Nuclear Engineering 150: Midterm 2 Study Guide

Disclaimer: This is not an official study guide. Stuff ~~might~~ **is** wrong. Use the lecture notes and book!

Note: Everything in this guide is from the text () or lecture, or office hours and should be cited as completely as possible.

Contents

1 Neutron Slowing Down

1.1 Lethargy

- Lethargy (u) is a measure of the amount that a neutron has slowed down *relative* to an energy E_0. It is important to note that this is a relative measure, it only tells us about the neutron's energy when compared to our reference value.[1, Lec. 9]

$$u = \ln\left(\frac{E_0}{E}\right) = \ln(E_0) - \ln(E)$$

- As the neutron slows down, relative to E_0, the value of lethargy goes *up*. This is why we call it a measure of the slowing down. This is why we call it lethargy because it's like a measure of the neutron's sleepiness or something; some nuclear engineer obviously thought he was being super cute.[1, Lec. 9]

- Every collision causes a decrease in neutron energy (and increase in neutron lethargy). If the neutron goes from E_i to E_f after a scattering event, we can solve for the difference in the initial and final energies.

$$\Delta u = \ln\left(\frac{E_0}{E_f}\right) - \ln\left(\frac{E_0}{E_i}\right) = \ln\left(\frac{E_i}{E_f}\right)$$

  This is also called the logarithmic energy loss.[1, Lec.9]

1.1.1 Average Logarithmic Energy Loss per collision

- If we want to know the average logarithmic energy loss per collision, we can take the average. We call that squiggle (written ξ). We will use the energy loss per collision from the last midterm, and denote the original energy E and the final energy E':

$$\xi = \overline{\ln\left(\frac{E}{E'}\right)} = \int_{\alpha E}^{E} dE' \ln\left(\frac{E}{E'}\right) p(E \to E')p = \int_{\alpha E}^{E} dE' \ln\left(\frac{E}{E'}\right) \frac{1}{(1-\alpha)E}$$

Finally, you get:

$$\xi = 1 + \frac{\alpha}{1-\alpha}\ln(\alpha)$$

Note that this doesn't depend on the original energy E. This makes sense, because it's just an average value, we integrated over all the possible energies. In the end, it's just a function of what it's colliding with, because:

$$\alpha = \left(\frac{A-1}{A+1}\right)^2$$

[1, Lec. 9]

- Note that for hydrogen ($A = 1$), the value of α is 0. This makes ξ undefined (ln(0) is undefined), so we just set ξ for hydrogen at 1 (there might be a mathematical way to justify this but who cares). This is the highest ξ can be:

$$\xi = 1 = \ln\left(\frac{E}{E'}\right) \to E' = \frac{E}{e}$$

- You might be saying "But study guide, can't the neutron lose *all* of its energy in a collision with a proton? Like billiard balls?" and I'd say "who says billiard balls, they're pool balls, get it together." Remember this is the *average* energy lost per collision so its between the maximum lost (all of it) and the minimum lost (none of it).

- The value of ξ gets smaller and smaller with increasing A; bouncing a ping-pong ball off a basketball won't do much to slow the ping-pong ball down.

$$\xi \approx \frac{2}{A + 2/3}, \text{for } A > 10$$

Smaller $A \to$ larger ξ (more effective at slowing). [1, Lec. 9]

- We can use this to figure out the average number of elastic collisions required to go from an energy E_1 to E_2 (Note that we use higher numbers for lower energies, this is pretty standard notation in neutronics; this is why E_0 was our *highest* relative energy when defining lethargy).

$$n = \frac{\Delta u}{\xi} = \frac{\ln\left(\frac{E_1}{E_2}\right)}{\xi}$$

This is just the total logarithmic difference of our two energies, divided by the logarithmic amount of energy lost in each collision (distance divided by rate).[1, Lec. 9]

1.1.2 ξ for Molecules

- If you have a molecule, you sum over the individual values of ξ, weighted by their cross-sections. This is to take into account that there's a different probability that the neutron will scatter off of the different elements in the molecule.

$$\bar{\xi} = \frac{1}{\Sigma_s}\sum_i \xi_i \Sigma_{si}$$

Where Σ is the total scattering cross-section for the molecule and Σ_{Si} is the scattering cross-section for each element i.[1, Lec. 9]

- Remember to get the total scattering cross-section for the molecule, you just sum based on how many of each you have. For example, for water:

$$\Sigma_s^{H_2O} = N_{H_2O}(2\sigma^H + \sigma^O)$$

The number densities should all cancel out, letting you just use microscopic cross-sections.

