

Package ‘FastLORS’

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Type Package

Title Joint Modeling for eQTL Mapping in R

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Author Jacob Rhyne, Jessie Jeng, and Eric Chi

Maintainer Jacob Rhyne <jdrhyne2@ncsu.edu>

Description This package applies FastLORS to perform eQTL mapping for gene expression and SNP data. It can also be used to apply the LORS method of Yang et al. (2013). The package also contains two pre-screening methods to reduce the number of SNPs before joint modeling: (1) HC-Screening: a method that selects the top SNPs based on their higher criticism statistics (Rhyne et al. 2018) and (2) LORS-Screening: which fits a marginal estimate and selects the top SNPs per each gene.

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VignetteBuilder knitr

R topics documented:

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Fast_LORS	<i>Fast_LORS</i>
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Description

Fast_LORS is a function for solving the LORS optimization problem in Can Yang et al. (2013) through the proximal gradient method

Usage

```
Fast_LORS(Y, X, rho, lambda, maxiter = 5000, eps = 2.2204e-16,
          tol = 1e-04, verbose = FALSE)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs
rho	parameter for enforcing sparsity of coefficient matrix
lambda	parameter for enforcing low-rank structure of hidden factor matrix
maxiter	maximum number of iterations
eps	constant used when checking the convergence. Ensures no division by 0.
tol	tolerance level for convergence
verbose	chooses whether details should be printed to console. Default is FALSE.

Value

B	The estimated coefficients
mu	The estimated intercept
L	The estimated matrix of hidden factors
f_val_vec	The objective function values
res_vec	The relative change in objective function values
iter	The number of iterations

Examples

```
##Example

## Generate some data
n <- 50
p <- 200
q <- 100
k <- 10
set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)
L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
  B[activeSNPs[i], genes_influenced] <- 2
}
E <- matrix(rnorm(n*q),n,q)
Y <- X %*% B + L + E

rho <- runif(1,3,5)
lambda <- runif(1,3,5)

## Usage
Fast_LORS(Y, X, rho, lambda)
```

Fast_LORS_Tuning

Fast_LORS_Tuning

Description

Fast_LORS_Tuning is a function used perform parameter tuning using FastLORS instead of LORS

Usage

```
Fast_LORS_Tuning(Y, X, rho, lambda, Training, Validation, maxiter = 5000,
  eps = 2.2204e-16, tol = 1e-04, B = NULL, mu = NULL, L = NULL)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs
rho	parameter used to enforce sparsity of B
lambda	parameter used to enforce low-rank structure of L
Training	Boolean matrix for training data
Validation	Boolean matrix for validation data
maxiter	maximum number of iterates
eps	a small constant to prevent dividing by zero when checking relative change in function values.

tol	tolerance threshold for convergence
B	an estimate for matrix of coefficients. Default is NULL.
mu	an estimate for the intercept. Default is NULL.
L	an estimate for the hidden factors Default is NULL.

Value

B	matrix of coefficients
L	matrix of hidden factors
mu	vector of intercepts
Err	Residual sum of squares of validation data
f_vals	Objective function values
res_vec	Relative change in objective function values
iter	Number of iterations

GetMaxRho

*GetMaxRho***Description**

GetMaxRho is a function used to determine the maximum value as a candidate for rho. See the parameter tuning section of Yang et al. (2013) Note: This function is adapted from the LORS MATLAB implementation

Usage

```
GetMaxRho(X, Y, L, Omega0)
```

Arguments

X	matrix of SNPs
Y	gene expression matrix
L	matrix of hidden factors
Omega0	Boolean matrix of observed entries

Value

MaxRho The maximum value of rho to be used in parameter tuning

Examples

```
##Example

## Generate some data
n <- 50
p <- 200
q <- 100
k <- 10
```

```

set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)
L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
  B[activeSNPs[i], genes_influenced] <- 2
}
E <- matrix(rnorm(n*q),n,q)
Y <- X %*% B + L + E
Omega0 <- !(is.na(Y))

## Usage
GetMaxRho(X, Y, L, Omega0)

