Classical Dynamics and Special Relativity (PH21201) - Mini Project - Chaos: An Introduction

Chua's circuit with a Quintic Nonlinearity

Team Members

- Jessica John Britto [20PH20014]
- Pranay Pandey [20PH20026]
- Shreya Bhatt [20PH20037]

Basic Circuit and Equations

Using KVL at the node 1,

$$I_{C_1} = I_R - I_N$$
$$\therefore C_1 \frac{dV_1}{dt} = \frac{1}{R}(V_{C2} - V_{C1}) - g(V_{C_1})$$

Using KVL at the node 2,

$$I_{C_2} = I_R - I_L$$
$$\therefore C_2 \frac{dV_2}{dt} = \frac{1}{R}(V_{C_2} - V_{C_1}) + I_L$$

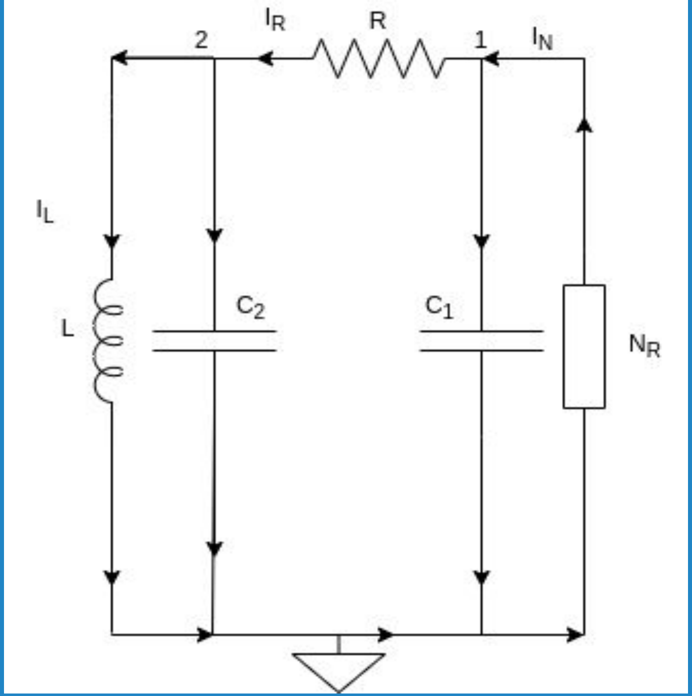
System of nonlinear differential equations

$$\dot{x} = \frac{1}{RC_1}(y - x) - \frac{1}{C_1} g(x)$$
$$\dot{y} = \frac{1}{RC_2}(y - x) + \frac{1}{C_2} z$$
$$\dot{z} = -\frac{1}{L} y$$

Rewriting the above equations

$$\alpha = \frac{C_2}{C_1}, \beta = \frac{R^2 C_2}{L}, \tau = \frac{t}{RC_2}, z' = Rz$$

$$\dot{x} = \alpha(y - h(x))$$
$$\dot{y} = y - x + z$$
$$\dot{z} = -\beta y$$



Basic Chua’s Circuit

Calculation of Jacobian Matrices

Differential Equations-

$$\dot{x} = \alpha(y - x - g(x))$$
$$\dot{y} = x - y + z$$
$$\dot{z} = -\beta y$$

Finding Fixed Points-

$$\dot{x} = 0,\ \dot{y} = 0,\ \dot{z} = 0$$
$$\dot{z} = 0 \Rightarrow y_\star = 0$$
$$\dot{y} = 0 \Rightarrow x_\star = -z_\star$$
$$\dot{z} = 0 \Rightarrow x_\star + g(x) = 0$$
$$\dot{x} = \alpha[y - x - (x^5 + (c-1)x)]$$
$$\dot{x} = \alpha(y - x - x^5 - cx + x)$$
$$\dot{x} = \alpha(y - x^5 - cx)$$
$$\dot{x} = \alpha(y - f(x))$$

$$f(x) = x^5 + cx$$
$$f(x_\star) = x_\star + g(x_\star) = 0$$
$$x_\star^5 + cx_\star = 0 \Rightarrow x_\star = 0 \ or\ x_\star = (-c)^{\frac{1}{4}}$$

Fixed Points when c<0 and c>0-

$\therefore$, for $c \geq 0$, one fixed point exists at (0, 0, 0)

for $c < 0$, three fixed points exist at -

$$\begin{cases} \rightarrow (0,0,0) & O \\ \rightarrow \left(|c|^{\frac{1}{4}}, 0, -|c|^{\frac{1}{4}}\right) & +P \\ \rightarrow \left(-|c|^{\frac{1}{4}}, 0, |c|^{\frac{1}{4}}\right) & -P \end{cases}$$

For small perturbations-

$$x = x_\star + \Psi$$
$$y = y_\star + \eta$$
$$z = z_\star + \epsilon$$

$$\begin{bmatrix} \dot{\Psi} \\ \dot{\eta} \\ \dot{\epsilon} \end{bmatrix} = \begin{bmatrix} -\alpha(5x_\star^4 + c) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix} \begin{bmatrix} \Psi \\ \eta \\ \epsilon \end{bmatrix}$$

Stability Matrix-

$$J = \begin{bmatrix} -\alpha(5x_\star^4 + c) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$$

→ **For fixed point O, J=M_0 -**

$$M_o = \begin{bmatrix} -\alpha c & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$$

