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Computational Physics Lab Report-2

Aim:

1. Consider the logistic map: $x_{n+1} = Ax_n(1 - x_n)$, where, x_n is the nth iteration of x for a starting value of $0 \le x \le 1$. Here A is a constant.
   a. Write a code to generate the logistic map. Start by varying the value of A to observe the following:
      - With A between 0 and 1, the value of x_n will eventually go to zero, independent of the initial value of x.
      - With A between 1 and 2, the population will quickly approach the value (A-1)/A independent of the initial x value.
      - With A between 2 and 3, x_n will also eventually approach the value (A−1/A), but first will fluctuate around that value for some time.
      - With A between 3 and 3.449, from almost all initial conditions x_n will approach permanent oscillations between two values.

      Plot x_n vs n for all these conditions for a value of n>50.
   b. For an initial value x=0.3, vary the value of A from 0.5 to 3.99 in total 250 steps. For each value of A, note the values of x_n for n=150, then make a plot of A vs. x_n and see the bifurcation and chaos. Now change the initial value of x. Do you see any change in the plot?
   c. For A = 3.0, choose two points x and x' close to 1 where, x' = x + 0.01 and iterate. Plot $\log(|x_n - x'_n| / 0.01)$ as a function of n. See if it is approaching a straight line for high n. Check for other values around 3, and you will see error dropping to a platue and eventually to 0 soon enough. This happens because at A=3, near the bifurcation memory, the systems approaches equilibrium in a dramatically slow manner.
2. Root finding 1: How many real roots do the polynomial $f(x) = 2x^3 - 5$ have? Find the roots using the method of bisection.

Tools Used: Jupyter Notebook, Python, NumPy, Pandas, Matplotlib.

Theory:

1. Bifurcation theory is the mathematical study of changes in the qualitative or topological structure of a given family of curves, such as the integral curves of a family of vector fields, and the solutions of a family of differential equations.
2. Chaos theory is the study of deterministic difference (differential) equations that display sensitive dependence upon initial conditions (SDIC) in such a way as to generate time paths that look random.
3. The logistic map is a polynomial mapping (equivalently, recurrence relation) of degree 2, often cited as an archetypal example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations.
4. Method of bisection: The bisection method is a root-finding method that applies to any continuous function for which one knows two values with opposite signs. The

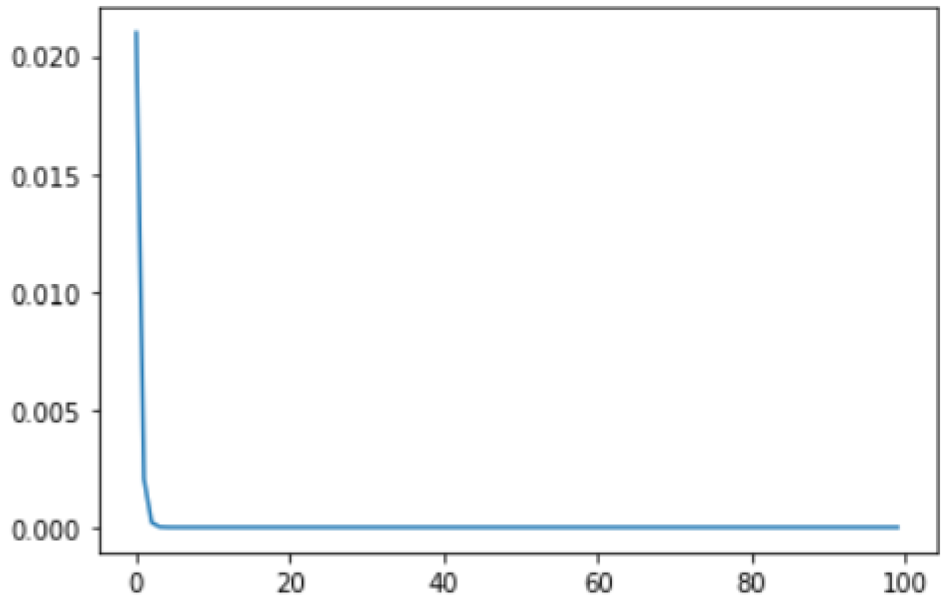
method consists of repeatedly bisecting the interval defined by these values and then selecting the subinterval in which the function changes sign, and therefore must contain a root.

