Nuclear Engineering 101: Final Study Guide

Disclaimer: This is not an official study guide. Stuff ~~might~~ **is** wrong. Use the lecture notes and book!

Note: Everything in this guide is from the text (Krane) or lecture, or office hours and should be cited as completely as possible.

Contents

1 Reactions

1.1 General Information

- The reaction:

$$a + X \rightarrow Y + b$$

Can be written in reaction notation as:

$$X(a, b)Y$$

[1, pp. 378-379]

- A microscopic cross section (σ) represents the "relative probability for the reaction to occur." It can be used in the following equation:

$$R = (\rho d)_{target} \times I_{beam} \times \sigma_{reaction}$$

Where R is the reaction rate (in reactions/sec/cm^2); (ρd) is the density (in g/cm^3) times the width of the target (in cm), also known as the areal density (this needs to be converted to atoms/cm^2); I_{beam}

is the incident particle flux (in atoms/sec/cm^2); and $\sigma_{reaction}$ is the microscopic cross section of the reaction occurring. This is only valid when very little of the beam reacts (small σ) and everything moves in straight lines. Note that is it common practice to drop the 1/cm^2 from the reaction rate and beam current (I) by assuming that the target is exactly 1 cm^2 in size. This can also be formulated as:

$$R = NdI_{beam}\sigma_{reaction}$$

In this case, N is explicitly the number density of the target material (in atoms/cm^3) and d is the width of the target (in cm). This is exactly equivalent to the last equation, but ρd has been converted into number density.

It can also be expressed in terms of flux as:

$$R = n\phi\sigma$$

Where n is the number density of target atoms (in atoms/cm^3), ϕ is the flux in (atoms/sec/cm^2) and σ is the same. This will give a reaction rate in units of reactions/cm^3. [2, Lec. 25]

> **Current vs. Flux:** In general, the incident beam current I is used when you have a beam of particles on a target, hence it is formulated with the number density of the target and the width of the target. This is also why the reaction rate is reactions/sec/area of the target. Flux is generally used when you have a bunch of particles *inside* a material causing reactions. Then you are more interested in the number density of the material, and your reaction rate is in reactions/sec/unit volume.
>
> In the end, it doesn't really matter if you get the units right. Dimensional analysis will save you.

- Microscopic cross sections are generally given in units of barns. 1 barn = 10^{-24} cm^2.
- The cross section is not always constant over angle (it rarely is). So the *differential cross section* is used:

  $$\frac{d\sigma}{d\Omega}$$

  What is confusing, is this is just a number, in units of barns/steradian. It's representing the fact that some small number of particles ($d\sigma$) will strike our small detector ($d\Omega$). It is dependent on the angle of scatter (θ) and the polraization of the radiation (ϕ). Generally we assume there is no affect due to polarization (things are randomly polarized).

  We can find the size of our detector $d\Omega$ in steradians, which is related to the area of our detector (dA) and the distance from the target (r) by:

  $$d\Omega = \frac{dA}{r^2}$$

  Then, if we know the differential cross section at the angle of our detector, we can multiply to get the reaction cross section for our detector:

  $$\sigma_{det} = d\Omega\frac{d\sigma}{d\Omega}$$

  This represents something **very specific**. This is the probability that incoming particles striking the target will then be detected by our detector. Based on the size of our detector ($d\Omega$) and our a priori knowledge of the number of particles that will be seen in a small area ($\frac{d\sigma}{d\Omega}$). The value of that differential cross section will probably vary with angle, so you have to know the differential cross section for the angle where your detector is to even use this. More rigorously, you'd integrate over the area of the detector and $\frac{d\sigma}{d\Omega}$ may vary over the integral:

  $$\sigma_{det} = \int_{detector}\frac{d\sigma}{d\Omega}d\Omega$$

  Or, you can get the total σ by integrating over the whole angle space.

- Conserved quantities in reactions:
  - Total energy
  - Linear momentum
  - Angular momentum
  - Parity $(-1)^l$ (except in weak interactions)
- **Kinematics** For a reaction, $X(a,b)Y$:

$$Q = (m_X + m_a - m_Y - m_b)c^2$$
$$Q = T_Y + T_b - T_X - T_a$$

- Exothermic Q>0 :

$$m_X + m_a > m_Y + m_b$$
$$T_Y + T_b > T_X + T_a$$

- Endothermic Q<0 :

$$m_X + m_a < m_Y + m_b$$
$$T_Y + T_b < T_X + T_a$$

- Reaction reaches excited states of Y:

$$Q_{ex} = (m_X + m_a - m_{Y*} - m_b)c^2 = Q_0 - E_{ex}$$

- Compound nucleus:

$$Q = -T_a = (m_X + m_a - m_{C*})c^2 - E_{ex}$$

1.2 Photo-nuclear Interactions

- A photon interacts with the nucleus directly. For this to happen, we need to have an energy level at the energy of the incoming photon. [2, Lec 25]
- There are three types of photo-nuclear interactions:
  - Spontaneous emission: if the nucleus is in an energy level, it can release a photon to de-excite. This is an intrinsic property of the level.
  - Resonant absorption: if the incoming photon is at the exact same energy of an energy level, it can be absorbed and the nucleus excited to that state. The energy of the state is the resonance energy.
  - Stimulated emission: One photon goes in, two photons come out. This occurs if the incoming photon is at the resonant value. This is the principle by which lasers work (on the atomic scale), but it has not been seen for nuclei.

  [2, Lec 25]
- The cross section for this to occur (σ_0) is a function of the nucleus' angular momentum, and internal conversion (IC) factors (α). As the probability of IC rises, photon capture becomes more rare, it's hard to make a nucleus capture photons when it wants to eject electrons. [2, Lec 25]
- The width of the emitted state:

$$\Gamma = \frac{\hbar}{\tau}$$

  The longer the mean lifetime (τ), the more well defined the energy level's value is. [2, Lec 25]

1.2.1 Resonance Absorption

- The resonance energy can be affected by any recoil that will result from the capture. This is because the nucleus needs both enough energy to be in its new excited state, **and** enough energy to recoil; so the incoming photon needs to have a little bit more than the expected resonance energy. As shown in Figure 1, the resonance energy has been shifted up by the recoil energy E_R from the expected value ΔE. [2, Lec 25]

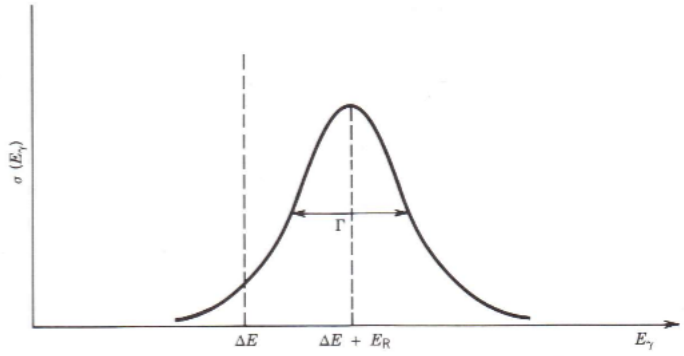

Figure 1: The resonance absorption peak is normally at ΔE, it is shifted *up* by the recoil energy E_R, Krane figure 10.23. [1].

- This has an exactly opposite effect on the emission spectrum. The absorption spectrum was shifted *up* because the incoming photon needed extra energy to recoil the nucleus. The emission spectrum is shifted *down* because the recoil takes some of the energy of the emitting photon. [2, Lec. 25]
- **Doppler broadening:** thermal motion makes the nuclei move back and forth, so the incoming photons energy looks doppler shifted either higher or lower. Therefore, a photon with energy just off the resonance may actually be absorbed because the relative motion can shift its energy to the resonance value. A wider energy range can now be absorbed by the resonance, so the peak gets wider or *broadens*. [1, pp. 363]
- Doppler broadening can can cause overlap between the emission and absorption peaks. [2, Lec. 25]
- **Mossbauer Effect:** If a nuclei is in a crystal lattice, its recoil will be inhibited by the fact that *its stuck in a lattice.* You're not just causing one nuclei to move, but all the ones around it, this makes the recoil energy very low, and therefore minimizes the shift in resonance energy by recoil. This allows you to nail down the *actual* resonance energy. [2, Lec 25]

1.2.2 Giant Dipole Resonance

High energy photons can "ionize" the entire nucleus by creating dipole motion between all the protons and all the neutrons. This means that all the protons moving together and all the neutrons are moving together, and these two groups are in a resonance with each other. This only occurs when the incoming photon energy is very high, >12 MeV. [2, Lec 25]

I don't really understand this so please explain it to me if you can.