2 Reactor Criticality

2.1 Neutron Population

- We generally view the neutron population in our reactor in "generations." This is an artificial construct, but is a good way to understand the inner mechanics of a reactor. We model our fission chain reaction as generating a whole bunch of neutrons at once (a generation) that then go on and live their lives. Ultimately, the neutrons will either leak, be absorbed, go on to cause more fission, etc. The fissions will then create the next generation of neutrons.

- Leakage and absorption are examples of loss mechanisms, and fission is a production mechanism. Overall, if we could take the whole loss rate $L(t)$ and the whole production rate $P(t)$, we could calculate the change in neutrons between generations:

$$\frac{dn(t)}{dt} = P(t) - L(t)$$

2.1.1 Multiplication Factor

- The multiplication factor is the ratio of neutrons in the current generation, to those in the last generation.[1, Lec. 10]

$$k \equiv \frac{\text{Number of neutrons in this generation}}{\text{Number of neutrons in the last generation}}$$

- There are three possible situations[1, Lec. 10]:

  $k < 1$: Subcritical; there are less neutrons in this generation than last, population is decreasing

  $k = 1$: Critical: there are the same number of neutrons in this generation as the last, the neutron population is steady.

  $k > 1$: Supercritical: there are more neutrons in this generation than the last, population is increasing.

- Alternatively, we can also define k using the production rate of neutrons and the loss rate:

$$k \equiv \frac{P(t)}{L(t)}$$

  You can see how this has the same three situations discussed above.[1, Lec. 10]

2.1.2 Neutron Population Lifetime

- Using this loss rate $L(t)$, we can figure out how long our generation would survive (with no production). At a given time t, we have $n(t)$ neutrons, so using number over loss rate:

$$\ell \equiv \frac{n(t)}{L(t)}$$

  This is our neutron generation lifetime.[1, Lec 10.]

- We can combine this with our value of k above by examining the change in our neutron population over time using the equation from above:

$$\frac{dn(t)}{dt} = P(t) - L(t) = \left[\frac{P(t)}{L(t)} - 1\right] L(t) = [k-1]\frac{L(t)}{n(t)}n(t) = \frac{k-1}{\ell}n(t)$$

  We can solve this using an initial condition $n(0) = n_0$:

$$n(t) = n_0 \text{Exp}\left(\frac{k-1}{\ell}t\right)$$

2.2 Four and Six Factor Formulas

The four and six factors are values that multiply the number of neutrons in the last generation (n) to get the number of neutrons in the next generation. For this, we only assume that there are two populations of neutrons, fast and thermal.

2.2.1 Those Six Factors

Start with n fast neutrons created from fission.

Fast non-leakage probability There is a chance these fast neutrons will leak out of the reactor. The chance that this *doesn't* happen is P_{FNL}, or the **fast non-leakage probability**. This means that nP_{FNL} fast neutrons are left, and $n(1 - P_{\text{FNL}})$ leak out of the reactor.

Resonance escape probability The fast neutrons that don't leak will collide with the moderator and slow down. While slowing down, they must pass through the resonance region where there is an enhanced chance of capture in the fuel. The probably of *not* being absorbed while slowing down to thermal energies is p, the **resonance escape probability**. Therefore, npP_{FNL} fast neutrons reach thermal energies.

Thermal non-leakage probability These thermal neutrons can also escape, just like fast neutrons. Again, the chance this *doesn't* happen is the **thermal non-leakage probability**, P_{TNL}. This gives us $npP_{\text{FNL}}P_{\text{TNL}}$ thermal neutrons that don't leak out.

Thermal utilization factor If a thermal neutron doesn't leak out, it has to go somewhere: it gets absorbed. But, just because the thermal neutron is absorbed, doesn't mean that it's absorbed in the fuel, or *utilized*. It could be absorbed anywhere else in the reactor. This gives rise to the **thermal utilization factor**:

$$f = \frac{\Sigma_a^{\text{fuel}}}{\Sigma_a^{\text{fuel}} + \Sigma_a^{\text{non-fuel}}} = \frac{\propto \text{Probability of absorption in fuel}}{\propto \text{Probability of absorption in anything}}$$

Remember that cross-sections are characteristic of (proportional to) probabilities, so dividing cross sections gives us an actual probability. All f is is the probability that given a neutron is absorbed, it is absorbed in fuel. We knew that our thermal neutrons were absorbed, so we can just tack this onto the end of our growing expression to get the number of thermal neutrons absorbed in the fuel material: $fnpP_{\text{FNL}}P_{\text{TNL}}$.

