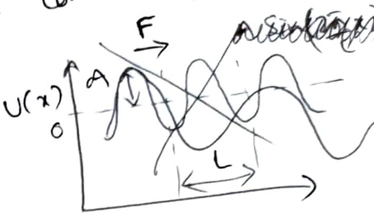
fokker-planck equation

$$\frac{\partial P(x,t)}{\partial t} = D_0 \frac{\partial}{\partial x}\left(e^{-\beta U(x)} \frac{\partial}{\partial x} e^{\beta U(x)} \right) p(x,t)$$

The above equation results from the continuity equation



$$\frac{d\, p(x,t)}{dt} = -\frac{\partial}{\partial x} J(x,t)$$

where the probability flux (current)

$$J(x,t) = -D_0\, e^{-\beta U(x)} \frac{\partial}{\partial x} e^{\beta U(x)} p(x,t)$$

$U(x) = V(x) - FL$

Since $U(x)$ is a periodic function

$$U(x+L) = U(x) - FL$$

~~As~~

~~similarly~~ The same follows the probability density and the current

$$\hat{J}(x+L,t) = \hat{J}(x,t)$$

$$\hat{p}(x+L,t) = \hat{p}(x,t)$$

where

$$\hat{p}(x,t) = \sum_n p(nL+x,t)$$

$$\hat{J}(x,t) = \sum_n J(nL+x,t)$$

$n \in \mathbb{Z}$

Here $\hat{p}(x,t)$ is normalized in any interval $(x, x+L)$ provided that $p(x,t)$ is normalized, e.g., $\int_{-\infty}^{\infty} p(x,t)\, dx = 1$

In the steady state limit the probability current is a constant,

$$\hat{J}(x,t) \xrightarrow{t\to\infty} \hat{J}$$

$$\Rightarrow \quad \hat{J} = -D_0\, e^{-\beta U(x)} \frac{d}{dx} e^{\beta U(x)} \hat{P}_{st}(x)$$

$$\int_x^{x+L} \frac{\hat{J}}{D_0} e^{\beta U(x')} dx' = -\int_x^{x+L} \frac{d}{dx'} e^{\beta U(x')} \hat{P}_{st}(x')\, dx'$$

$$\frac{\hat{J}}{D_0} \int_x^{x+L} e^{\beta U(x')} dx' = \hat{P}_{st}(x) \left[1 - e^{-\beta F L}\right] e^{\beta U(x)}$$

$$\frac{\hat{J}}{D_0} e^{-\beta U(x)} \int_x^{x+L} e^{\beta U(x')} dx' = \left(1 - e^{-\beta F L}\right) \hat{P}_{st}(x) \rightarrow \circledast$$

Again integrate over $0 - L$

$$\Rightarrow \text{we get} \quad \frac{\hat{J}}{D_0} \int_0^L e^{-\beta U(x)} dx \int_x^{x+L} e^{\beta U(x')} dx' = \left(1 - e^{-\beta F L}\right) \underbrace{\int_0^L \hat{P}_{st}(x)\, dx}_{1}$$

$$\Rightarrow \quad \hat{J} = \frac{D_0 \left(1 - e^{-\beta F L}\right)}{\int_0^L e^{-\beta U(x)} dx \int_x^{x+L} e^{\beta U(x')} dx'} \rightarrow \circledast'$$

The general relation between the stationary probability current and the steady state particle current $\langle \dot{x} \rangle$ ~~is~~ (or $\langle v \rangle$)

$$\langle v \rangle = \int_0^L \hat{J}\, dx = \hat{J} L$$

Then the particle current is

$$\boxed{\langle v \rangle = \frac{D_0 L \left(1 - e^{-\beta F L}\right)}{\int_0^L e^{-\beta U(x)}\, dx \int_x^{x+L} e^{\beta U(x')}\, dx'}}$$

Now, substituting $\circledast'$ in $\circledast$ we can calculate the steady state probability ~~density~~ distribution (density)

$$\Rightarrow \boxed{\hat{P}_{st}(x) = \frac{e^{-\beta U(x)} \int_x^{x+L} e^{\beta U(x')}\, dx'}{\int_0^L e^{-\beta U(x)}\, dx \int_x^{x+L} e^{\beta U(x')}\, dx'}}$$