```

HC_Screening

HC_Screening HC_Screening is a function to apply the HC-Screening screening method of Rhyne et al. (2018) (In progress) Note: HC-Screening ranks SNPs by their higher criticism statistics and selects the top n, where n is the number of samples

Description

HC_Screening HC_Screening is a function to apply the HC-Screening screening method of Rhyne et al. (2018) (In progress) Note: HC-Screening ranks SNPs by their higher criticism statistics and selects the top n, where n is the number of samples

Usage

```
HC_Screening(Y, X)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs

Value

selectedSNPs The SNPs selected by HC-Screening

Examples

```

## Example

## Generate some data
n <- 50
p <- 200
q <- 100
k <- 10
set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)

```

```

L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
  B[activeSNPs[i], genes_influenced] <- 2
}
E <- matrix(rnorm(n*q),n,q)
Y <- X %*% B + L + E

## Usage
HC_Screening(Y, X)

```

InitialEst

InitialEst InitialEst is a function to build an initial estimate for B**Description**

InitialEst InitialEst is a function to build an initial estimate for B

Usage

```
InitialEst(Y, X, lambda = NULL)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs
lambda	tuning parameter

Value

B An initial estimate of the coefficient matrix

Examples

```

## Example
## Generate some data
n <- 50
p <- 200
q <- 100
k <- 10
set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)
L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
  B[activeSNPs[i], genes_influenced] <- 2
}

```

```
}  
E <- matrix(rnorm(n*q),n,q)  
Y <- X %*% B + L + E  
  
Init_est <- InitialEst(Y,X)
```

linspace	<i>linspace</i>
----------	-----------------

Description

linspace is a function to space a sequence linearly from x1 to x2

Usage

```
linspace(x1, x2, n = 100)
```

Arguments

x1	a starting point
x2	an ending point
n	length of sequence

Value

a linear spaced sequence from x1 to x2 of length n

Examples

```
linspace(100,10,5)
```

logspace	<i>logspace</i>
----------	-----------------

Description

logspace is a function to space a sequence evenly on the log scale from x1 to x2

Usage

```
logspace(x1, x2, n = 50)
```

Arguments

x1	a starting point
x2	an ending point
n	length of sequence

Value

a sequence from x1 to x2 of length n spaced evenly on the log scale

Examples

```
##Example
logspace(100,10,5)
```

LORS0	<i>LORS0</i>
-------	--------------

Description

LORS0 is a function for solving the LORS optimization problem through the method described in Can Yang et al. (2013). This function is adapted from the authors MATLAB implementation

Usage

```
LORS0(Y, X, rho, lambda, maxiter = 1000, eps = 2.2204e-16, tol = 1e-04,
      verbose = FALSE)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs
rho	parameter for enforcing sparsity of coefficient matrix
lambda	parameter for enforcing low-rank structure of hidden factor matrix
maxiter	maximum number of iterations
eps	constant used when checking the convergence. Ensures no division by 0.
tol	tolerance level for convergence
verbose	chooses whether details should be printed to console. Default is FALSE.

Value

B	The estimated coefficients
mu	The estimated intercept
L	The estimated matrix of hidden factors
f_val_vec	The objective function values
res_vec	The relative change in objective function values

Examples

```
##Example

#' ## Generate some data
n <- 50
p <- 200
q <- 100
k <- 10
set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)
L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
  B[activeSNPs[i], genes_influenced] <- 2
}
E <- matrix(rnorm(n*q),n,q)
Y <- X %*% B + L + E

rho <- runif(1,3,5)
lambda <- runif(1,3,5)
LORS0(Y, X, rho, lambda)
```

LORS2

LORS2

Description

LORS2 is a function used in parameter tuning in LORS. See the parameter tuning section described in Can Yang et al. (2013). This function is adapted from the authors MATLAB implementation

Usage