Observations and Graphs:

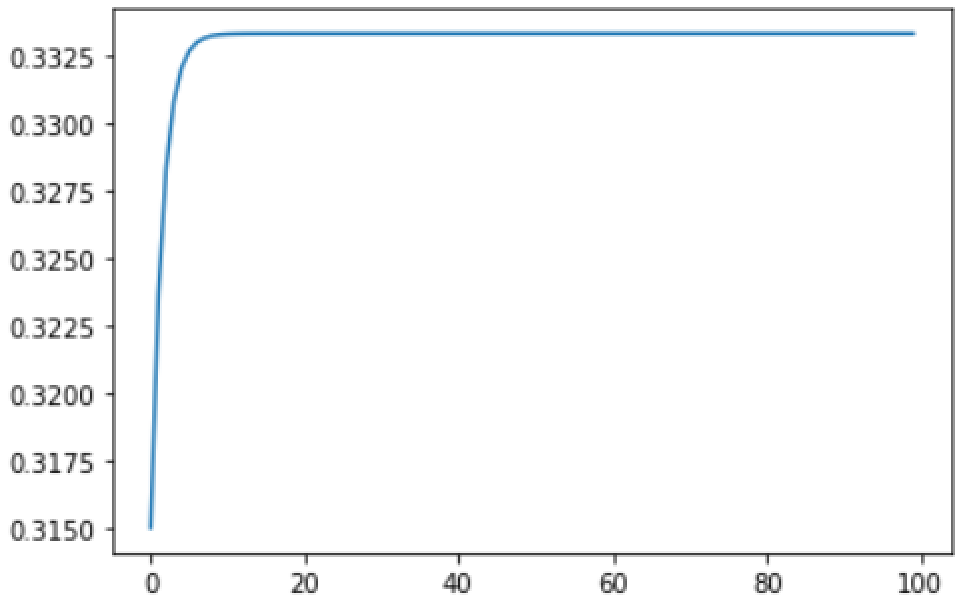
For problem-1:

Part a:

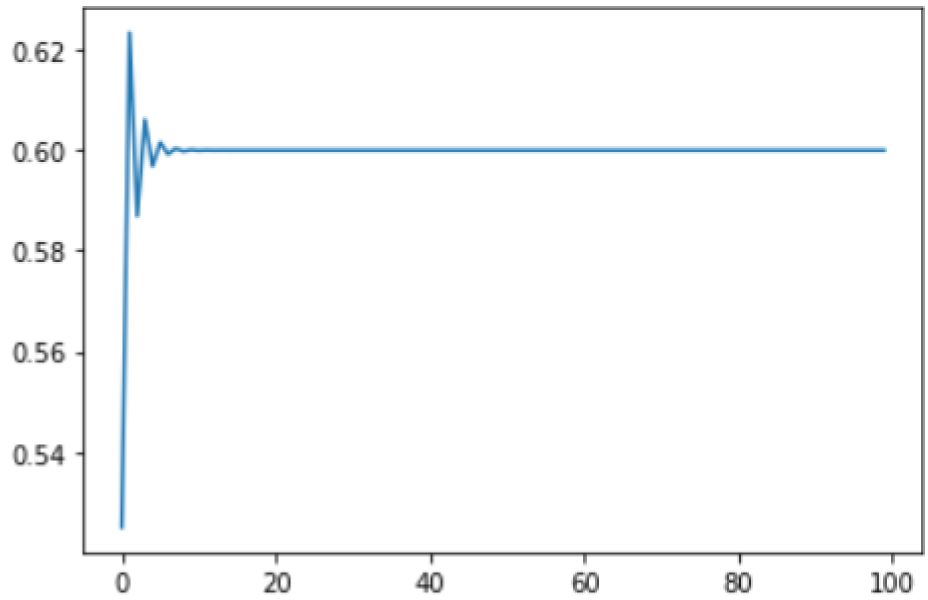
When **A takes a value between 0 and 1**, clearly, the value of x_n goes to zero eventually, independent of the initial value of x.