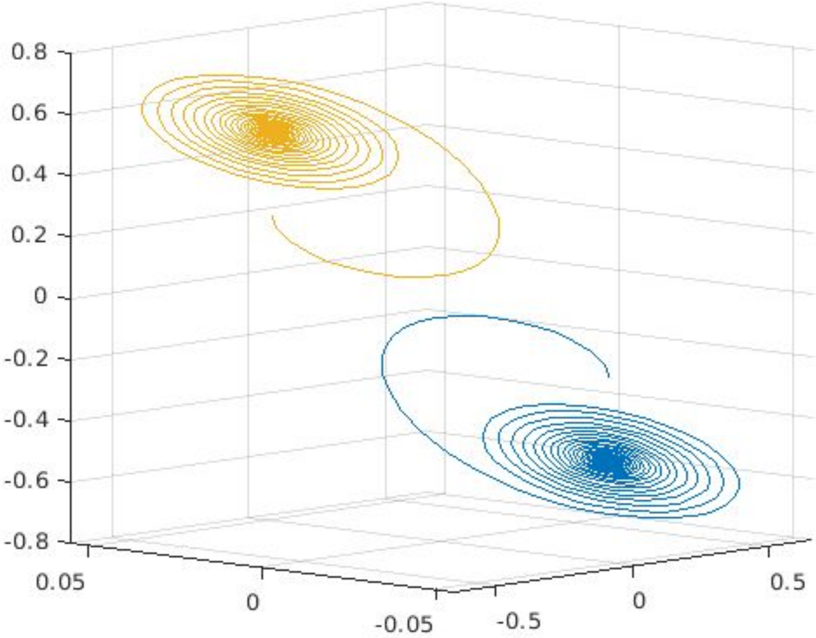
→ **For fixed points, ±P, J=$M_{\pm P}$ -**

$$M_{\pm P} = \begin{bmatrix} 4\alpha c & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$$

Bifurcation Diagrams

Depending on the eigenvalues, we can determine the type of fixed point, from the roots of the characteristic equation Det[sI - M] = 0 of matrices mentioned in previous slide. This will depend on the parameter values .

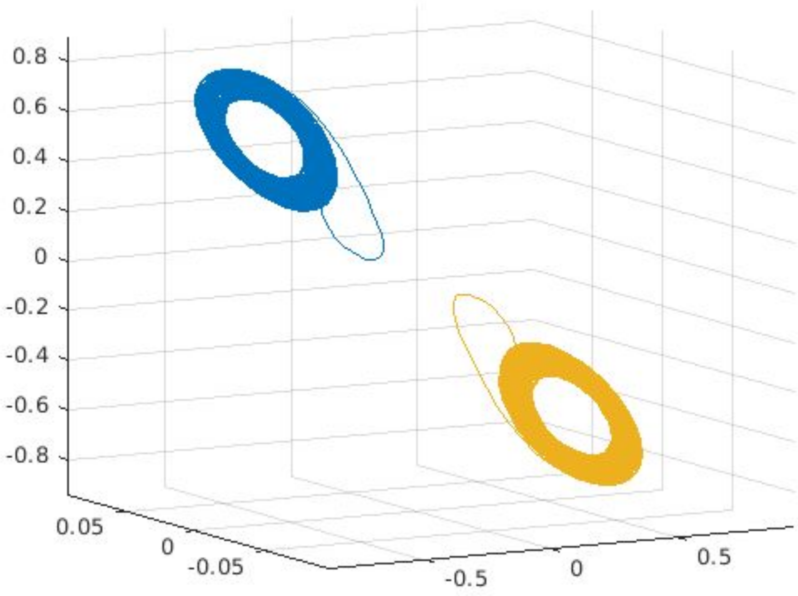
Bifurcation with respect to Alpha



Type of phase plot-

When **alpha is 6**, beta is 16, c is -0.142. 2 stable fixed points, symmetric about the origin. 1 unstable fixed point at origin.

Bifurcation with respect to Alpha



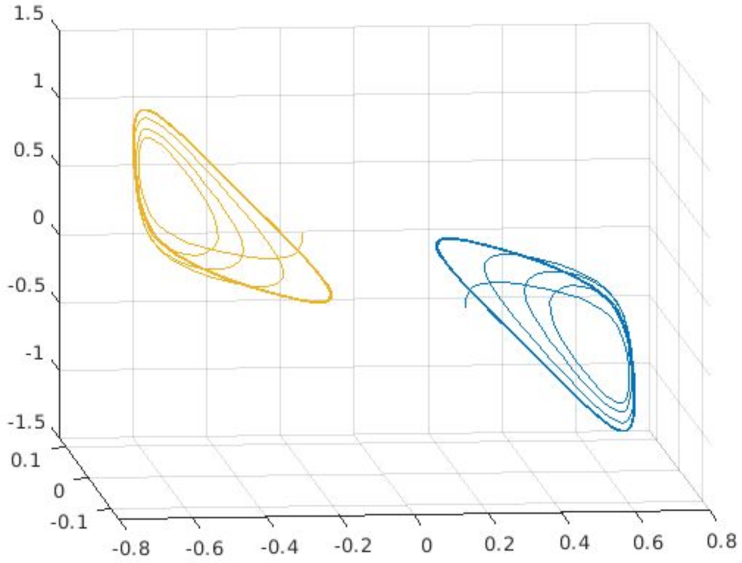
Type of phase plot-

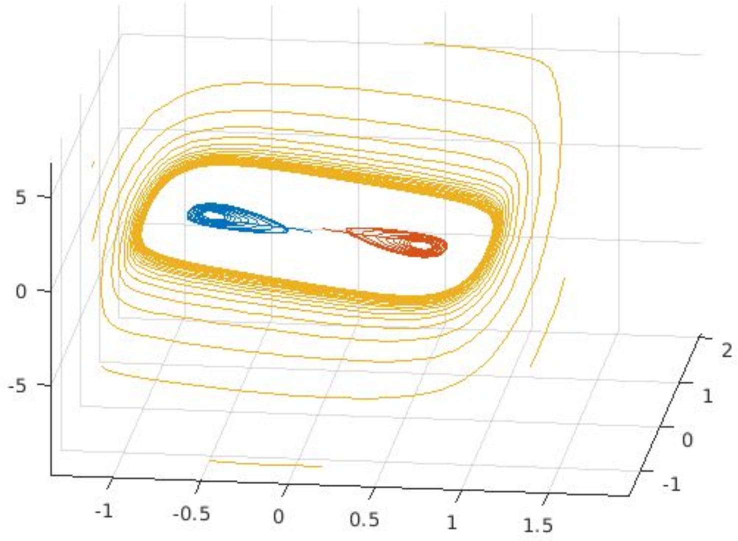
When **alpha is 7.2**, beta is 16, c is -0.142. 2 asymmetric orbits with period (n), 1 unstable fixed point at origin.

