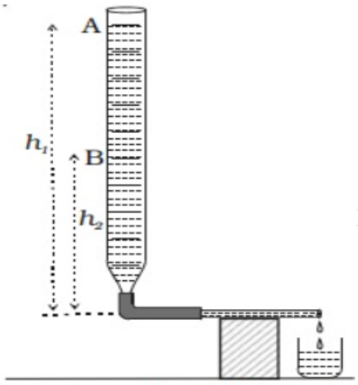
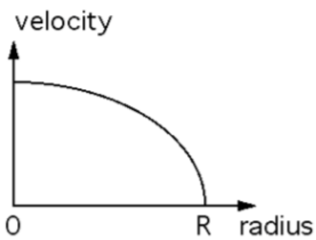


Fig. **Determination of coefficient of viscosity by Poiseuille's flow**

We know empirically that the velocity gradient should look like this:



At the centre

- $r=0$
- $\frac{dv}{dr} = 0$
- v is at its maximum.

At the edge

- $r=R$
- $v=0$

From the velocity gradient equation above, and using the empirical velocity gradient limits, integration can be made to get an expression for the velocity.

$$\frac{dv}{dr} = \left(\frac{\Delta P}{2\eta L}\right)\cdot r$$

rewriting

$$\int_v^0 dv = \left(\frac{\Delta P}{2\eta L}\right)\cdot\int_r^R r\,dr$$

$$v\left(r\right) = \left(\frac{\Delta P}{4\eta L}\right)\left[R^2 - r^2\right]$$

This has a parabolic form as expected.

Now the equation of continuity giving the volume flux for a variable speed is:

$$\frac{dV}{dt} = \int v \bullet dA$$

Substituting the velocity profile equation and the surface area of the moving cylinder:

$$\frac{dV}{dt} = \int v \cdot dA = \int_0^R \left(\frac{\Delta P}{4\eta L}\right)\left[R^2 - r^2\right] \cdot (2\pi r dr)$$
$$= \left(\frac{\pi \cdot \Delta P}{2\eta L}\right)\int_0^R \left(R^2 r - r^3\right)dr$$
$$= \left(\frac{\pi \cdot \Delta P}{2\eta L}\right)\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$
$$= \frac{\pi \cdot \Delta P \cdot R^4}{8\eta L}$$

Poiseuille's equation-

$$\frac{dV}{dt} = \frac{\pi \cdot \Delta P \cdot R^4}{8\eta L}$$

From Poiseuille's equation, on plotting $\frac{dV}{dt}$ vs ΔP, the slope of the plot is $\frac{\pi R^4}{8\eta L}$, hence η can be calculated.

Observations:

- Temperature of the experimental liquid (water)= 35˚C.
- Least count of meter scale = 0.1 cm.
- Least count of vernier calipers of travelling microscope = 0.001 cm.
- Least count of manostat = 0.1 cm.

Table-1-

Measure the length (L) of the capillary tube: using meter scale.

No. of observations	Measured length (cm)	Mean (L) cm
01	15.2	15.2
02	15.3	
03	15.2	

Table-2-

Measuring the radius of the capillary tube: using travelling microscope

SI No.	Reading of one end of tube (R_1)			Reading of one end of tube (R_2)			Radius, r = (R_1- R_2) cm	Mean, r (cm)
	Main scale (cm)	Vernier (cm)	Total (cm)	Main scale (cm)	Vernier (cm)	Total (cm)		
1	9.65	0.007	9.657	9.55	0.003	9.553	0.052	0.0512
2	9.65	0.001	9.651	9.55	0.006	9.556	0.0475	
3	9.65	0.020	9.670	9.55	0.012	9.562	0.054	

Table-3-

Determination of the volume of liquid collected per second for a pressure difference