1.3 Coulomb Scattering

1.3.1 Elastic (Rutherford)

- A particle approaches the nucleus at a distance b (the impact parameter) and scatters off the coulomb potential. The particle follows a hyperbolic path. [1, pp. 396]

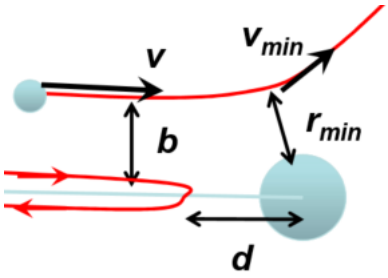

Figure 2: Geometry for elastic coulomb scatter. [2, Lec 24]

- Based on the geometry in Figure 2:
  Incoming particle (at long distances):

$$V = 0 \text{ (far away, the incoming particle has negligible potential energy)}$$
$$T = \frac{1}{2}mv^2$$
$$\ell = mvb$$

  Target particle:

$$V = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{d}$$
$$T = 0$$
$$\ell = 0$$

  Incoming particle (at the minimum distance r_{min}):

$$V = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0}\frac{1}{r_{min}}$$
$$T = \frac{1}{2}mv_{min}^2$$
$$\ell = mv_{min}r_{min}$$

  No energy or angular momentum is transferred to the target nucleus (hence elastic scattering). [2, Lec 24]

1.3.2 Inelastic (Coulex)

- **Coulomb Excitation:** inelastic Coulomb scattering. An incoming particle scatters off the potential of a target and leaves some energy behind. This "Coulex" reaction can excite nuclei up rotational bands. [2, Lec. 24]
- Reaction Q-value is to create the final products at rest.
  - Center of Mass Frame: products are at rest, $Q = Q$.
  - Lab Frame: products are *not* at rest. Threshold energy for reaction is:

$$E_{\text{threshold}} = Q\left(\frac{m_a + m_x}{m_x}\right)$$

    for a(X,Y)b.

1.4 Direct Reactions

- Occur on time frames of $10^{-21} - 10^{-22}$s. [2, Lec 25]
- These are collisions off the nucleons on the surface. They occur fast enough and with high enough energy that the incoming particle actually "sees" the individual protons and neutrons instead of the nucleus as a whole. This happens because the wavelength of the incoming particle is inversely proportional to momentum, and therefore energy:
$$\lambda = \frac{h}{p}$$
So higher energy particles can interact with individual nucleons. [2, Lec 25]
- Different incoming angles affect the cross sections for reactions. This is because the incoming particle is a wave, and the nucleus is a big wave made up of little waves, so you can have constructive and destructive interference as the two interact. They can add up to make a reaction or less likely based on angle. [2, Lec 25]

1.4.1 Kinematics

For an incoming particle a, outgoing particle b and product, as shown in Figure 3 The momentum is conserved:

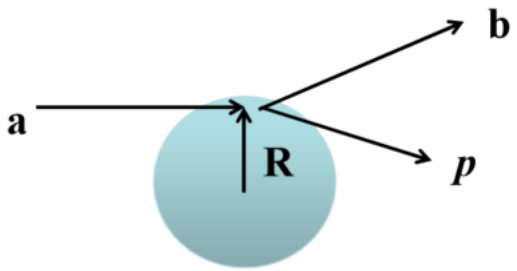

Figure 3: Direction reaction kinematics diagram.

$$\vec{p}_{product} = \vec{p}_a - \vec{p}_b$$

Based on the radius R that the incoming particle comes in, there is some amount of angular momentum ℓ transferred to the product (spinning up the nucleus):

$$\ell = Rp$$

Using conservation of energy, you can also relate the final product momentum (p) to the incoming and outgoing particles:

$$p^2 = p_a^2 + p_b^2 - 2p_a 2p_b \cos\theta$$

If you solve for ℓ you can figure out what J^π the product will be left in. In summary:

1. Figure out the momentum of the product using the momenta of the products and the angle of collision (that p^2 formula).
2. Solve for the angular momentum transfer ℓ using R ($\ell = Rp$).
3. Add $\ell \pm \frac{1}{2}$ to the original J^π of the target to get the final spin.
4. Parity change will go $\Delta\pi = (-1)^\ell$ where that ℓ is the angular momentum transfer from step 2.

[2, Lec 25]

1.5 Compound Reactions

- Reactions with a definite intermediate state:

$$a + X \rightarrow C^* \rightarrow Y + b$$

  Where C^* represents the compound nucleus. [1, pp. 416]

- Works best for particles with low incident energy (10-20 MeV), to reduce the chance the particle can escape with its energy and identity. [1, pp. 416]

- Occur on time frames of $10^{-16} - 10^{-18}$ seconds. The time-scale for decay is large compared to the time-scale for formation. This means that nuclei don't "remember" how they were formed; the formation process has no affect on what eventually happens to the nuclei (decay, etc). Only total energy and angular momentum information is retained. [2, Lec 26]