Bifurcation with respect to Alpha

Type of phase plot-

When **alpha is 8.9**, beta is 16, c is -0.142. 2 asymmetric orbits across origin with period (n), 1 unstable fixed point at origin, 1 Large stable orbit.

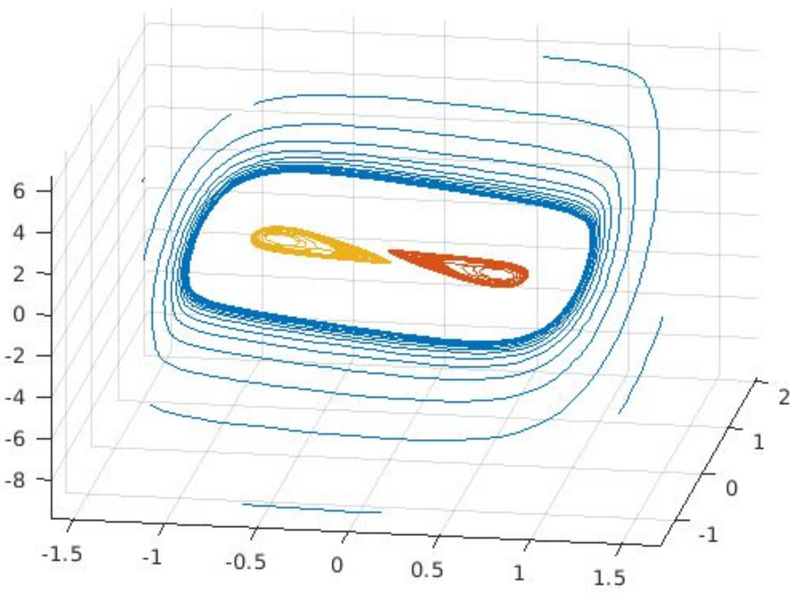


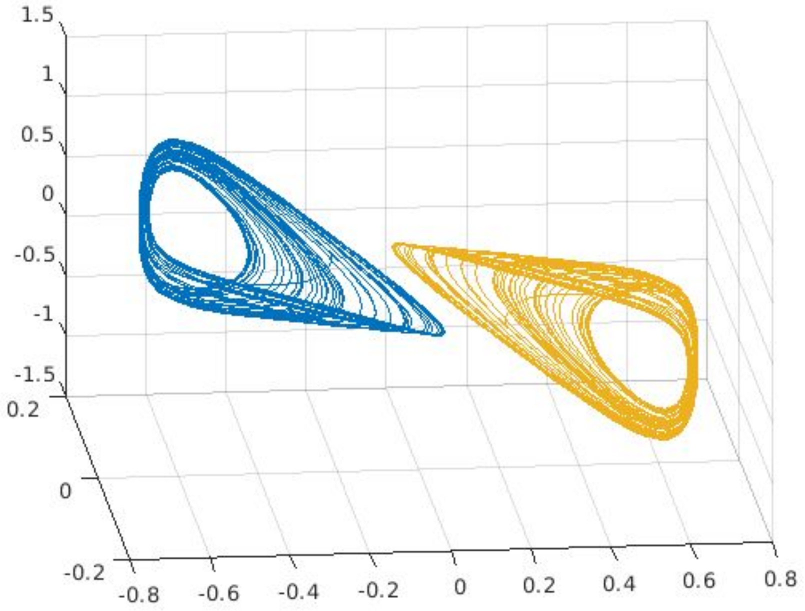


Bifurcation with respect to Alpha

Type of phase plot-

When **alpha is 9.8**, beta is 16, c is -0.142. 2 Chua's spiral attractors across origin, 1 unstable fixed point at origin, 1 Large stable orbit.