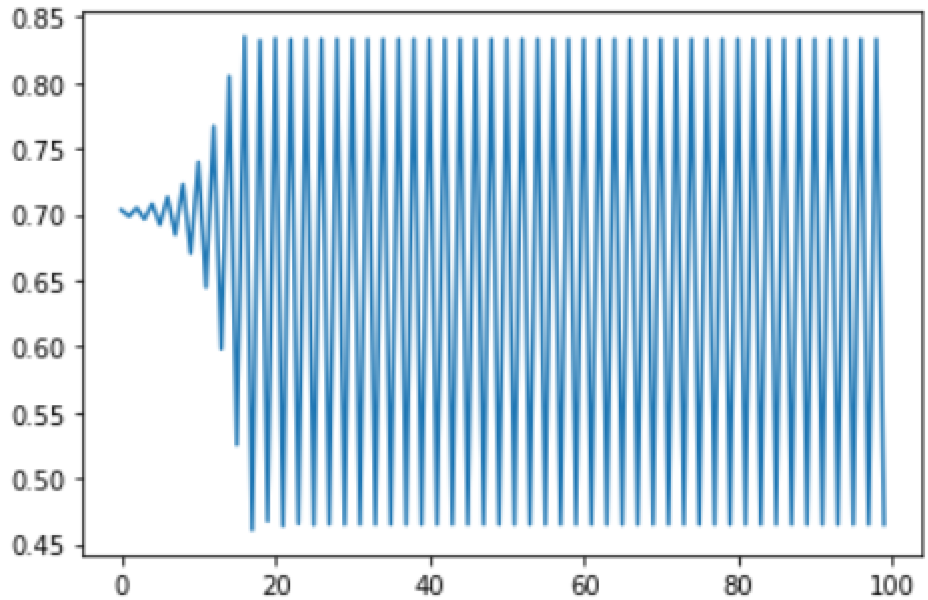
When **A takes a value between 1 and 2**, clearly, the population will quickly approach the value (A-1)/A independent of the initial x value.



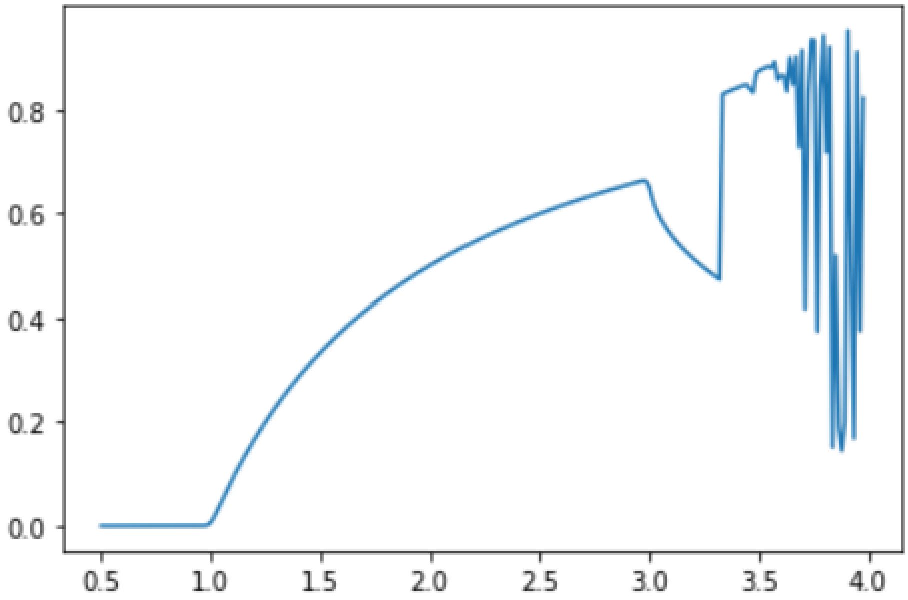
When **A takes a value between 2 and 3**, clearly, the value of x_n eventually approaches the value (A−1)/A, but first will fluctuate around that value for some time.



