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ELECTROMAGNETISM LABORATORY

EXPERIMENT-1

Aim:

1. To study the current-voltage relationship in an ac circuit containing an inductor and an external resistor in series.
2. To study the variation of the resistance of the inductor with the frequency of the ac source and hence to determine the inductance.
3. To draw the phasor diagram and hence to determine the inductance and the resistance of the inductor.

Theory:

In figure. 3.EM14(i), a series combination of an external resistance R and an inductor (coil) of inductance L and resistance r is connected to an ac source of angular frequency ω and RMS voltage V. If I is the RMS current flowing in the circuit and V_R is the RMS voltage across the resistance R, then

$$I=\frac{V_R}{R} \quad \cdots (i)$$

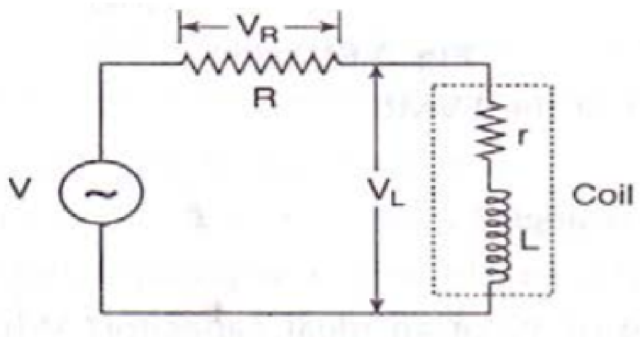


Fig. 3.EM14(i)

If V_L is the RMS voltage across the inductor, then $V_L = \sqrt{r^2 + X_L^2} \cdot I \quad \rightarrow (ii)$ Where X_L is the inductive resistance: $X_L = \omega L = 2\pi f L \quad -> \quad (iii)$, f being the frequency in Hz. The resistance r accounts for the losses in the inductor. The impedance of the inductor is

$$Z_L = \frac{V_L}{I} = \sqrt{r^2 + X_L^2} \quad \rightarrow (iv)$$

1. The current I can be determined from Eq. (i) by measuring the voltage V_R and knowing the resistance R. If the voltage V_1 across the inductor is also measured, then at a given frequency $V_L \propto I$.Thus the plot of V_L along the x-axis and I along the y-axis will be a straight line passing through the origin [vide Eq.(ii)]. This shows that the current-voltage relationship in an ac circuit containing an inductor obeys Ohm's law. The slope of the V_L - I plot gives $1/Z_L$. Let Z_1 and Z_2 be the values of Z_1 for two frequencies f_1 (= ω_1/ 2π) and f_2 (= ω_2/ 2π). Then $Z^2_1 = r^2 + \omega^2_1 L^2$ and $Z^2_2 = r^2 + \omega^2_2 L^2$. These two equations give

$$L = \frac{1}{2\pi} \cdot \sqrt{\frac{Z_2^2 - Z_1^2}{f_2^2 - f_1^2}} \rightarrow (v)$$

and $$r = \sqrt{\frac{Z_1^2 f_2^2 - Z_2^2 f_1^2}{f_2^2 - f_1^2}} \quad \text{... (vi)}$$

[The quantity $Z^2_1 f^2_2 - Z^2_2 f^2_1$ is quite small, and can turn out to be negative due to experimental errors. In that case, r cannot be found from Eq. (vi).]

2. By measuring V_R and V_L at different frequencies, Z_L can be calculated from Eqs. (i) and (iv) at those frequencies. If Z_L (along the y-axis) is plotted against f (along the x-axis), we get the graph of Fig. 3.EM14(ii). The intercept of the curve on the Z_L-axis gives the resistance r. At high frequencies, $X_L >> r$, so that $Z_L = X_L$. Hence the graph at high frequencies is almost linear, the slope of which is $2\pi L$. Thus from the slope, the inductance L can be found.