1.6 Resonance Reactions

- There is a range of incident particle energy called the resonance region. In this area, there are discrete levels in the compound nucleus that line up with the incident particle's energy. [1, pp. 424]

- The capture cross section for these energies is high and their life-times are small. They usually only decay by rejecting the incident particle (scattering) or γ-decay. [1, pp. 424]

- This happens because at these energies, the amplitude of the incoming particle wave function lines up really well with the amplitude of the wave-function of the nucleus. Just like in decays, processes are more likely when wave-functions line up. Quantum mechanics! [1, pp. 424] (The excitement in quantum mechanics was *not* drawn from Krane, although I do not doubt that he is totally into it, nerd)

2 Neutron Physics

- Free neutrons are unstable, they will β-decay into protons with a half-life of 10.6 minutes. [1, pp. 444]

- Neutron energies are listed in Table 1.

	Energy
Thermal	≈ 0.025 eV
Epithermal	≈ 1 keV
Fast	100 keV - 10 MeV

Table 1: Neutron Energies [1, pp.445]

2.1 Attenuation

- As neutrons move through a material, they are absorbed and scattered, which will reduce the overall intensity of the beam. We consider scattered neutrons to be gone because they generally scatter away from the beam, so we don't see them anymore. Also, we usually define intensity with a given energy, so scattering lowers their energy and they leave our intensity.

- The loss of intensity (I) of neutrons of a *given energy* in a distance dx of material:

$$dI = -I\sigma_t n dx$$

  Where σ_t is the total cross section of the material (absorption plus scattering) and n is the number of atoms per unit volume of the material. To make this useful, solve:

$$I = I_0 e^{-\sigma_t n x}$$

Where x is the distance the neutrons are traveling through the material. Remember, this isn't a decrease in the total number of neutrons, just the ones in our beam with the amount of energy we shot them into the material with. The scattering will make lower-energy neutrons, so they aren't actually gone for reals. [1, pp. 448]

- You can also use this if you are given the flux ϕ in atoms/sec/cm^2:

$$\phi = \phi_0 e^{-\sigma_t n x}$$

2.2 Collisions

- The main process by which neutrons slow down (lose energy, also called the process of moderation) is collisions.
- In an elastic collision with a nucleus of atomic mass A, a neutron with incoming energy E will have final energy:

$$\frac{E'}{E} = \frac{A^2 + 1 + 2A\cos\theta}{(A+1)^2}$$

[1, pp.448]

- The maximum energy loss is from a head-on collision ($\theta = 180°$), where the above equation reduces to:

$$\left(\frac{E'}{E}\right)_{min} = \left(\frac{A-1}{A+1}\right)^2$$

This is the "min" value because it's the minimum final energy (E') that the neutron can have after the collision. As A gets larger, the energy transferred goes down, the neutron just kind of grazes off. The equation is maximized for $A = 1$ (hydrogen) which is why water is so good at moderating. [1, pp. 448]

- We usually use the average energy lost after each collision (actually, the log of that amount), represented by squiggle (ξ) (This is actually the greek letter *xi* but no one knows how to pronounce that. If you do, shut up it's squiggle now).

$$\xi = \left[\log \frac{E}{E'}\right]_{av}$$

This can be related to the final energy after the neutron after n collisions:

$$\log E'_n = \log E - n\xi$$

This is good for problems where you want to figure out how many collisions are required to thermalize a neutron. There is a complex equation for the value of ξ assuming isotropic scattering:

$$\xi = 1 + \frac{(A-1)^2}{2A} \log \frac{A-1}{A+1}$$

For hydrogen ($A = 1$), $\xi = 1$. [1, pp.449-450]

2.3 Capture

- Most of the time, neutron capture on most massive nuclei form compound nuclei. [2, Lec 26]
- Neutron capture immediately makes a nucleus with energy:

$$E = S_n + E_n$$

Where S_n is the neutron separation energy and E_n is the energy of the incoming neutron. [2, Lec 27]