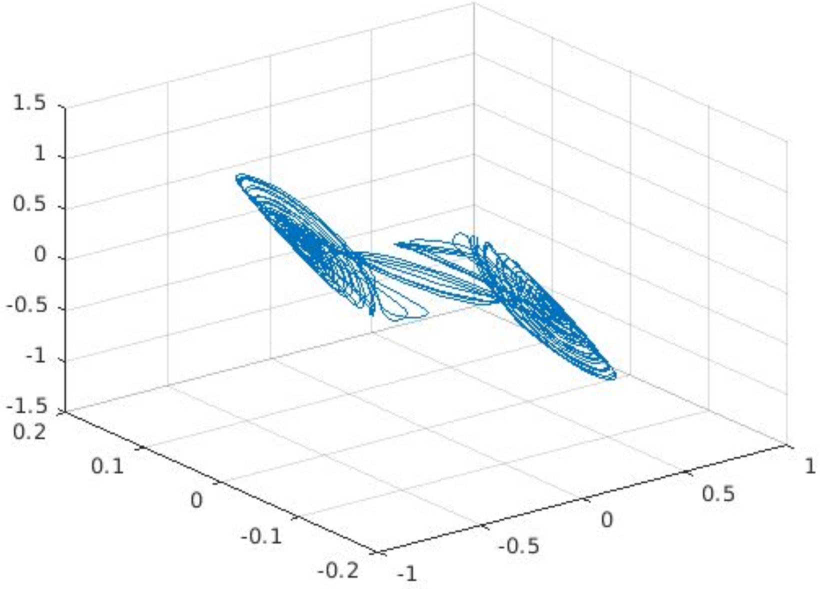


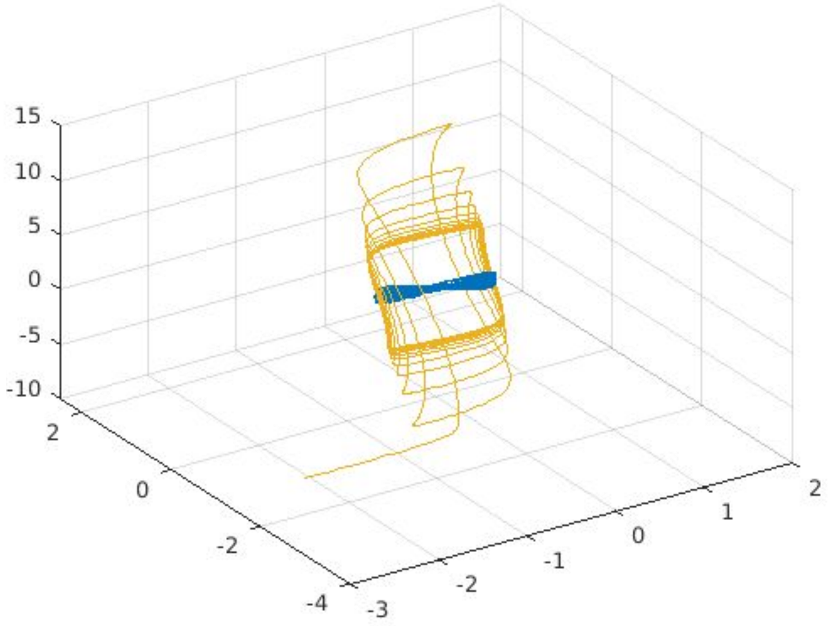


Bifurcation with respect to Alpha

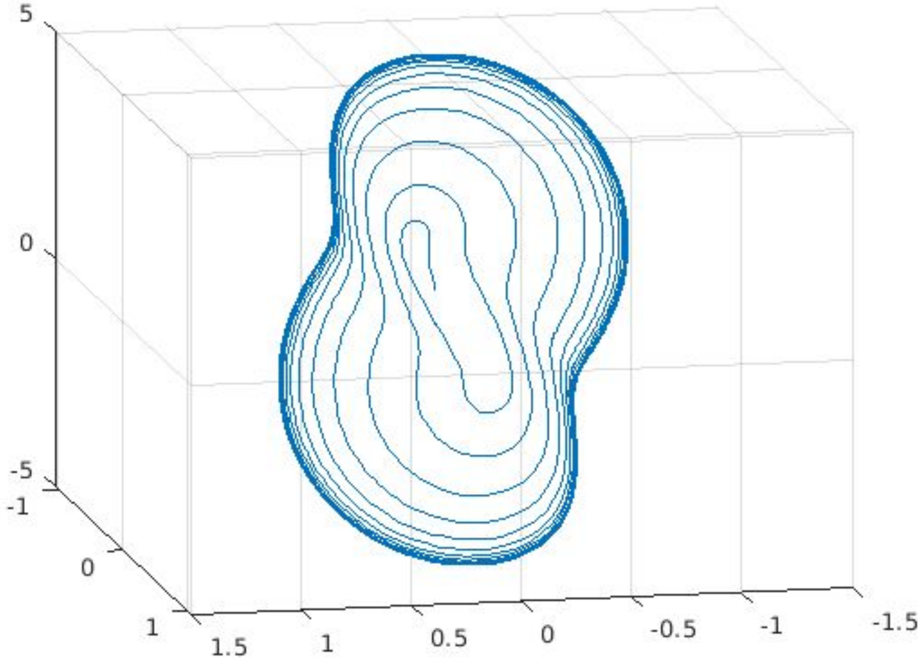
Type of phase plot-

When **alpha is 11.0**, beta is 16, c is -0.142. Chua's double scroll attractor, 1 unstable fixed point at the origin, 1 stable large orbit.





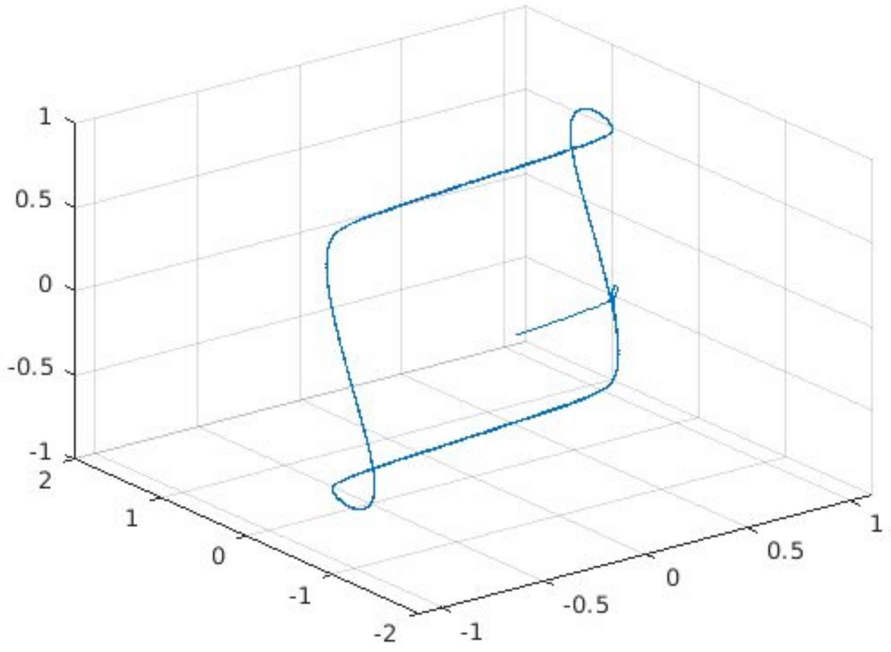
Bifurcation with respect to Alpha



Type of phase plot-

When **alpha is 17**, beta is 16, c is -0.142. 1 stable attractor, 1 unstable fixed point at origin.