```
LORS2(Y, X, L, Omega1, Omega2, B, rho, lambda, tol, maxIter = 1000)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs
L	matrix of hidden factors
Omega1	Boolean matrix for training data
Omega2	Boolean matrix for validation data
B	a matrix of coefficients for the SNPs
rho	parameter for enforcing sparsity of coefficient matrix
lambda	parameter for enforcing low-rank structure of hidden factor matrix
tol	tolerance level for convergence
maxIter	the maximum number of iterations

Value

B	The estimated coefficients
mu	The estimated intercept
L	The estimated matrix of hidden factors
Err	The residual sum of squares on the validation set

Examples

```
##Example

#' ## Generate some data
n <- 50
p <- 200
q <- 100
k <- 10
set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)
L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
  B[activeSNPs[i], genes_influenced] <- 2
}
E <- matrix(rnorm(n*q),n,q)
Y <- X %*% B + L + E

Omega0 <- !(is.na(Y))
mask <- matrix(runif(nrow(Y)*ncol(Y)) > 0.5, nrow = nrow(Y), ncol = ncol(Y))
Omega1 <- Omega0 & mask
Omega2 <- Omega0 & !mask
rho <- runif(1,3,5)
lambda <- runif(1,3,5)
tol <- 1e-4

## Usage
LORS2(Y, X, L, Omega1, Omega2, B, rho, lambda, tol)
```

LORSscreen

LORSscreen LORSscreen is a function to solve the LORS-Screening optimization problem in Yang et al. (2013)

Description

LORSscreen LORSscreen is a function to solve the LORS-Screening optimization problem in Yang et al. (2013)

Usage

```
LORSscreen(Y, X, lambda, tol)
```

Arguments

Y	gene expression matrix
X	a SNP
lambda	tuning parameter
tol	a tolerance level

Value

B the estimated coefficients for the SNP
 L the estimated hidden factors
 mu the estimate for the intercept

Examples

```
##Example

## Generate some data
n <- 50
p <- 200
q <- 100
k <- 10
set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)
L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
  B[activeSNPs[i], genes_influenced] <- 2
}
E <- matrix(rnorm(n*q),n,q)
Y <- X %*% B + L + E

## Usage to build initial estimate

Bhat_initial <- c()
for(SNP_col in 1:ncol(X)){
  X1 <- matrix(X[,SNP_col], ncol = 1)
  LS <- LORSscreen(Y, X1, lambda = 0.1, 0.01)
  B_row <- LS$B
  Bhat_initial <- rbind(Bhat_initial, B_row)
}
```

LORS_Screen_Parallel *LORS_Screen_Parallel* LORS_Screen_Parallel is a function used to run LORS-Screening on a subset of the columns of X. Can be used to perform LORS-Screening in parallel on a cluster.

Description

LORS_Screen_Parallel LORS_Screen_Parallel is a function used to run LORS-Screening on a subset of the columns of X. Can be used to perform LORS-Screening in parallel on a cluster.

Usage

```
LORS_Screen_Parallel(Y, X, chunk)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs
chunk	a group of columns to run the screening on. Done in batches of 1000.

Value

myB	matrix of coefficients from LORS-Screening
lambda	tuning parameter used in LORS-Screening

Examples

```
##Example

## Generate some data
n <- 20
p <- 50
q <- 30
k <- 4
set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)
L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
  B[activeSNPs[i], genes_influenced] <- 2
}
E <- matrix(rnorm(n*q),n,q)
Y <- X %*% B + L + E

## Usage
LORS_Screen_Parallel(Y, X, chunk = 1)
```

ParamTuneParallel	<i>ParamTuneParallel</i> ParamTuneParallel is a function used to run the parameter tuning of either FastLORS or LORS in parallel
-------------------	--

Description

ParamTuneParallel ParamTuneParallel is a function used to run the parameter tuning of either FastLORS or LORS in parallel

Usage

```
ParamTuneParallel(Y, X, fold, seed = 123)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs
fold	fold in cross-validation
seed	random seed used to create training and validation sets

