Nuclear Engineering 101: Midterm 1 Study Guide

Disclaimer: This is not an official study guide. Stuff might is wrong. Use the lecture notes and book!

Note: Everything in this guide is from the text (Krane) or lecture, or office hours and should be cited as completely as possible.

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1 Nuclear Properties

• The mean nuclear radius is given by:

$$R = R_0 A^{1/3}$$

where A is the nuclear mass and R_0 is a constant (generally 1.2 fm). [1, pp. 48]

• Nuclear binding energy is given by:

$$B(Z, A) = [Zm(^{1}H) + Nm(n) - m(^{A}_{Z}X)]c^{2}$$

where $\operatorname{m}({}_{Z}^{A}X)$ is the *atomic* mass of the atom, and $\operatorname{m}({}^{1}H)$ is the atomic mass of hydrogen. Make sure you use atomic, to ensure the masses of the electrons cancel out. [2, Lec 2]

1.1 Semi-empirical Mass Formula

- Factors that determine the amount of binding energy:
 - The strong nuclear force is short range.
 - Nucleons on the surface have less neighbors.
 - Protons repel each other.
 - Symmetry is important ($Z \approx N$).
 - Pairing is important (protons and neutrons like to pair up)

These are all contained in the Bethe-Weizsäcker Formula. [2, Lec 3]

- The Semi-empirical mass formula (Bethe-Weiszäcker Formula) is called that because it is based on physics, but uses empirical measurements to get the constants. Therefore, it is not derived from first principles, but is just a model of what we have observed. [1, Lec 3] This means it's *wrong* and just represents our best estimate of what is happening.
- The semi-empirical mass formula:

$$B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}} \frac{(A-2Z)^2}{A} + \delta(A, Z)$$

Binding Energy = Volume term - surface term - Coulomb Term - Symmetry term + Pairing term

Where a_n are constants adjusted to make the calculated binding energy match experimental masses. The pairing term:

$$\delta = \begin{cases} +a_P A^{-3/4} & \text{for Z and N even} \\ -a_P A^{-3/4} & \text{for Z and N odd} \\ 0 & \text{for A odd} \end{cases}$$

2 Radioactive Decay

• The decay constant λ (units \sec^{-1}) is the probability per unit time that an atom will decay. The number of atoms decaying per time is given by:

$$\frac{dN}{dt} = -\lambda N$$

Where N is the total number of radioactive nuclei present. This can be solved to get:

$$N(t) = N_0 e^{-\lambda t}$$

Where N_0 is the initial amount of nuclei presents when t = 0. [1, pp. 161]

• The half-life is the time for half the initial number of nuclei to decay:

$$t_{1/2} = \frac{0.693}{\lambda}$$

The mean lifetime τ is a bit different and is average amount of time before a radioactive nuclei will decay:

$$\tau = \frac{1}{\lambda}$$

I don't know when you'd actually use this. [1, pp. 161]

References

- [1] Kenneth S. Krane. Introductory Nuclear Physics. John Wiley & Sons, Inc., 3rd edition, 1988.
- [2] Lee Bernstein. Nuclear engineering class lectures. Fall 2015.