Using the formula given below, volume per second can be found -

$$\frac{dV}{dt} = \frac{\pi \cdot \Delta P \cdot R^4}{8\eta L}$$

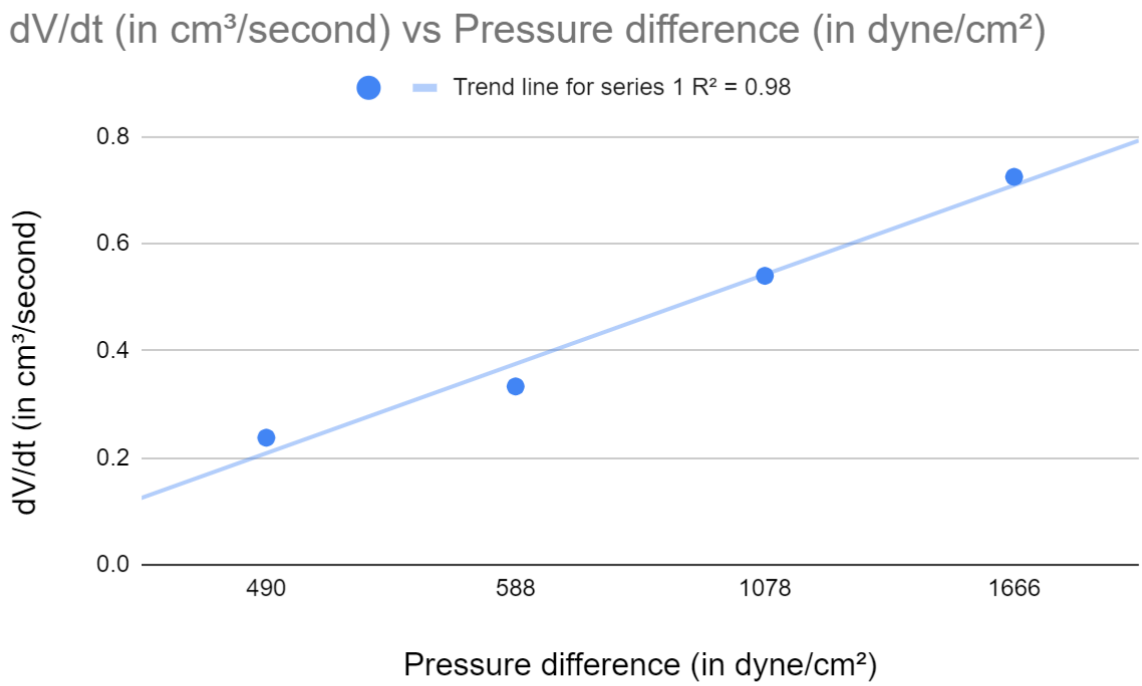
L_1 = Reading of the liquid level in the right arm of thermanostat (cm)

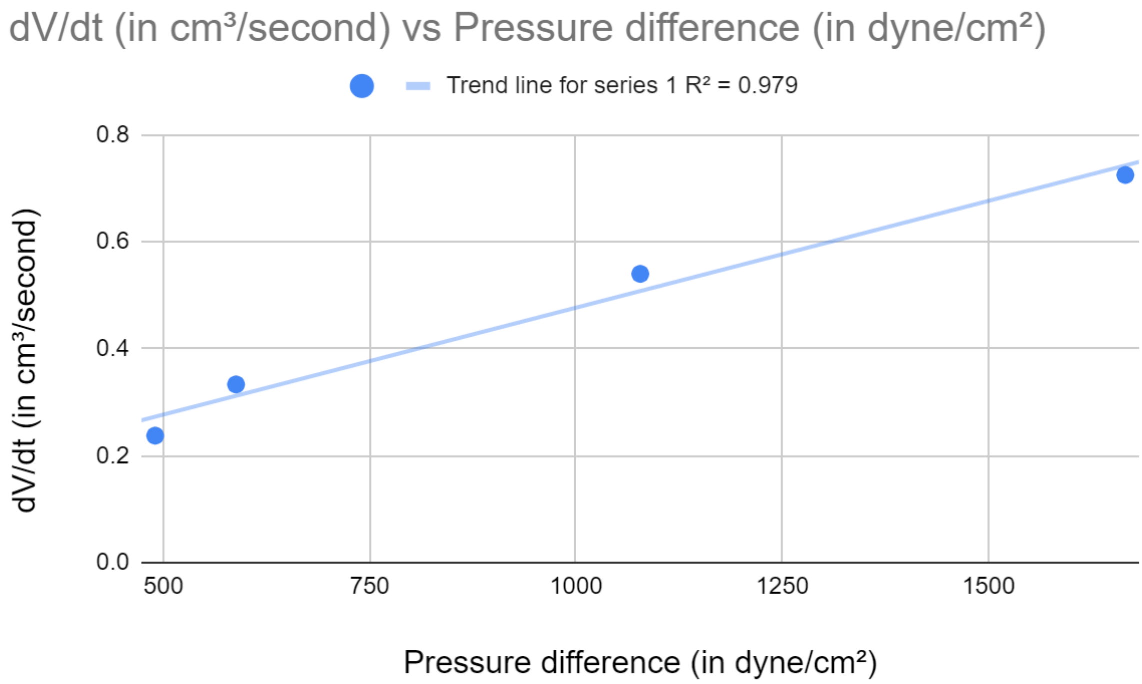
L_2 = Reading of the liquid level in the left arm of manostat (cm)