- Neutron capture rates are higher with higher Q values. The Q value in this case is just equal to the neutron separation energy, S_n, so the higher S_n, the higher the chance of capturing a neutron. This is why capture rates near the "valley of stability" are high, because S_n is large. [2, Lec 27]
- Why does this happen? Nuclei like to go to states with lots of available transitions. The larger the Q-value, the more options the nuclei has for the decay that follows. Therefore, the higher the Q value (S_n), the higher the chance of neutron capture. [2, Lec 27]
- You can figure out the energy and spin parity of the nuclei immediately following neutron capture. The final spin of the nuclei is:
$$I' = I + \ell + s$$
and the parity change:
$$\Delta\pi = (-1)^\ell$$
If the neutron is thermal, the captured neutron will probably have no angular momentum ("s-wave capture"). So $I' = I \pm \frac{1}{2}$; unless $I = 0$, in which it is always $I' = \frac{1}{2}$. Parity doesn't change because $\ell = 0$. [1, pp.463]
- After capture, the excited capture state will γ-decay down into all the accessible excited states of the compound nucleus. Many of them are accessible because S_n is usually pretty high. Accessible means any states that the excited state can go to via $E1$ radiation, described below. [1, pp. 463]
- After capturing a neutron, the primary transition that follows is dominated by Electric Dipole, $E1$ ($\Delta I = 0$ or 1 and $\Delta\pi = -1$). The transition will populate all of these possible states. Magnetic dipole radiation and higher multipole radiation are usually present, but are usually far less intense than $E1$. [2, Lec 27]

  Example: If neutron capture results in a compound nucleus in an energy state (at S_n) with $J^\pi = 2^+$, the following primary transition will populate states: $1^-, 2^-, 3^-$.
- In summary:
  - A neutron with energy E_n is captured and makes a compound nucleus with energy $E = E_n + S_n$.
  - The compound nucleus has a spin parity related to it's old spin-party (I) and the spin and angular momentum of the neutron.
  - This excited compound nucleus will γ-decay via $E1$ (electric dipole) radiation down to many other states. (Primary γ-rays)
  - Unless it went down to the ground state, more decays will occur. (Secondary γ-rays)

3 Fission

- Fission occurs because a heavy nucleus splitting into two smaller fragments results in a net increase in binding energy, due to the drop in the binding energy per nucleon curve after Iron. Going to this more tightly bound system means energy must be released. [1, pp. 479]
- Energy released by fission is primarily (approx 80%) in the kinetic energy of the fragments that fly apart due to Coulomb repulsion. [1, pp.479]
- Fission is inhibited by the Coulomb barrier, the height of which is roughly equal to the energy released in fission. If fission puts the two products energy just below the height of the Coulomb barrier, there is a decent probability that it will occur (due to tunneling). This is *spontaneous fission*. [1, pp. 481]
- *Induced fission* occurs after the absorption of some energy, such as a low-energy neutron or photon. If the intermediate state is now above the Coulomb barrier, fission may occur (it competes with other types of decays). If it is still below the barrier, other decays modes will be more likely. Note that fission is never guaranteed, it just becomes a competitive form of decay in certain circumstances. [1, pp. 481]

- The mass distribution of particles emitted by fission of ^{235}U forms two peaks at A =95 and A =140, these nuclei will be very rich in neutrons. Some of these extra neutrons are emitted almost immediately, and are called prompt neutrons. [1, pp. 485]
- Fission fragments may decay via β-decay and then emit more neutrons. These are called delayed neutrons. This can take a few seconds. [1, pp. 485]
- The excitation energy of the compound nucleus formed by absorbing a thermal neutron can be found (using ^{235}U as an example):

$$m(^{236}\mathrm{U}^*) = m(^{235}\mathrm{U}) + m_n$$
$$E_{ex} = [m(^{236}\mathrm{U}^*) - m(^{236}\mathrm{U})]$$

  This assumes that the incoming neutron has 0 energy, which is a good assumption for thermal neutrons. If the excitation energy is higher than the activation energy (required for fission), then the excited nucleus can fission. The original nucleus (^{235}U in this case) is considered *fissile.* If the excitation energy is less than the activation energy, than a thermal neutron will not cause fission. In that case a neutron with **some** energy is required to bump up the excitation energy above activation. [1, pp. 488-489]

3.1 Shell Model Effects

- A nucleus that is stretched into an ellipse has a different total binding energy, because the Coulomb force is now different. If the energy goes up when it gets stretched, it will keep going and the nucleus will readily fission. This is described with a distortion parameter (ϵ). [1, pp. 494]
- Shell model comes into play here. Normally, a single energy level has one angular momentum and one energy, and a bunch of nucleons in it that all have those properties. In reality, the nucleons can have a range of angular momenta with the same energy (degeneracy) but they all look the same to us. When we deform a nucleus, this goes out the window and those angular momenta actually matter. Now, the energy of each nucleon depends on its angular momentum and the energy level splits out into a bunch of other levels. This is shown in Figure 4, note that some go up and some go down, and overall the effect is more pronounced as it gets more deformed (this is the Nilsson model, more info is in the study guide for midterm 2). [2, pp. 151-153]

