Presentation-5

July 19, 2022

Matrix for Simon's Oracle

For single & two-qubit systems

For a **single qubit**, the oracle outputs the following-

1. $U_f |0\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$
2. $U_f |1\rangle = |0\rangle$

and its matrix is $U_{f1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$

Similarly, for two qubits, U_f gives -

1. $U_f |00\rangle = \frac{1}{2}(|11\rangle + |00\rangle + |01\rangle + |10\rangle)$
2. $U_f |10\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$
3. $U_f |01\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$
4. $U_f |11\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

Matrix for Simon's Oracle

For single & two-qubit systems [Continued]

The matrix for the two-qubits is - $U_{f2} = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

Taking the tensor product of U_{f1} gives -

$$U'_{f2} = U_{f1} \otimes U_{f1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{bmatrix}$$

Matrix for Simon's Oracle

For single, two, three-qubits systems

For **three qubits**, the oracle outputs-

1. $U_f\,|000\rangle = \frac{1}{2\sqrt{2}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle)$
2. $U_f\,|001\rangle = \frac{1}{2}(|000\rangle + |010\rangle + |100\rangle + |110\rangle)$
3. $U_f\,|010\rangle = \frac{1}{2}(|000\rangle + |001\rangle + |100\rangle + |101\rangle)$
4. $U_f\,|011\rangle = \frac{1}{2}(|000\rangle + |011\rangle + |100\rangle + |111\rangle)$
5. $U_f\,|100\rangle = \frac{1}{2}(|000\rangle + |001\rangle + |010\rangle + |011\rangle)$
6. $U_f\,|101\rangle = \frac{1}{2}(|000\rangle + |010\rangle + |101\rangle + |111\rangle)$
7. $U_f\,|110\rangle = \frac{1}{2}(|000\rangle + |001\rangle + |110\rangle + |111\rangle)$
8. $U_f\,|111\rangle = \frac{1}{2}(|000\rangle + |011\rangle + |101\rangle + |110\rangle)$

Matrix for Simon's Oracle

For single, two, three-qubits systems [Continued]

And its matrix is - $U_{f3} = \begin{bmatrix} \frac{1}{2\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2\sqrt{2}} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2\sqrt{2}} & \frac{1}{2} & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2\sqrt{2}} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

Matrix for Simon's Oracle

For single, two, three-qubits systems [Continued]

$|0\rangle \longrightarrow 0, 1$ and $|1\rangle \longrightarrow 0$
$|00\rangle \longrightarrow 0, 1, 2, 3,\ |01\rangle \longrightarrow 0, 2,\ |10\rangle \longrightarrow 0, 1$ and $|11\rangle \longrightarrow 0, 3$
$|000\rangle \longrightarrow 0, 1..7,\ |001\rangle \longrightarrow 0, 2, 4, 6,\ |010\rangle \longrightarrow 0, 1, 4, 5,$
$|011\rangle \longrightarrow 0, 3, 4, 7,\ |100\rangle \longrightarrow 0, 1, 2, 3,\ |101\rangle \longrightarrow 0, 2, 5, 7,$
$|110\rangle \longrightarrow 0, 1, 6, 7$ and $|111\rangle \longrightarrow 0, 3, 5, 6$
$|0000\rangle \longrightarrow 0, 1..15,\ |0001\rangle \longrightarrow 0, 2, ..14,$
$|0010\rangle \longrightarrow 0, 1, 4, 5, 8, 9, 12, 13,\ |0011\rangle \longrightarrow 0, 3, 4, 7, 8, 11, 12, 15,$
$|0100\rangle \longrightarrow 0, 1, 2, 3, 8, 9, 10, 11,\ ,$
$|1101\rangle \longrightarrow 0, 2, 5, 7, 9, 11, 12, 14, \ldots$

Matrix for Simon's Oracle

For single, two, three-qubits systems [Continued]

The oracle's solutions corresponds to -

1. If a bit in the input string is zero, then corresponding to its index, it can be 0 or 1.
2. If a bit is 1 and the input has odd number of 1's, then 2n non-zero bits of the strings can be 1, the remaining one non-zero bit string must be zero.
3. If a bit is 1 and the input has even number of 1's, then the corresponding non-zero bit strings must be 1.

The input bit string has to be reversed to follow Qiskit's ordering. The occurrence of the first non-zero string bit of the reversed input string must be stored in a variable [called *flagbit*].
Its oracle is implemented by first copying the qubits of input register to that of the second register using CNOT gates, i.e, $q1_a \longrightarrow q2_a$. Then, whenever, we have a non-zero string bit occuring in the reversed string, we perform a $CNOT$ operation with control as input register qubit whose index corresponds to *flagbit* and target as second register qubit whose index corresponds to the index of the occurrence of a non-zero bit string in the reversed input string.

When input is "011", we get the following circuit -

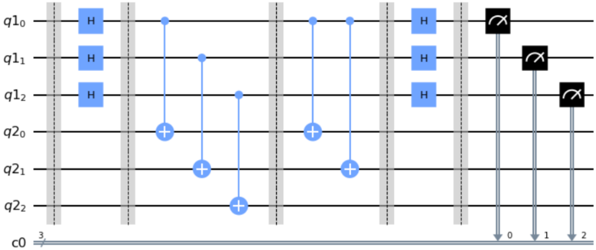

Figure: Circuit for Simon's Algorithm