ρ = Density of water 1gm.cm^3, g=980 cm/s^2

SI No.	L_1(cm)	L_2(cm)	h = (L_1- L_2) (cm)	Pressure difference (dyne/cm^2)	Time of collection (sec)	Vol. of water (cm^3)	Vol. per sec. (cm^3/sec)
1	28.8	29.3	0.5	490	80	19	0.2375
2	28.9	29.5	0.6	588	60	20	0.3333
3	28	29.1	1.1	1078	50	27	0.5400
4	27.6	29.3	1.7	1666	40	29	0.7250

Graph-





Scale-

- X-axis: 1 unit = 250 $dyne\ cm^{-2}$
- Y-axis: 1 unit = 0.2 $cm^{3}\ s^{-1}$

Calculation-

From the graph, its clear that the slope is 0.0003995333671 and we know that the slope of $\frac{dV}{dt}$ vs ΔP is $\frac{\pi R^4}{8\eta L}$.

Therefore, $\frac{\pi R^4}{8\eta L} = 0.0003995333671$

$$\Rightarrow \frac{\pi R^4}{8\times0.0003995333671L} = \eta \quad -\ (1)$$

$$L = 15.2cm,\ R = r = 0.0512\ cm$$

Putting in these values in equation-1, we get -

$$\eta = \frac{\pi(0.0512)^4}{8\times0.0003995333671\times15.2} = 4.441430409 \times 10^{-4}\ dyne - second\ cm^{-2}$$

Therefore, $\eta = 4.44 \times 10^{-4}\ dyne - second\ cm^{-2}$

Error Analysis-

$$\frac{dV}{dt} = \frac{\pi \cdot \Delta P \cdot R^4}{8\eta L}$$

$$\frac{\Delta\eta}{\eta} = \frac{\Delta L}{L} + 4\frac{\Delta R}{R} + \frac{\Delta h}{h} + \frac{\Delta V}{V} + \frac{\Delta t}{t}$$

ΔL=0.1cm, ΔR= 0.001 cm, ΔV= 1 cm^3, Δt= 0.01 sec, Δh= 0.1 cm

L = 15.2 cm, h = 1.7 cm, R = 0.0512 cm, t = 40 seconds.

$$\frac{\Delta\eta}{\eta} = \frac{0.1}{15.2} + \frac{0.1}{1.7} + \frac{0.01}{40} + \frac{0.004}{0.0512} + \frac{0.1}{15.2} = 0.1782602354$$

$$\frac{\Delta\eta}{\eta} \times 100 = 17.826\%$$

$$\Delta\eta = 0.1782602354 \times 4.44 \times 10^{-4} = 0.00007917304303 \approx 0.75\ (dyne - second\ cm^{-2})$$

$$\Delta\eta = 0.00007917304303 \approx 0.000079(dyne - second\ cm^{-2})$$

$$\Delta\eta = 0.79 \times 10^{-4}(dyne - second\ cm^{-2})$$

Therefore, the maximum percentage error in η is 17.826%.

Result-

1. For liquid at 35°C, the coefficient of viscosity of a liquid in a capillary tube (η) = $(4.44 \pm 0.79) \times 10^{-4} dyne - second\ cm^{-2}$.
2. For liquid at 35°C, the maximum percentage error in the coefficient of viscosity of a liquid in a capillary tube (η) is 17.826%.