Value

myParams	lambda and a sequence of rho values to use in parameter tuning
Training	Training Data
Validation	Validation Data

Examples

```
##Example

## Generate some data
n <- 20
p <- 50
q <- 30
k <- 4
set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)
L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
  B[activeSNPs[i], genes_influenced] <- 2
}
E <- matrix(rnorm(n*q),n,q)
Y <- X %*% B + L + E

## Usage
ParamTuneParallel(Y, X, fold = 1)
```

prox_1

prox_1

Description

prox_1 is the soft-thresholding function. Let the entries of \mathbf{b} be b_j . The function subtracts τ from b_j for $b_j > \tau$, sets b_j to 0 where $\text{abs}(b_j) < \tau$, and adds τ where $b_j < -\tau$.

Usage

```
prox_1(b, tau)
```

Arguments

- b a matrix or vector
- tau the value to to apply soft thresholding with

Value

prox_b the soft-thresholding function applied to b with threshold tau

rankHC	<i>rankHC</i> rankHC is a function used to rank a Bhat matrix by higher criticism statistics
--------	--

Description

rankHC rankHC is a function used to rank a Bhat matrix by higher criticism statistics

Usage

rankHC(Bhat_standardized)

Arguments

- Bhat_standardized
 a standardized coefficient matrix

Value

- index The indices of the sorted Higher Criticism values
- HC_vec The higher criticism statistics for the standardized Bhat matrix

Run_LORS	<i>Run_LORS</i>
----------	-----------------

Description

Run_LORS is a function used to run either FastLORS or LORS

Usage

```
Run_LORS(Y, X, method = "FastLORS", screening = "LORS-Screening",
  tune_method = "FastLORS", seed = 123, maxiter = 10000,
  eps = 2.2204e-16, tol = 1e-04, cross_valid = TRUE)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs
method	chooses with modeling method to run
screening	Either "LORS-Screening", "HC-Screening", or "None". The default method, LORS-Screening, is recommended if the number of SNPs is large. HC-Screening of Rhyne et al. (2018) is under development but is included here as an option.
tune_method	chooses whether FastLORS should be used for parameter tuning or the original LORS procedure should be used. Default is FastLORS
seed	random seed to be used for setting training and validation set. Default is 123.
maxiter	maximum number of iterations
eps	constant used when checking the convergence. Ensures no division by 0.
tol	tolerance level for convergence
cross_valid	chooses whether cross-validation should be used in parameter tuning. Default is TRUE.

Value

LORS_Obj or Fast_LORS_Obj	A list produced from LORS or FastLORS containing (1) B: estimate of the coefficient matrix (2) L: estimate of the matrix of hidden factors (3) mu: estimate of the vector of intercepts (4) f_val_vec: objective function values and (5) res_vec: relative change in objective function values
selectedSNPs	The SNPs selected by the screening method
screening_time	The time (in seconds) spent on screening step
param_time	The time (in seconds) spent on the parameter tuning step
model_time	The time (in seconds) spent on the joint modeling step
total_time	The time (in seconds) spent on the screening, parameter tuning, and joint modeling steps
rho	The value of rho chosen through parameter tuning
lambda	The value of lambda chosen through parameter tuning

Examples

```
##Example

## Generate some data
n <- 20
p <- 50
q <- 30
k <- 4
set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)
L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
}
```

```

B[activeSNPs[i], genes_influenced] <- 2
}
E <- matrix(rnorm(n*q),n,q)
Y <- X %*% B + L + E

## Usage
Run_LORS(Y, X, method = "FastLORS")

```

Run_LORS_Screening	<i>Run_LORS_Screening</i> Run_LORS_Screening is a function to to apply the LORS-Screening Algorithm in Yang et al. (2013)
--------------------	---

Description

Run_LORS_Screening Run_LORS_Screening is a function to to apply the LORS-Screening Algorithm in Yang et al. (2013)

Usage

```
Run_LORS_Screening(Y, X, lambda = NULL)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs
lambda	tuning parameter

