Stabilizer Codes and Quantum Error-Correcting Codes - Chapters 1 and 2

Introduction and Basics of QEC

Introduction

Using KVL at the node 1,

$$I_{C_1} = I_R - I_N$$

$$\therefore C_1\frac{dV_1}{dt} = \frac{1}{R}(V_{C2} - V_{C1}) - g(V_{C_1})$$

Using KVL at the node 2,

$$I_{C_2} = I_R - I_L$$

$$\therefore C_2\frac{dV_2}{dt} = \frac{1}{R}(V_{C_2} - V_{C_1}) + I_L$$

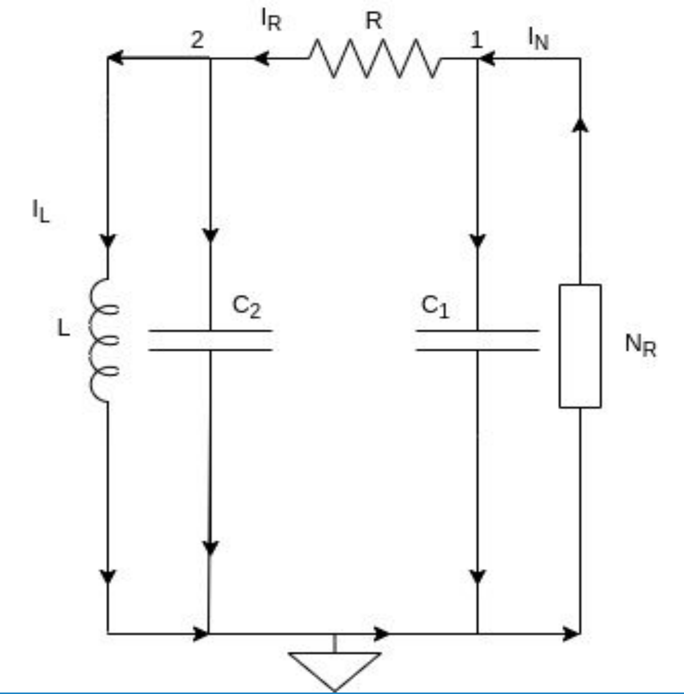
System of nonlinear differential equations

$$\begin{aligned}\dot{x} &= \frac{1}{RC_1}(y-x) - \frac{1}{C_1}g(x)\\ \dot{y} &= \frac{1}{RC_2}(y-x) + \frac{1}{C_2}z\\ \dot{z} &= -\frac{1}{L}y\end{aligned}$$

Rewriting the above equations

$$\alpha = \frac{C_2}{C_1}, \beta = \frac{R^2C_2}{L}, \tau = \frac{t}{RC_2}, z' = Rz$$

$$\begin{aligned}\dot{x} &= \alpha(y - h(x))\\ \dot{y} &= y - x + z\\ \dot{z} &= -\beta y\end{aligned}$$



Basic Chua's Circuit

Introduction-

1. What are tractable and intractable class of problems?
2. Proving the Church-Turing Thesis is incorrect for Quantum computers.
3. Reason for exponential power of Quantum computers.
4. Why no new approach for QEC?

Introduction to Quantum Mechanics

Introduction-

1. What is a qubit and its representation?
2. What are entangled states?
3. Significance of Pauli Matrices and measurement of qubits
4. Inner product

Differential Equations-

$$\dot{x} = \alpha(y - x - g(x))$$
$$\dot{y} = x - y + z$$
$$\dot{z} = -\beta y$$

Finding Fixed Points-

$$\dot{x} = 0,\ \dot{y} = 0,\ \dot{z} = 0$$
$$\dot{z} = 0 \Rightarrow y_\star = 0$$
$$\dot{y} = 0 \Rightarrow x_\star = -z_\star$$
$$\dot{z} = 0 \Rightarrow x_\star + g(x) = 0$$
$$\dot{x} = \alpha[y - x - (x^5 + (c-1)x)]$$
$$\dot{x} = \alpha(y - x - x^5 - cx + x)$$
$$\dot{x} = \alpha(y - x^5 - cx)$$
$$\dot{x} = \alpha(y - f(x))$$

$$f(x) = x^5 + cx$$
$$f(x_\star) = x_\star + g(x_\star) = 0$$
$$x_\star^5 + cx_\star = 0 \Rightarrow x_\star = 0 \ or\ x_\star = (-c)^{\frac{1}{4}}$$

Fixed Points when c<0 and c>0-

$\therefore$, for $c \geq 0$, one fixed point exists at (0, 0, 0)

for $c < 0$, three fixed points exist at -

$$\begin{cases} \rightarrow (0,0,0) & O \\ \rightarrow \left(|c|^{\frac{1}{4}}, 0, -|c|^{\frac{1}{4}}\right) & +P \\ \rightarrow \left(-|c|^{\frac{1}{4}}, 0, |c|^{\frac{1}{4}}\right) & -P \end{cases}$$

For small perturbations-

$$x = x_\star + \Psi$$
$$y = y_\star + \eta$$
$$z = z_\star + \epsilon$$

$$\begin{bmatrix} \dot{\Psi} \\ \dot{\eta} \\ \dot{\epsilon} \end{bmatrix} = \begin{bmatrix} -\alpha(5x_\star^4 + c) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix} \begin{bmatrix} \Psi \\ \eta \\ \epsilon \end{bmatrix}$$

Stability Matrix-

$$J = \begin{bmatrix} -\alpha(5x_\star^4 + c) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$$

→ For fixed point O, J=M_0 -

$$M_o = \begin{bmatrix} -\alpha c & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$$

→ For fixed points, ±P, J=$M_{\pm P}$ -

$$M_{\pm P} = \begin{bmatrix} 4\alpha c & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$$

Introduction to Classical Coding Theory

Introduction-

1. What are the three steps in communication?
2. Types of communication channel
3. Channel Capacity definition and error-correcting codes with an example
4. Hamming distance, generator, parity check matrices and syndrome - definitions
5. Hamming bounds - sphere of packing bounds