Bifurcation with respect to Beta



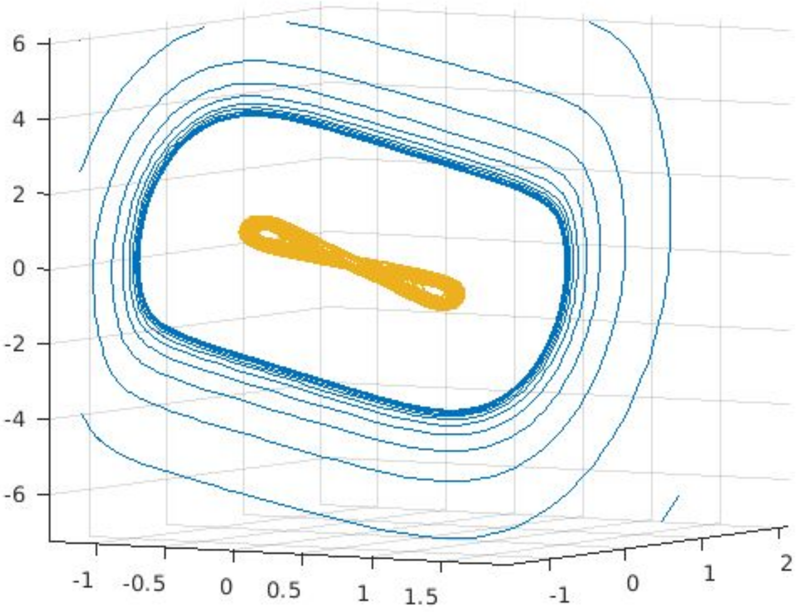
Type of phase plot-

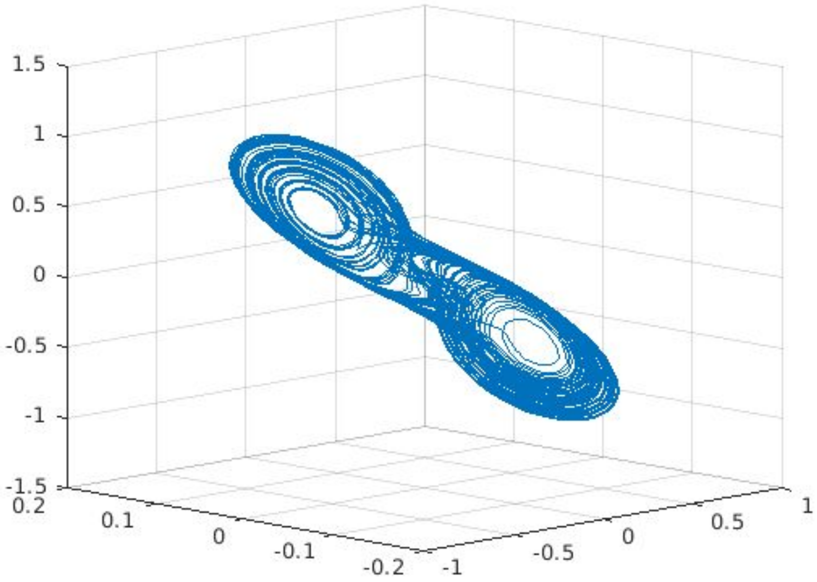
When alpha is 11, **beta is 0.1**, c is -0.142. 1 large stable orbit, and unstable fixed point at origin.

Bifurcation with respect to Beta

Type of phase plot-

When alpha is 11, **beta is 12**, c is -0.142. Chua's double scroll attractor, unstable fixed point at origin, 1 large stable orbit.

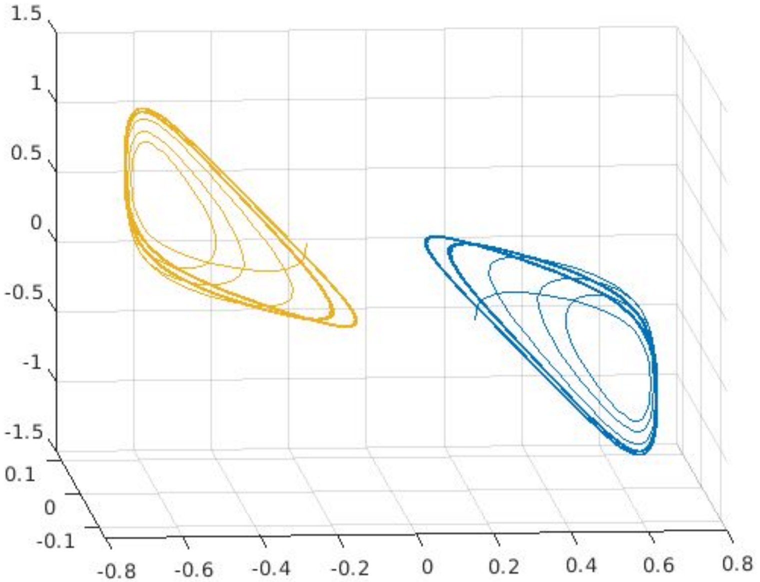


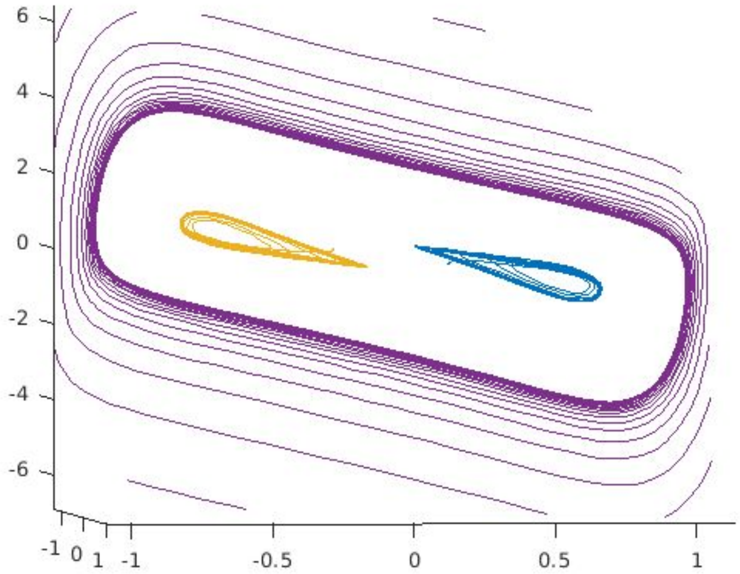


Bifurcation with respect to Beta

Type of phase plot-

When alpha is 11, **beta is 16**, c is -0.142. 2 Chua's spiral attractors, 1 unstable fixed point at origin, 1 large stable orbit.