Value

selectedSNPs the SNPs selected by LORS-Screening

Examples

```

##Example

## Generate some data
n <- 50
p <- 200
q <- 100
k <- 10
set.seed(123)
X <- matrix(rbinom(n*p,1,0.5),n,p)
L <- matrix(rnorm(n*k),n,k) %*% t(matrix(rnorm(q*k),q,k))
B <- matrix(0, ncol(X), ncol(L))
activeSNPs <- sort(sample(c(1:nrow(B)), 20))
for(i in 1:length(activeSNPs)){
  genes_influenced <- sort(sample(c(1:ncol(B)),5))
  B[activeSNPs[i], genes_influenced] <- 2
}
E <- matrix(rnorm(n*q),n,q)
Y <- X %*% B + L + E

selectedSNPs <- Run_LORS_Screening(Y, X)

```

S	<i>S</i> <i>S</i> is a function used internally in rankHC. It calculates empirical cdf's.
---	---

Description

S *S* is a function used internally in rankHC. It calculates empirical cdf's.

Usage

```
S(t, my_matrix)
```

Arguments

t	cutoff value of empirical cdf
my_matrix	a coefficient matrix

Value

The empirical distribution function of the coefficients evaluated at t

softImpute	<i>softImpute</i>
------------	-------------------

Description

softImpute is a function from Mazudmer et al. (2010). It solves the problem $\min \|X - Z\|_{\Omega} + \alpha \|Z\|_{\text{Nuclear}}$ and is used in parameter tuning for LORS. Note: This function is adapted from the LORS MATLAB implementation

Usage

```
softImpute(X, Z, Omega0, Omega1, Omega2, alpha0, maxRank)
```

Arguments

X	a (possibly) incomplete matrix
Z	the target matrix
Omega0	Boolean matrix of observed entries
Omega1	Boolean matrix of training entries
Omega2	Boolean matrix of validation entries
alpha0	initial tuning parameter
maxRank	maximum rank of the solution

Value

Z	Estimate of the target matrix
Err	Squared Error of the difference between X and Z on the validation set
rank_alpha	The rank of the estimates
znorm	The sum of the soft-thresholded singular values of the estimates
Alpha	The tuning parameters used

standardizeBhat	<i>standardizeBhat</i> standardizeBhat is a function used to standardize a coefficient matrix
-----------------	---

Description

standardizeBhat standardizeBhat is a function used to standardize a coefficient matrix

Usage

```
standardizeBhat(Y, X, Bhat)
```

Arguments

Y	gene expression matrix
X	matrix of SNPs
Bhat	a coefficient matrix

Value

A standardized estimate of the coefficient matrix.

survival	<i>survival</i> survival is a function used internally in rankHC. It calculates the survival function of the standard normal distribution
----------	---

Description

survival survival is a function used internally in rankHC. It calculates the survival function of the standard normal distribution

Usage

```
survival(t)
```

Arguments

t	a cutoff value
---	----------------

Value

$1 - \Pr(Z \leq t)$ where Z is a standard normal random variable

svd_st	<i>svd_st</i>
--------	---------------

Description

svd_st is a function for performing soft-thresholded singular value decomposition of a matrix X

Usage

```
svd_st(X, lambda)
```

Arguments

X	a matrix
lambda	the value to to apply soft thresholding with

Value

L the soft-thresholded singular value decomposition of X.

SVT	<i>SVT</i>
-----	------------

Description

SVT is a function to perform soft-thresholded singular value decomposition. It is used to get an initial estimate for L. Note: This function is adapted from the LORS MATLAB implementation

Usage

```
SVT(Y, lambda)
```

Arguments

Y	gene expression matrix
lambda	a tuning parameter

Value

L the singular value decomposition of Y, soft-thresholded with lambda.

Examples

```
##Example
set.seed(123)

Y <- matrix(rnorm(50*100, 7,1), nrow = 50, ncol = 100)
lambda <- runif(1,3,5)
SVT(Y, lambda)
```

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