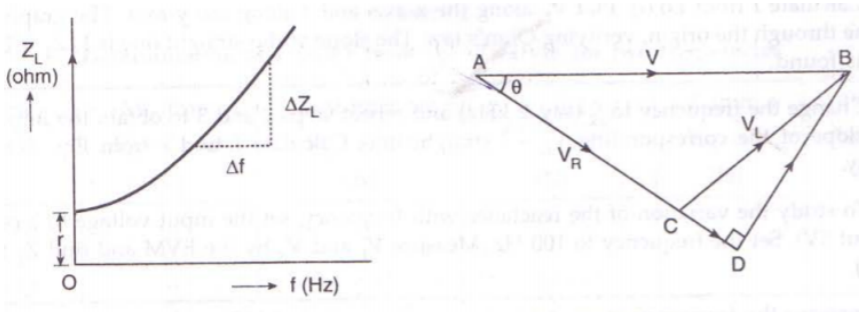


Fig. 3.EM14(ii) Fig. 3.EM14(iii)

3. The phasor diagram for the voltages is shown in Fig. 3.EM14(iii). The phasors V, V_R and V_L are represented (with a proper choice of scale) by the sides AB, AC, and CB of the triangle ABC. If BD is perpendicular to AC (extended), CD and DB give the phasors Ir and IX_L respectively. Since AC gives the phasor IR, we have
$$\frac{AC}{DB} = \frac{IR}{IX_L} = \frac{R}{2\pi f L}$$
Hence the inductance
$$L = \frac{R}{2\pi f} \cdot \frac{DB}{AC}$$
Again $$\frac{AC}{CD} = \frac{IR}{Ir} = \frac{R}{r}$$
So that $$r = R \cdot \frac{CD}{AC}$$
The phase angle **θ** by which the current I lags the source voltage V is given by $\boldsymbol{\theta} = \arctan\left(\frac{DB}{AD}\right)$
So, L, r and **θ** can be found from the phasor diagram using Eqs. (vii), (viii) and (ix), respectively.

Apparatus and Accessories:

1. An air-core coil (L =30 mH, r =50Ω)
2. Some carbon resistors or a resistance box (0-1000Ω)
3. An audio oscillator of low output impedance
4. An electronic voltmeter (EVM)

Experimental Results:

TABLE-01

$V_R - V_L\ (DATA);\ R = 47\Omega$

Frequency of ac source$f = 1\ kHz = f_1$; Least count of the voltmeter is $\Delta V = 0.001\ V$ and least count of function generator $\Delta f = 0.1\ Hz$

S. No.	Input voltage peak to peak V_{PP} (V)	Input voltage in r.m.s V_{rms} (Volts)	Voltage across R, V_R(r.m.s) (Volts)	Voltage across L, V_L(r.m.s) (Volts)	$I = \frac{V_R}{R}$ (amp)
01	0.5	0.177	0.046	0.131	0.0009787234043
02	1.0	0.353	0.086	0.270	0.001829787234
03	1.5	0.530	0.104	0.426	0.002212765957
04	2.0	0.707	0.124	0.580	0.002638297872
05	2.5	0.883	0.150	0.734	0.003191489362
06	3.0	1.060	0.176	0.888	0.003744680851
07	3.5	1.237	0.204	1.038	0.004340425532
08	4.0	1.414	0.225	1.192	0.004787234043
9	4.5	1.590	0.244	1.346	0.005191489362
10	5.0	1.767	0.267	1.500	0.005680851064

TABLE-02

$V_R - V_L\ (DATA);\ R = 47\Omega$

Frequency of ac source, $f = 2\ kHz = f_1$; Least count of the voltmeter is $\Delta V = 0.001\ V$ and least count of function generator $\Delta f = 0.01\ Hz$

S. No.	Input voltage peak to peak V_{PP} (V)	Input voltage in r.m.s V_{rms} (Volts)	Voltage across R, V_R(r.m.s) (Volts)	Voltage across L, V_L(r.m.s) (Volts)	$I = \frac{V_R}{R}$ (amp)
01	0.5	0.177	0.025	0.146	0.0005319148936
02	1.0	0.353	0.048	0.298	0.001021276596
03	1.5	0.530	0.071	0.458	0.001510638298
04	2.0	0.707	0.094	0.612	0.002
05	2.5	0.883	0.116	0.765	0.002468085106
06	3.0	1.060	0.138	0.926	0.002936170213
07	3.5	1.237	0.160	1.128	0.003404255319
08	4.0	1.414	0.183	1.225	0.003893617021
9	4.5	1.590	0.205	1.384	0.004361702128