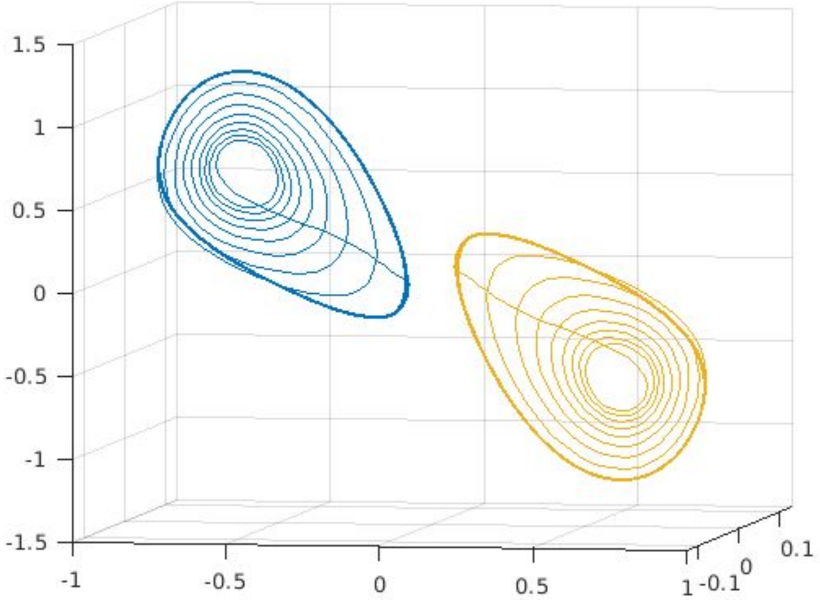


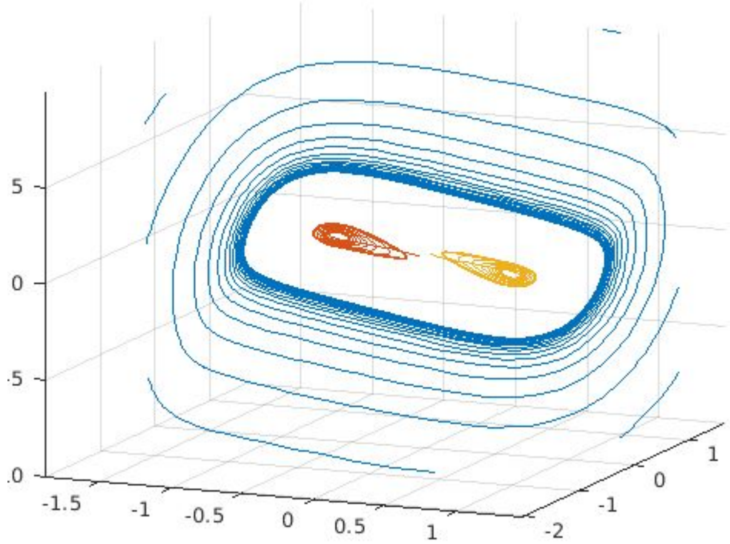


Bifurcation with respect to Beta

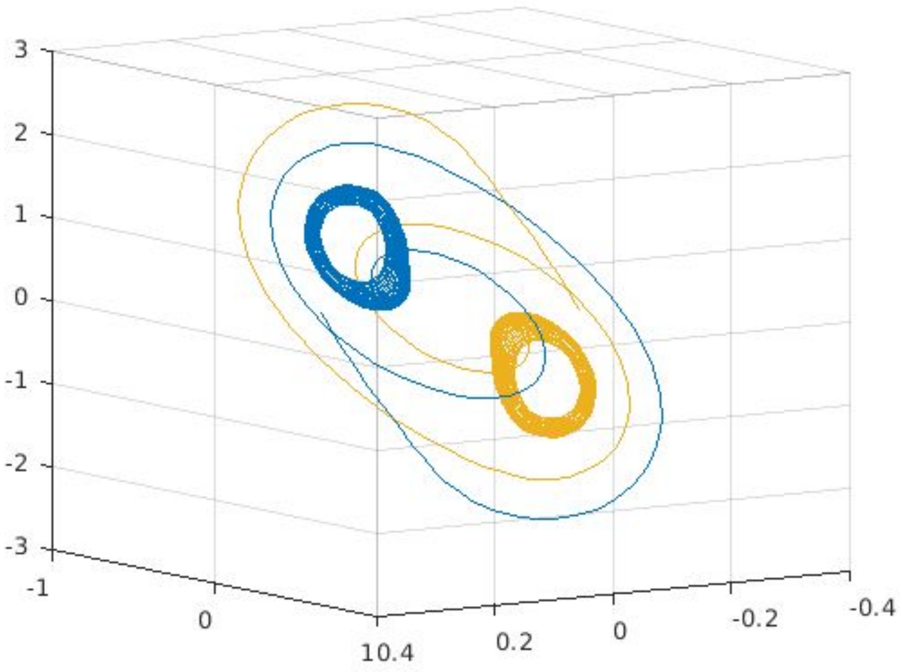
Type of phase plot-

When alpha is 11, **beta is 21**, c is -0.142. 2 Chua's spiral attractors, 1 unstable fixed point at origin, 1 large stable orbit.





