

Jessie George
HW1

Problem 1

1. Successor Function

Rule 1: $\$| \rightarrow |a$

Rule 2: $a| \rightarrow |a$

Rule 3: $a\# \rightarrow |$

Example:

$\$||| \#$

$|a|| \#$ By Rule 1

$||a| \#$ By Rule 2

$|||a\#$ By Rule 2

$||||$ By Rule 3

2. Double Function

Rule 1: $\$| \rightarrow |a$

Rule 2: $a| \rightarrow ||a$

Rule 3: $a\# \rightarrow |$

Example:

$\$||| \#$

$|a|| \#$ By Rule 1

$|||a| \#$ By Rule 2

$|||||a\#$ By Rule 2

$|||||$ By Rule 3

3. Addition Function

Rule 1: $\$| \rightarrow |a$

Rule 2: $a| \rightarrow |a$

Rule 3: $a\&| \rightarrow |a$

Rule 4: $|a\# \rightarrow |$

Example

$\$||\&||| \#$

$|a|\&||| \#$ By Rule 1

$||a\&||| \#$ By Rule 2

$|||a|| \#$ By Rule 3

$||||a| \#$ By Rule 2

$|||||a\#$ By Rule 2

$|||||$ By Rule 4

Problem 2

1. Rewrite system

Rule 1: $0+0 \rightarrow 0$

Rule 2: $0+1 \rightarrow 1$

Rule 3: $1+0 \rightarrow 1$

Rule 4: $0+2 \rightarrow 2$

Rule 5: $2+0 \rightarrow 2$

Rule 6: $1+1 \rightarrow 2$

Rule 7: $1+2 \rightarrow 0$

Rule 8: $2+1 \rightarrow 0$

Rule 9: $2+2 \rightarrow 1$

Rule 10: $(0) \rightarrow 0$

Rule 11: $(1) \rightarrow 1$

Rule 12: $(2) \rightarrow 2$

2. Examples

Input: $((1+2)+0)$

$((1+2)+0) = ((0)+0)$ By rule 7

$((0)+0) = (0+0)$ By rule 10

$(0+0) = (0)$ By rule 1

$(0) = 0$ By rule 10

Output: 0

Input: $(1+(2+2))$

$(1+(2+2)) = (1+(1))$ By rule 9

$(1+(1)) = (1+1)$ By rule 11

$(1+1) = (2)$ By rule 6

$(2) = 2$ By rule 12

Output: 2

3. Yes it's unique.

Problem 3

1.

$$r^+ = rr^*$$

2.

$$r^* = \varepsilon | r | rr | rrr | \dots$$

$$r^+ = r | rr | rrr | \dots$$

Problem 4

Digit = $0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

Sign = $+ | -$

Regular expression

$(\epsilon | \text{Sign})(\text{Digit})^*(\text{Digit})^*(\epsilon | ((\text{E})(\epsilon | \text{Sign})(\text{Digit})^*(\epsilon | (.(\text{Digit})^+))))$

Problem 5

1. All strings of "a"s and "b"s
2. Binary numbers starting with 1 and ending with 001 or 011

Problem 6

1. $(a^*(bc)^*d^*)^*$
2. $(a^*(\epsilon | b)(\epsilon | c | cc | ccc | cccc))^*$