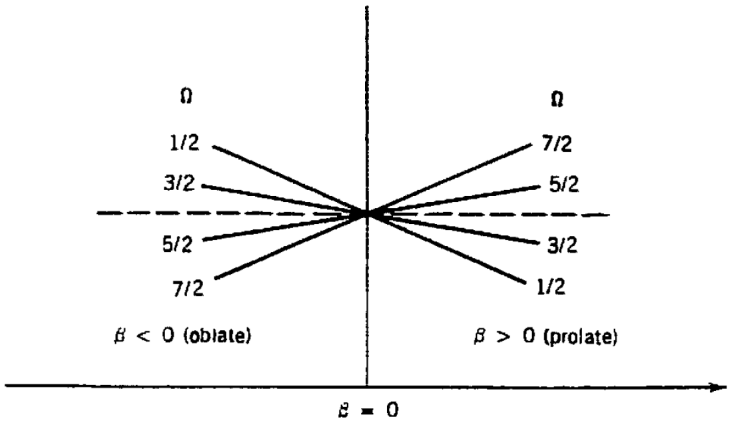

Figure 4: Level splitting with deformation. [1, pg. 153]

- What does this have to do with fission?
  1. As the nucleus becomes more deformed ($\epsilon \uparrow$), if the nucleus is not stable under fission, its binding energy is going up like ϵ^2 as it runs towards fission. The system total energy E_T is going up with distortion.

2. Some valence nucleon energies go up and some go down (as discussed above). If the valence nucleons happen to be in a state above the dotted line (where their energy goes *up* with deformation), they will add to E_T as the nucleus gets more deformed and it goes up faster. Woohoo fission here we come.
3. This doesn't last forever. Eventually, as $E_T \uparrow$ and it keeps getting more deformed, nucleons will start dropping into energy states where their energy goes *down* with deformation. This happens because the *real* diagram is super complicated and messy, (it's Figure 5.29 in Krane if you want to look at it).
4. Once enough of the valence electrons are in a state where their energy goes *down* with deformation, they start driving the total energy E_T down.
5. Then you have to keep deforming until valence nucleons go to another state where their energy goes up with deformation.
6. This makes a little valley of stability at the top of the energy vs. deformation diagram. This is shown in Figure 5. [1, pp. 493-495]

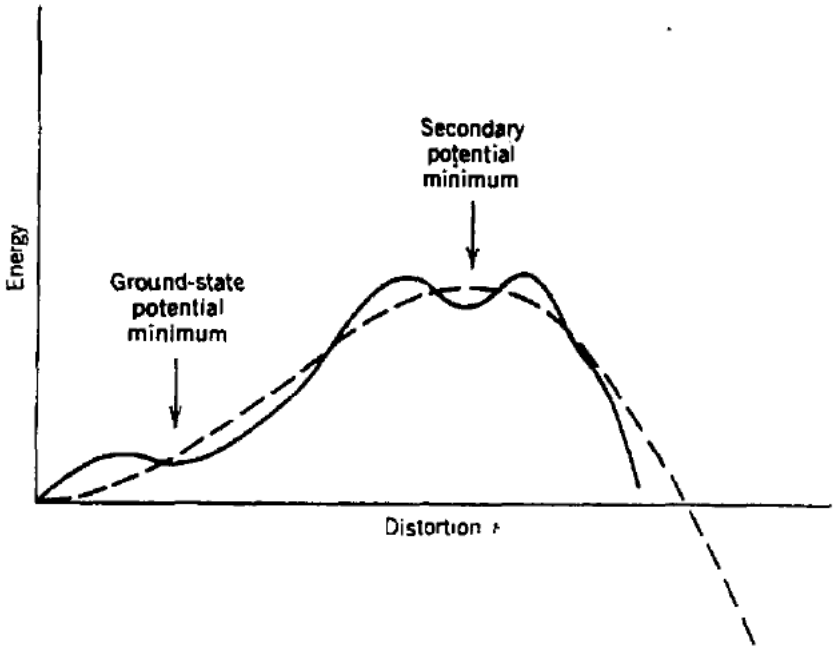

Figure 5: Energy with distortion. The solid line shows the change due to shell structure. [1, pp. 496]

- This actually helps us cause fission, now we don't have to excite the nucleus all the way to the top of the barrier, we can just get to the bottom of that secondary well to make fission likely. [1, pp. 495]
- The nucleus can hang out in this well. Just like any other potential well there are bound states with energy levels, this is called a *fission isomer*. They generally have rotation bands (because they are deformed) and can either decay down to the ground state, or (usually) spontaneously fission. These are shape isomers, because the shape change required to go down to the ground state makes that transition much less likely. [1, pp. 495-496]