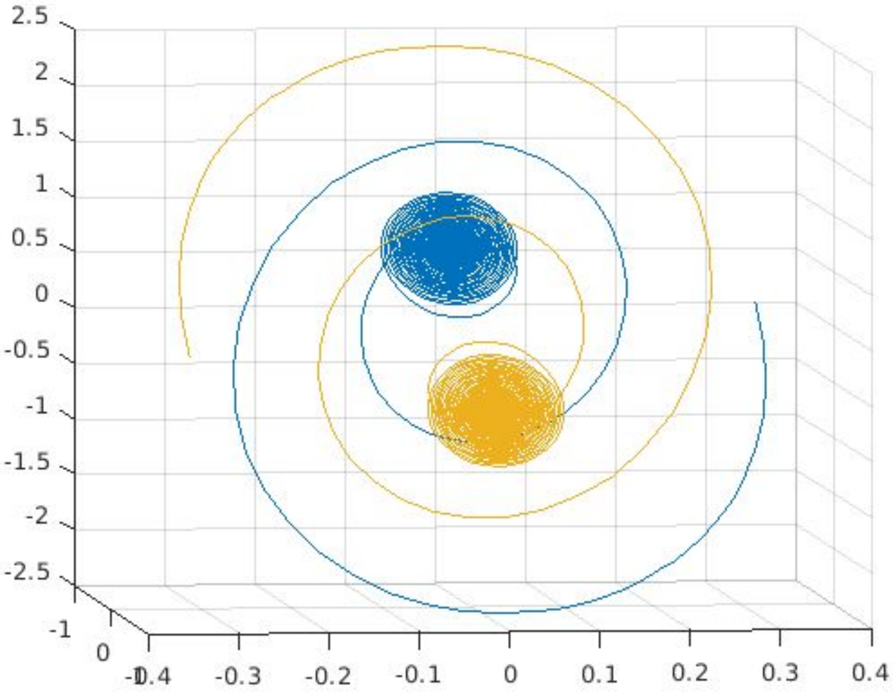
Bifurcation with respect to Beta



Type of phase plot-

When alpha is 11, **beta is 25**, c is -0.142. 2 asymmetric orbits with period (n), 1 unstable fixed point at origin.