10	5.0	1.767	0.221	1.547	0.00470212766

TABLE-03

Determination of L and r from V_L-I curves for two frequencies

Frequencies f (Hz)	ΔV_L from V_L-I plot (V)	ΔI from V_L-I plot (A)	$Z_L = \frac{\Delta V_L}{\Delta I}\Omega$	L from Eq. (v) (H)	r from Eq. (vi)* (Ω)
1 k (=f_1)	0.612	0.00214893617	284.7920792	0.0149512932	268.8687828
2k (=f_3)	0.628	0.001914893617	327.9555556		

*r cannot be found from Eq. (vi) if $Z_1^2 f_2^2 - Z_2^2 f_1^2$ turns out to be negative due to experimental errors.

TABLE-04

Variation of ZL with frequency

Given R = 47 Ω and applied input voltage peak to peak V_{PP} = 5 volts i.e. V_{rms} = 1.768 volts

Frequencies f (Hz)	V_R(r.m.s) (Volts)	V_L(r.m.s) (Volts)	$I = \frac{V_R}{R}$ (amp)	$Z_L = \frac{V_L}{I}\Omega$
100	1.196	0.570	0.02544680851	22.39966555
200	0.909	0.857	0.01934042553	44.31133113
300	0.745	1.025	0.01585106383	64.66442953
400	0.638	1.132	0.01357446809	83.39184953
500	0.562	1.205	0.01195744681	100.7740214
600	0.504	1.267	0.01072340426	118.1527778
700	0.459	1.310	0.009765957447	134.1394336
800	0.422	1.343	0.008978723404	149.5758294
900	0.390	1.379	0.00829787234	166.1871795
1000	0.364	1.408	0.007744680851	181.8021978
1100	0.340	1.424	0.007234042553	196.8470588
1200	0.320	1.450	0.006808510638	212.96875
1300	0.298	1.471	0.006340425532	232.0033557
1400	0.289	1.478	0.00614893617	240.366782
1500	0.276	1.491	0.005872340426	253.9021739
1600	0.263	1.501	0.005595744681	268.2395437

1700	0.250	1.517	0.005319148936	285.196
1800	0.240	1.529	0.005106382979	299.4291667
1900	0.232	1.531	0.004936170213	310.1594828
2000	0.222	1.547	0.004723404255	327.518018
2100	0.214	1.552	0.004553191489	340.8598131
2200	0.206	1.561	0.004382978723	356.1504854
2300	0.200	1.566	0.004255319149	368.01
2400	0.193	1.574	0.004106382979	383.3056995
2500	0.187	1.583	0.003978723404	397.8663102
2600	0.181	1.589	0.00385106383	412.6132597
2700	0.176	1.594	0.003744680851	425.6704545
2800	0.170	1.600	0.003617021277	442.3529412
2900	0.166	1.604	0.003531914894	454.1445783
3000	0.161	1.607	0.003425531915	469.1242236
3100	0.157	1.611	0.003340425532	482.2738854
3200	0.153	1.615	0.003255319149	496.1111111
3300	0.149	1.619	0.003170212766	510.6912752
3400	0.145	1.623	0.003085106383	526.0758621
3500	0.142	1.626	0.003021276596	538.1830986
3600	0.138	1.630	0.002936170213	555.1449275
3700	0.135	1.633	0.002872340426	568.5259259
3800	0.131	1.637	0.002787234043	587.3206107
3900	0.128	1.640	0.002723404255	602.1875
4000	0.126	1.642	0.002680851064	612.4920635
4100	0.123	1.645	0.002617021277	628.5772358
4200	0.120	1.648	0.002553191489	645.4666667
4300	0.118	1.650	0.002510638298	657.2033898
4400	0.116	1.652	0.002468085106	669.3448276
4500	0.112	1.656	0.002382978723	694.9285714
4600	0.110	1.658	0.002340425532	708.4181818
4700	0.108	1.660	0.00229787234	722.4074074
4800	0.106	1.662	0.002255319149	736.9245283
4900	0.104	1.664	0.002212765957	752
5000	0.102	1.666	0.002170212766	767.6666667