3.2 Reactors

- The neutron reproduction factor k_∞ is the net change in thermal neutrons from one generation to the next. The ∞ is because we assume we're in an infinite reactor where no neutrons escape (leak). If

$k_\infty = 1$, we have the same number of neutrons each generation, the reactor is *critical*. If $k_\infty > 1$, the reactor is *supercritical* and if $k_\infty < 1$ the reactor is *subcritical*. [1, pp. 501]

- Fission events of ^{235}U create approximately 2.5 fast neutrons per fission. $\nu \approx 2.5$. [1, pp. 501]
- We know how many fast neutrons are made per fission, but not every thermal neutron absorbed causes fission. Sometimes it is just absorbed. To correct for this we use η:

$$\eta = \nu \frac{\sigma_f}{\sigma_f + \sigma_a}$$

Where σ_f is the cross section for fission, and σ_a is the cross section for absorption that doesn't cause fission. [1, pp.502]

> Note that the notation used by Krane is confusing. Fission is *an absorption event*. Usually, **but not here**, the total absorption cross section σ_a is defined as the sum of fission and other events (such as γ-decay): $\sigma_a = \sigma_f + \sigma_\gamma$. I'm going to stick with Krane's notation here. So remember, σ_a is **absorption that doesn't cause fission** and does *not* include σ_f.

- The difference between η and ν can be confusing:
  - ν: The number of fast neutrons per *fission*.
  - η: The number of fast neutrons per *thermal neutron absorbed* by fuel. This will always be lower than ν because some amount of neutrons will be absorbed and not cause fission (σ_a above).
- If we have N thermal neutrons in the last generation that all get absorbed in our fuel, we will now have $N\eta$ new fast neutrons.
- Rarely, these fast neutrons will actually cause fission (in ^{238}U), which leads to a sliiiight increase in our total fast neutrons. We call this the *fast fission factor* (ϵ). It's usually small, like 1.03. [1, pp.502]
- So now, we have $N\eta\epsilon$ fast neutrons.
- The fission cross section is low for fast neutrons, so we want to reduce their energy (slow or moderate) them down to thermal energies where the fission cross section is high. Unfortunately, as they slow down, they have to go through the resonance region where there is a high probability of absorption that won't cause fission (by ^{238}U for example). The probability that a given fast neutron manages to reach thermal without being absorbed is called the *resonance escape probability* (p). Krane gives a value of 0.9. [1, pp. 503]
- Now, of our $N\eta\epsilon$ fast neutrons, $N\eta\epsilon p$ have all slowed down to thermal energy.
- Just because a neutron is thermal doesn't mean it will be absorbed in our fuel. Reactors are made up of a ton of steel and moderator (water or graphite), and other stuff. The neutrons might get absorbed in that instead of the fuel. So we use the *thermal utilization factor* (f):

$$f = \frac{\sigma_a^{\text{fuel}} + \sigma_f^{\text{fuel}}}{\sigma_a^{\text{all}} + \sigma_f^{\text{all}}}$$

Where all means all the materials, including fuel. This is the fraction of neutrons absorbed in the fuel instead of absorbed somewhere else. [1, pp. 503]

- Now, of our thermal neutrons, $N\eta\epsilon p$, we have $N\eta\epsilon pf$ that are actually absorbed in our fuel (remember we are talking about an infinite reactor, so they can't escape).

- Finally, we can get k_∞. Last generation we had N neutrons absorbed in our fuel, and now we have $N\eta\epsilon pf$ neutrons in this generation absorbed in our fuel. k_∞ is just the ratio of neutrons this generation to last generation:
$$k_\infty = \frac{N\eta\epsilon pf}{N} = \eta\epsilon pf$$
This is the *four factor formula*. [1, pp. 503]

- Effects that change one of the factors may affect the criticality of the reactor (by changing k_∞).
  - In a water-cooled reactor, if the temperature of the water goes up:
    * Water is the moderator, it is used to slow down neutrons because hydrogen is great at removing neutron energy.
    * As temperature goes up, water gets *less* dense.
    * Less dense water means collisions happen less frequently, so neutrons slow down slower and travel further.
    * If it's harder to slow down to thermal energies, they spend more time in the resonance region, so there is a higher probability they will be resonantly absorbed.
    * The resonance *escape* probability goes *down*, and overall k_∞ goes down.