When **A takes a value between 3 and 3.449**, from almost all initial conditions x_n will approach permanent oscillations between two values.



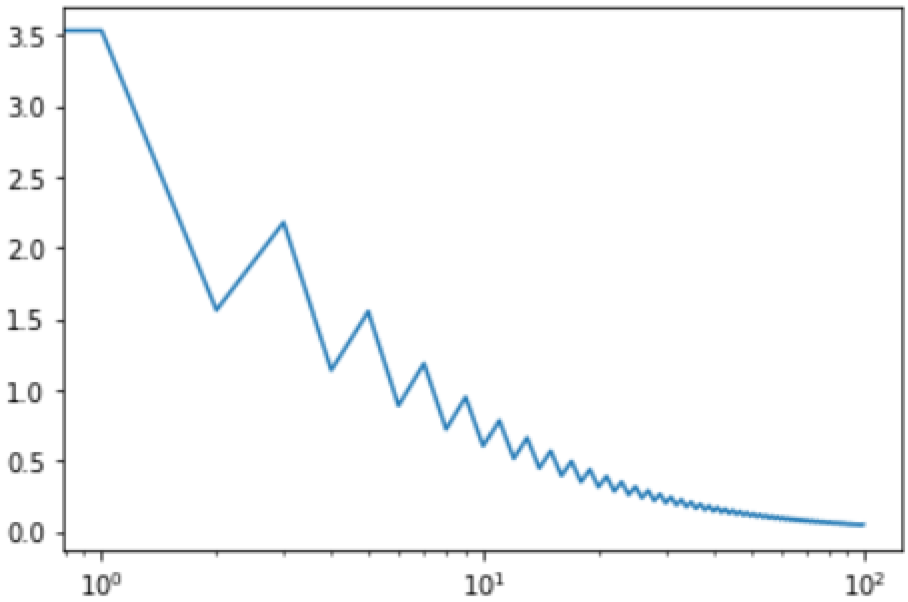
<u>Part b:</u>
On changing the initial value of x, the plot remains the same, indicating that it is independent of the initial value of x.



The plot above represents the various cases that were obtained in Part-a. We can observe bifurcation and chaos in this plot above.

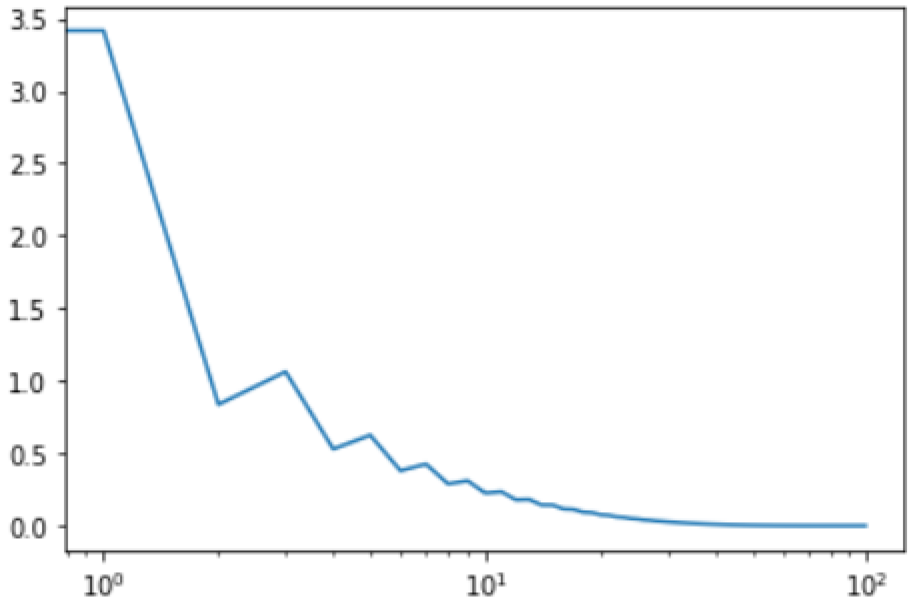
<u>Part c:</u>

When A is 3.0, we obtain the following **plot of log($|x_n - x'_n|$ / 0.01) vs n** and it's clear that it approaches a straight line for higher values of n.