TABLE-05

Determination of L and r from Z_L-f plot for any three frequencies

From the straight portion of the curve			Intercept on the Z_L axis = r (Ω)
ΔZ_L (Ω)	Δf (Hz)	$L = \frac{1}{2\pi}.\frac{\Delta Z_L}{\Delta f}(H)$	
145.7158202	1000	0.023191393	29

TABLE-06

Given R = 47 Ω and applied input voltage peak to peak V_{PP} = 5 volts i.e. V_{rms} = 1.768 volts

Least count of voltmeter is $\Delta V = 0.001\ V$ and least count of function generator $\Delta f = 0.1\ Hz$

S. No.	Frequency f (Hz)	Input voltage in r.m.s V_{rms} (Volts)	Voltage across R, V_R(r.m.s) (Volts)	Voltage across L, V_L(r.m.s) (Volts)
01	1000	1.768	0.364	1.408
02	2000	1.768	0.222	1.547
03	3000	1.768	0.161	1.607
04	4000	1.768	0.126	1.642

Choose of the scale for the phasor diagram: 1 cm =0.1 Volts

S. No.	Frequency f (Hz)	Input voltage AB (≡ V_{rms}) (cm)	Voltage across R, AC (≡ V_R) (cm)	Voltage across L, CB (≡V_L) (cm)
01	1000	17.68	3.64	14.08
02	2000	17.68	2.22	15.47
03	3000	17.68	1.61	16.07
04	4000	17.68	1.26	16.42

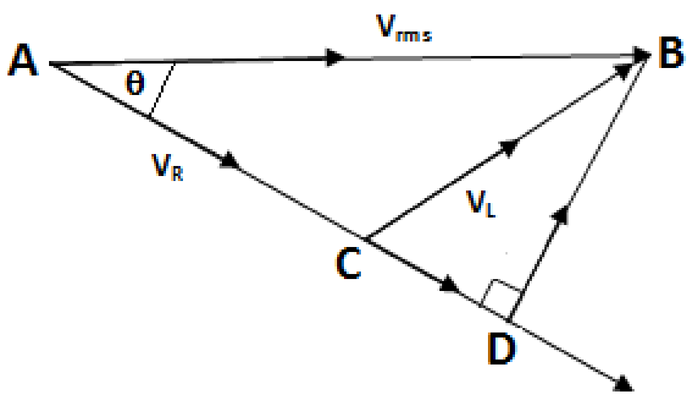


TABLE-07

Determination L, r and θ from phasor diagram:

$R = 47\ \Omega$

S. No.	f (Hz)	AC (cm)	DB (cm)	CD (cm)	L from Eq. (vii) (H)	r from Eq. (viii) (ohm)	θ from Eq. (ix) (degrees)
01	1000	3.64	2.332261207	13.88549451	0.004795280532	179.2907258	7.580279984
02	2000	2.22	1.567928964	15.39033784	0.002642905453	325.8314767	5.087889579
03	3000	1.61	1.33	16.3	0.0021	475.84	4.31
04	4000	1.26	1.5	16.1	0.0022	601.12	4.9

Calculations:

L, r and θ can be determined from the following equations of the phasor diagram-

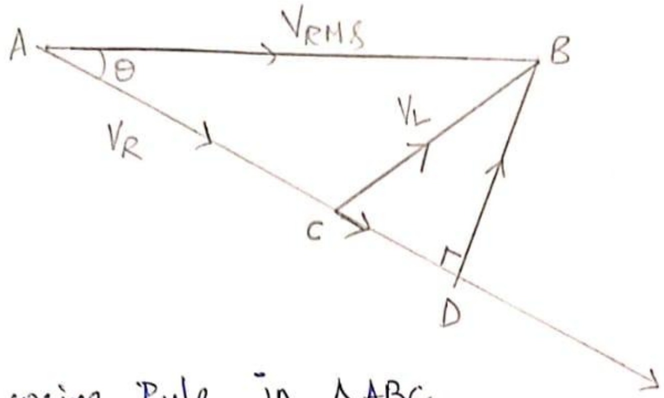
$$\frac{AC}{DB} = \frac{IR}{IX_L} = \frac{R}{2\pi f L}$$