4 Fusion & Plasmas

4.1 Kinematics

- The following fusion reaction:
$$a + X \to Y + b$$
will have product particles with energies that sum to the Q value of the reaction:
$$\frac{1}{2}m_b v_b^2 + \frac{1}{2}m_Y v_Y^2 = Q$$
Which gives:
$$\frac{1}{2}m_b v_b^2 = \frac{Q}{1 + m_b/m_Y}$$
$$\frac{1}{2}m_Y v_Y^2 = \frac{Q}{1 + m_Y/m_b}$$
[1, pp. 531]

4.2 Energy

- For the reaction to occur, the two reactants must have enough energy to overcome the Coulomb barrier given by:
$$V_c = \frac{e^2}{4\pi\epsilon_0}\frac{Z_a Z_X}{R_a + R_X}$$
You can generally assume each particle has half of this energy initially.

- The rate of a nuclear reaction depends on the product of the cross section for the reaction to occur σ and the speed of the particles v. Normally, for thermal neutron reactions outside the resonance region $\sigma \propto 1/v$, so σv is constant, this isn't true in fusion reactions. [1, pp. 532]

- In thermonuclear fusion, at the high temperatures, the cross section σ is not proportional to $1/v$, as seen in Figure 6. So σv will not be constant. The value of σv is averaged over all speeds and energies to get: $\langle \sigma v \rangle$. [1, pp. 533]

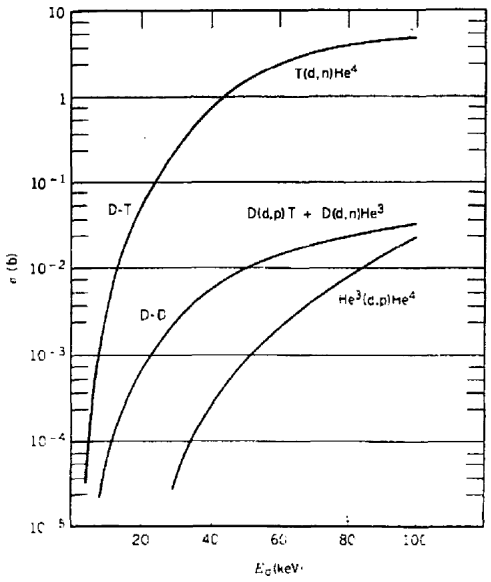

Figure 6: Cross section for fusion reactions. E_d is the energy of the deuteron in the fusion, which will be proportional to the speed of the particle. [1, pp. 532]

- The energy released in fusion is given by:

$$E_f = \frac{1}{4} n^2 \langle \sigma \nu \rangle Q \tau$$

  This assumes that we have n density of particles causing fusion (like D and T in a fuel pellet) and that there are equal amounts of both (the individual densities are $\frac{1}{2}n$). Q is the energy released in the reaction, τ is the length of time the plasma is confined and $\langle \sigma v \rangle$ is the average cross section times particle velocity. [1, pp. 541]

- The energy per unit volume required to heat the plasma is:

$$E_{\text{th}} = 3nkT$$

  Where n is the density of the particles, k is the boltzman constant and T is the plasma temperature. [1, pp. 541]

- Overall, if we heat the plasma using E_{th} to a temperature T and confine it at that temperature for time τ, we get E_f out. We have an overall net gain if $E_f > E_{\text{th}}$:

$$\frac{1}{4} n^2 \langle \sigma v \rangle Q \tau > 3nkT$$

$$n\tau > \frac{12kT}{\langle \sigma v \rangle Q}$$

  [1, pp. 541-542]

4.2.1 Losses

- The primary way that a plasma loses energy is *bremsstrahlung*. When two particles Coulomb scatter in a plasma, they accelerate and release radiation. This is bad for our plasma because we want to keep as much of the energy in the particles as possible to keep them at a high enough energy to continue fusion. [1, pp. 539]

- These bremsstrahlung losses are proportional to Z^2, so it is beneficial to use lighter nuclei:

$$P_{\text{br}} = 0.5 \times 10^{-36} Z^2 n_{\text{ion}} n_e (kT)^{1/2}$$

  where P is in W/m^3, n_{ions} and n_e are the number densities of the ions and electrons in the plasma, respectively, and Z is the charge of the ions. It's probably most important just to know that it's less for lighter nuclei (due to Z^2), and that there is a temperature at which the gains from fusion exceed the bremsstrahlung losses: 4 keV for D-T and 40 keV for D-D. [1, pp. 540]

- There are other losses, such as synchrotron radiation from the particles orbiting in the magnetic field, but these are negligible compared to the bremsstrahlung. [1, pp. 539]

References

[1] Kenneth S. Krane. Introductory Nuclear Physics. John Wiley & Sons, Inc., 3rd edition, 1988.

[2] Lee Bernstein. Nuclear engineering class lectures. Fall 2015.