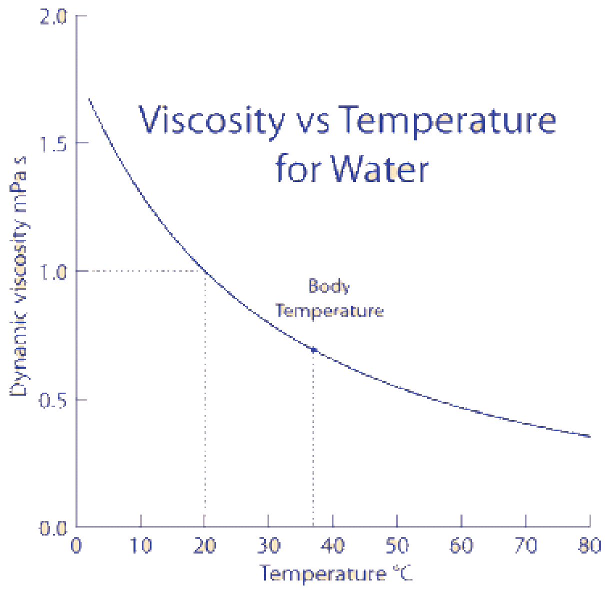
Precautions-

1. The temperature of the laboratory must be noted as the value of viscosity is very sensitive to the temperature.
2. The presence of impurities in the liquid or the immersed tubes can alter the viscosity, hence cleanliness is desired.
3. Bubbles should be avoided in the liquid column.
4. The capillary tube has to be placed horizontally to avoid the effect of gravity.
5. The glassware must be handled with extreme care.
6. There should not be too much fluctuation of the surrounding temperature.
7. To reduce statistical error in measurements, at least 3-5 readings must be taken.
8. Zero error must be noted in the measuring instruments.
9. Parallax and back-lash errors during measurement must be avoided.
10. A magnifying glass can be used to avoid errors while noting the readings from the travelling microscope.

Discussions-

1. Viscosity is a measure of a fluid's resistance to flow. The SI unit of viscosity is poiseiulle (PI). Its other units are newton-second per square metre (N-s m^{-2}) or pascal-second (Pa s).
2. Coefficient of viscosity - It is defined as the ratio of applied shear stress to velocity gradient in a fluid flow.

3. Viscosity arises when there is relative motion between layers of the fluid. More precisely, it measures resistance to flow arising due to the internal friction between the fluid layers as they slip past one another when fluid flows. Viscosity can also be thought of as a measure of a fluid's thickness or its resistance to objects passing through it.
4. A fluid with large viscosity resists motion because its strong intermolecular forces give it a lot of internal friction, resisting the movement of layers past one another. On the contrary, a fluid with low viscosity flows easily because its molecular makeup results in very little friction when it is in motion. Gases also exhibit viscosity, but it is harder to notice in ordinary circumstances.
5. Viscosity of a liquid/gas has a strong dependence on temperature. Hence, it's a good practice to note down the temperature of the laboratory where the experiment is being performed. Throughout this experiment, it has been assumed that the temperature of the laboratory is the same as that of the liquid used in the experiment.
6. Viscosity of a liquid decreases when temperature increases while the viscosity of gases increases with increase in temperature. Given below is the graph for the dynamic viscosity of water with respect to temperature.



7. The importance of viscosity can be understood from the following examples-
    a. The knowledge of coefficient of viscosity and its variation with temperature helps us to choose a suitable lubricant for specific machines. In light machinery thin oils (example, lubricant oil used in clocks) with low viscosity is used. In heavy machinery, highly viscous oils (example, grease) are used.
    b. The knowledge of coefficient of viscosity of organic liquids is used to determine their molecular weights.
8. The viscosity of liquids decreases rapidly with an increase in temperature, and the viscosity of gases increases with an increase in temperature. Thus, upon heating, liquids flow more easily, whereas gases flow more slowly. Also, viscosity does not change as the amount of matter changes, therefore it is an intensive property.