DB can be found from this equation $\cos\theta = \dfrac{AB^2 + AC^2 - BC^2}{2 \cdot AB \cdot AC}$ and $DB = \sin\theta \cdot AB$

L can be found from this equation $L = \dfrac{R}{2\pi f} \cdot \dfrac{DB}{AC}$

CD can be found from this equation $\dfrac{AC}{CD} = \dfrac{IR}{Ir} = \dfrac{R}{r}$

r and θ can be found from the following equations $r = R \cdot \dfrac{CD}{AC}$ and $\theta = \arctan\left(\dfrac{DB}{AD}\right)$



Using cosine Rule in ΔABC,

$$\cos\theta = \frac{V_{RMS}^2 + V_R^2 - V_L^2}{2 V_{RMS} V_R} \quad -(1)$$

$|V_{RMS}| = AB$

$|V_R| = AC$

$|V_L| = BC$

In ΔABD, $\sin\theta = \frac{DB}{AB}$

$\Rightarrow DB = \sin\theta\,(AB)$

$\therefore$, for set-1, $f = 1000\,Hz$, $AB = 17.68\,cm$, $AC = 3.64\,cm$,

$BC = 14.08\,cm$

$$\cos\theta = \frac{(17.68)^2 + (3.64)^2 - (14.08)^2}{2(17.68)(3.64)}$$

$\theta = 7.5803^\circ$

$DB = \sin\theta\,(17.68) = 2.33\ cm$

$$L = \frac{R}{2\pi f} \cdot \frac{DB}{AC} = \frac{47}{2\pi(1000)} \cdot \frac{(2.33)}{3.64}$$

$L = 4.7882026 \times 10^{-3}\,H$

$L = 0.0047882026\,H$

From ΔBCD, $BC^2 = CD^2 + BD^2$

$CD^2 = BC^2 - BD^2$

$CD = 13.8858\,cm$

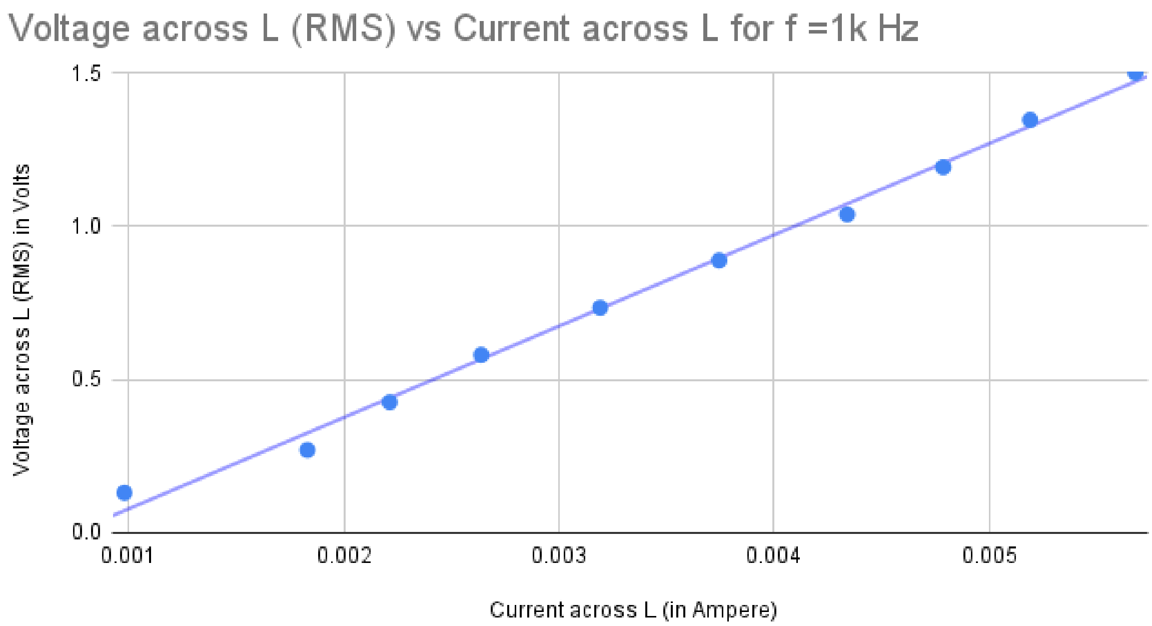
$$r = R\left(\frac{CD}{AC}\right) = 47 \times \frac{13.8858}{3.64} = 179.2956$$