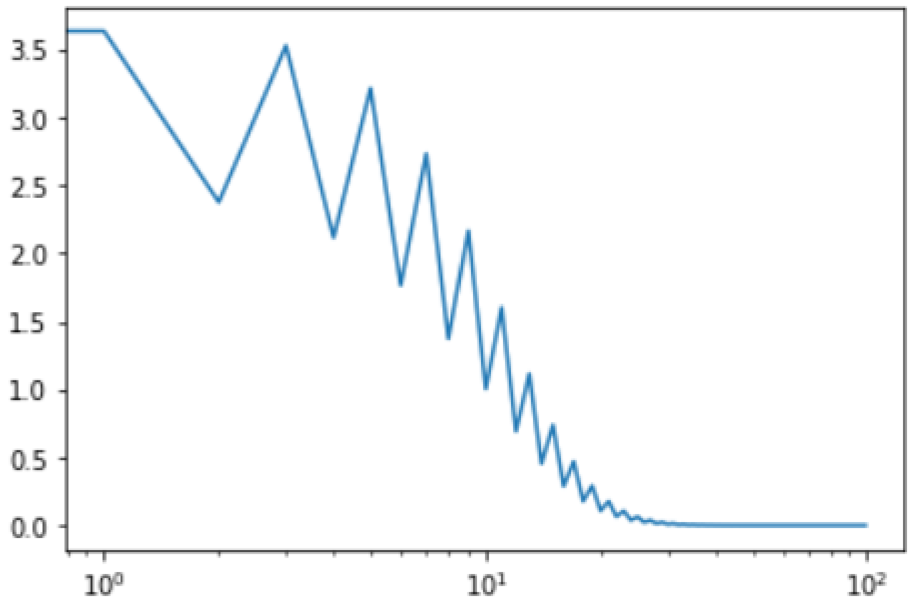


For values of A around 3, we get the following graphs when A is 2.9 and 3.1 respectively, and it's clear that the error drops to a plateau and eventually to 0 soon enough. This happens because at A=3, near the bifurcation memory, the system approaches equilibrium in a dramatically slow manner.

When A=2.9



When A=3.1



<u>For problem-2:</u>
When the interval is between -1 and 3, the value of the approximate root obtained is **1.357177734375,** and the value of the function at this point is **-0.0003434240643400699**, which is clearly close to zero. The number of real roots for this function is **one.**

We define a user-defined error limit since using the method of bisection, we do not always get the correct root within a finite number of steps for a particular interval, hence avoiding the recursive function going on for an infinite number of steps.

<u>Algorithm of the code-</u>

A user-defined error **e is set to 0.001.**

A function called k is defined which takes the parameters as a and b, where a and b are the endpoints of the interval.

t = (a+b)/2 → t is the root of the given function

If conditions-

- If a or b is a root of the function, then the root (a or b) is returned.
- Else if f(a)*f(b)>0, then the set of intervals chosen is incorrect as the two endpoints of the interval must be on either side of the local minima/maxima, only then we can take end points of the intervals which contain the root.
- If the above conditions are satisfied, then we execute the following statements-

```
if abs(f(t))>e:
    if f(t)==0:
        return t
    elif not(f(t)==0):
        if f(a)*f(t)<0:
            b=t
            return k(a,b)
        elif f(b)*f(t)<0:
            a=t
            return k(a,b)
elif abs(f(t))<e:
    return t
```
.