# Assessing the Effectiveness of Acceleration Methods for Deterministic Neutron Transport Solvers

## Building a new tool for developers.

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ANS Summer Meeting: Acceleration Methods

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A majority of our limited time and effort (and funding) should be dedicated to designing new and better acceleration methods, **not implementing and analyzing results**.

# Outline

1. Why acceleration methods?
2. Analysis and implementation challenges
3. Design paradigm
4. Status and future work

# Steady-state Boltzman Transport Equation

Our problem of interest is the time-independent transport equation on a domain of interest $\mathbf{r} \in V$ [3],

$$\left[\hat{\Omega} \cdot \nabla + \Sigma_t(\mathbf{r}, E)\right] \psi(\mathbf{r}, E, \hat{\Omega}) \\ = \int_0^\infty dE' \int_{4\pi} d\hat{\Omega}' \Sigma_s(\mathbf{r}, E' \to E, \hat{\Omega}' \to \hat{\Omega}) \psi(\mathbf{r}, E', \hat{\Omega}') \\ + Q(\mathbf{r}, E, \hat{\Omega}) \ ,$$

with a given boundary condition,

$$\psi(\mathbf{r}, E, \hat{\Omega}) = \Gamma(\mathbf{r}, E, \hat{\Omega}), \quad \mathbf{r} \in \partial V, \quad \hat{\Omega} \cdot \hat{n} < 0$$

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Gauss-Seidel source iteration $\mathbf{\Psi}_{(k+1)} = \mathbf{L}^{-1}\mathbf{M}\left[\mathbf{S}\mathbf{\Phi}_{(k)} + \frac{1}{k}\mathbf{F}\mathbf{\Phi}_{(0)}\right]$

Power iteration $\mathbf{\Psi}_{(k+1)} = \mathbf{L}^{-1}\mathbf{M}\left[\mathbf{S}\mathbf{\Phi}_{(0)} + \frac{1}{k}\mathbf{F}\mathbf{\Phi}_{(k)}\right]$

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Power iteration can converge arbitrarily slowly as the dominance ratio $k_1/k_0$ approaches unity.

Motivates the development of **acceleration methods** to address these issues.

- Source Iteration: Diffusion two-grid method (TG).
- Power Iteration: Nonlinear diffusion acceleration (NDA).

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with total computational work

$$W = \sum_{\ell=1}^{N} w_{(\ell)} \ .$$

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**Note**

The same amount of error needs to be removed for the problem to converge, regardless of the method used to remove it.

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## What do we need?

A coding framework designed with the developer end-user in mind, that is portable and reproducible.

# Defining work

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In general, we use inversions of the transport matrix – explicitly or implicitly (*sweeps*) – as a unit of work.

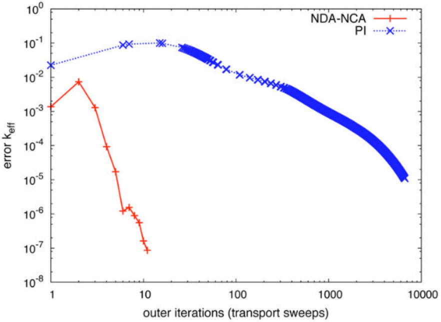


**Figure 1:** NDA convergence vs standard power iteration [7]

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This becomes complicated as our acceleration methods become more complex, and take on more work.

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## What do we need?

Additional, good data that enables us to assess the effectiveness of our method.

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- Combined or complex schemes may invalidate assumptions.
- Implementation and reproducibility can be difficult.

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1. relieves some of the burden of implementing a novel acceleration method,
2. provides a controlled environment for measuring the effectiveness of the novel method, and,
3. provides tools for verifying the basis for effectiveness.

# Designed for implementation

**Goal 1**

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```
SystemInitializerI
------------------
+ Initialize(System&)
: void
```

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- Heavy usage of polymorphism.
- Comprehensive testing coverage.

# Controlled testing environment

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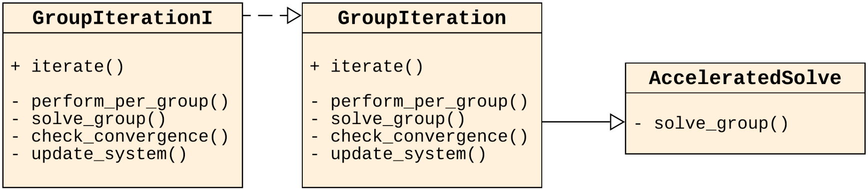

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- Enables a true comparison of the accelerated solve to a control solve.
- Makes the modifications *portable*.
- Enables us to compare the implementation of the method to dis-aggregate the computer science from the method itself.

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BART will include the ability to *instrument* a solve to gather enough data to draw useful conclusions about the effectiveness of acceleration schemes.

- Storage of solve parameters (eigenvalues, fluxes).
- Storage of hierarchy of iterations.
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Adding new instrumentation must be easy!

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- Uses the Google Protocol Buffers file format for cross-sections.

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- Future methods: transport two-grid acceleration (TTG), and a combination of NDA and TTG.
- Instrumentation in development: *in-situ* stepwise Fourier Analysis.

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- Implementing and analyzing acceleration methods has practical challenges.
- We are developing a new code aimed at helping developers of new methods.
- We hope that the code will ease the burden of coding up new methods, and help provide good, useful data for understanding if and why the methods are worthwhile to implement in production level codes.

Thank you

# References


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# Outline

5 Backup Slides

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Backup