Important Note: The above formulation assumes you have a homogeneous reactor. That is, the flux in the fuel and non-fuel materials are the same, and the volume of the fuel is the same as the volume of the non-fuel materials. If this isn't the case, you have to do more. For example, if we have only fuel and moderator of different volumes and with different flux:

$$f = \frac{\Sigma_a^{\text{fuel}} V^{\text{fuel}} \phi^{\text{fuel}}}{\Sigma_a^{\text{fuel}} V^{\text{fuel}} \phi^{\text{fuel}} + \Sigma_a^{\text{mod}} V^{\text{mod}} \phi^{\text{mod}}}$$

Or, we can divide through by $V^{\text{fuel}}\phi^{\text{fuel}}$ to get:

$$f = \frac{\Sigma^{\text{fuel}}}{\Sigma^{\text{fuel}} + \Sigma_a^{\text{mod}} \frac{V^{\text{mod}}}{V^{\text{fuel}}} \frac{\phi^{\text{mod}}}{\phi^{\text{fuel}}}}$$

The important thing to note here is that if $V^{\text{fuel}} > V^{\text{mod}}$ then the fraction is > 1 and f can be higher than for the homogenous case.

Reproduction Factor Finally, we the number of neutrons that are actually absorbed in fuel and we've almost come full circle. Now we just need to figure out how many neutrons we get out of those absorptions to start the next generation. For this, we use the **reproduction factor**:

$$\eta = \frac{\nu \Sigma_f}{\Sigma_f + \Sigma_a}$$

This isn't exactly equal to the average number of neutrons from fission (ν) because sometimes an absorption event doesn't result in fission. In this case, Σ_a is absorption events that *do not* include fission, sometimes shown as Σ_γ. η depends on the fuel, and ranges from 1.34 for natural uranium to 2.08 for pure uranium 235. Now, we have the number of fission neutrons due to thermal fission: $\eta f n p P_{\text{FNL}} P_{\text{TNL}}$.

The difference between η and ν can be confusing:

- ν: The number of fast neutrons per *fission*.
- η: The number of fast neutrons per *thermal neutron absorbed* by fuel. This will always be lower than ν because some amount of neutrons will be absorbed and not cause fission (Σ_a above).

Fast Fission Factor Some of the fast neutrons from the beginning might have caused fission before thermalizing. To account for this, we use a **fast fission factor**. This is essentially a correction to what we ended up with after the last step (the number of fission neutrons due to thermal fission):

$$\epsilon = \frac{\text{Number of fission neutrons due to fast and thermal fission}}{\text{Number of fission neutrons due to thermal fission}}$$

So in the end, we have the total number of neutrons due to both fast and thermal fission from our original population of neutrons n: $\epsilon \eta f n p P_{\text{FNL}} P_{\text{TNL}}$. The value of ϵ ranges from 1.00 to 1.10.

2.2.2 The Six-Factor Formula

Now, we can use this with our definition of the multiplication factor:

$$k \equiv \frac{\text{Number of neutrons in this generation}}{\text{Number of neutrons in the last generation}}$$
$$k = \frac{\epsilon \eta f n p P_{\text{FNL}} P_{\text{TNL}}}{n}$$
$$k = \epsilon \eta f p P_{\text{FNL}} P_{\text{TNL}}$$

This is our six-factor formula. This allows us to multiply these factors together and determine if the reactor is critical ($k = 1$), subcritical ($k < 0$) or supercritical ($k > 0$).

2.2.3 The Four-Factor Formula

If we have an infinite reactor, there is no leakage: $P_{\text{FNL}} = P_{\text{TNL}} = 1$. Therefore, our equation reduces down to four factors. We call this k_∞ because it's for an infinite reactor.[1, Lec. 10]

$$k_\infty = \epsilon p f \eta$$

Leakage is impossible to avoid unless your reactor is infinite, so k_∞ represents the upper bound of k.[1, Lec. 10]

References

[1] Jasmine Vujic. Nuclear engineering class lectures. Spring 2015.