Bifurcation with respect to Beta

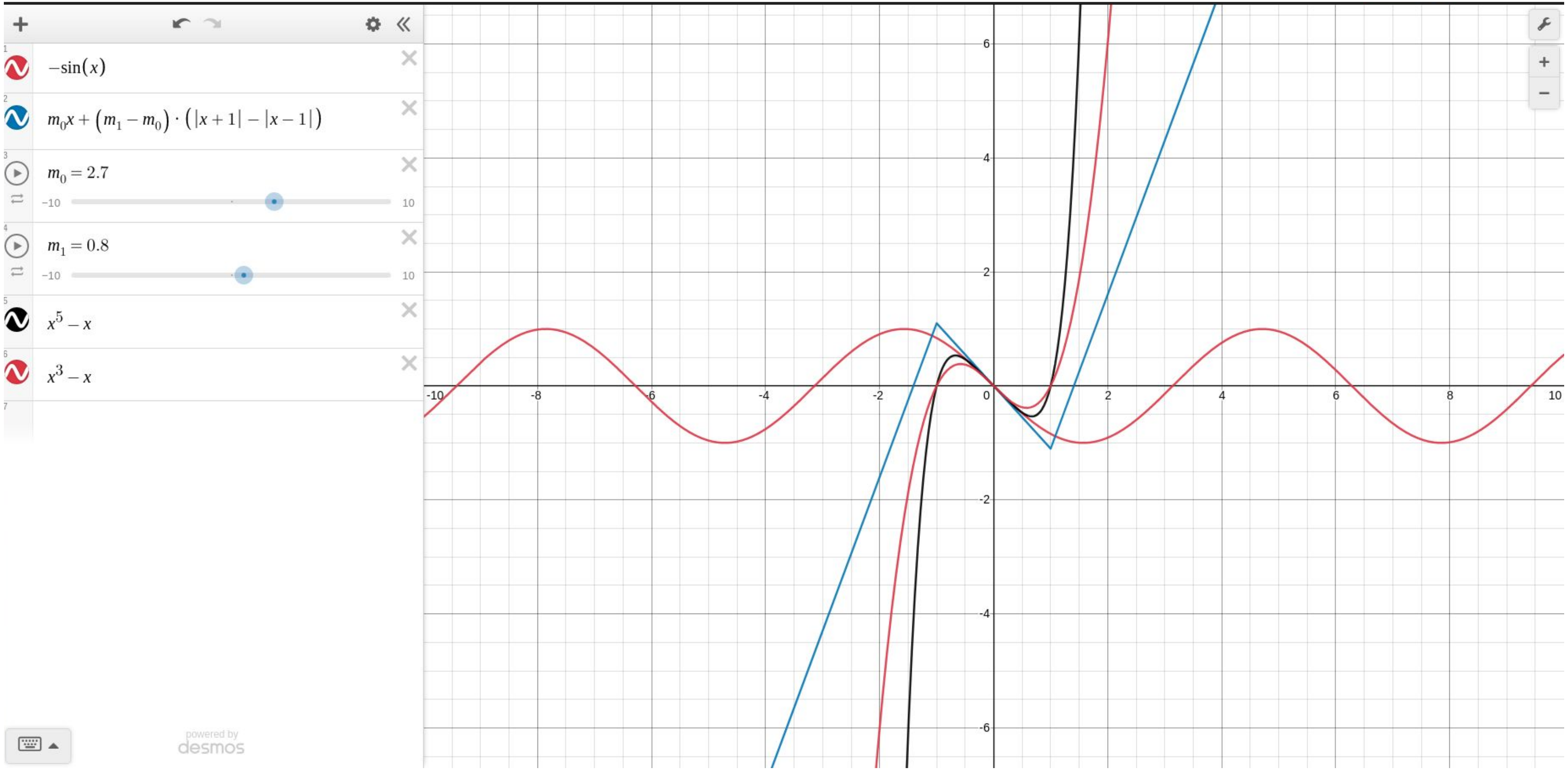


Type of phase plot-

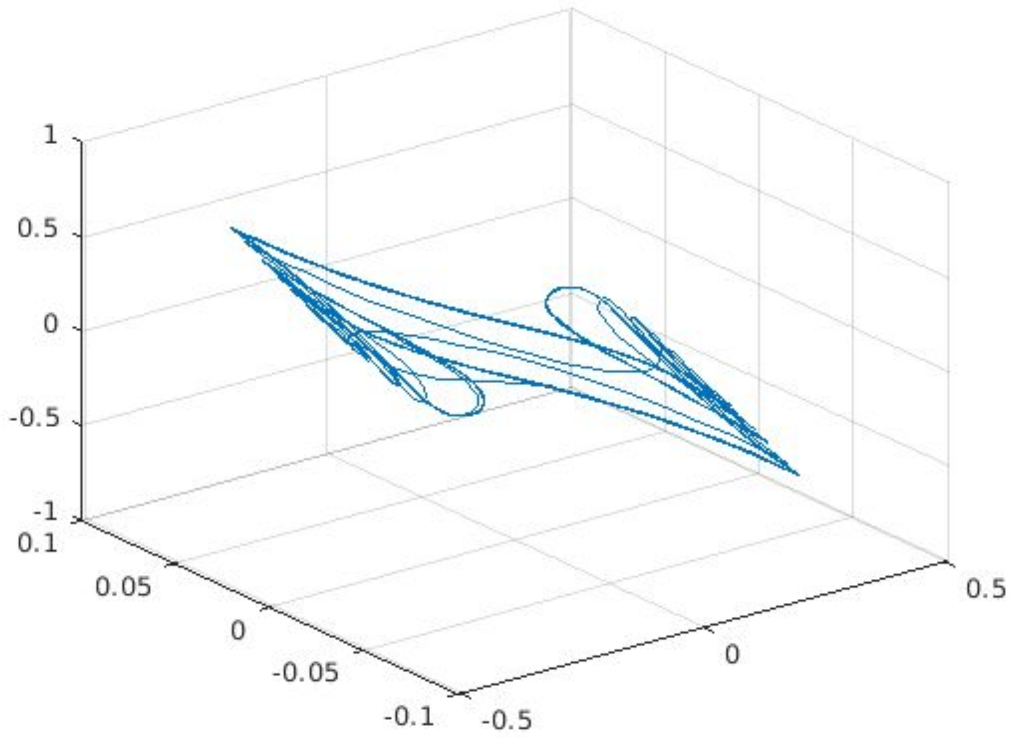
When alpha is 11, **beta is 40**, c is -0.142. 2 stable fixed points, symmetric about the origin. 1 unstable fixed point at origin.

Other nonlinearities which exhibit chaos and parallels with normal chua’s circuit

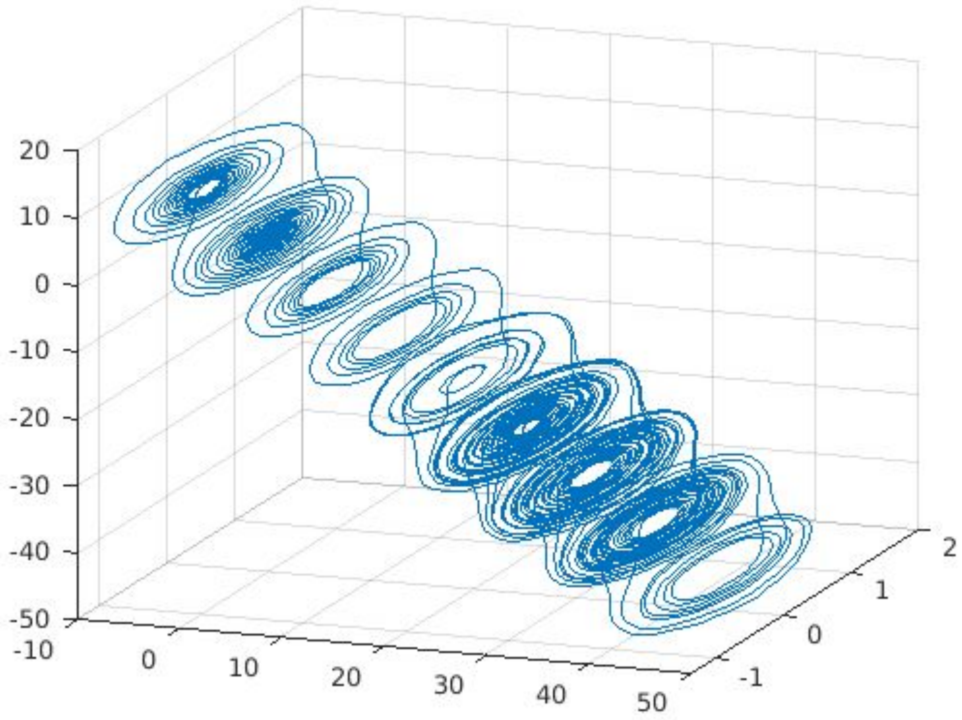
Functions matching the general shape of the piecewise function in the original circuit are shown to exhibit chaos. These include odd-powered polynomials, sin(x), piecewise combinations of other functions, etc. We can observe the chaotic phenomenon as shown in subsequent slides -



x^3-x



piecewise sin(x)



MATLAB Code

Chua.m

```
function out = chua(t,in)
x = in(1);
y = in(2);
z = in(3);
alpha  = 11;
beta   = 16;
c = -0.142;
h= x^5+c*x
xdot = alpha*(y-h);
ydot = x - y+ z;
zdot  = -beta*y;
out = [xdot ydot zdot]';
```

https://www.chuacircuits.com/matlabsim.php

Chua_sim.m

```
[t,y] = ode45(@chua,[0 300],[-0.03 -0.03 -0.03]);
plot3(y(:,1),y(:,2),y(:,3))
grid
```

CONCLUSION

- Applications in Communication and biomedical.
- Beneficial to study effects of smooth nonlinear resistors on Chua’s Circuit.
- Studied the chaos in Chua’s Circuit with Quintic nonlinearity and results obtained were similar to that of basic Chua Circuit.

References

- Implementation of Chua’s circuit with a Cubic Nonlinearity - Guo Qun Zhong Implementation of Chua's circuit with a cubic nonlinearity - Circuits and Systems I: Fundamental Theory and Applications, IE
- Chua’s Equation with Cubic nonlinearity - Anshan Huang, Ladislav Pivka, Chai Wah Wu, Martin Franz
- NPTEL - Chaos, Fractals and Dynamical Systems Lecture 7 - Prof.S.Banerjee IIT Kharagpur
- Python Applications - Dynamical Systems
- https://www.chuacircuits.com/matlabsim.php
- https://adipandas.github.io/posts/2021/03/fixed-point-high-dim/