# Title of Qualification Exam Talk

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# Outline

# Steady-state Boltzman Transport Equation

Our problem of interest is the time-independent transport equation for a critical system on a domain of interest $\mathbf{r} \in \mathbb{D}$,$E \in \mathbb{E}$ [1]

$$\left[\hat{\Omega} \cdot \nabla + \Sigma_t(\mathbf{r}, E)\right] \psi(\mathbf{r}, E, \hat{\Omega}) \\ = \int_0^\infty dE' \int_{4\pi} d\hat{\Omega}' \Sigma_s(\mathbf{r}, E' \to E, \hat{\Omega}' \to \hat{\Omega}) \psi(\mathbf{r}, E', \hat{\Omega}') \\ + Q(\mathbf{r}, E, \hat{\Omega})$$

with a given boundary condition,

$$c_0 \psi(\mathbf{r}, \hat{\Omega}, E) + c_1 \frac{\partial \psi(\mathbf{r}, \hat{\Omega}, E)}{\partial \mathbf{r}} = f(\mathbf{r}, \hat{\Omega}, E), \quad \hat{n} \cdot \hat{\Omega} < 0, \mathbf{r} \in \partial \mathbb{D}$$

# Energy discretization

Introduce a discretization of the energy domain $\mathbb{E}$ into $G$ non-overlapping elements, such that

$$E_h = \{E_1, E_2, \ldots, E_G\}\,, \quad \mathbb{E} = \bigcup_{g=1}^{G} E_g$$

Assume that the energy-dependent angular flux can be separated into a group angular flux and a energy function within each of these groups

$$\psi(\mathbf{r}, E, \hat{\Omega}) \approx \psi_g(\mathbf{r}, \hat{\Omega}) f_g(E), \quad E \in E_g$$

Finally, assume that

$$\int_{E_{g'} \in E_h} f_g(E) dE = \delta_{g,g'}$$

# Single-group steady-state transport equation

$$\left[\hat{\Omega} \cdot \nabla + \Sigma_t(\mathbf{r}, E)\right] \psi_g(\mathbf{r}, \hat{\Omega})$$
$$= \sum_{g'=1}^{G} \int_{4\pi} d\hat{\Omega}' \Sigma_s(\mathbf{r}, E_{g'} \to E_g, \hat{\Omega}' \to \hat{\Omega}) \psi_{g'}(\mathbf{r}, \hat{\Omega}') + Q_g(\mathbf{r}, \hat{\Omega})$$

# Transport equation second-order forms

Consider the mono-energetic form of the transport equation, using the scattering operator $S\psi(\mathbf{r},\hat{\Omega}) = \int_{4\pi} d\hat{\Omega}'\Sigma_s(\mathbf{r},\hat{\Omega}' \to \hat{\Omega})\psi(\mathbf{r},\hat{\Omega}')$:

$$\left[\hat{\Omega}\cdot\nabla + \Sigma_t(\mathbf{r})\right]\psi(\mathbf{r},\hat{\Omega}) = S\psi(\mathbf{r},\hat{\Omega}) + Q \tag{1}$$

Substitute $-\hat{\Omega}$ for $\hat{\Omega}$, add to Eq. (1), and divide by two to get a function of even- and odd-parity angular fluxes.

$$\hat{\Omega}\cdot\nabla\psi^- + \Sigma_t\psi^+ = S^+\psi^+ + Q^+$$

where,

$$\psi^+ = \frac{1}{2}\left(\psi(\hat{\Omega}) + \psi(-\hat{\Omega})\right)$$
$$\psi^- = \frac{1}{2}\left(\psi(\hat{\Omega}) - \psi(-\hat{\Omega})\right)$$

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# References

[1] E. E. Lewis and W.F. Miller, Jr.
*Computational Methods of Neutron Transport*.
American Nuclear Society, 1993.