$\therefore, r = 179.2956273\,\Omega$

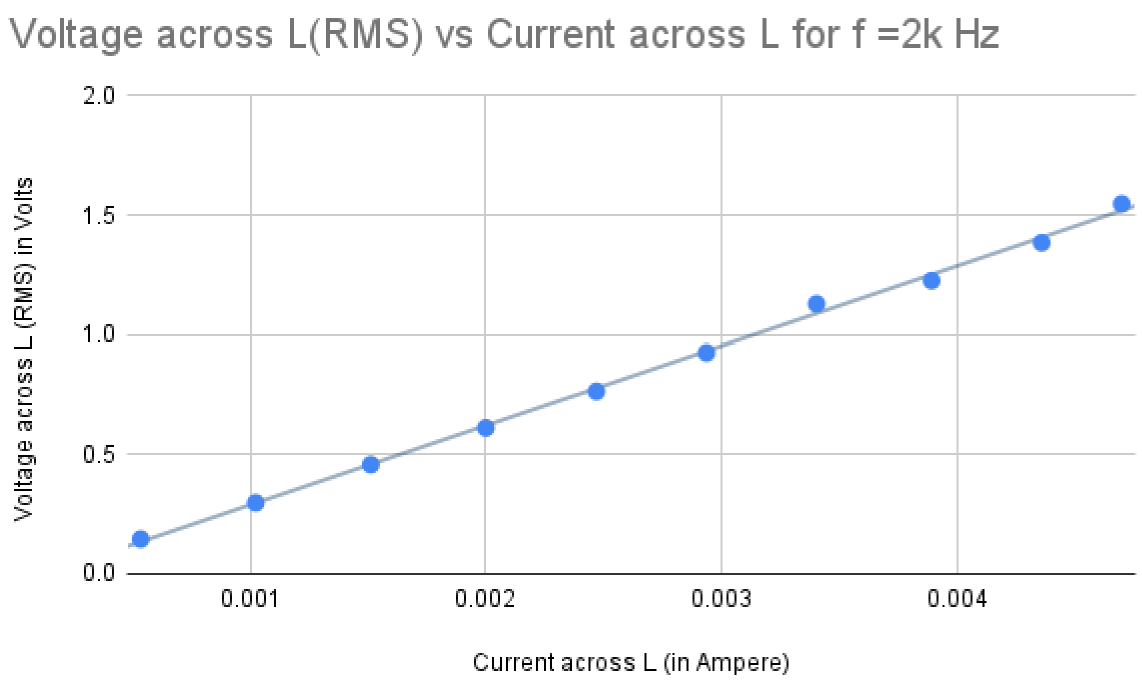
Similarily, the same process can be applied to the remaining sets to find L, r and θ

Graphs:

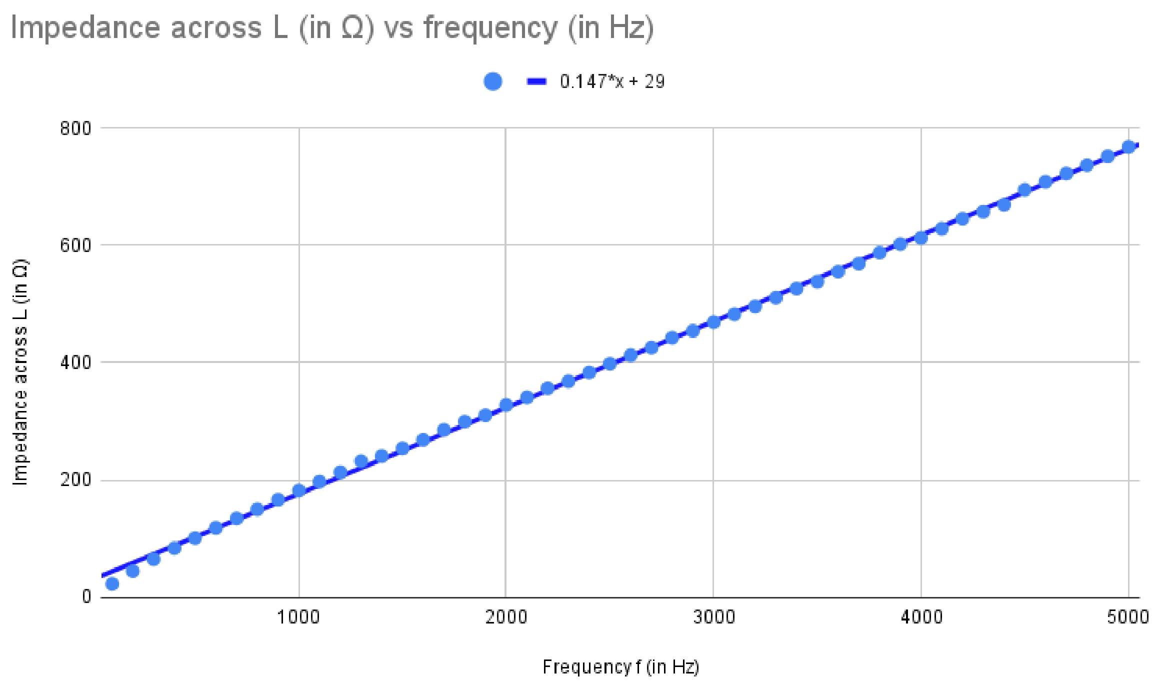
Plot of V_L vs I of Table-1:



Plot of V_L vs I of Table-2:



<u>Plot of Z_L vs f of Table-5:</u>



Equation of the graph when Z_L vs f of Table-5 is plotted - Z_L=0.147*f + 29

Proportional error in L:

We have $V_L = 2\pi f L I = 2\pi f L \frac{V_R}{R}$

$$L = \frac{R}{2\pi f L} \cdot \frac{V_L}{V_R}$$

or ,

Since R is given, there will be no error due to R, hence the proportional error in L is

$$\frac{\delta L}{L} = \frac{\delta f}{f} + \frac{\delta V_R}{V_R} + \frac{\delta V_L}{V_L},$$

Where δf, δV_R and δV_L are the errors in f, V_R , and V_L respectively; these errors correspond to the smallest scale divisions of the respective measuring instruments. Substituting the values of $\frac{\delta f}{f}, \frac{\delta V_R}{R}, and \frac{\delta V_L}{L}$ for a particular measurement, the proportional error $\frac{\delta L}{L}$ can be calculated.

<u>Calculations-</u>

For $f = 2k\ Hz$, V_R=0.116 V, V_L=0.765 V, δV=0.001V, δf=0.01 Hz, the proportional error in L is-

$$\frac{\delta L}{L} = \frac{0.01}{2000} + \frac{0.001}{0.116} + \frac{0.001}{0.765}$$

Therefore, the proportional error $\frac{\delta L}{L}$ is 0.99%

Precautions and Discussion:

1. If the amplitude of the output voltage of the oscillator changes with the frequency it should be adjusted.
2. The resistance R must be noninductive.
3. The connecting wires should be straight and short.
4. Instead of an EVM, a CRO can be used.
5. In the plot of Z_L vs frequency of Table-5, we do not obtain a curve in lower frequencies, i.e, starting from 100Hz in our data, as a curve will be obtained if we have data for frequencies starting from 10Hz. Hence, we obtain a straight line when Z_L vs f is plotted.